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- **Strain-displacement relation From cylinderical to spherical coordinate system:**

As the relation b/w cylinderical & spherical coordinates is:

$$r = \rho \sin\phi \quad , \quad z = \rho\cos\phi \quad , \quad \theta = \theta$$

where

$$\rho = \sqrt{r^2+z^2} \quad , \quad \theta = \tan^{-1}(y/x) \quad , \quad \phi = \arc\cos\left(\frac{z}{\rho}\right)$$

Partial derivatives for the above equations

$$\frac{\partial}{\partial r} = \frac{\partial \rho}{\partial r} \cdot \frac{\partial}{\partial \rho} + \frac{\partial \phi}{\partial r} \frac{\partial}{\partial \phi}$$

$$= \sin\phi \frac{\partial}{\partial \rho} + \frac{r^2}{\sqrt{r^2-z^2}\,\rho^{3/2}} \cdot \frac{\partial}{\partial \phi}$$

$$\frac{\partial}{\partial z} = \frac{\partial \rho}{\partial z} \cdot \frac{\partial}{\partial \rho} + \frac{\partial \phi}{\partial z} \cdot \frac{\partial}{\partial \phi}$$

$$= \cos\phi \frac{\partial}{\partial \rho} + \frac{rz}{\sqrt{r^2-z^2}\,\rho^{3/2}} \cdot \frac{\partial}{\partial \phi}$$

Now

$$u_r = u_\rho \sin\phi + u_\phi \frac{rz}{\sqrt{r^2-z^2}\,\rho^{3/2}} \quad ; \quad u_z = u_\rho\cos\phi + u_\phi \frac{rz}{\sqrt{r^2-z^2}\,\rho^{3/2}} \quad ;$$

$$u_\theta = u_\theta$$

Calculating $e_\rho = \frac{\partial u_r}{\partial r}$.

$$\hat{e}_\rho = \sin\phi \left[\frac{\partial}{\partial \rho} \left(u_\rho \sin\phi + u_\phi \frac{r^2}{\sqrt{r^2 - z^2} \cdot \rho^{3/2}} \right) \right] + \frac{r^2}{\sqrt{r^2 - z^2} \cdot \rho^{3/2}} \frac{\partial}{\partial \phi} \left[u_\rho \sin\phi + u_\phi \frac{r^2}{\sqrt{r^2 - z^2}\, \rho^{3/2}} \right]$$

$$= \left[\frac{\partial u_\rho}{\partial \rho} \sin^2\phi + \frac{\partial u_\phi}{\partial \rho} \cdot \frac{r^2 \sin\phi}{\rho^{3/2} \sqrt{r^2 - z^2}} + \frac{u_\phi r^2}{\sqrt{r^2 - z^2}} \cdot \frac{\sin\phi}{\rho^{-5/2}} + \frac{\partial u_\rho}{\partial \phi} \cdot \frac{\sin\phi\, r^2}{\sqrt{r^2 - z^2} \cdot \rho^{3/2}} + \frac{r^2 \partial u_\rho \cos\phi}{\sqrt{r^2 - z^2}} + \frac{\partial u_\phi}{\partial \phi} \cdot \frac{r^4}{\rho^3 (r^2 - z^2)} \right]$$

$$\hat{e}_\rho = \frac{\partial u_\rho}{\partial \rho} \sin^2\phi + \left(\frac{\partial u_\phi}{\partial \rho} \cdot \frac{1}{\rho^{3/2}} + \frac{u_\phi}{\rho^{-5/2}} + \frac{\partial u_\rho}{\partial \phi} \cdot \frac{1}{\rho^{3/2}} \right) \frac{r^2 \cdot \sin\phi}{\sqrt{r^2 - z^2}} + \left(-u_\rho \cos\phi + \frac{\partial u_\phi}{\partial \phi} \cdot \frac{1}{\rho^3} \frac{r^2}{\sqrt{r^2 - z^2}} \right) \frac{r^2}{\sqrt{r^2 - z^2}}$$

$$\hat{e}_\phi = \frac{\partial u_z}{\partial z}.$$

$$\hat{e}_\phi = \cos\phi \frac{\partial}{\partial \rho} \left[u_\rho \cos\phi + u_\phi \cdot \frac{rz}{\rho^{3/2} \sqrt{r^2 - z^2}} \right] + \frac{rz}{\sqrt{r^2 - z^2}\, \rho^{3/2}} \frac{\partial}{\partial \phi} \left[u_\rho \cos\phi + u_\phi \frac{rz}{\sqrt{r^2 - z^2}\, \rho^{3/2}} \right]$$

$$= \frac{\partial u_\rho}{\partial \rho} \cos^2\phi + \frac{\partial u_\phi}{\partial \rho} \cdot \frac{rz}{\rho^{3/2}} \frac{\cos\phi}{\sqrt{r^2 - z^2}} + \frac{u_\phi r z}{\sqrt{r^2 - z^2}} \cdot \frac{\cos\phi}{\rho^{-5/2}} + \frac{\partial u_\rho}{\partial \phi} \frac{\cos\phi\, rz}{\rho^{3/2} \sqrt{r^2 - z^2}} - \frac{\sin\phi \cdot rz\, u_\phi}{\sqrt{r^2 - z^2}\, \rho^{3/2}} + \frac{\partial u_\phi}{\partial \phi} \frac{r^2 z^2}{(r^2 - z^2) \rho^3}$$

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$$\partial\phi = \frac{2\partial u_\rho}{\partial \rho}\cos^2\phi + \left(\frac{\partial u_\phi}{\partial \rho}\frac{1}{\rho^{3/2}} + \frac{u_\phi}{\rho^{-5/2}} + \frac{\partial u_\rho}{\partial \phi}\cdot\frac{1}{\rho^{3/2}}\right)$$

$$\frac{\cos\phi \cdot rz}{\sqrt{r^2-z^2}} + \left(\frac{\partial u_\phi}{\partial \phi}\frac{rz}{\sqrt{r^2-z^2}\cdot\rho^{3/2}} - \frac{u_\rho \sin\phi}{\rho^{3/2}}\right)\frac{rz}{\sqrt{r^2-z^2}}.$$

Therefore we have strain-displacement relation, it becomes;

$$e_\rho = \frac{\partial u_r}{\partial r}, \quad e_\phi = \frac{1}{\rho}\left(u_r + \frac{\partial u_z}{\partial \phi}\right)$$

$$e_\theta = \frac{1}{\rho\sin\phi}\left(\frac{\partial u_\theta}{\partial \theta} + \sin\phi\, u_r + \cos\phi\, u_z\right)$$

$$e_{\rho\phi} = \frac{1}{2}\left(\frac{1}{\rho}\frac{\partial u_r}{\partial \phi} + \frac{\partial u_z}{\partial \rho} - \frac{u_z}{\rho}\right)$$

$$e_{\phi\theta} = \frac{1}{2\rho}\left(\frac{1}{\sin\phi}\cdot\frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial \phi} - \cot\phi\, u_\theta\right)$$

$$e_{\theta\rho} = \frac{1}{2}\left(\frac{1}{\rho\sin\phi}\cdot\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial \rho} - \frac{u_\theta}{\rho}\right)$$