

Control systems

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Assignment-1

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Problem Statement

23. Find the transfer function, $G(s) = \frac{X_1(s)}{F(s)}$, for the translation mechanical system shown in Figure P2.9.

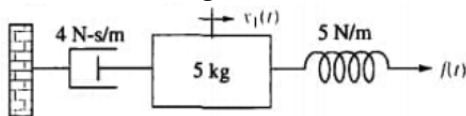


FIGURE P2.9

Solution

$X_2(t)$ is the displacement of the right member of the spring as the force is applied to the spring.

Writing the equation of motion

forces act on the block are left direction due to viscous damper, left due to block, left due to displacement

$X_1(t)$ on spring and right due to displacement $X_2(t)$ on spring.

so the equation of motion of the block is

$$M \frac{d^2 X_1(t)}{dt^2} + f_v \frac{dX_1(t)}{dt} + KX_1(t) = KX_2(t) \quad (3.1)$$

substituting the values we get following equation

$$5 \frac{d^2 X_1(t)}{dt^2} + 4 \frac{dX_1(t)}{dt} + 5X_1(t) = 5X_2(t) \quad (3.2)$$

The laplace transform of the equation of motion is

$$5S^2 X_1(S) + 4S X_1(S) + 5X_1(S) - 5X_2(S) = 0 \quad (3.3)$$

$$X_1(S)(5S^2 + 4S + 5) - 5X_2(S) = 0 \quad (3.4)$$

Forces act on the spring are right direction due to displacement $X_1(t)$ on spring and left due to displacement $X_2(t)$ on spring and force acting right on the spring

the equation of motion of spring

$$KX_2(t) = F(t) + kX_1(t) \quad (3.5)$$

substituting the values we get

$$5X_2(t) = F(t) + 5X_1(t) \quad (3.6)$$

The laplace transform of the equation of motion is

$$5X_2(S) = F(S) + 5X_1(S) \quad (3.7)$$

adding equation (3.4) and (3.7) we get

$$X_1(S)(5S^2 + 4S + 5) = F(S) + 5X_1(S) \quad (3.8)$$

solving the equation (3.8) we get

$$X_1(S)(5S^2 + 4S + 5) - 5X_1(S) = F(S) \quad (3.9)$$

$$X_1(S)(5S^2 + 4S + 5 - 5) = F(S) \quad (3.10)$$

$$X_1(S)(5S^2 + 4S) = F(S) \quad (3.11)$$

Transfer Function

$$\text{T.F} = \frac{X_1(S)}{F(S)}$$

from equation (3.11) we get

$$\frac{X_1(S)}{F(S)} = \frac{1}{S(5S+4)}$$

$$\frac{X_1(S)}{F(S)} = \frac{\frac{1}{5}}{S(S+\frac{4}{5})}$$

$$\text{therefore, T.F} = \frac{\frac{1}{5}}{S(S+\frac{4}{5})}$$