Science 1

Predator Prey Model :

→ Prey : y
→ Predator : x

• Assumptions :

1) Prey birth $\propto$ size of population
2) Prey death $\propto$ size of Pd & Py pop.
3) Pd birth $\propto$ size of Pd & Py pop

→ Rate of change of a population = Birth Rate - Death Rate $\}$ Population balance Eqn.

→ $x' = \dfrac{dx}{dt} = \underbrace{\alpha x}_{\text{birth}} - \underbrace{\beta x y}_{\text{death}}$ → $y' = \dfrac{dy}{dt} = \delta x y - \gamma y$

→ $\alpha, \beta, \gamma, \delta > 0$ are constant parameters

→ To find the steady states, we set the diff. eqn to 0,

$$\alpha x - \beta x y = 0 \qquad \hat{y} = \frac{\alpha}{\beta}$$

$$\delta x y - \gamma y = 0 \qquad \hat{x} = \frac{\gamma}{\delta}$$

→ steady states : $\underbrace{(0,0)}_{\text{trivial}}$, $\underbrace{\left(\frac{\gamma}{\delta}, \frac{\alpha}{\beta}\right)}_{\text{equilibrium}}$

$$\text{Jacobian}(J) = \begin{array}{c} \\ \left[\begin{matrix} \alpha - \beta y & -\beta x \\ \delta y & \delta x - \gamma \end{matrix}\right] \\ \text{diff: w.r.t } x \quad \text{w.r.t } y \end{array} \begin{array}{l} \to \text{Prey} \\ \to \text{Pred.} \end{array}$$

$$J\Big|_{(\gamma/\delta,\ \alpha/\beta)} = \begin{bmatrix} 0 & -\gamma\beta/\delta \\ \delta\alpha/\beta & 0 \end{bmatrix}$$ → Eigen values are imaginary