Computer Experiments: Probability

1. (A) Generate random samples for rolling a fair (i.e. there's an equal probability for each value of the dice) 6-faced dice for N times.

First, roll the dice for N=10 times (help randi)

Plot the histogram of the outcome values (help hist, help bar)

(B) Normalize the histogram values with N so that we get the estimated probability density function (pdf) for the dice (in this case the sum of histogram bins equal to one).

Plot the normalized histogram, i.e., the estimated pdf

Define the true pdf for the fair dice and plot it on top of the estimated pdf (help hold)

(C) Are the true pdf and the estimated pdf equal? Repeat the process for a few of times and observe the new samples:

Is the estimated pdf (i.e., the rolled dice values) always the same?

(D) Increase the number of trials to N=10000 and repeat the process for a few times

Is the estimated pdf (i.e., the rolled dice values) changing?

2. (A) Generate random samples for rolling a weighted dice, where the probability to get the value 6 is 1/3 and the rest of the values are equally probable.

First, roll the dice for N=10 times (help randi, rand)

Plot the normalized histogram (i.e., the pdf estimate)

(B) Define the true pdf for the weighted dice and plot it on top of the estimated pdf
Besides using the plot-function, try also

stairs (help stairs)

stem (help stem)

3. (A) Considering $X \sim (\mu, \sigma^2)$, define the probability $P(a<X<b)$ for a=1 and b=2 (help qfunc)
By using the same principle, calculate the probability $P(X<a)$ and $P(X>b)$

(B) Now, make sure that the sum of above probabilities equal to 1. In other words, check numerically that $P(-\infty<X<\infty)= P(X<a)+P(a<X<b)+ P(X>b)=1$

4. Estimate the PMF for a geom(0.25) random variable for A = 1,2,..., 20 using a computer simulation and compare it to the true PMF.

Also, estimate the CDF from your computer simulation.

Determine the average value for a large number of realizations.

What is the probability that X > 4?

5. Use a computer simulation to generate realizations of a Pois(A) random variable with A = 5 by approximating it with a bin(100,0.05) random variable. What is the average value of X?

6. An urn contains 3 red balls, 3 black balls, and 3 white balls. If 6 balls are chosen with replacement, how many of each color is most likely?

Hint: You will need a computer to evaluate the probabilities.

7. Determine the probability that in a class of 23 students two or more students have birthdays on January 1. Use a computer simulation to verify your result.

8. An urn contains 4 red balls and 2 black balls. Two balls are chosen at random and without replacement. What is the probability of obtaining one red ball and one black ball in any order?

Verify your results by enumerating all possibilities using a computer evaluation.

9. If a discrete random variable has the PMF $p_X[k] = 1/4$ for $k = -1$ and $p_X[k] = 3/4$ for $k=1$, find the mean and variance.

Perform a computer simulation to estimate its mean and variance. How does it compare to the true mean and variance?

10. An $N\times1$ random vector X has $E_{X_i}[X_i] = \mu$, and $Var(X_i) = i\sigma^2$ for i = 1,2,..., N. The components of X are independent. Does the sample mean random variable converge to μ, as N becomes large? Carry out a computer simulation for this problem and explain your results.

فرمت گزارش:

گزارش بایستی به زبان فارسی و در قالب PDF باشد. در گزارش تحلیل و نتیجه‌گیری خود را در رابطه با هر تمرین به شکل مختصر بیان فرمایید.

فایل گزارش خود را به شکل «شماره دانشجویی_Report1» نام‌گذاری نمایید(مانند91234567_Report1).

فرمت کدها:

برای هر تمرین بایستی فایل کد جداگانه در محیط MATLAB تهیه شود.

هر فایل کد خود را به شکل «شماره دانشجویی_CE1_k» نام‌گذاری فرمایید که k بیانگر شماره تمرین می‌باشد. (مانند CE1_3_91234567 برای تمرین سوم).

نحوه تحویل:

فایل‌های کد و گزارش خود را که طبق فرمت‌های فوق تهیه شده‌اند، در قالب یک فایل فشرده در سایت درس بارگذاری نمایید.

فایل فشرده را به شکل «شماره دانشجویی_CE1» نام‌گذاری فرمایید(مانند91234567_CE1).

آخرین موعد تحویل: 4 آبان ماه