Probability & Random Processes

Modules:
- M1 : Basics of Prob.
- M2 : Discrete RV's
- M3 : continuous RV's
- M4 : Tail Bounds & Limit Thms
- M5 : Random Processes

Grading:
- Assignments : 15%
- Q1, Q2 : 15%
- In class Quiz : 5%
- Midsem : 20%
- endsem : 30%

Ref Books:
1) Papoulis
2) Bertsekas
3) Grimmett

1/8/25 Lecture 1

Module 1 :
1) Approaches to Defining Prob
2) Prob space
3) continuity of Prob
4) conditional Prob, Baye's Theorem & Total Prob Thm
5) Review of counting

• Classical Approach :

$$P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total possible no. of outcomes}}$$

→ Issues with this def :
1) not equally likely
2) Does not apply for infinite outcomes

• Frequency Approach :

$$P(E) = \frac{\text{No. of times E occurs}}{\text{Total No. of times event is performed}} = \lim_{n\to\infty} \frac{n_E}{n}$$

→ Issues :
1) Infinite does not make sense
2) May not guarantee convergence

• Axiomatic Approach :

$(\Omega, \mathcal{F}, P) \longrightarrow$ Probability space

Ω → sample space, $\mathcal{F}$ → Event space, P → probability

Set Theory

1) $A \setminus B = \{x \in A \text{ s.t } x \notin B\}$
2) $A \cup B = \{x \in A \text{ or } x \in B\}$

Countably Infinite set : The elements of the set can be listed
- If there is a bij. b/w the set & $\mathbb{N}$.
- Has $\mathbb{N}$ cardinality.

→ Prove that, $\mathbb{Q} \cap [0,1]$ is countably infinite
→ let $p/q \in \mathbb{Q}$ & $p/q \in [0,1]$

$0 < p/q < 1$

Uncountably Infinite set : Injection $\mathbb{N} \to S$

Exercise: Prove that $\{0,1\}^\infty$ is uncountably infinite
↳ sol: cantors diag. argument

$B_n = \{n, n+1, n+2 \dots\}$

We know $\left(\bigcup_{i=1}^{\infty} A_i\right)^c = \bigcap_{i=1}^{\infty} A_i^c$

Assume $x \in A_i \quad i \in \mathbb{N}$

c ↳ $x \notin A_i \quad i \in \mathbb{N}$

$\therefore x \in A_i^c \quad i \in \mathbb{N} \to x \in \bigcap_{i=1}^{\infty} A_i^c$