

Probability & Random Processes

Modules:

- M1: Basic of Prob.
- M2: Discrete RV's
- M3: Continuous RV's
- M4: Tail Bounds & Limit Thms
- M5: Random Processes

Grading:

- Assignment: 15%
- Q1, Q2: 15%
- In class Quiz: 5%
- Midsem: 30%
- Endsem: 30%

Ref Books:

- 1) Papoulis
- 2) Bertsekas
- 3) Grimmett

1/12/25 Lecture 1

Module 1: 1) Approaches to Defining Prob

- 2) Prob Space
- 3) Continuity of Prob
- 4) Conditional Prob, Baye's Theorem & Total Prob Thm
- 5) Review of Counting

• Classical Approach:

$$P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total possible no. of outcomes}}$$

→ Issues with this def:

- 1) Not equally likely
- 2) Does not apply for infinite outcomes

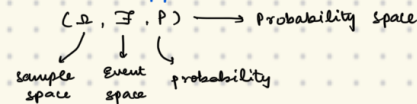
• Frequency Approach:

$$P(E) = \frac{\text{No. of times E occurs}}{\text{Total No. of times event is performed}} = \lim_{n \rightarrow \infty} \frac{u_E}{n}$$

→ Issues:

- 1) Infinite does not make sense
- 2) May not guarantee convergence

• Axiomatic Approach:



Set Theory

- 1) $A \setminus B = \{ \omega \in A \text{ s.t. } \omega \notin B \}$
- 2) $A \cup B = \{ \omega \in A \text{ or } \omega \in B \}$

Countably Infinite set: The elements of the set can be listed

→ If there is a bij. b/w the set & \mathbb{N} .

→ Has \aleph cardinality.

→ Prove that, $\mathbb{Q} \cap [0, 1]$ is countably infinite

→ let $r/n \in \mathbb{Q}$ & $p/q \in [0, 1]$

$$0 < p/q < 1$$

Uncountably Infinite set: Injection $\mathbb{N} \rightarrow \mathbb{R}$

Exercise: Prove that $\{0, 1\}^{\infty}$ is uncountably infinite

→ Sol: Cantor's diag. argument

$$B_N = \{u_1, u_{+1}, u_{+2}, \dots\}$$

We know $(\bigcup_{i=1}^{\infty} A_i)^c = \bigcap_{i=1}^{\infty} A_i^c$

Assume $\omega \in A_i \quad i \in \mathbb{N}$

→ $\omega \notin A_i \quad i \in \mathbb{N}$

$\therefore \omega \in A_i^c \quad i \in \mathbb{N} \rightarrow \omega \in \bigcap_{i=1}^{\infty} A_i^c$