IT252 Homework 9

Problem 1.
a) What is the FFT of (1,0,0,0) ? What is the appropriate value of ω in this case? And what is the inverse-FFT of (1,0,0,0) ?

b) What is the FFT of (1,0,1,-1) ? What is the appropriate value of ω in this case? And what is the inverse-FFT of (1,0,1,-1) ?

Problem 2. What is the sum of the nth roots of unity? $\omega^0_n + \omega^1_n + \omega^2_n + ... + \omega^{n-1}_n$?

Problem 3. Read and understand the Divide-and-Conquer based 'Recursive-FFT' algorithm from CLRS section 30.2.

Problem 4. Scheduling to minimize *weighted completion times.* You are given a set of n jobs all of which need to be scheduled on a single processor. The i^{th} job has a duration (or length l_i) for which it needs the processor, and also a priority (or weight w_i). Note that unlike previous scheduling problems, the jobs here do not have a fixed start and end time. Your task is to come up with a scheduling of the n jobs that minimizes the *weighted completion times* of the jobs. Design a greedy algorithm for this problem and prove that your algorithm is indeed correct.
[Example: for 3 jobs with lengths 1,2,3 and weights 9,8,7 respectively, the scheduling order (job1, followed by job2, followed by job3) has completion times 1, 3 and 6. (Job1 finishes at time 1, job2 at time 1+2 and job3 at time 3+3). Thus the *weighted completion time* for this particular schedule is 1.9 + 3.8 + 6.7 = 75. For the schedule (job3, job2, job1) the weighted completion time is 3.7 + 5.8 + 6.9 = 115.]

Problem 5. Give a linear-time algorithm that takes as input a tree and determines whether it has a *perfect matching:* a set of edges that touches each node exactly once.