Signal Processing

Logistics of SP

Textbooks : 1) Signal & Systems (OWN) → Oppenhei
2) DSP : Principle Algo's & Application (PM) → By John G, Proakis & Dimitris Manolakis

→ Lab sessions based on MATLAB.

Weightages

Q1 ⟶ 10
Q2 ⟶ 10
Ass ⟶ 20 ⟶ 8 assignments
Mid Exam ⟶ 25
End exam ⟶ 35

Lab
1) Submissions ⟶ 15
2) Mid Exam ⟶ 10
3) Final Exam / Project ⟶ 15

→ Total Marks : 140 [>130 → A]
→ demo & viva after lab , Tut Quizzes

Pre-requisits for SP → NESS :
1) LTI
2) Fourier series & Tranformation
3) Laplace Transform
4) Convolution → ROC
5) Discrete signals [More focus]

Fourier Analysis :

↳ works for periodic, aperiodic, continuous-time, discrete time.
→ FS : periodic & CT
→ FT : aperiodic & CT → special case of $\mathcal{L}$ $(s = j\omega)$
→ DTFT : aperiodic & DT
→ Discrete fourier Transform (DFT) ⟶ DT & finite length signals
$\xrightarrow{\text{algorithm}}$ FFT ↳ periodic signal of length N

Fourier Series:

synthesis : $x(t)$ ⟶ periodic (T)

$$x(t) = b_0 + \sum_{k=1}^{\infty} a_k \sin(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \cos(k\omega_0 t), \quad \omega_0 = \frac{2\pi}{T}$$

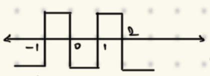
$$\xrightarrow[\text{formula}]{\text{complex}} \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

→ $a_k = \frac{1}{T}\int_{<T>} x(t)\sin(k\omega_0 t)\,dt$ & $b_k = \frac{1}{T}\int_{<T>} x(t)\cos(k\omega_0 t)\,dt$ $\Bigg\}$ $x(t) \longleftrightarrow \{a_k, b_k\}$ / $\{c_k\}$

→ $c_k = \frac{1}{T}\int_{<T>} x(t)\, e^{-jk\omega_0 t}\,dt$ for $k \neq 0$

→ b_0 : avg value of the periodic signal in the period

Sq. wave :



$\Big\}\, T = 2$

FS : $a_k = \frac{1}{2T}\int_{-1}^{1} x(t)\, e^{-jk\omega_0 t}\,dt$

$$= \frac{1}{2}\left[\int_{-1}^{0} e^{-jk\omega t}\,dt - \int_{0}^{1} e^{-jk\omega t}\,dt\right]$$

$$= \frac{1}{2}\left[-\frac{1}{jk\omega}\left[1 - e^{jk\omega}\right] + \frac{1}{jk\omega}\left[e^{-jk\omega} - 1\right]\right]$$

$$= \frac{1}{2jk\omega}\left[\cancel{-1} + e^{jk\omega} + e^{-jk\omega} \cancel{+1}\right] = \frac{1}{2jk\omega}(\cos(jk\omega) - 1)$$

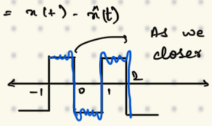
$b_k = 0 \;\forall k$ because it is a odd signal

Partial Reconstruction / Synthesis :

$$\hat{x}(t) = a_0 + \sum_{k=1}^{R} a_k \sin(k\omega_0 t) + \sum_{k=1}^{R} b_k \cos(k\omega_0 t)$$

Reconstruction error: $e(t) = x(t) - \hat{x}(t)$

→ Gibb's phenomena :



As we keep adding more n more terms the osc. get closer to point of disc. and wave becomes more flat.

→ $a_k \propto \frac{1}{k}$