

LECTURE 20

Convolutional Neural Networks (CNNs)

Building CNNsin Using Keras

Data Science, Fall 2023 @ Knowledge Stream
Sana Jabbar

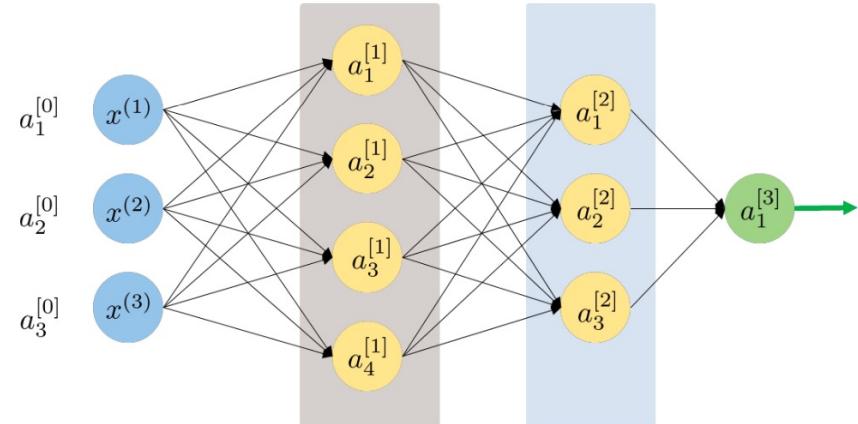
Review

Lecture 20

- Introduction to Neural Networks
- Neural network forward pass
- Activation functions
- Back Propagation

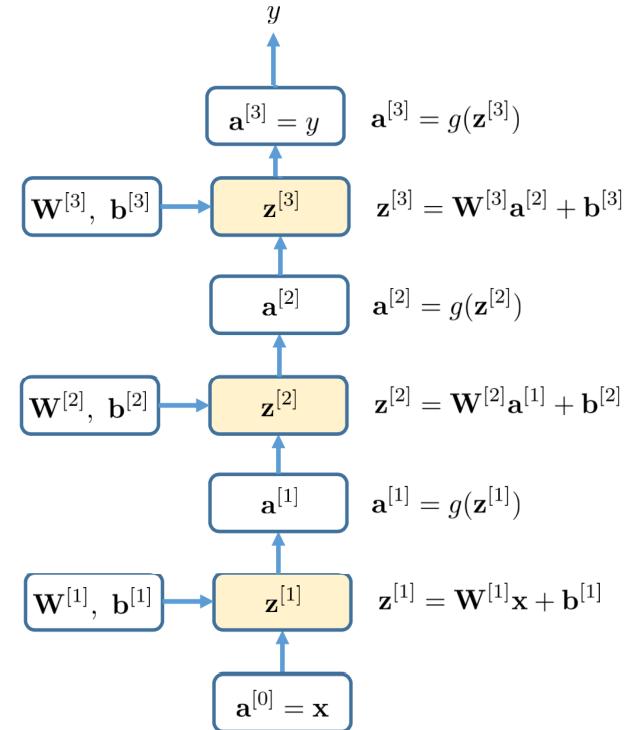
Back Propagation: Vectorization

- We compute loss function \mathcal{L} using forward pass.



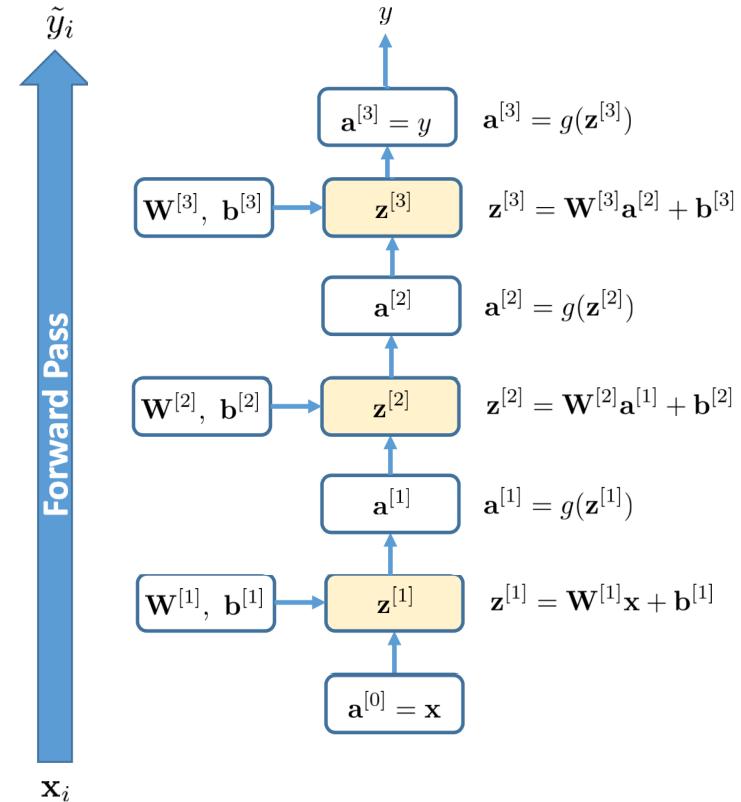
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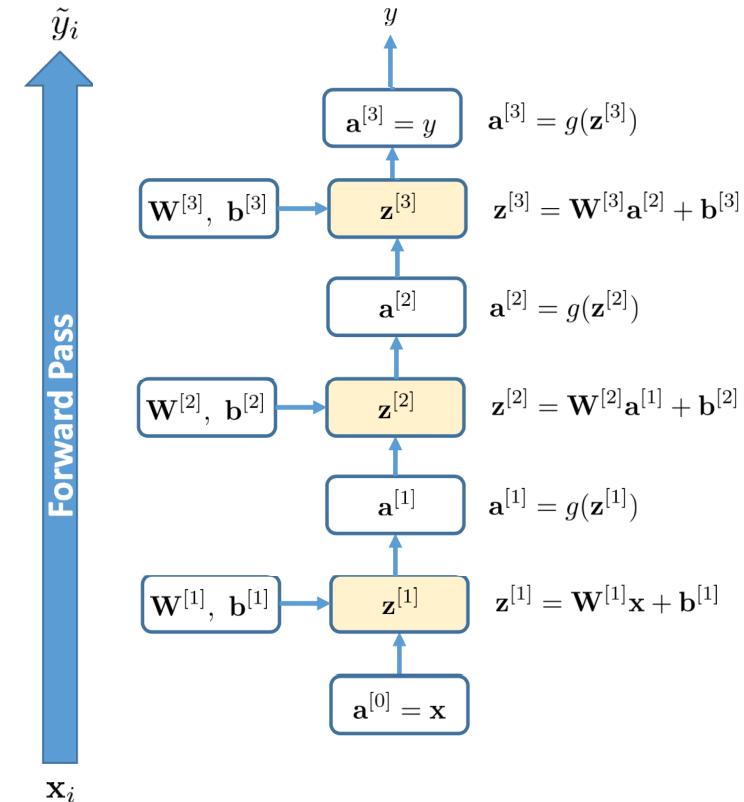
Back Propagation: Vectorization

- We compute loss function \mathcal{L} using forward pass.

- We update $\mathbf{W}^{[\ell]}$ and $\mathbf{b}^{[\ell]}$ using gradient descent as:

$$\mathbf{W}^{[\ell]} = \mathbf{W}^{[\ell]} - \alpha \frac{\partial \mathcal{L}}{\partial \mathbf{W}^{[\ell]}}$$

$$\mathbf{b}^{[\ell]} = \mathbf{b}^{[\ell]} - \alpha \frac{\partial \mathcal{L}}{\partial \mathbf{b}^{[\ell]}}$$



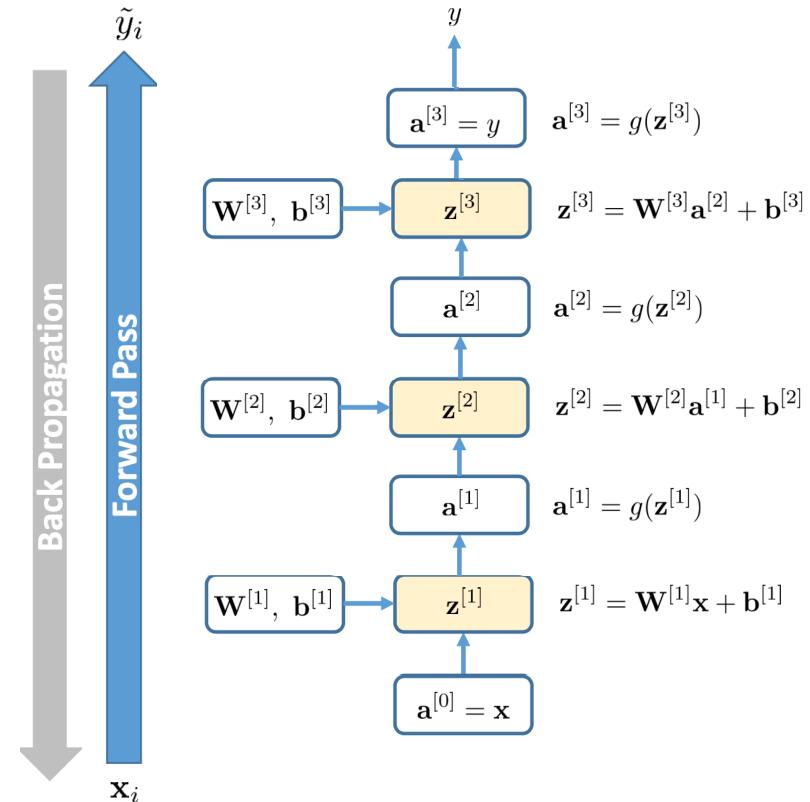
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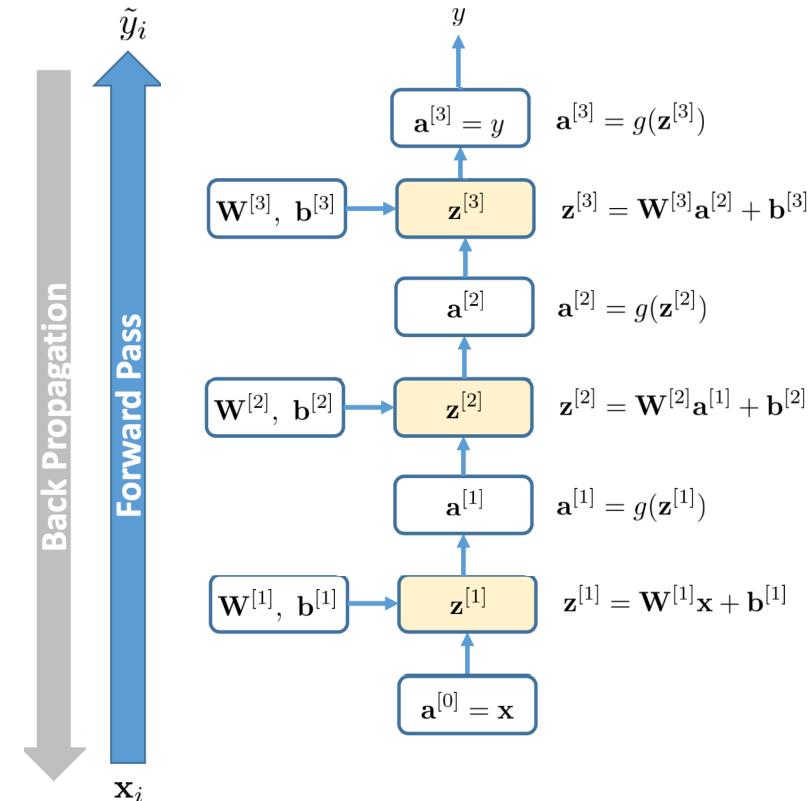
$$\mathbf{W}^{[\ell]} = \mathbf{W}^{[\ell]} - \alpha \frac{\partial \mathcal{L}}{\partial \mathbf{W}^{[\ell]}}$$

$$\mathbf{b}^{[\ell]} = \mathbf{b}^{[\ell]} - \alpha \frac{\partial \mathcal{L}}{\partial \mathbf{b}^{[\ell]}}$$

Partial Derivatives:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{[3]}} = \frac{\partial \mathcal{L}}{\partial \mathbf{a}^{[3]}} \frac{\partial \mathbf{a}^{[3]}}{\partial \mathbf{z}^{[3]}} \frac{\partial \mathbf{z}^{[3]}}{\partial \mathbf{W}^{[3]}}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{b}^{[3]}} = \frac{\partial \mathcal{L}}{\partial \mathbf{a}^{[3]}} \frac{\partial \mathbf{a}^{[3]}}{\partial \mathbf{z}^{[3]}} \frac{\partial \mathbf{z}^{[3]}}{\partial \mathbf{b}^{[3]}}$$



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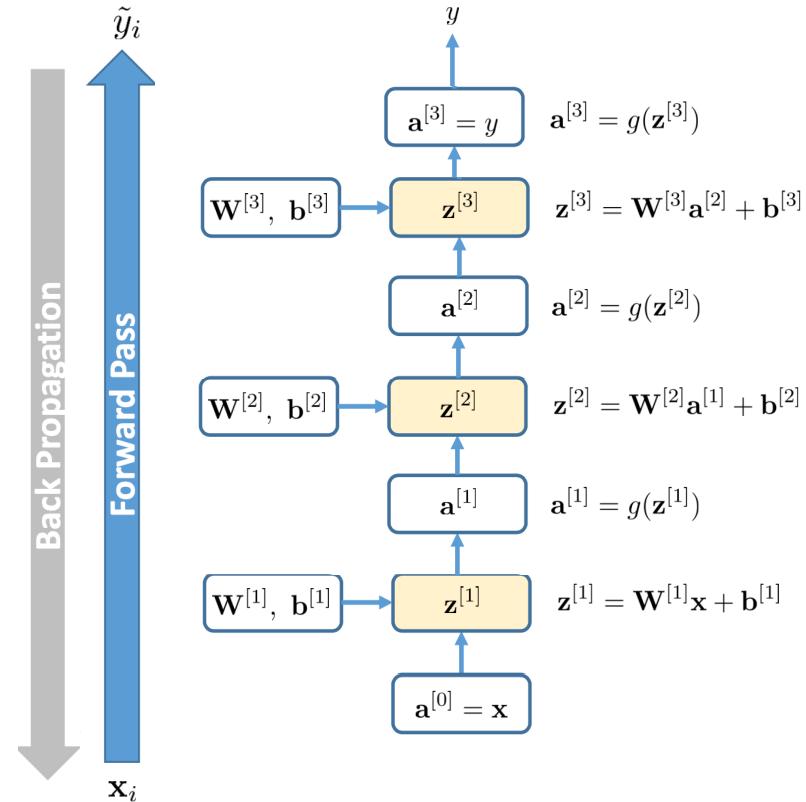
Partial Derivatives:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{[3]}} = \frac{\partial \mathcal{L}}{\partial \mathbf{a}^{[3]}} \frac{\partial \mathbf{a}^{[3]}}{\partial \mathbf{z}^{[3]}} \frac{\partial \mathbf{z}^{[3]}}{\partial \mathbf{W}^{[3]}}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{b}^{[3]}} = \frac{\partial \mathcal{L}}{\partial \mathbf{a}^{[3]}} \frac{\partial \mathbf{a}^{[3]}}{\partial \mathbf{z}^{[3]}} \frac{\partial \mathbf{z}^{[3]}}{\partial \mathbf{b}^{[3]}}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{[2]}} = \frac{\partial \mathcal{L}}{\partial \mathbf{a}^{[3]}} \frac{\partial \mathbf{a}^{[3]}}{\partial \mathbf{z}^{[3]}} \frac{\partial \mathbf{z}^{[3]}}{\partial \mathbf{a}^{[2]}} \frac{\partial \mathbf{a}^{[2]}}{\partial \mathbf{z}^{[2]}} \frac{\partial \mathbf{z}^{[2]}}{\partial \mathbf{W}^{[2]}}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{b}^{[2]}} = \frac{\partial \mathcal{L}}{\partial \mathbf{a}^{[3]}} \frac{\partial \mathbf{a}^{[3]}}{\partial \mathbf{z}^{[3]}} \frac{\partial \mathbf{z}^{[3]}}{\partial \mathbf{a}^{[2]}} \frac{\partial \mathbf{a}^{[2]}}{\partial \mathbf{z}^{[2]}} \frac{\partial \mathbf{z}^{[2]}}{\partial \mathbf{b}^{[2]}}$$



Back Propagation: Vectorization

- We compute loss function \mathcal{L} using forward pass.

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$$\mathbf{W}^{[\ell]} = \mathbf{W}^{[\ell]} - \alpha \frac{\partial \mathcal{L}}{\partial \mathbf{W}^{[\ell]}} \quad \mathbf{b}^{[\ell]} = \mathbf{b}^{[\ell]} - \alpha \frac{\partial \mathcal{L}}{\partial \mathbf{b}^{[\ell]}}$$

Partial Derivatives:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{[3]}} = \frac{\partial \mathcal{L}}{\partial \mathbf{a}^{[3]}} \frac{\partial \mathbf{a}^{[3]}}{\partial \mathbf{z}^{[3]}} \frac{\partial \mathbf{z}^{[3]}}{\partial \mathbf{W}^{[3]}}$$

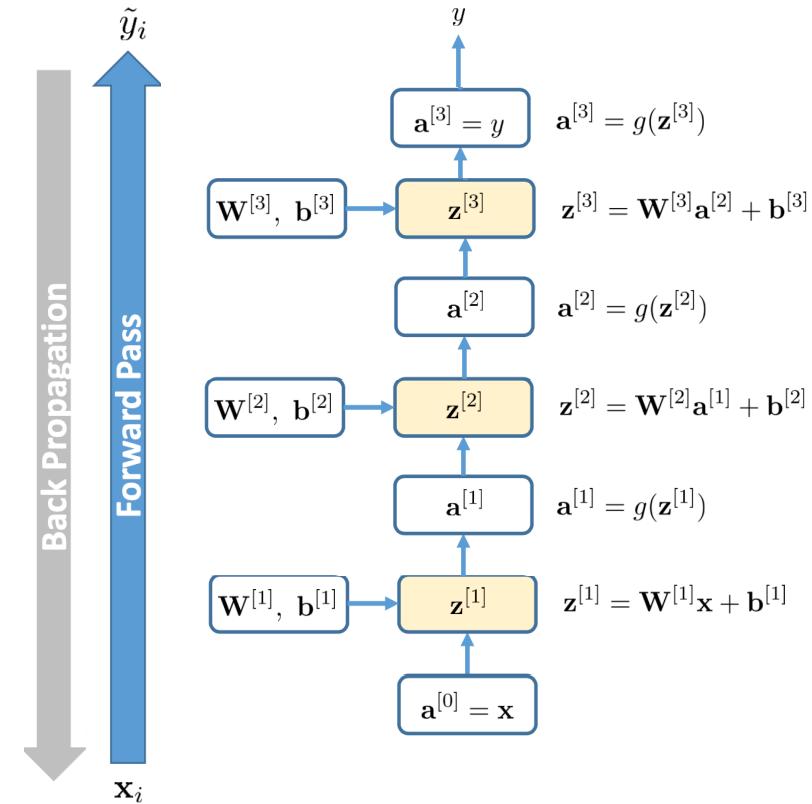
$$\frac{\partial \mathcal{L}}{\partial \mathbf{b}^{[3]}} = \frac{\partial \mathcal{L}}{\partial \mathbf{a}^{[3]}} \frac{\partial \mathbf{a}^{[3]}}{\partial \mathbf{z}^{[3]}} \frac{\partial \mathbf{z}^{[3]}}{\partial \mathbf{b}^{[3]}}$$

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$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{[1]}} = \frac{\partial \mathcal{L}}{\partial \mathbf{a}^{[3]}} \frac{\partial \mathbf{a}^{[3]}}{\partial \mathbf{z}^{[3]}} \frac{\partial \mathbf{z}^{[3]}}{\partial \mathbf{a}^{[2]}} \frac{\partial \mathbf{a}^{[2]}}{\partial \mathbf{z}^{[2]}} \frac{\partial \mathbf{z}^{[2]}}{\partial \mathbf{a}^{[1]}} \frac{\partial \mathbf{a}^{[1]}}{\partial \mathbf{z}^{[1]}} \frac{\partial \mathbf{z}^{[1]}}{\partial \mathbf{W}^{[1]}}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{b}^{[1]}} = \frac{\partial \mathcal{L}}{\partial \mathbf{a}^{[3]}} \frac{\partial \mathbf{a}^{[3]}}{\partial \mathbf{z}^{[3]}} \frac{\partial \mathbf{z}^{[3]}}{\partial \mathbf{a}^{[2]}} \frac{\partial \mathbf{a}^{[2]}}{\partial \mathbf{z}^{[2]}} \frac{\partial \mathbf{z}^{[2]}}{\partial \mathbf{a}^{[1]}} \frac{\partial \mathbf{a}^{[1]}}{\partial \mathbf{z}^{[1]}} \frac{\partial \mathbf{z}^{[1]}}{\partial \mathbf{b}^{[1]}}$$



Neural Networks In Keras

- `from keras.models import Sequential`
- `from keras.layers import Dense`
- `# built model`
- `model = Sequential([`
- `Dense(32, activation='relu', input_shape=(10,)),`
- `Dense(32, activation='relu'),`
- `Dense(1, activation='sigmoid'),`
- `])`
- `# compile your model`
- `model.compile(optimizer='sgd',`
- `loss='binary_crossentropy',`
- `metrics=['accuracy'])`

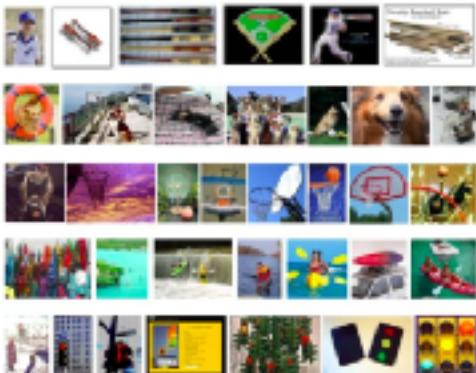
Neural Networks In Keras

- # now train your model using fit
- hist = model.fit(X_train, Y_train,
● batch_size=32, epochs=100,
● validation_data=(X_val, Y_val))

- # evaluate your model using model.evaluate
- model.evaluate(X_test, Y_test) [1]

Relentless research on visual recognition

Caltech 101



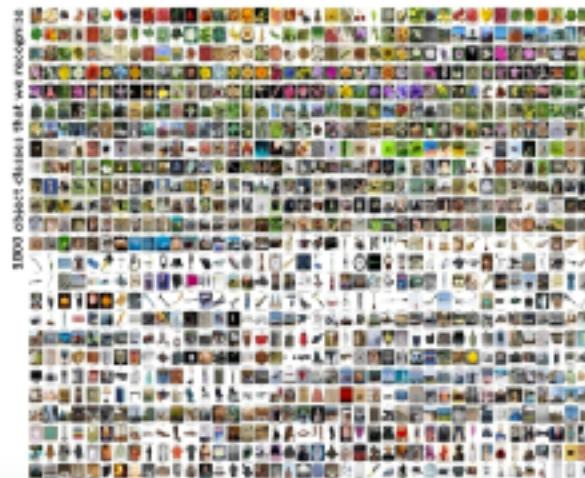
80 Million Tiny Images



PASCAL VOC



ImageNet



Convolution

Convolutional Neural Networks

- Automatic feature extraction.
- Highly accurate at image recognition & classification.
- Weight sharing.
- Minimizes computation.
- Ability to handle large datasets.
- Hierarchical learning

Digital Image



Pixel of an RGB image are formed from the corresponding pixel of the three component images

Digital Image



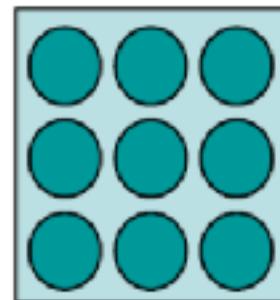
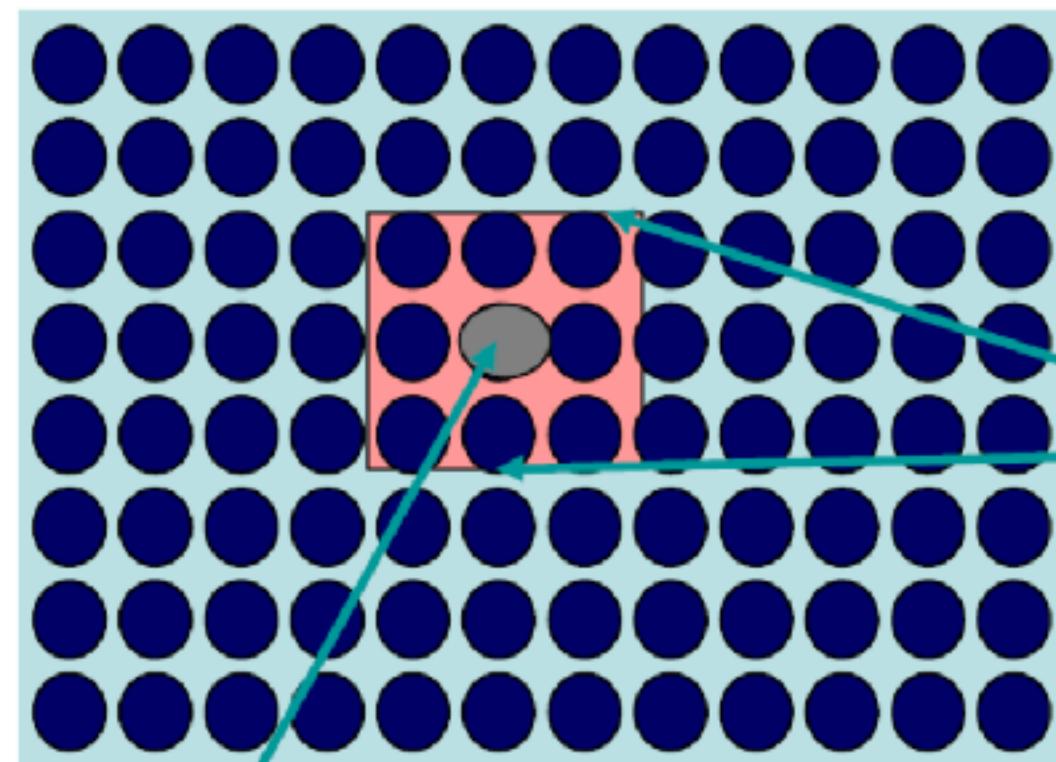
08 02 22 97 31 18 00 40 00 78 04 05 07 78 52 32 00 17 01 00
69 49 99 40 17 81 18 57 60 87 17 40 98 63 65 80 51 56 62 00
81 49 81 73 55 79 14 29 93 71 40 67 84 88 30 03 49 13 36 65
52 70 95 23 04 60 11 42 69 04 68 16 01 32 56 71 37 02 36 91
22 31 16 71 51 67 03 89 61 92 36 54 22 40 40 28 66 33 13 80
24 47 38 00 93 03 45 02 44 75 33 55 78 36 04 20 55 17 12 50
32 95 81 28 61 23 67 10 26 38 40 67 59 54 70 66 18 38 66 70
67 26 20 69 02 62 12 20 95 63 94 39 63 08 40 91 66 19 98 21
24 55 88 05 46 73 55 24 97 17 78 78 96 83 14 88 34 89 63 72
23 36 23 09 75 00 76 44 20 65 35 14 00 61 33 97 34 31 33 95
78 17 53 28 22 75 35 47 15 94 03 89 04 42 14 14 09 53 56 92
16 39 05 42 96 35 31 47 55 58 88 24 00 17 54 24 36 29 85 57
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58
19 80 81 68 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40
04 52 08 83 97 35 99 14 07 97 57 32 16 26 26 79 33 27 98 66
05 44 68 67 57 62 20 72 03 66 33 67 46 55 12 32 63 93 53 69
04 42 16 73 55 95 59 11 24 94 72 18 08 46 25 32 40 62 76 36
20 69 86 41 72 39 23 86 31 85 85 69 82 67 59 85 74 01 36 16
20 73 35 29 78 31 90 01 74 31 49 71 46 66 61 16 23 57 05 54
05 70 84 71 83 51 54 69 16 92 33 45 61 43 52 03 89 33 47 48

What the computer sees

image classification

82% cat
15% dog
2% hat
1% mug

Discrete 2D convolution

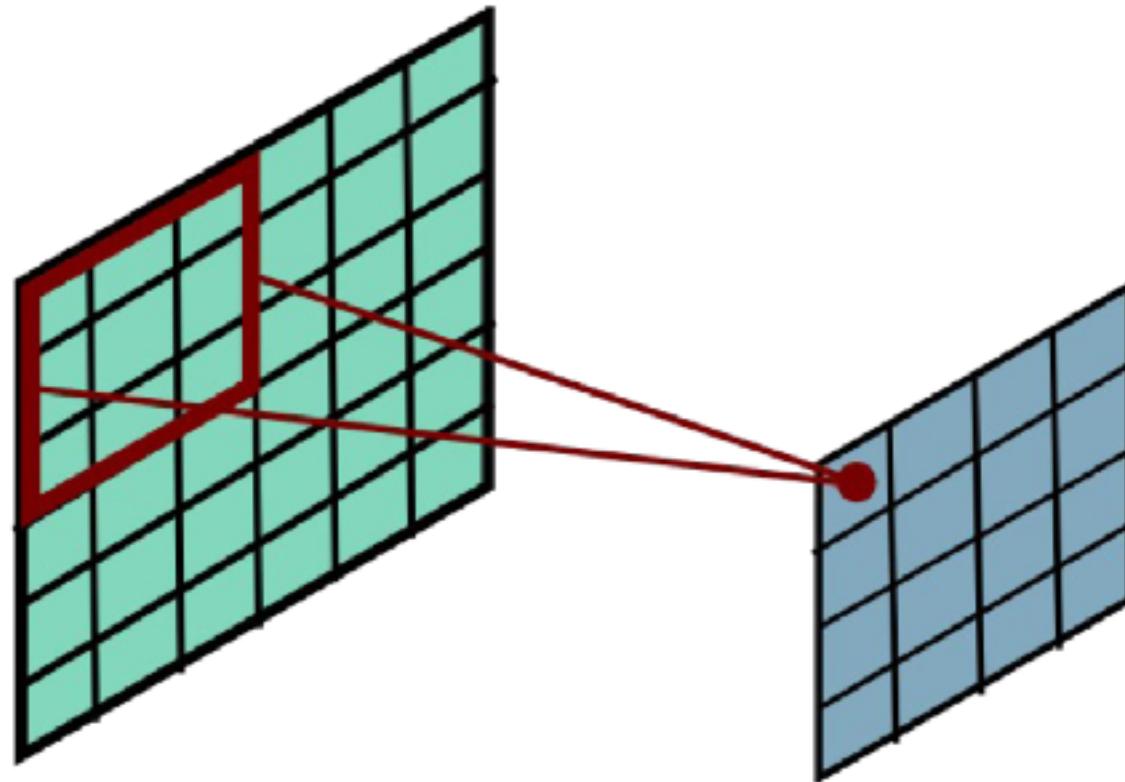


3X3
convolution
kernel

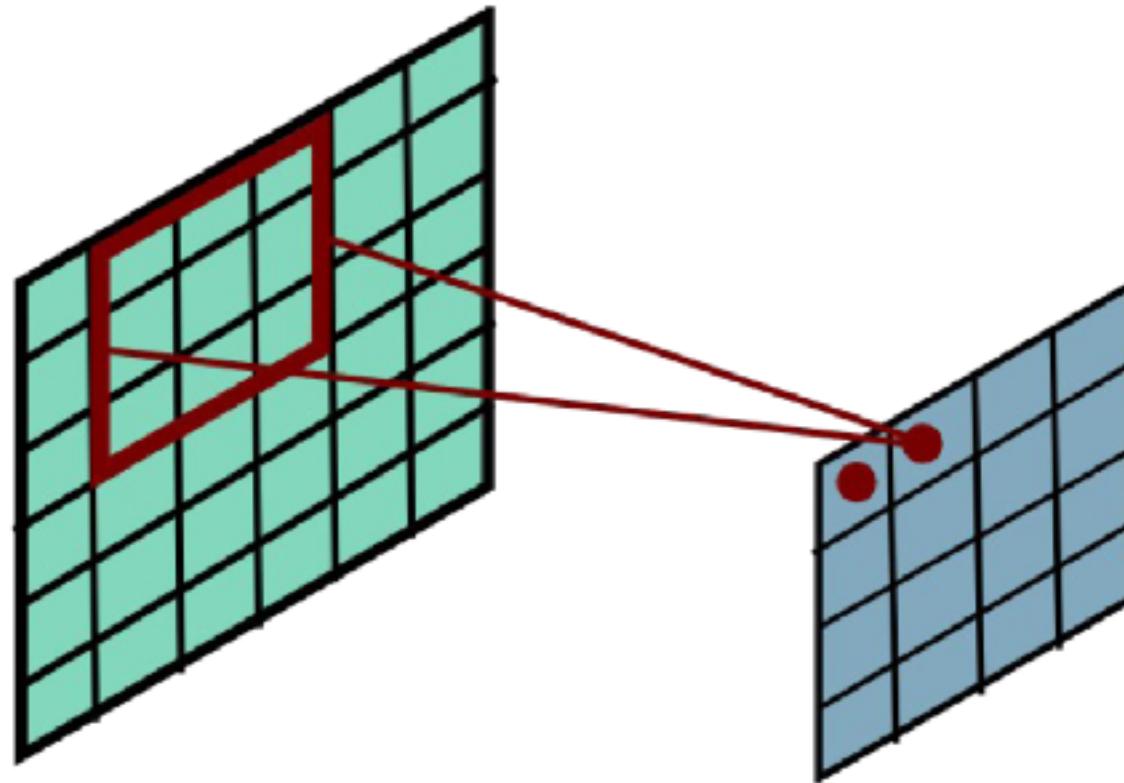
Pixels involved in
the convolution
process **for the**
single pixel p

Pixel p to be processed

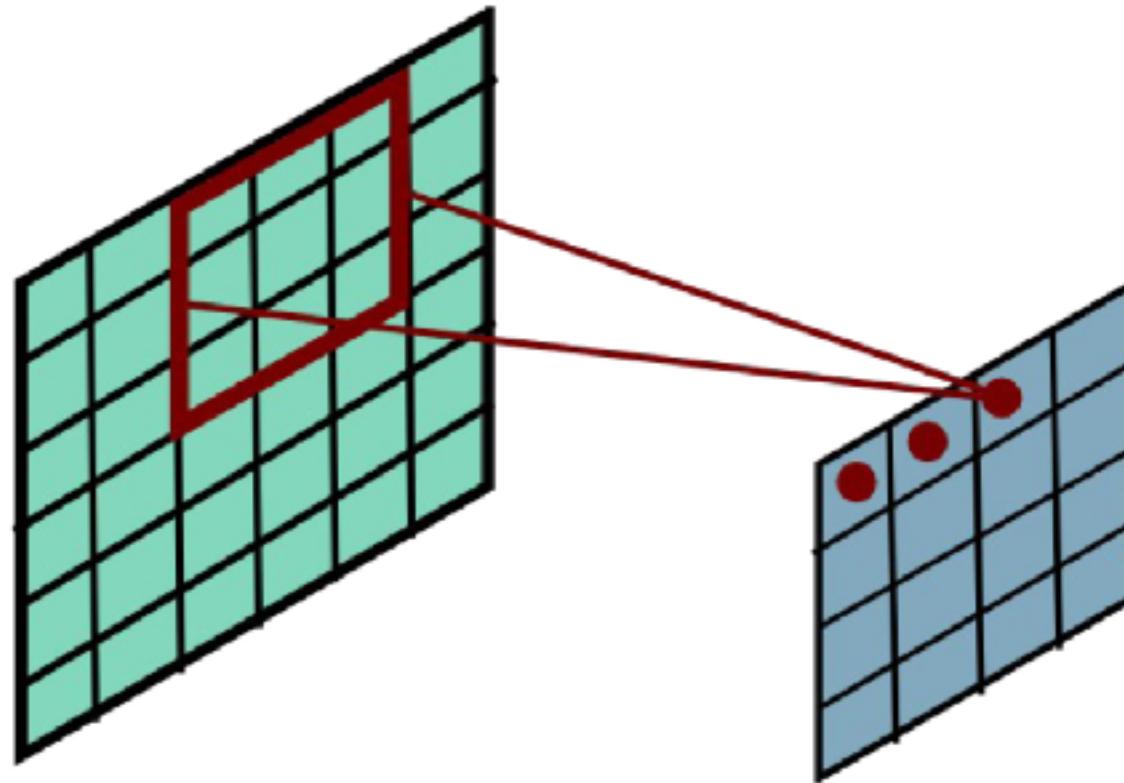
Convolution procedure



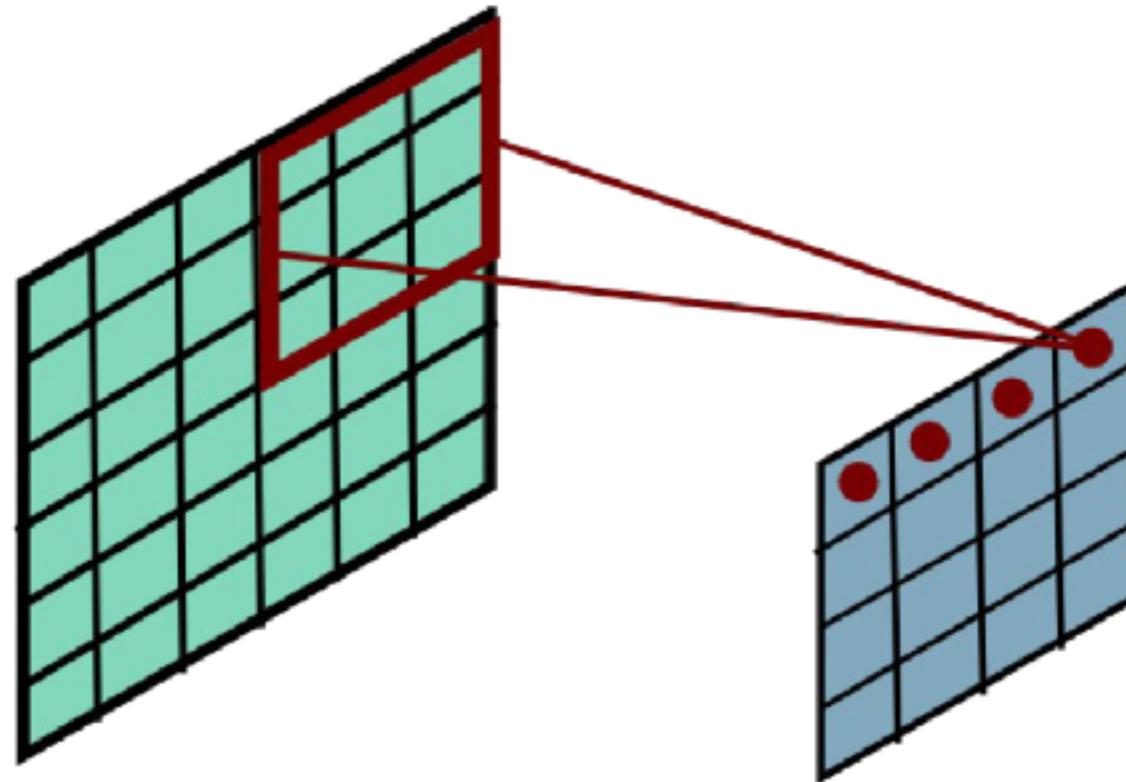
Convolution procedure



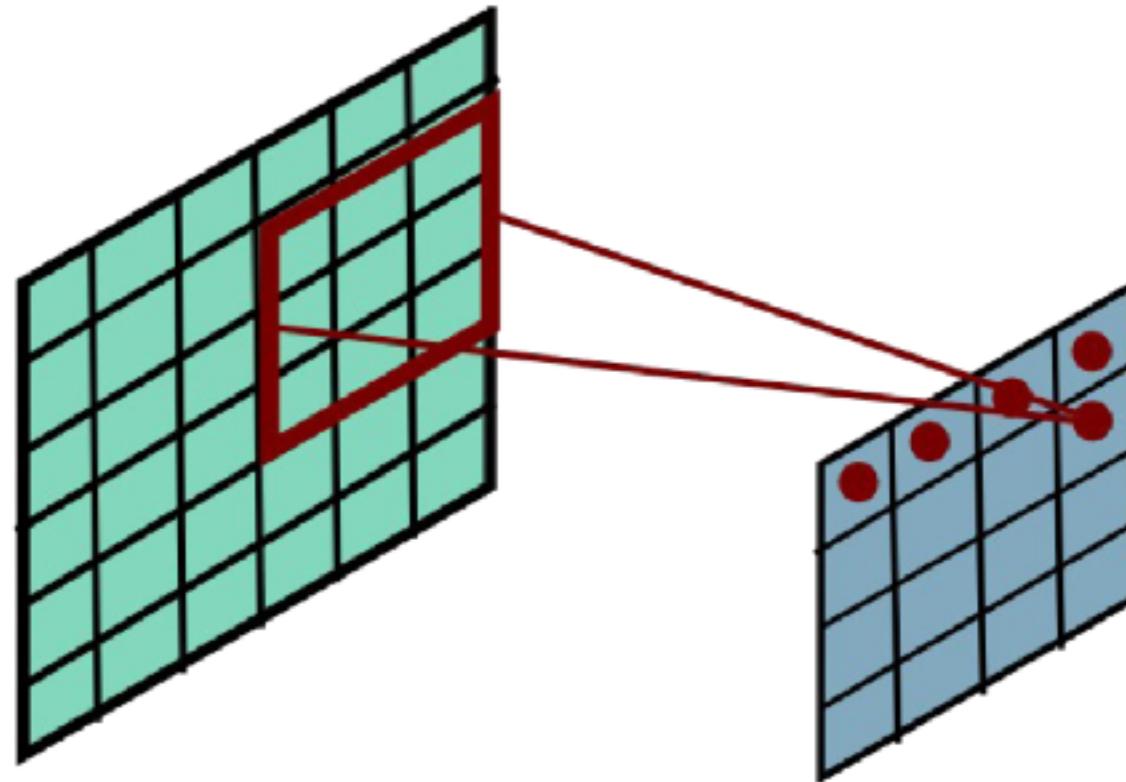
Convolution procedure



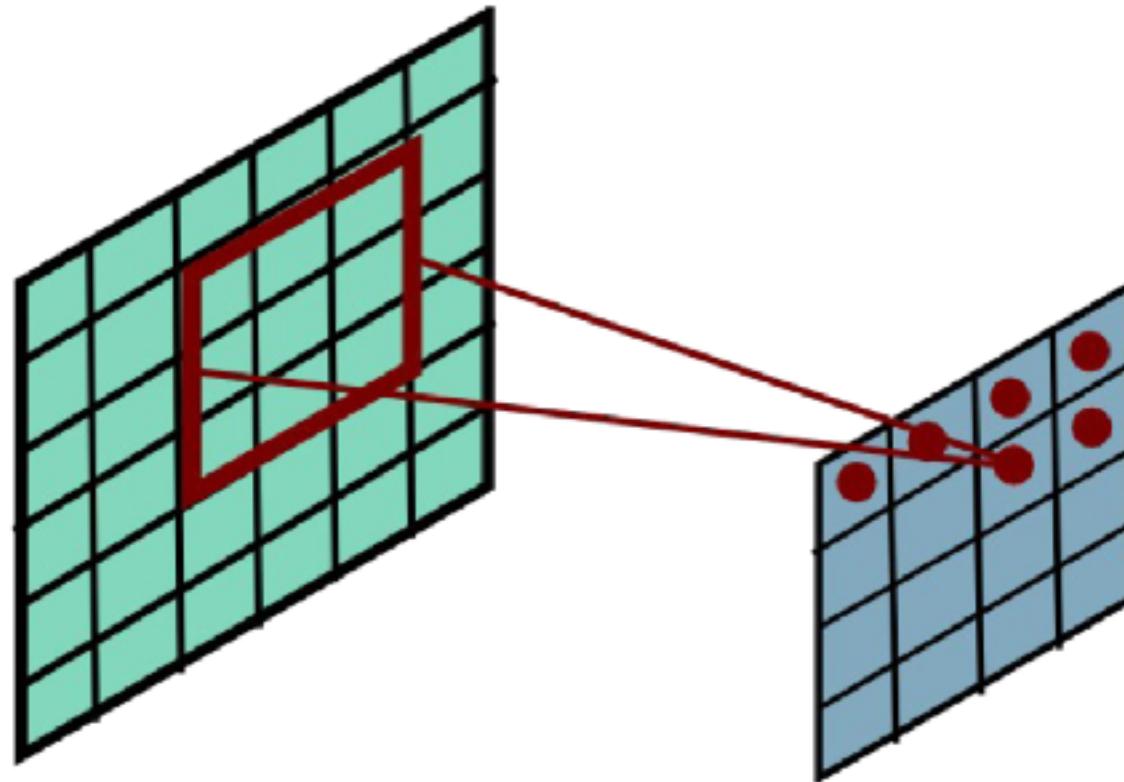
Convolution procedure



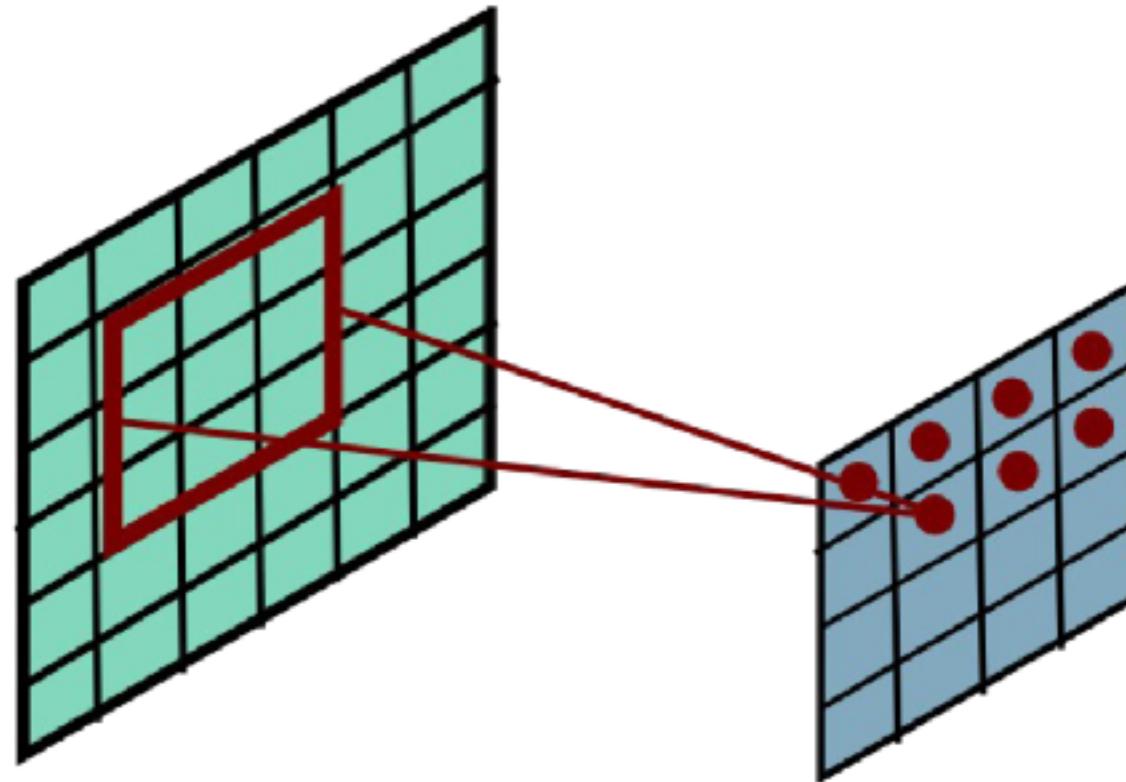
Convolution procedure



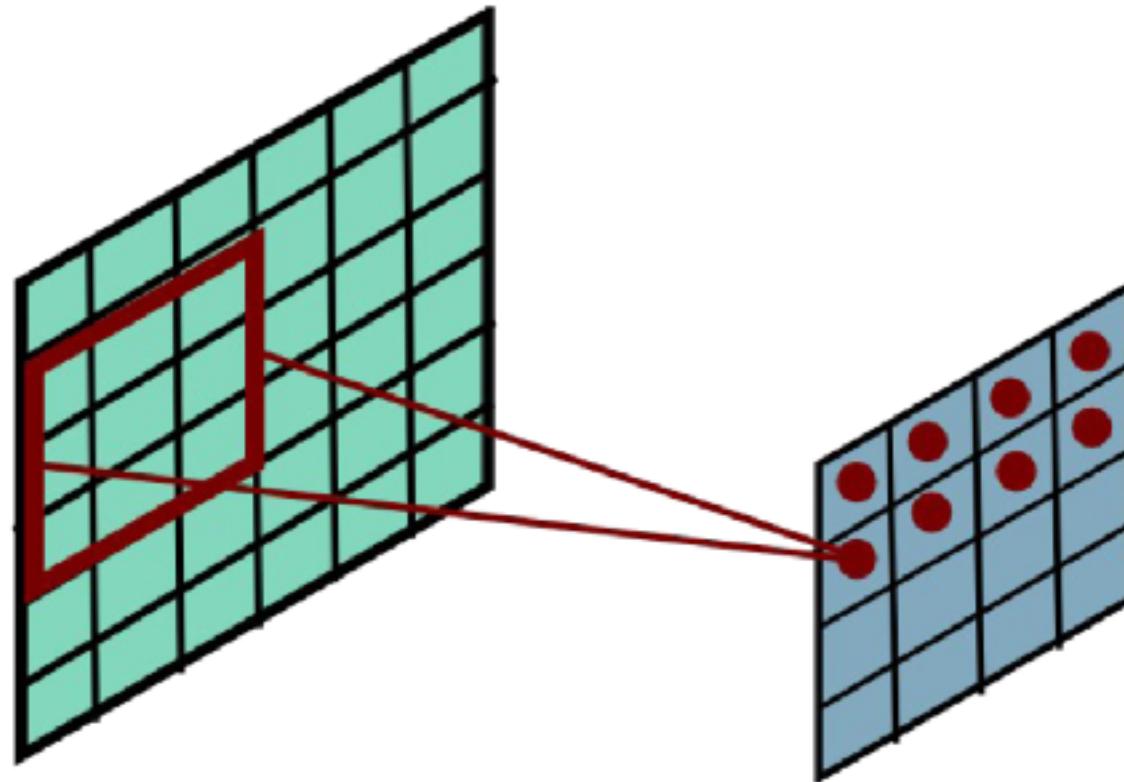
Convolution procedure



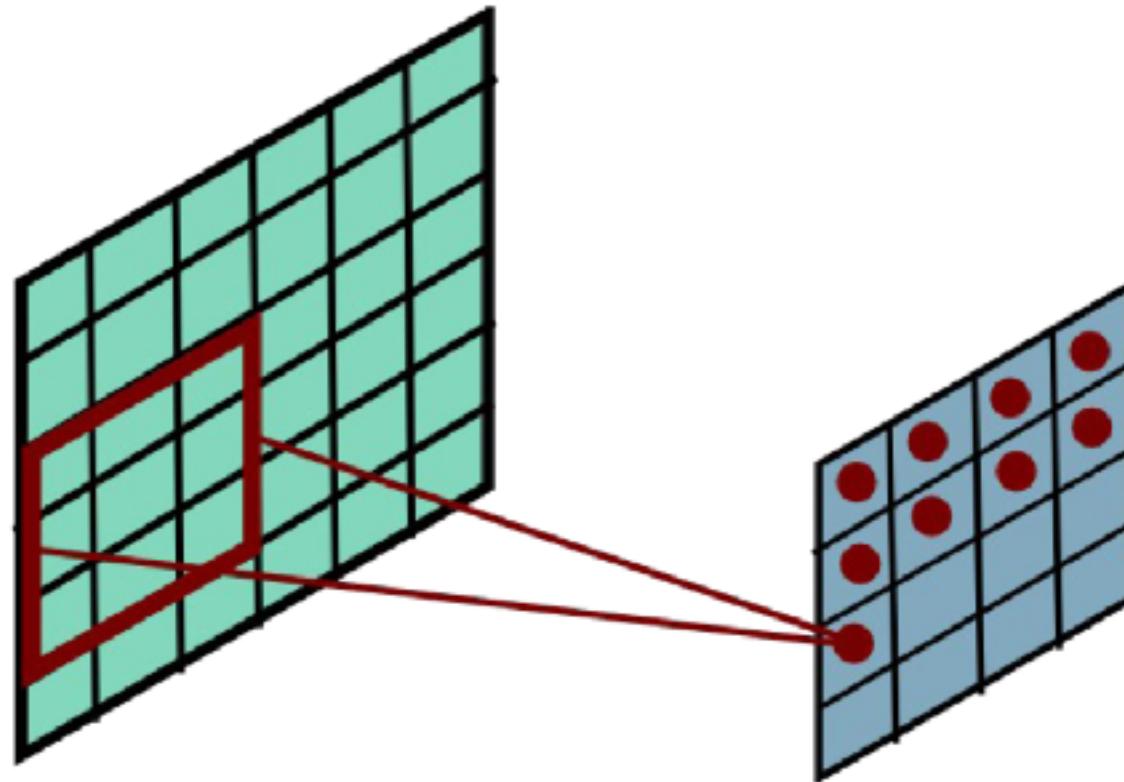
Convolution procedure



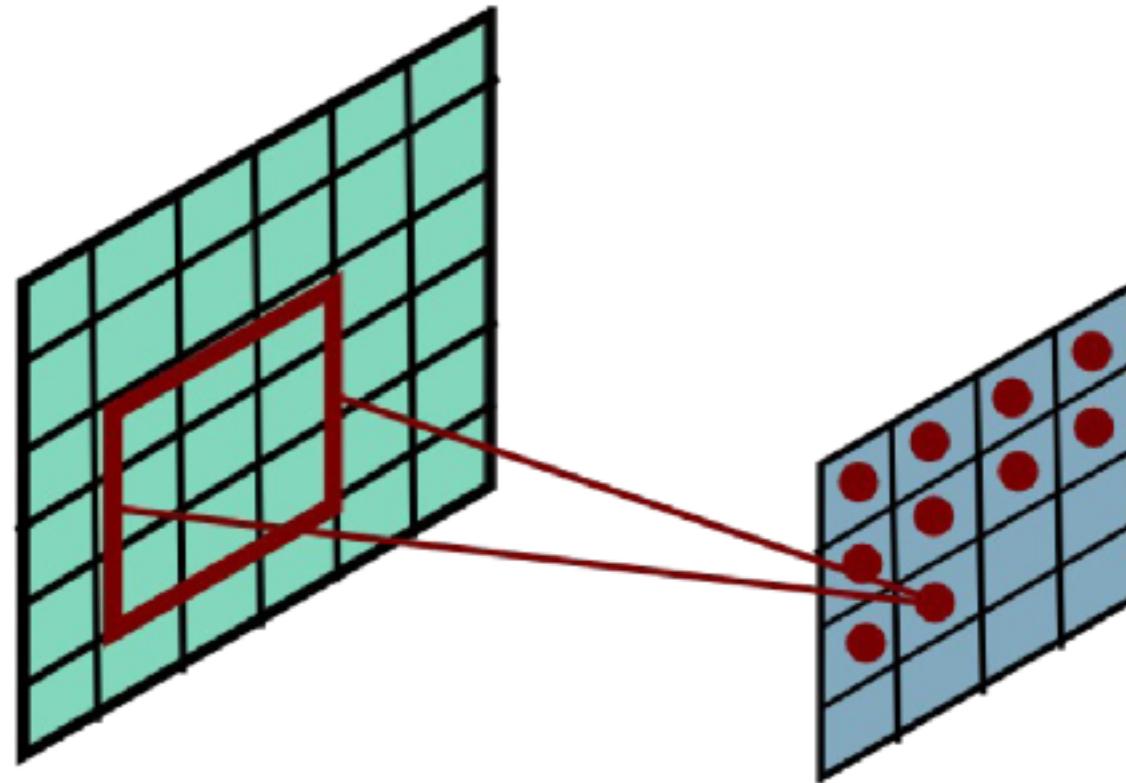
Convolution procedure



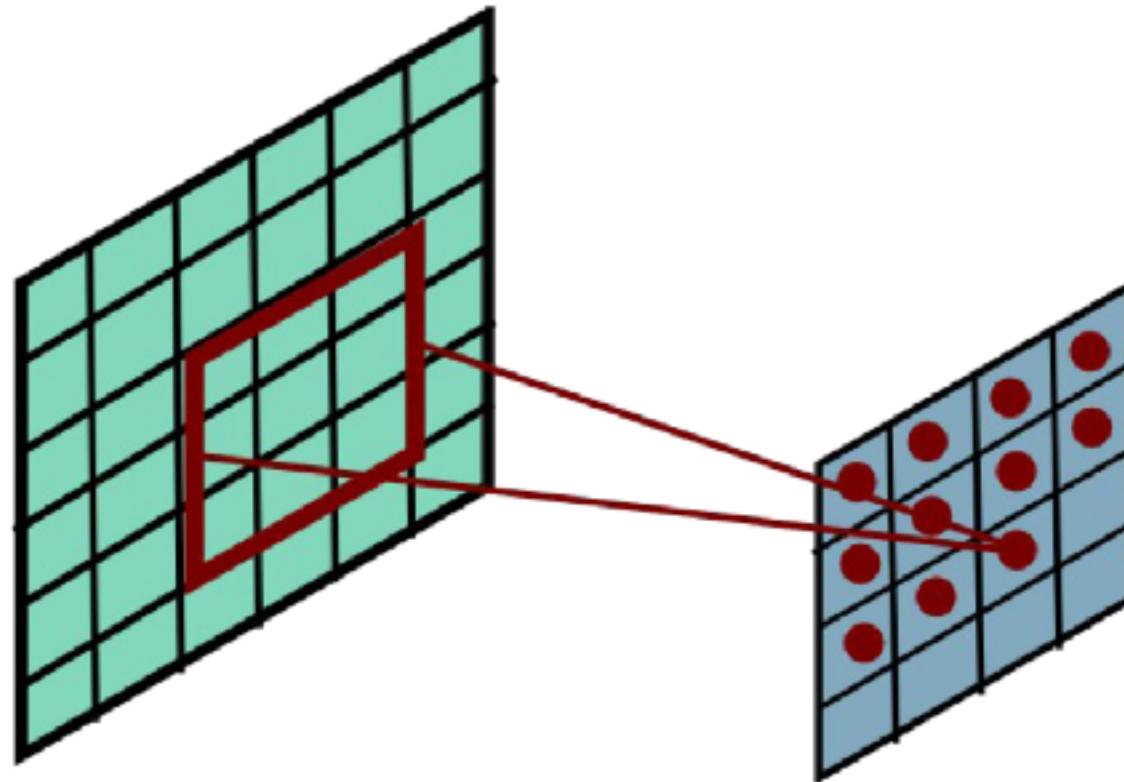
Convolution procedure



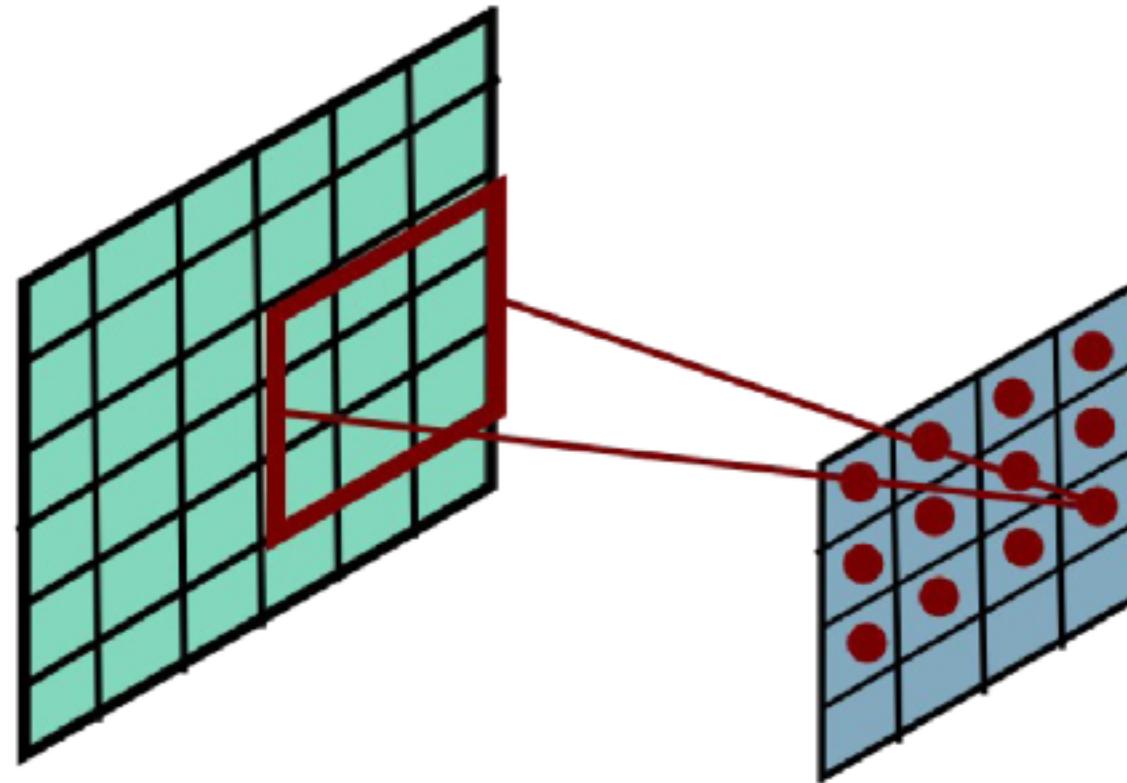
Convolution procedure



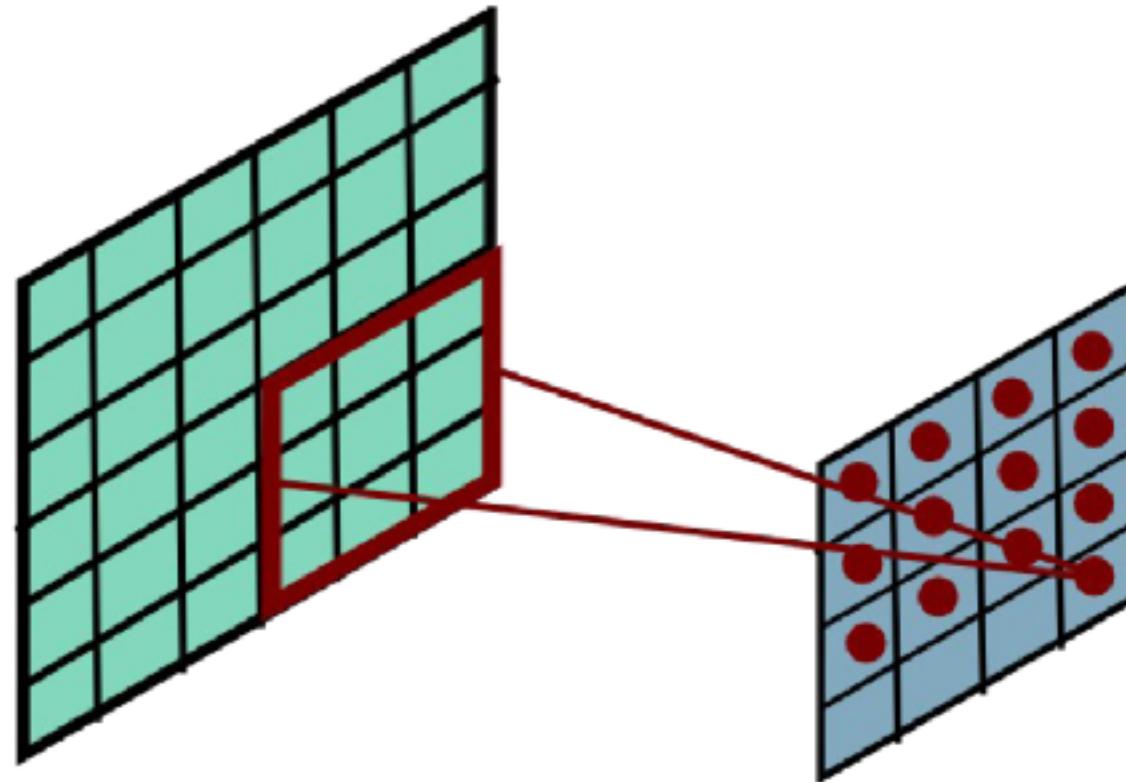
Convolution procedure



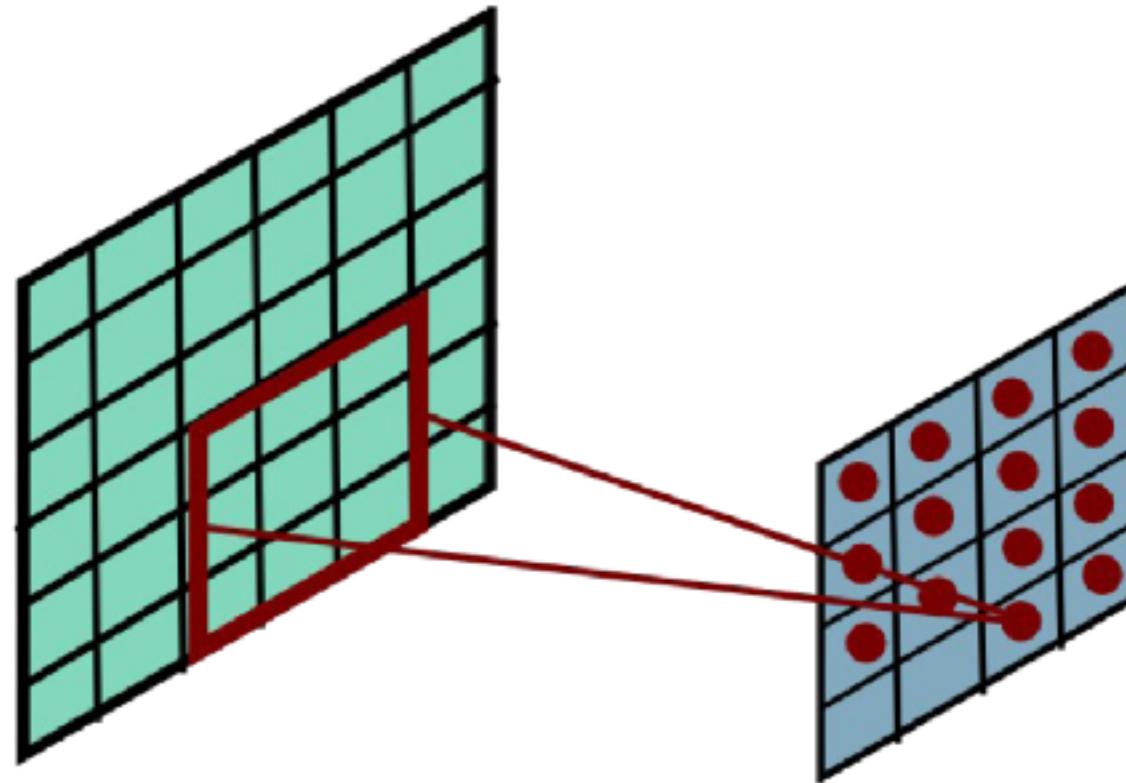
Convolution procedure



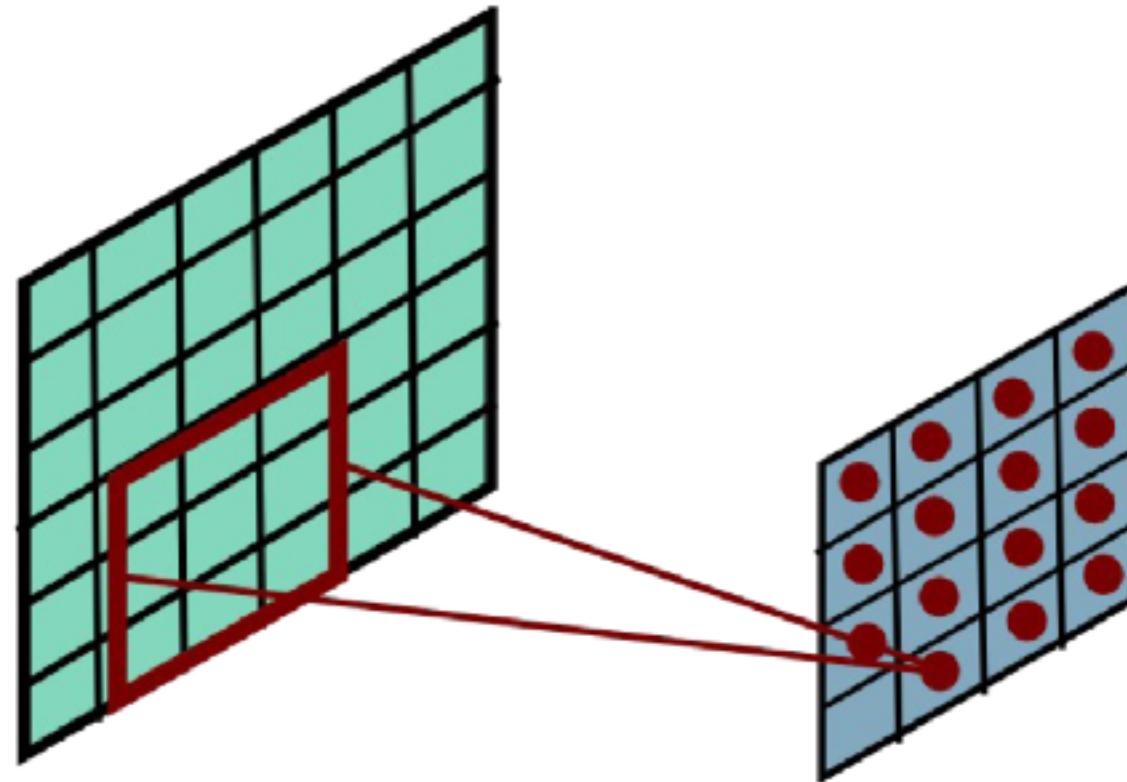
Convolution procedure



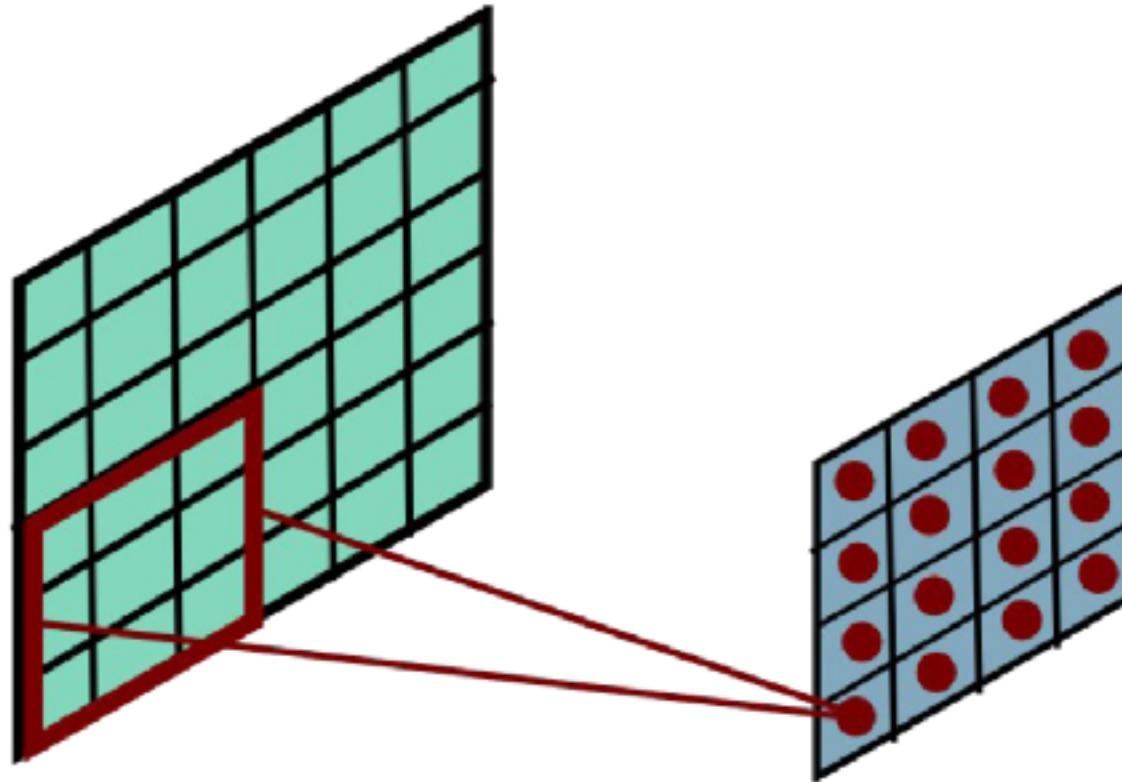
Convolution procedure



Convolution procedure

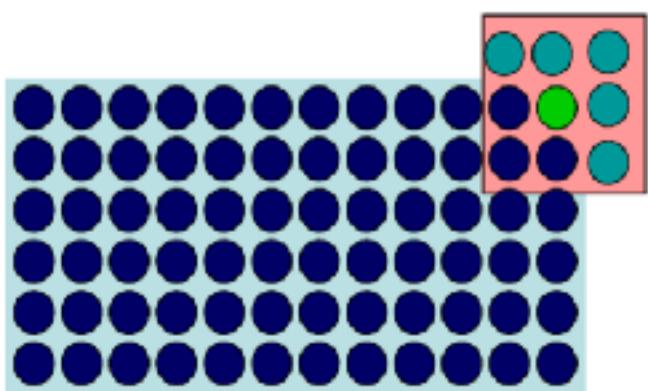


Convolution procedure



Convolution procedure

- ▶ Positioning
 - ▶ Place the **centre** of the window over the pixel position to be filtered
- ▶ Multiply
 - ▶ the original pixel values of the image by the filter values corresponding to their location
- ▶ Add
 - ▶ Add together the obtained multiplication results
 - ▶ The result of this addition is the **convolution result**
- ▶ Border
 - ▶ Mirror the border (Padding)



Numerical Example

Image	Filter	Conv Output																																																			
<table border="1"> <tr><td>1</td><td>1</td><td>5</td><td>7</td><td>3</td></tr> <tr><td>4</td><td>5</td><td>4</td><td>7</td><td>8</td></tr> <tr><td>9</td><td>1</td><td>3</td><td>1</td><td>6</td></tr> <tr><td>2</td><td>4</td><td>5</td><td>2</td><td>1</td></tr> <tr><td>1</td><td>3</td><td>1</td><td>2</td><td>1</td></tr> <tr><td>1</td><td>3</td><td>3</td><td>2</td><td>4</td></tr> </table>	1	1	5	7	3	4	5	4	7	8	9	1	3	1	6	2	4	5	2	1	1	3	1	2	1	1	3	3	2	4	<table border="1"> <tr><td>1</td><td>0</td><td>-1</td></tr> <tr><td>1</td><td>0</td><td>-1</td></tr> <tr><td>1</td><td>0</td><td>-1</td></tr> </table>	1	0	-1	1	0	-1	1	0	-1	<table border="1"> <tr><td>2</td><td>-8</td><td>-5</td></tr> <tr><td>3</td><td>0</td><td>-3</td></tr> <tr><td>3</td><td>3</td><td>1</td></tr> <tr><td>-5</td><td>4</td><td>3</td></tr> </table>	2	-8	-5	3	0	-3	3	3	1	-5	4	3
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3	0	-3																																																			
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$$1x1 + 1x0 + 5x-1 + 4x1 + 5x0 + 4x-1 + 9x1 + 1x0 + 3x-1 = 2$$

Numerical Example

Image	Filter	Conv Output
1 1 5 7 3	1 0 -1	
4 5 4 7 8	1 0 -1	
9 1 3 1 6	1 0 -1	
2 4 5 2 1		
1 3 1 2 1		
1 3 3 2 4		

$$1 \times 1 + 5 \times 0 + 7 \times -1 + 5 \times 1 + 4 \times 0 + 7 \times -1 + 1 \times 1 + 3 \times 0 + 1 \times -1 = -8$$

Numerical Example

Image					Filter			Conv Output		
1	1	5	7	3						
4	5	4	7	8						
9	1	3	1	6						
2	4	5	2	1						
1	3	1	2	1						
1	3	3	2	4						
					1	0	-1			
					1	0	-1			
					1	0	-1			
								2	-8	-5
								3	0	-3
								3	3	1
								-5	4	3

$$5x^1 + 7x^0 + 3x^{-1} + 4x^1 + 7x^0 + 8x^{-1} + 3x^1 + 1x^0 + 6x^{-1} = -5$$

Numerical Example

Image					Filter			Conv Output		
1	1	5	7	3						
4	5	4	7	8				2	-8	-5
9	1	3	1	6				3	0	-3
2	4	5	2	1				3	3	1
1	3	1	2	1				-5	4	3
1	3	3	2	4						

$$4x1 + 5x0 + 4x-1 + 9x1 + 1x0 + 3x-1 + 2x1 + 4x0 + 5x-1 = 3$$

Numerical Example

Image					Filter			Conv Output		
1	1	5	7	3						
4	5	4	7	8						
9	1	3	1	6						
2	4	5	2	1						
1	3	1	2	1						
1	3	3	2	4						

$$5x^1 + 4x^0 + 7x^{-1} + 1x^1 + 3x^0 + 1x^{-1} + 4x^1 + 5x^0 + 2x^{-1} = 0$$

Numerical Example

Image					Filter			Conv Output		
1	1	5	7	3						
4	5	4	7	8						
9	1	3	1	6						
2	4	5	2	1						
1	3	1	2	1						
1	3	3	2	4						
					1	0	-1			
					1	0	-1			
					1	0	-1			
								2	-8	-5
								3	0	-3
								3	3	1
								-5	4	3

$$4x^1 + 7x^0 + 8x^{-1} + 3x^1 + 1x^0 + 6x^{-1} + 5x^1 + 2x^0 + 1x^{-1} = -3$$

Numerical Example

Image	Filter	Conv Output
1 1 5 7 3	1 0 -1	
4 5 4 7 8	1 0 -1	
9 1 3 1 6	1 0 -1	
2 4 5 2 1		
1 3 1 2 1		
1 3 3 2 4		
		2 -8 -5
		3 0 -3
		3 3 1
		-5 4 3

$9 \times 1 + 1 \times 0 + 3 \times -1 + 2 \times 1 + 4 \times 0 + 5 \times -1 + 1 \times 1 + 3 \times 0 + 1 \times -1 = 3$

Numerical Example

Numerical Example

Image	Filter	Conv Output
1 1 5 7 3	1 0 -1	
4 5 4 7 8	1 0 -1	
9 1 3 1 6	1 0 -1	
2 4 5 2 1		2 -8 -5
1 3 1 2 1		3 0 -3
1 3 3 2 4		3 3 1
		-5 4 3

$3x1 + 1x0 + 6x-1 + 5x1 + 2x0 + 1x-1 + 1x1 + 2x0 + 1x-1 = 1$

Numerical Example

Image	Filter	Conv Output
1 1 5 7 3	1 0 -1	
4 5 4 7 8	1 0 -1	
9 1 3 1 6	1 0 -1	
2 4 5 2 1		2 -8 -5
1 3 1 2 1		3 0 -3
1 3 3 2 4		3 3 1
		-5 4 3
$2 \times 1 + 4 \times 0 + 5 \times -1 + 1 \times 1 + 3 \times 0 + 1 \times -1 + 1 \times 1 + 3 \times 0 + 3 \times -1 = -5$		

Numerical Example

Numerical Example

Image					Filter			Conv Output		
1	1	5	7	3						
4	5	4	7	8						
9	1	3	1	6						
2	4	5	2	1						
1	3	1	2	1						
1	3	3	2	4						
					1	0	-1			
					1	0	-1			
					1	0	-1			
								2	-8	-5
								3	0	-3
								3	3	1
								-5	4	3

$$5x^1 + 2x^0 + 1x^{-1} + 1x^1 + 2x^0 + 1x^{-1} + 3x^1 + 2x^0 + 4x^{-1} = 3$$

Smoothing Filter

Image						Filter			Conv Output			
						1	1	1				
128	128	128	52	52	52							
128	128	128	52	52	52	1	1	1	x1/9	128	103	77
128	128	128	52	52	52	1	1	1		103	94	86
52	52	52	128	128	128					77	86	94
52	52	52	128	128	128					103	128	
52	52	52	128	128	128							

128	128	128	52	52	52							
128	128	128	52	52	52	1	1	1	x1/9	128	103	77
128	128	128	52	52	52	1	1	1		103	94	86
52	52	52	128	128	128					77	86	94
52	52	52	128	128	128					103	128	
52	52	52	128	128	128							

Zero Padding - Edge Filter - Horizontal

Mirror Padding 1 for Filter Size 3x3

Image								Filter			Conv Output					
128	125	128	128	52	52	52	52									
126	128	126	128	52	128	52	128	1	1	1	0	0	0	0	0	0
128	125	128	128	52	52	52	52	0	0	0	-4	-2	-2	76	76	152
128	128	128	128	52	52	52	52	-1	-1	-1	225	225	76	-76	-228	-228
52	52	52	52	128	128	128	128				228	228	76	-76	-228	-228
52	52	52	52	128	128	128	128				0	0	0	0	0	0
52	52	52	52	128	128	128	128				0	0	0	0	0	0
52	52	52	52	128	128	128	128									
52	52	52	52	128	128	128	128									

Mirror Padding 1 for Filter Size 3x3

Image									Filter			Conv Output					
128	125	128	128	52	52	52	52										
126	128	126	128	52	128	52	128		1	1	1	0	0	0	0	0	0
128	125	128	128	52	52	52	52		0	0	0	-4	-2	-2	76	76	152
128	128	128	128	52	52	52	52		-1	-1	-1	225	225	76	-76	-228	-228
52	52	52	52	128	128	128	128					228	228	76	-76	-228	-228
52	52	52	52	128	128	128	128					0	0	0	0	0	0
52	52	52	52	128	128	128	128					0	0	0	0	0	0
52	52	52	52	128	128	128	128										

Mirror Padding 1 for Filter Size 3x3

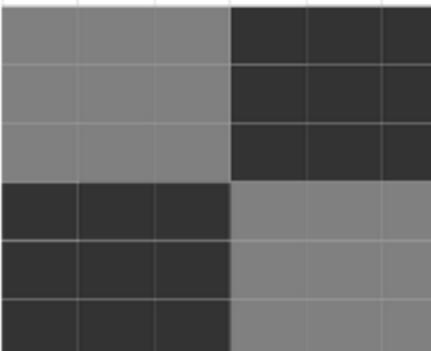
Mirror Padding 2 for Filter Size 5x5

	Image									Filter					Conv Output					
128	128	128	128	128	52	52	52	52	52	52	52	52	52	52	128	128	109	90	71	52
128	128	128	128	128	52	52	52	52	52	52	52	52	52	52	128	128	109	90	71	52
128	128	128	128	128	52	52	52	52	52	52	52	52	52	52	128	128	109	90	71	52
128	128	128	128	128	52	52	52	52	52	52	52	52	52	52	109	109	100	90	81	71
52	52	52	52	52	128	128	128	128	128	128	128	128	128	128	90	90	90	90	90	90
52	52	52	52	52	128	128	128	128	128	128	128	128	128	128	71	71	81	90	100	109
52	52	52	52	52	128	128	128	128	128	128	128	128	128	128	52	52	71	90	109	128
52	52	52	52	52	128	128	128	128	128	128	128	128	128	128						
52	52	52	52	52	128	128	128	128	128	128	128	128	128	128						

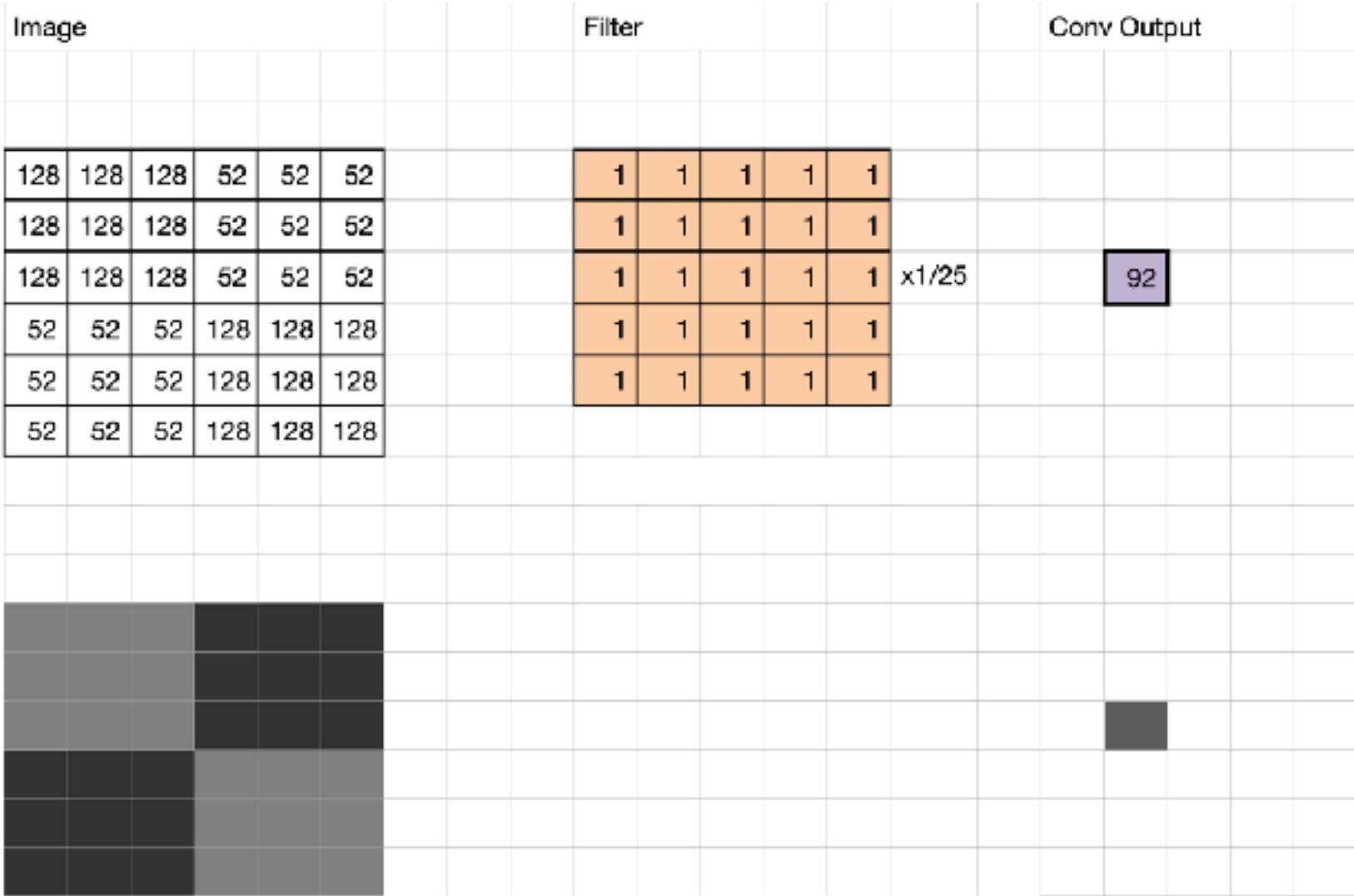
Stride 2, Padding 2

Stride 2, Padding 2

Stride = 2, Padding 0

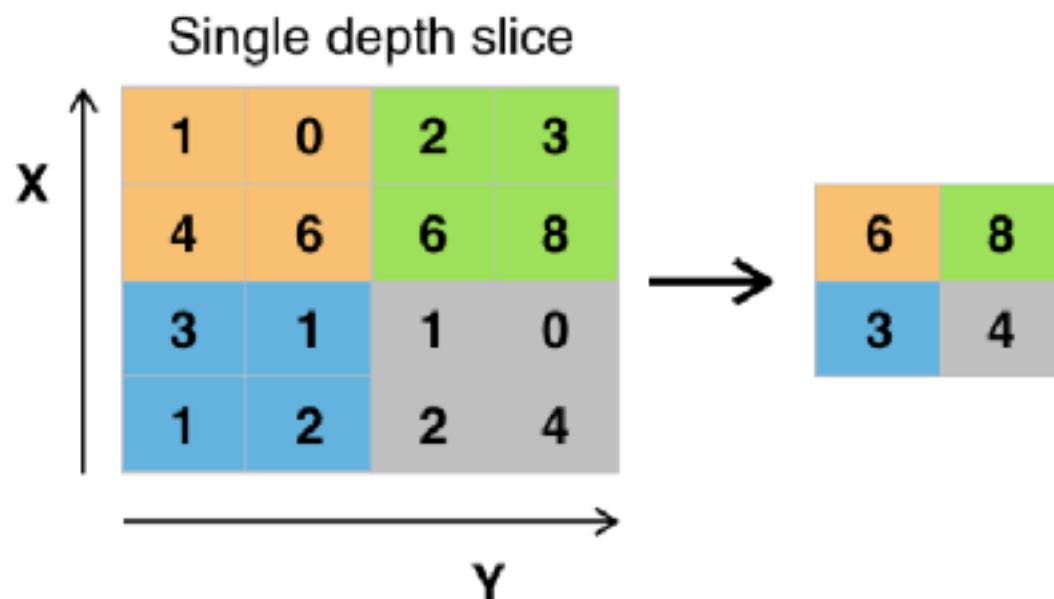
Image	Filter	Conv Output
128 128 128 52 52 52	1 1 1 1 1	
128 128 128 52 52 52	1 1 1 1 1	
128 128 128 52 52 52	1 1 1 1 1	x1/25
52 52 52 128 128 128	1 1 1 1 1	
52 52 52 128 128 128	1 1 1 1 1	
52 52 52 128 128 128	1 1 1 1 1	
		

Stride = 2, Padding 0



Pooling

- ▶ Input: 4x4
- ▶ Pool size 2x2
- ▶ Stride 2
- ▶ In case pools do not overlap



Today's Agenda

- ▶ Why we do convolution?
- ▶ How to define a convolution kernel?
- ▶ Filter Banks?

Filtering Examples



Original
Cameraman



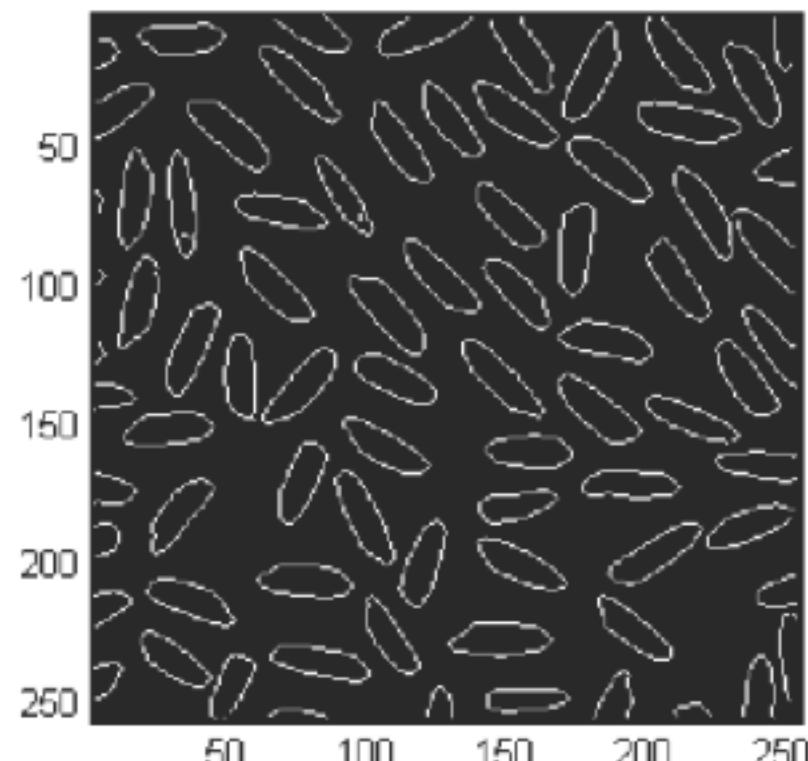
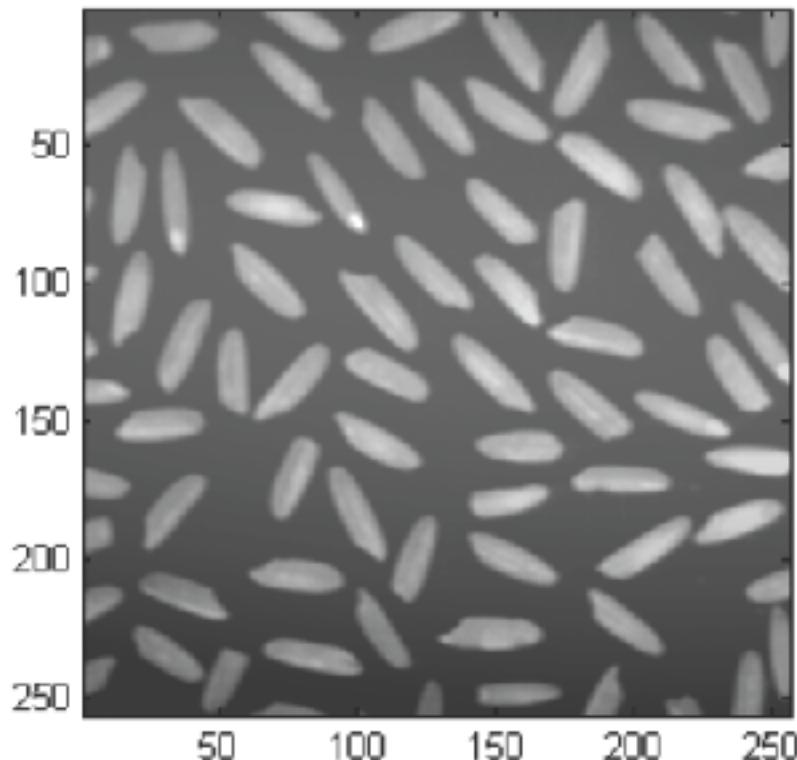
Cameraman blurred by convolution
Filter impulse response

$$\frac{1}{25} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & [0] & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

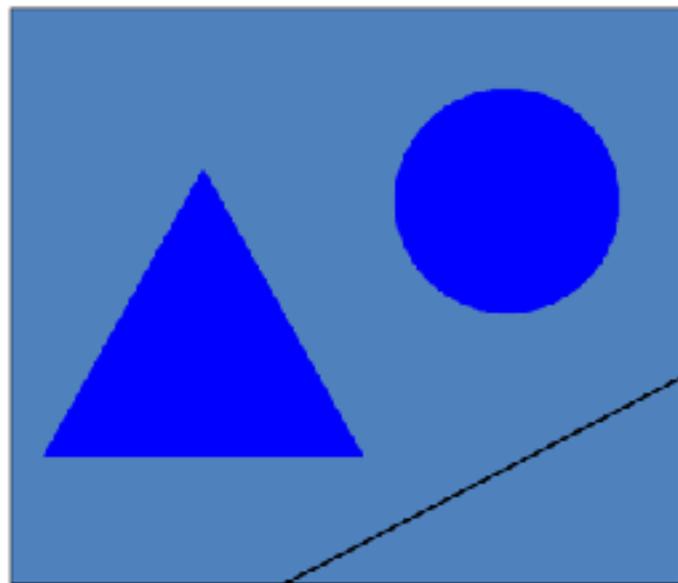
Noise Removal via fastNlMeansDenoising



Finding Shapes from Edges



Finding Shapes from Edges



Finding Edges - Differentiation

- ▶ Continuous form

$$\frac{\partial f}{\partial x} = \lim_{\Delta h \rightarrow 0} \frac{f(x + \Delta h) - f(x)}{\Delta h}$$

- ▶ Discrete form (put $\Delta h = 1$)

$$f_x = \frac{\partial f}{\partial x} = f(x+1) - f(x)$$

Discrete Derivatives in 2-D

$$f_x = \frac{\partial f}{\partial x} = f(x+1) - f(x) \quad f_y = \frac{\partial f}{\partial y} = f(y+1) - f(y)$$

$$h_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$h_y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



synthetic image



h_y output



h_x output

Vertical Edge Detection

$$h_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$



Original Image: Opera

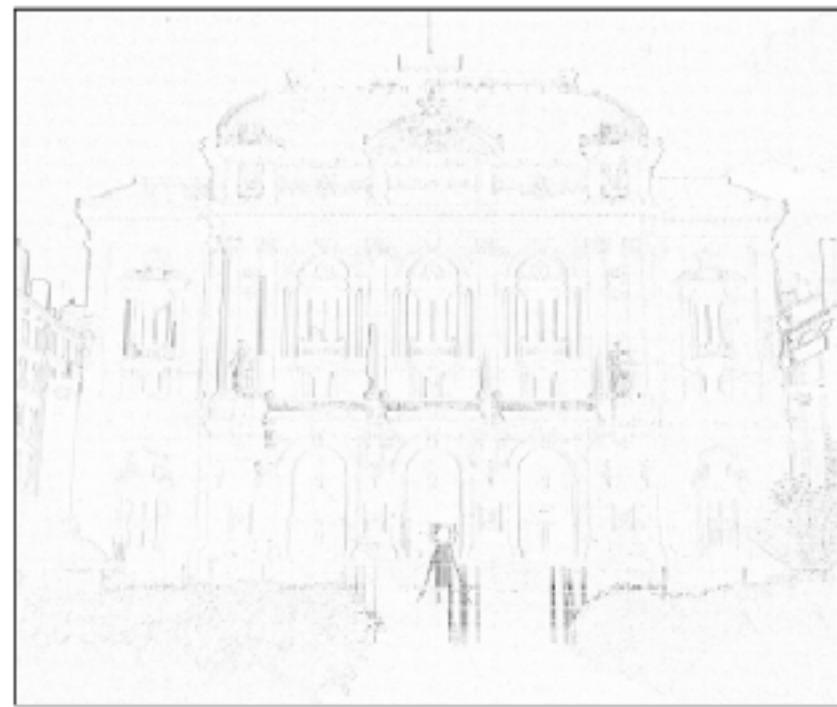


Vertical Edge Detection

$$h_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$



Original Image: Opera



Inverted Image for Visualisation

Horizontal Edge Detection

$$h_y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



Original Image: Opera

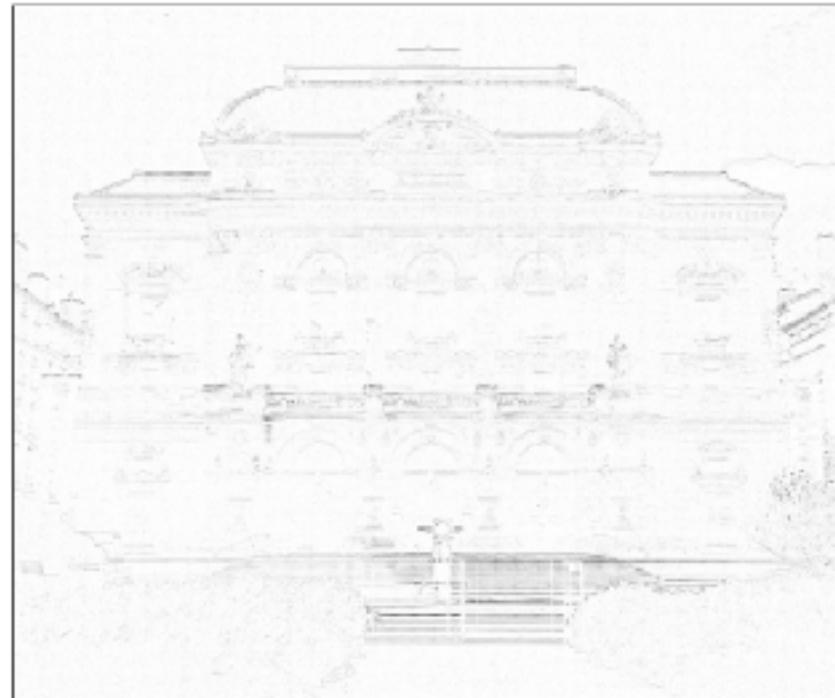


Horizontal Edge Detection



Original Image: Opera

$$h_y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



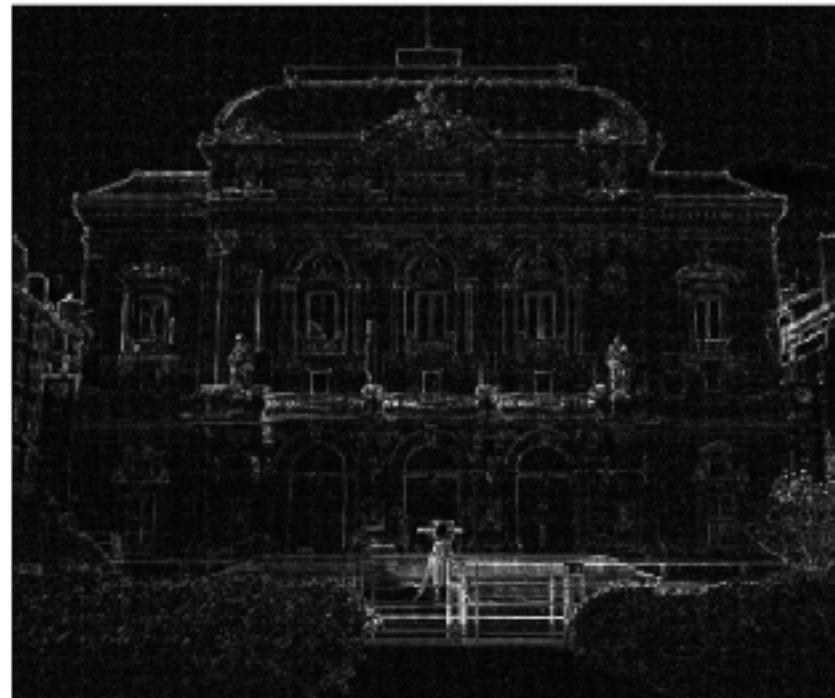
Inverted Image for Visualisation

Laplacian Filter

$$h = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$



Original Image: Opera

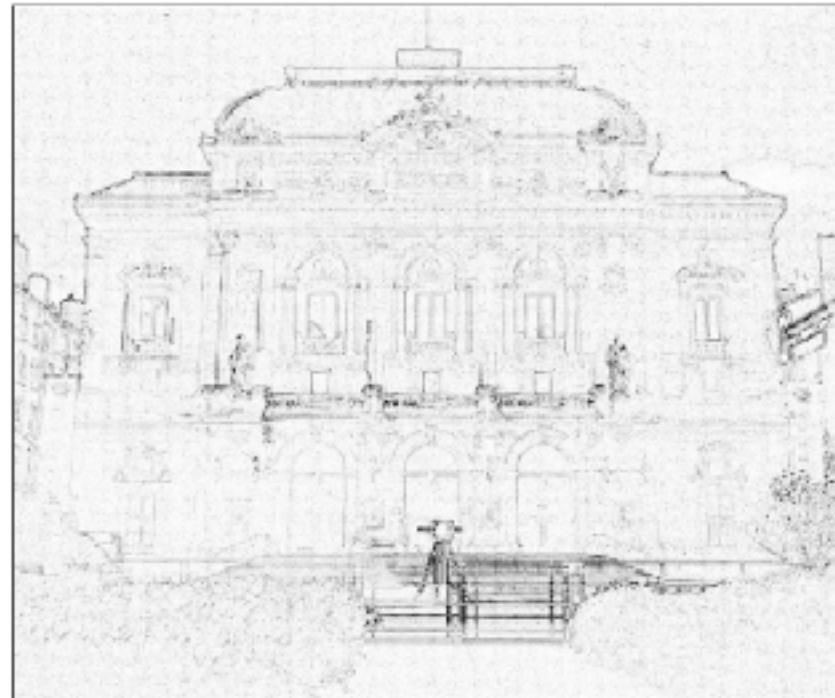


Laplacian Filter

$$h = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$



Original Image: Opera



Inverted Image for Visualisation

Sobel, Prewit & Roberts Operators

Sobel

$$h_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Prewit

$$h_x = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Roberts

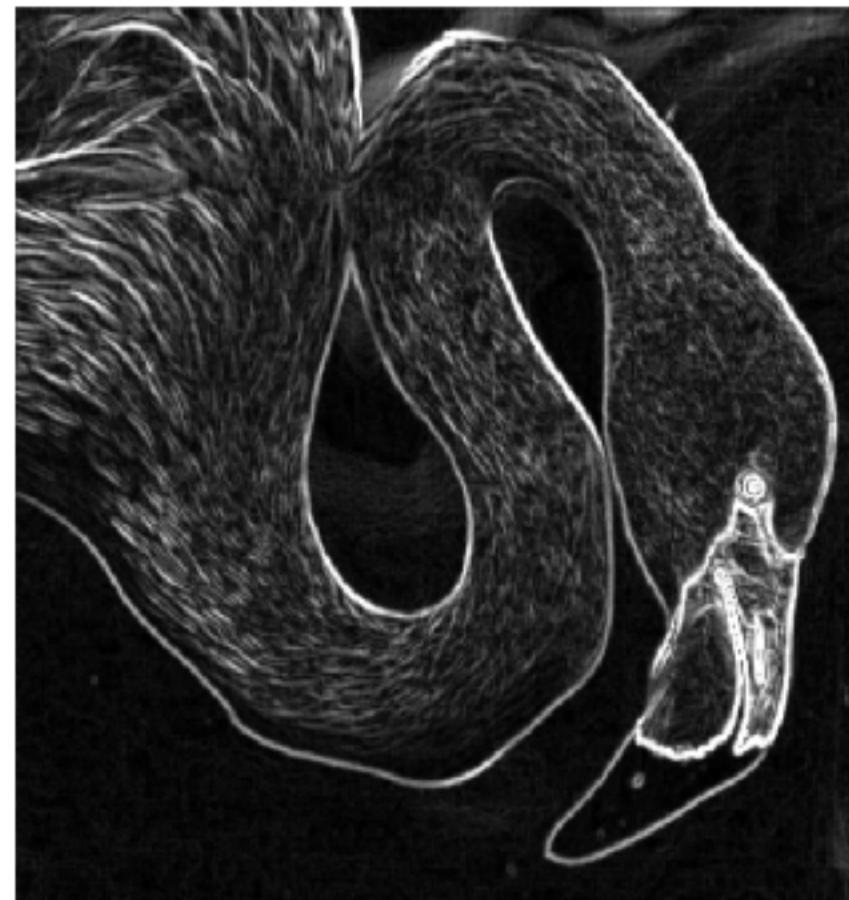
$$h_x = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$h_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$h_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$h_y = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Sobel



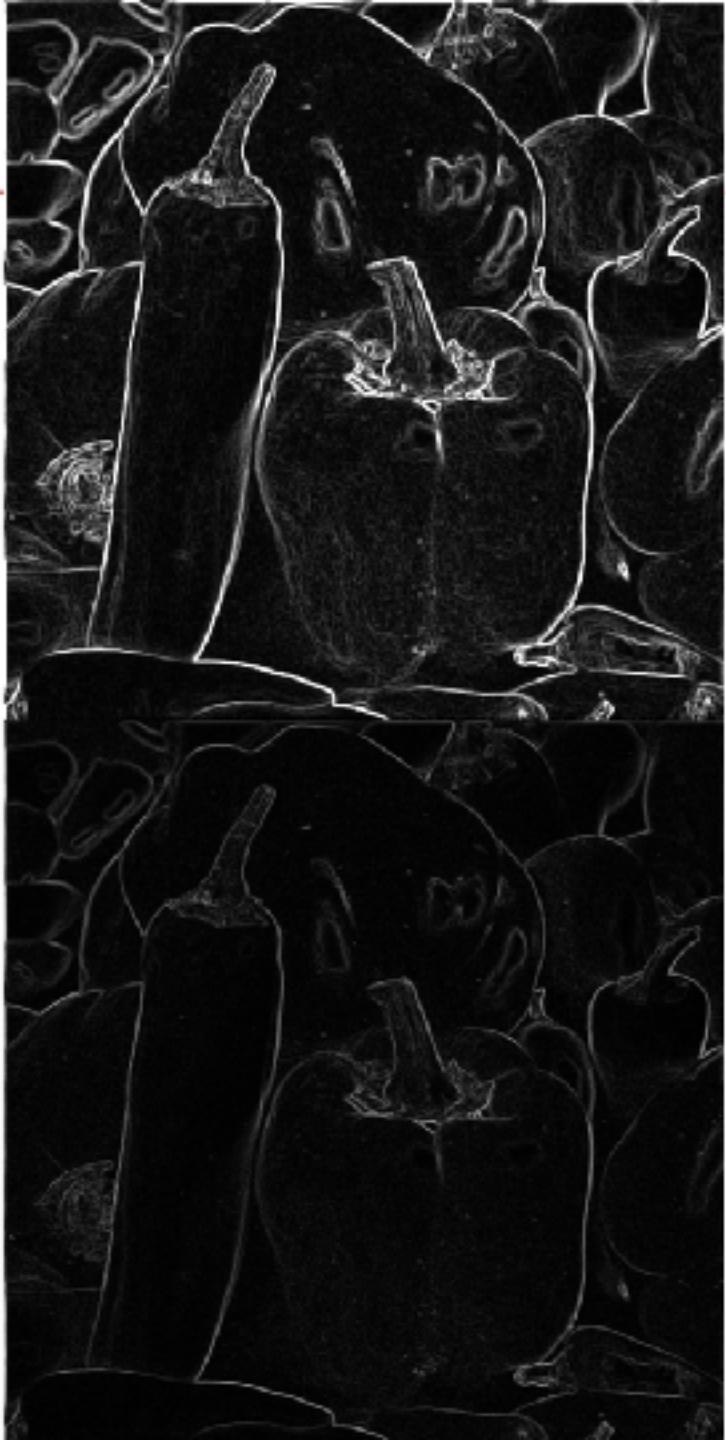
Original



Sobel

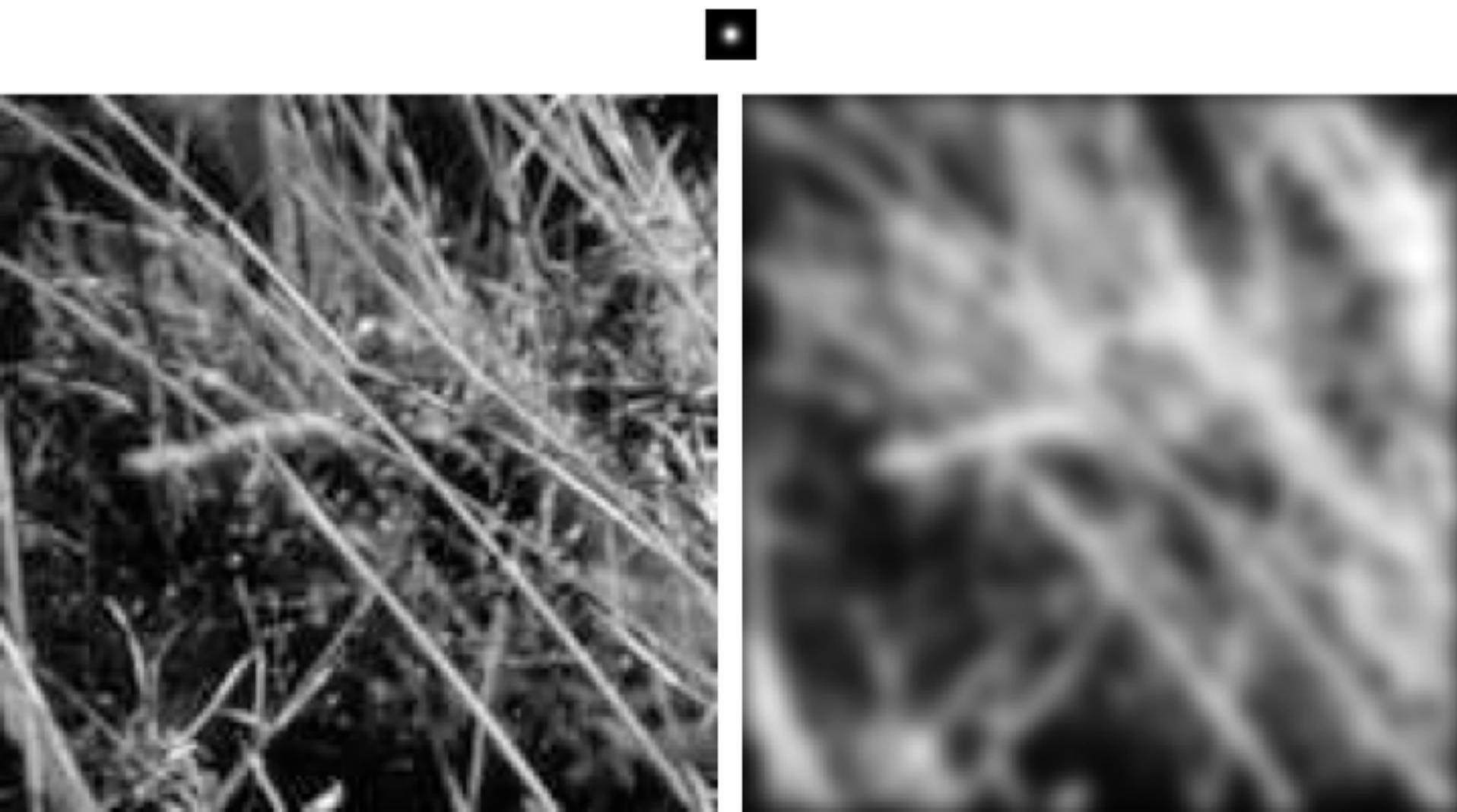


Roberts



Prewit

Smoothing with Gaussian filter



Practice with linear filters



Original

0	0	0
0	1	0
0	0	0

?



Practice with linear filters



Original

0	0	0
0	1	0
0	0	0



Filtered
(no change)



Practice with linear filters



Original

0	0	0
0	0	1
0	0	0

?



Practice with linear filters



Original

0	0	0
0	0	1
0	0	0



Shifted left
By 1 pixel



Practice with linear filters



Original

0	0	0
0	2	0
0	0	0

-

$\frac{1}{9}$	1	1	1
1	1	1	1
1	1	1	1

?

(Note that filter sums to 1)



Practice with linear filters



Original

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{matrix}$$

-

$$\frac{1}{9} \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$



Sharpening filter

- Accentuates differences with local average



Sharpening

- ▶ Derivative operators can highlight gray-level discontinuities
- ▶ The result of the derivative operator can be superimposed on the original image

$$g(x, y) = f(x, y) + h(x, y)$$

0	-1	0
-1	5	-1
0	-1	0

$$f = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad h = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Sharpening Example

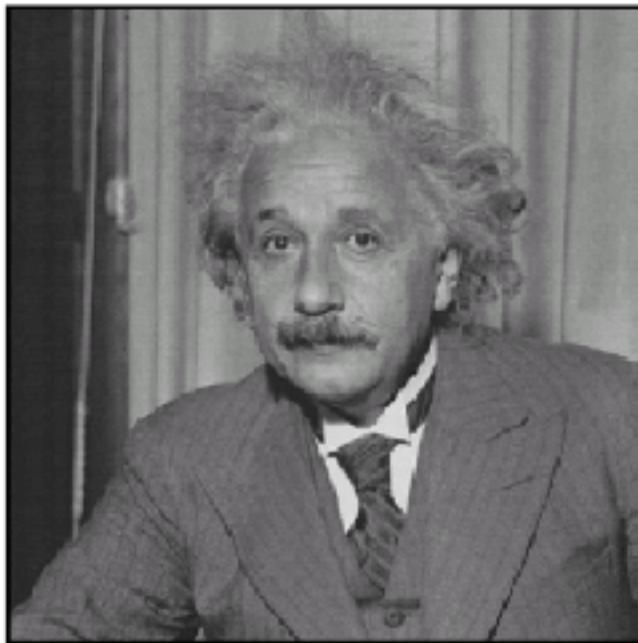
a b
c d

FIGURE 3.40

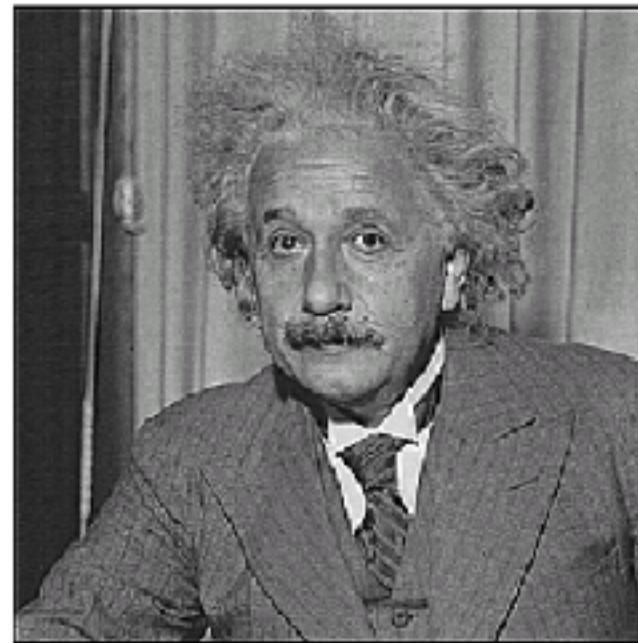
- (a) Image of the North Pole of the moon.
(b) Laplacian-filtered image.
(c) Laplacian image scaled for display purposes.
(d) Image enhanced by using Eq. (3.7-5).
(Original image courtesy of NASA.)



Sharpening



before

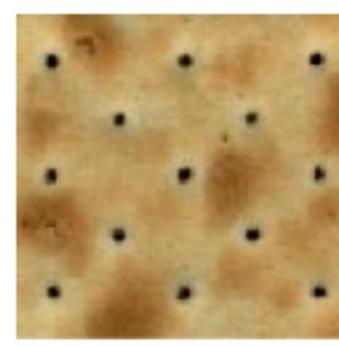
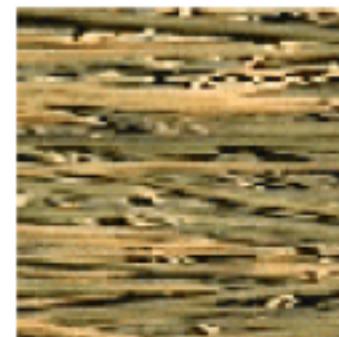


after



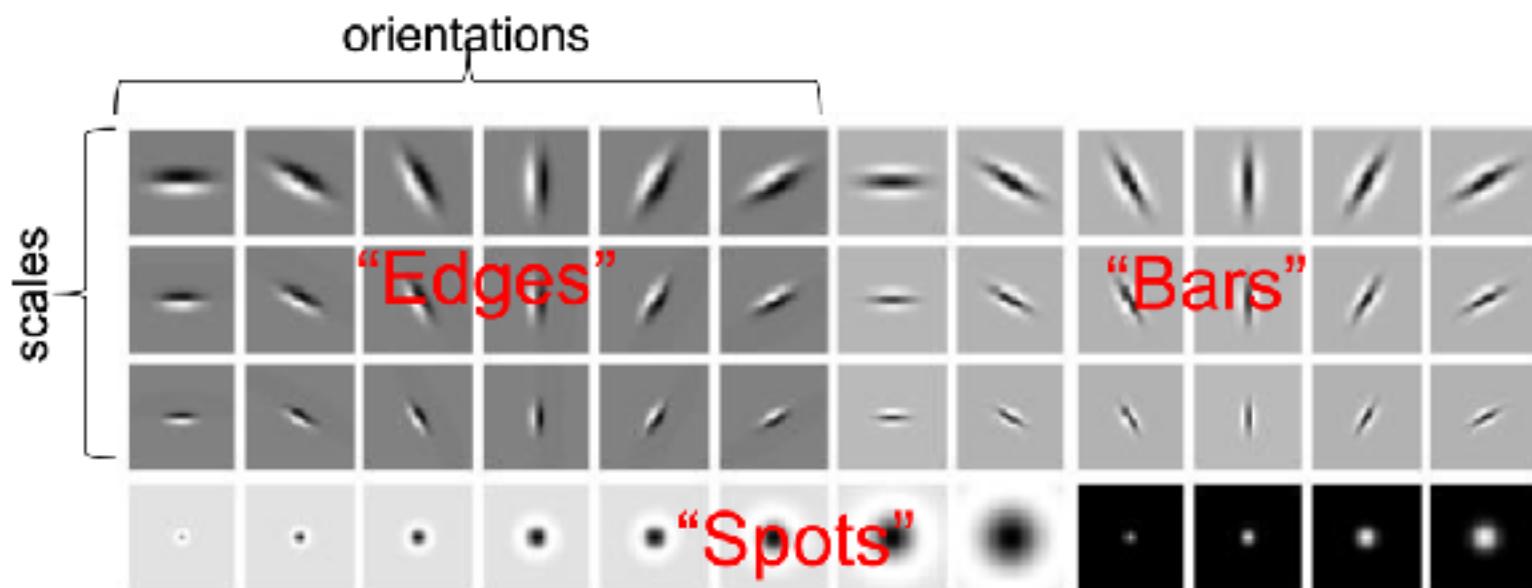
Image Classification using Filter Bank





Filter banks

- ▶ What filters to put in the bank?
 - ▶ Typically we want a combination of scales and orientations, different types of patterns.



Matlab code available for these examples: <http://www.robots.ox.ac.uk/~vgg/research/texclass/filters.html>

Filter bank

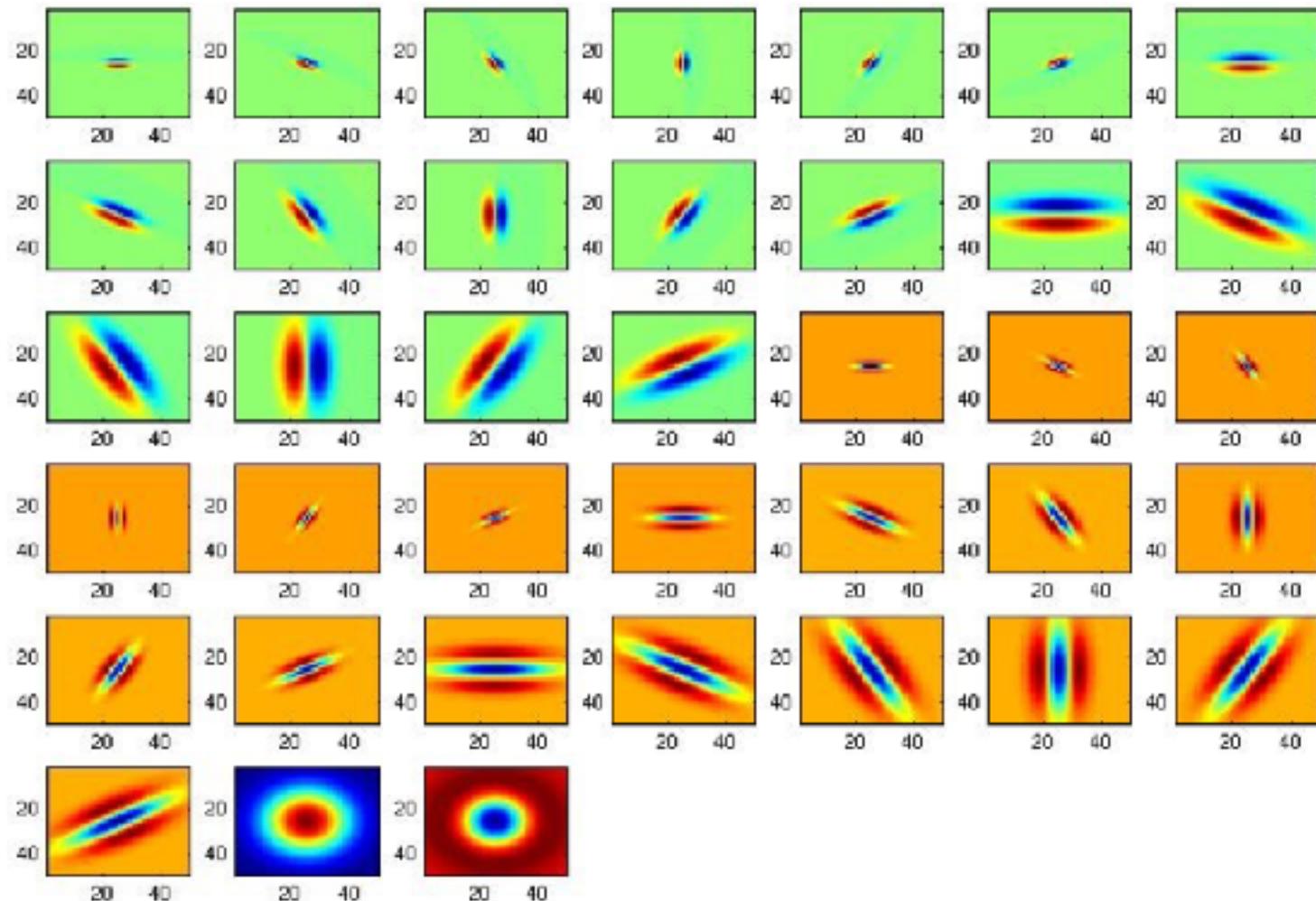
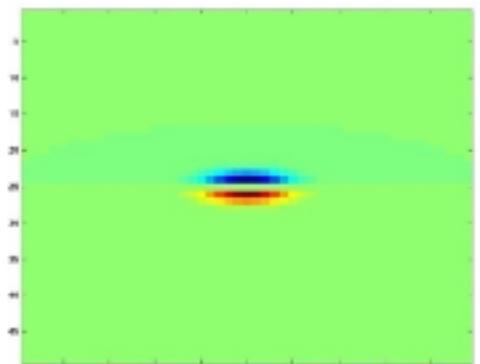
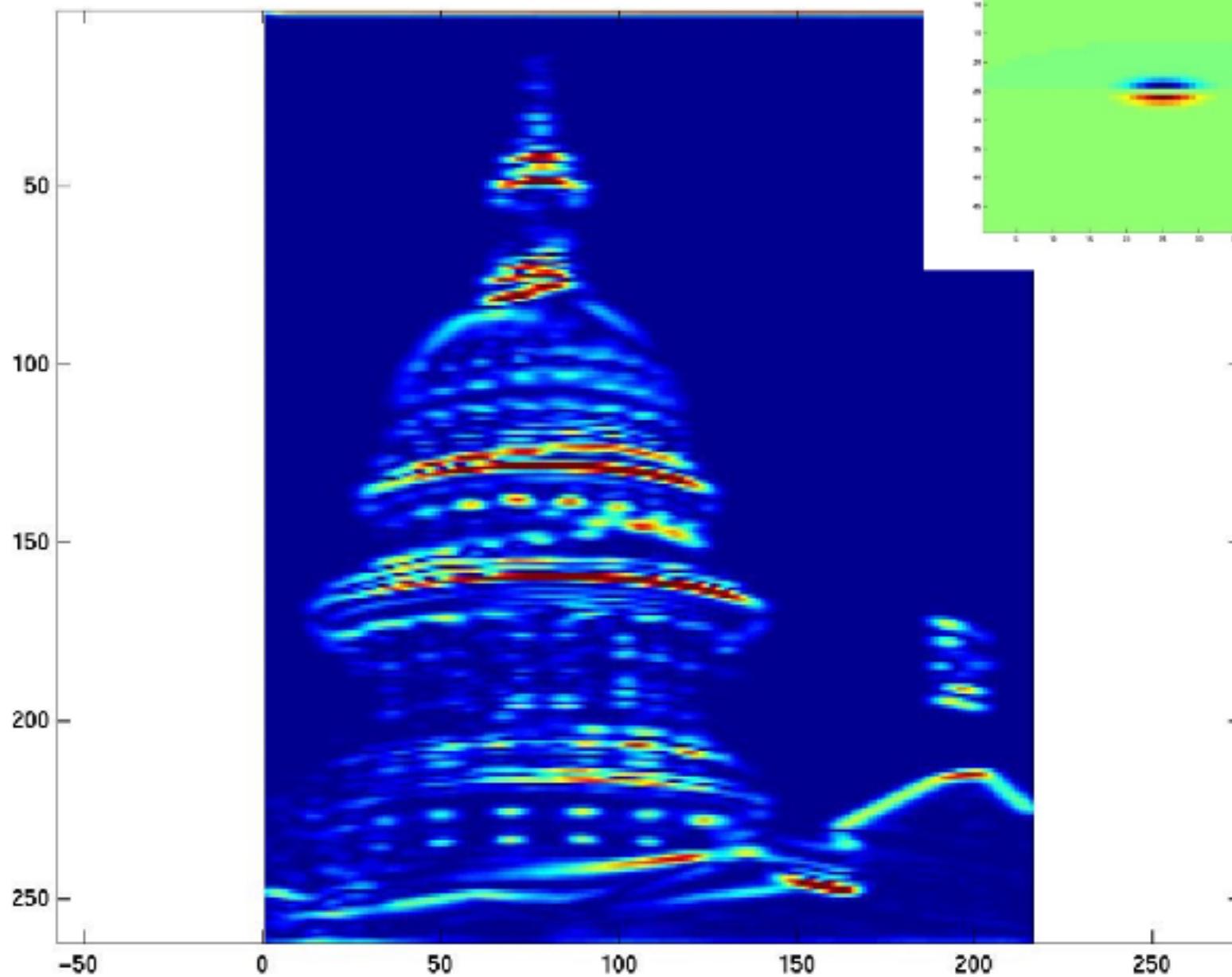
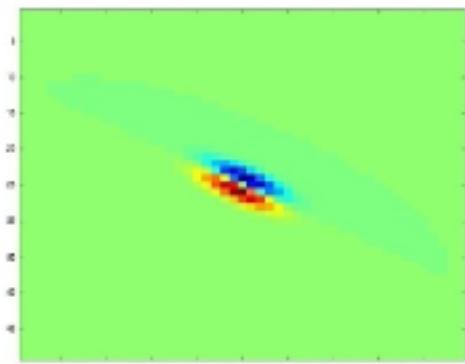
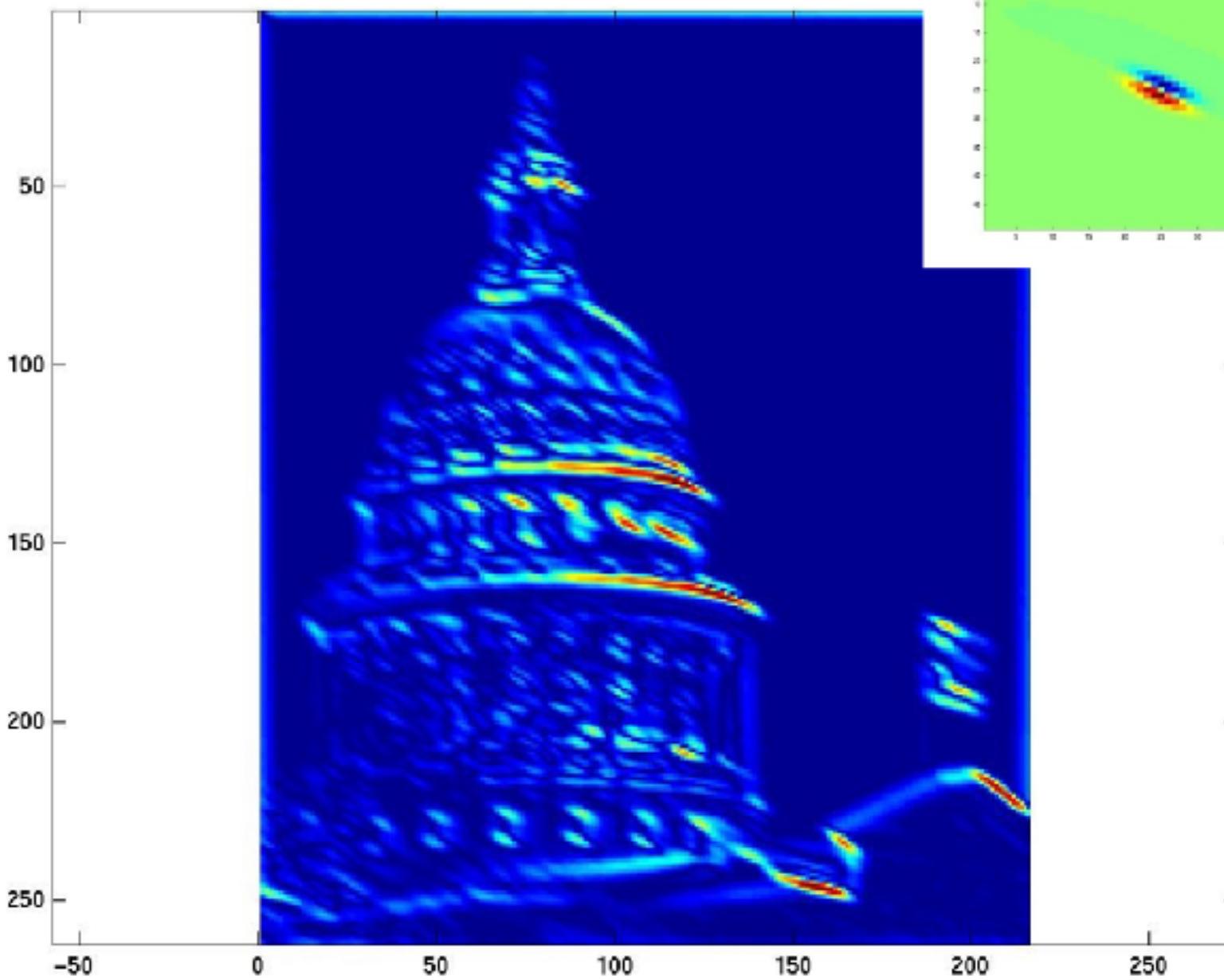


Image from <http://www.texaseplorer.com/austincap2.jpg>

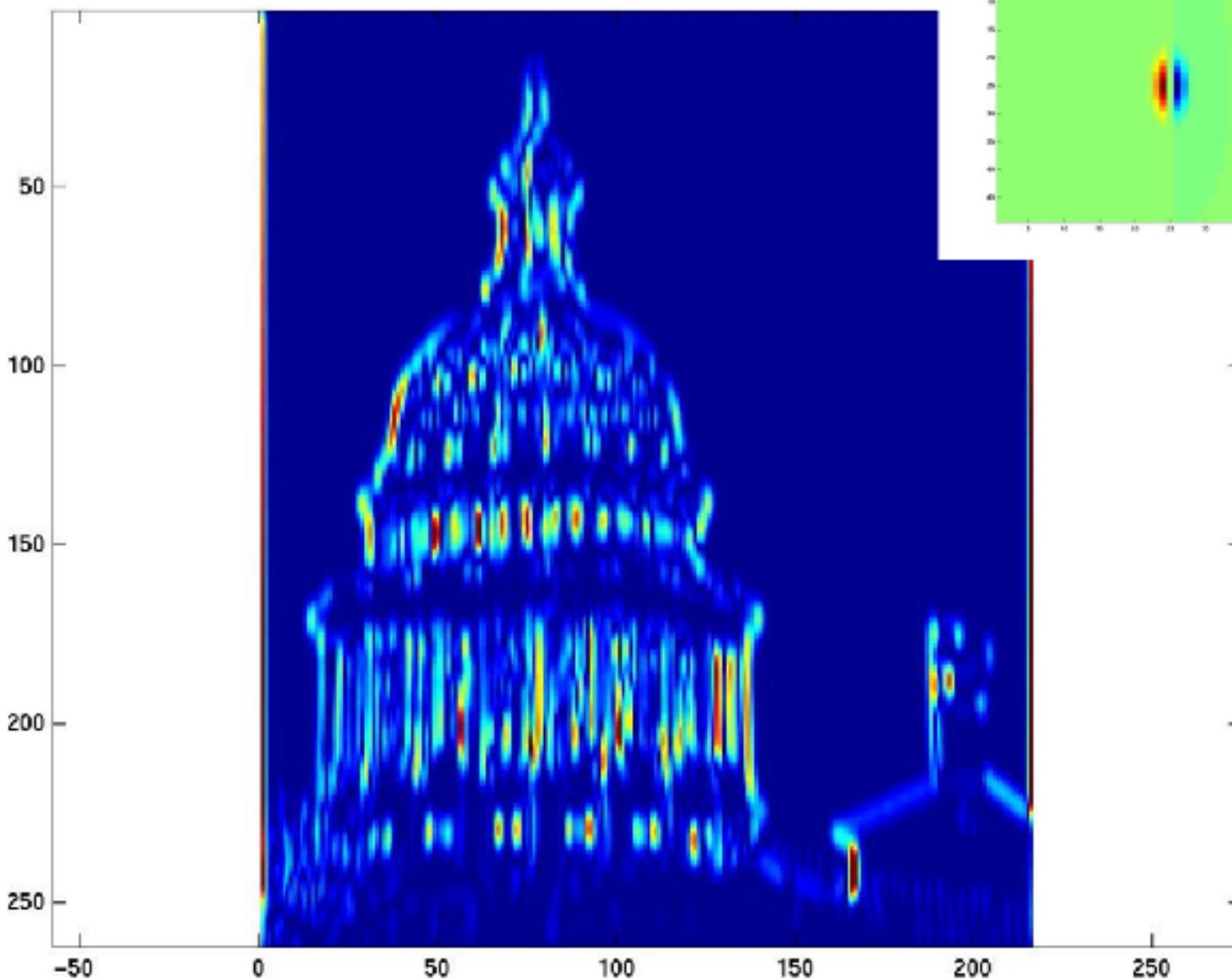


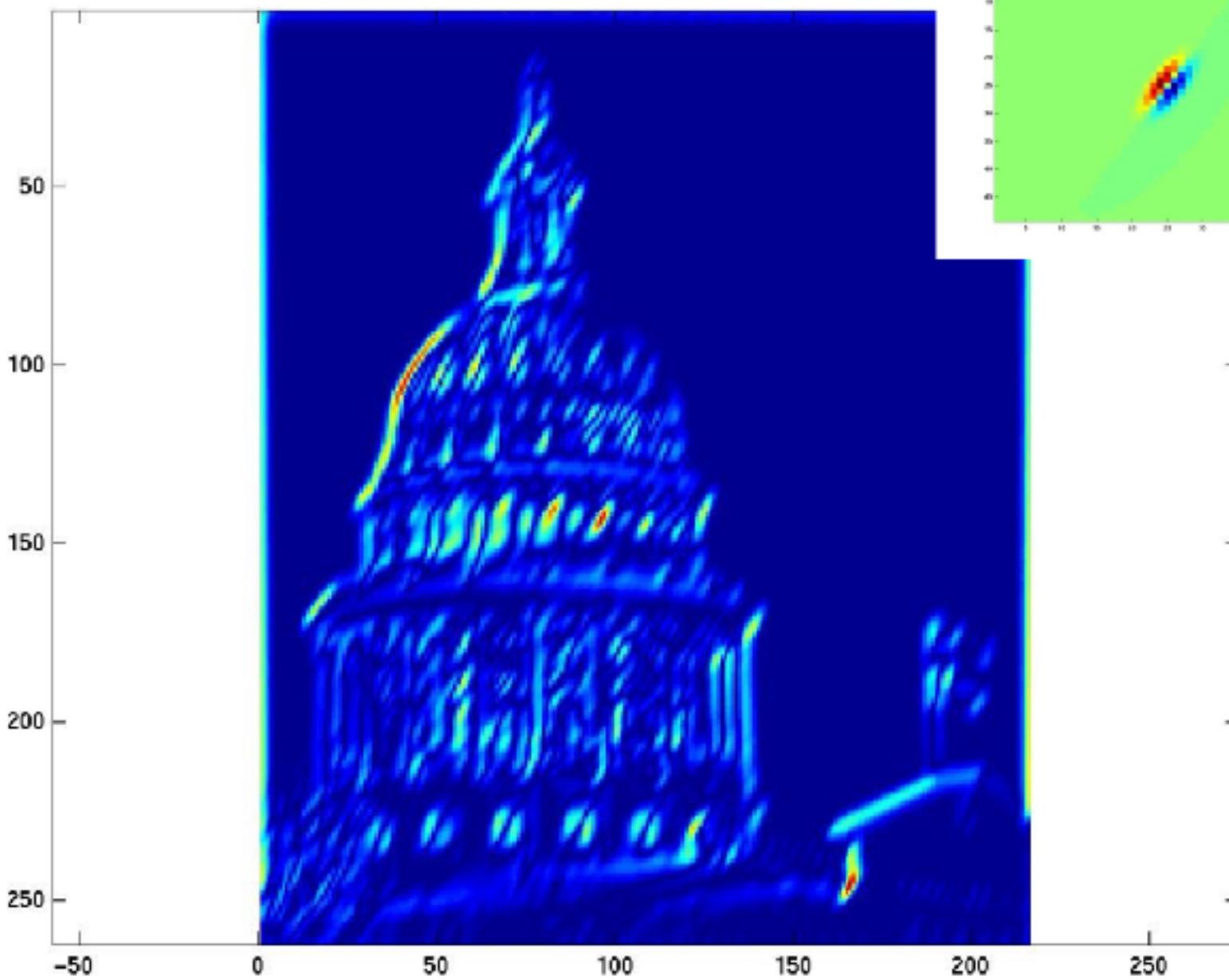
Kristen Grauman

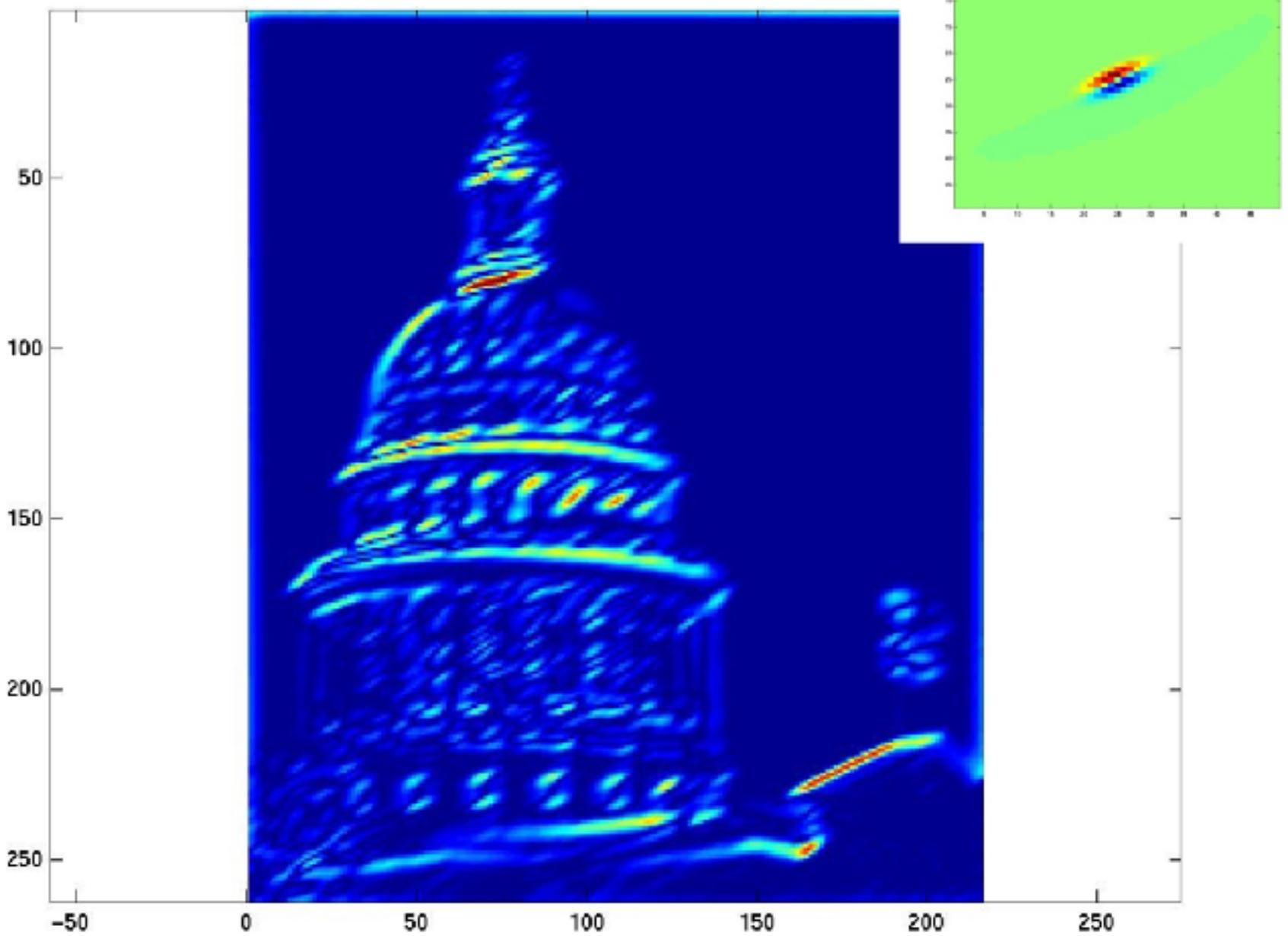


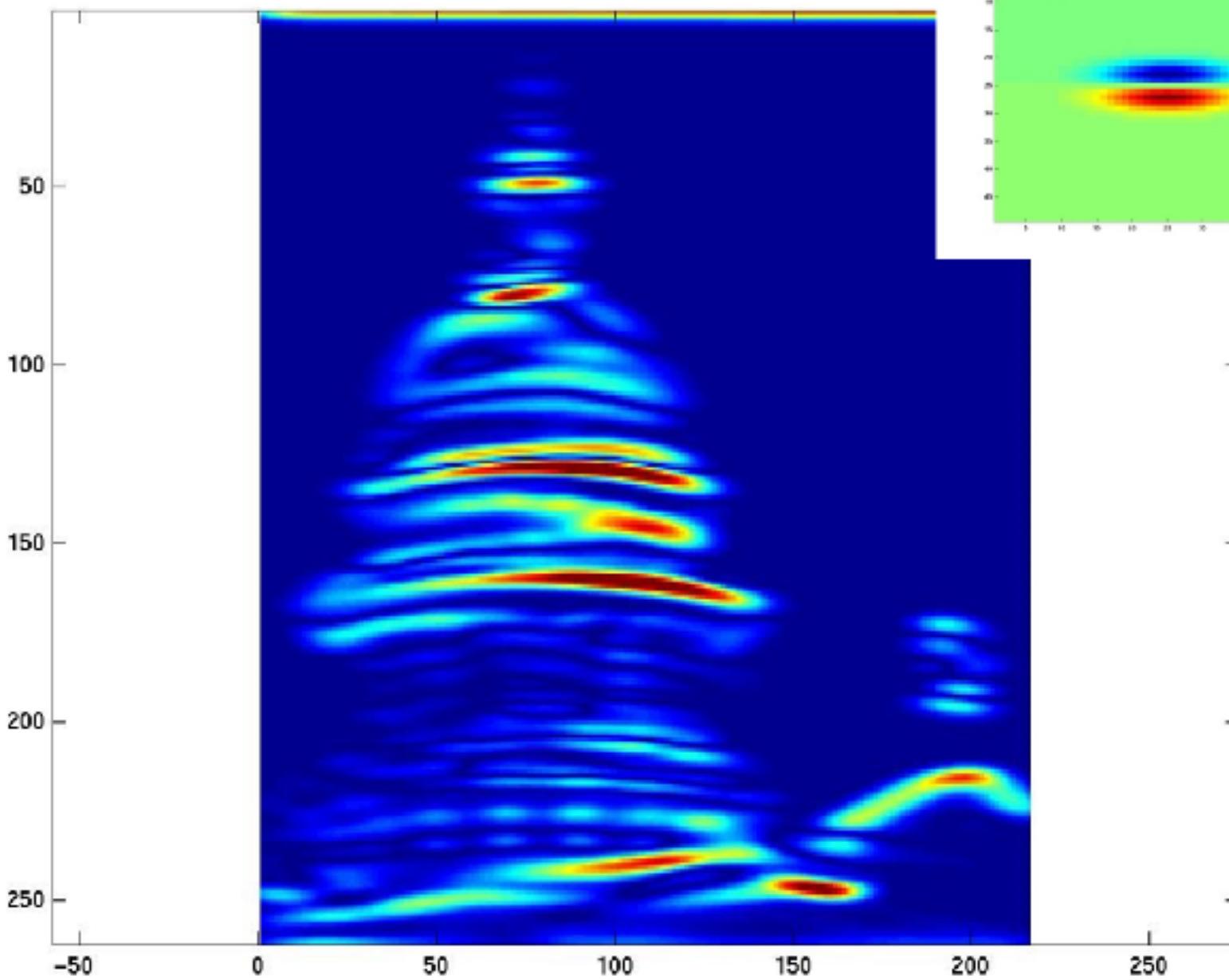


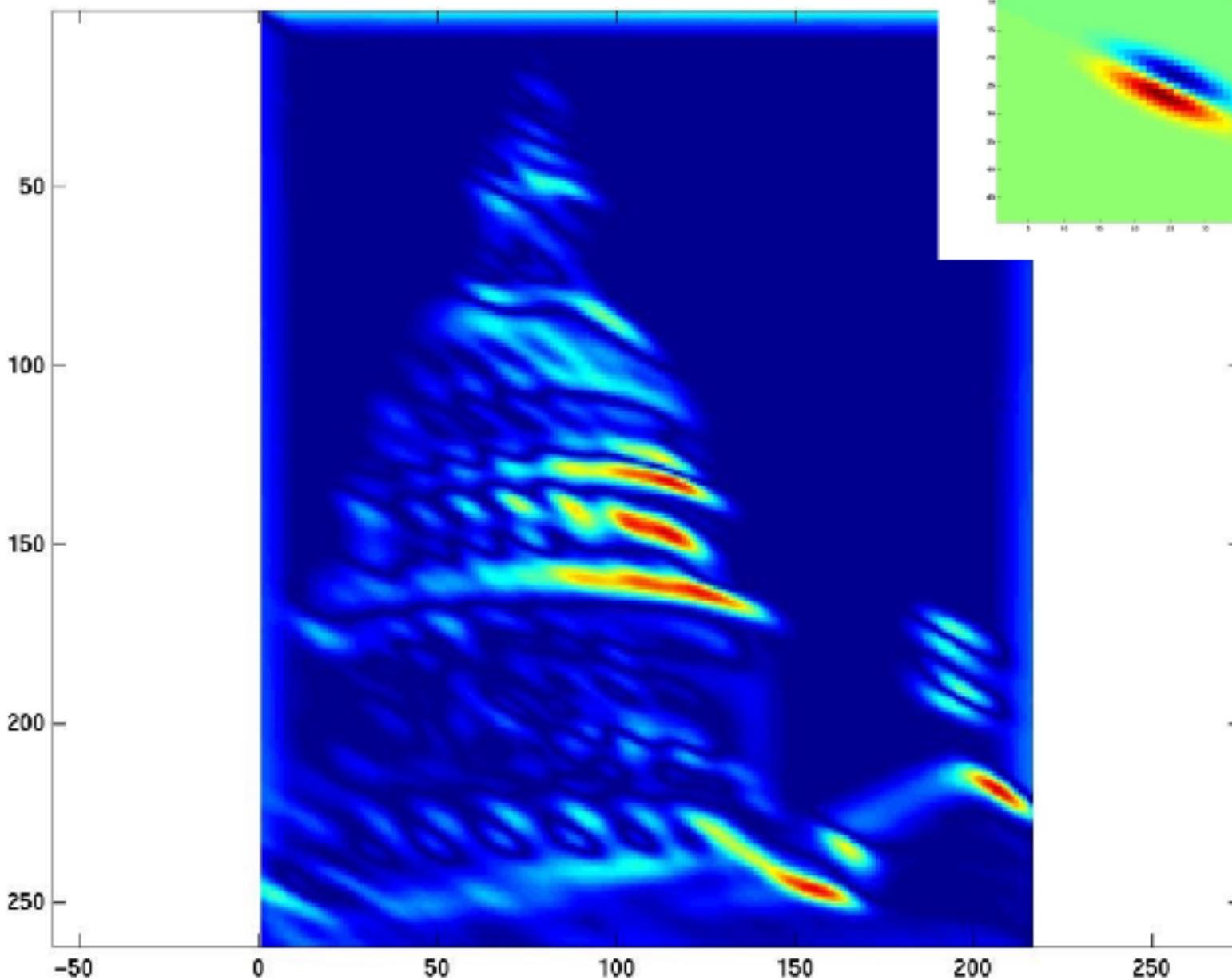


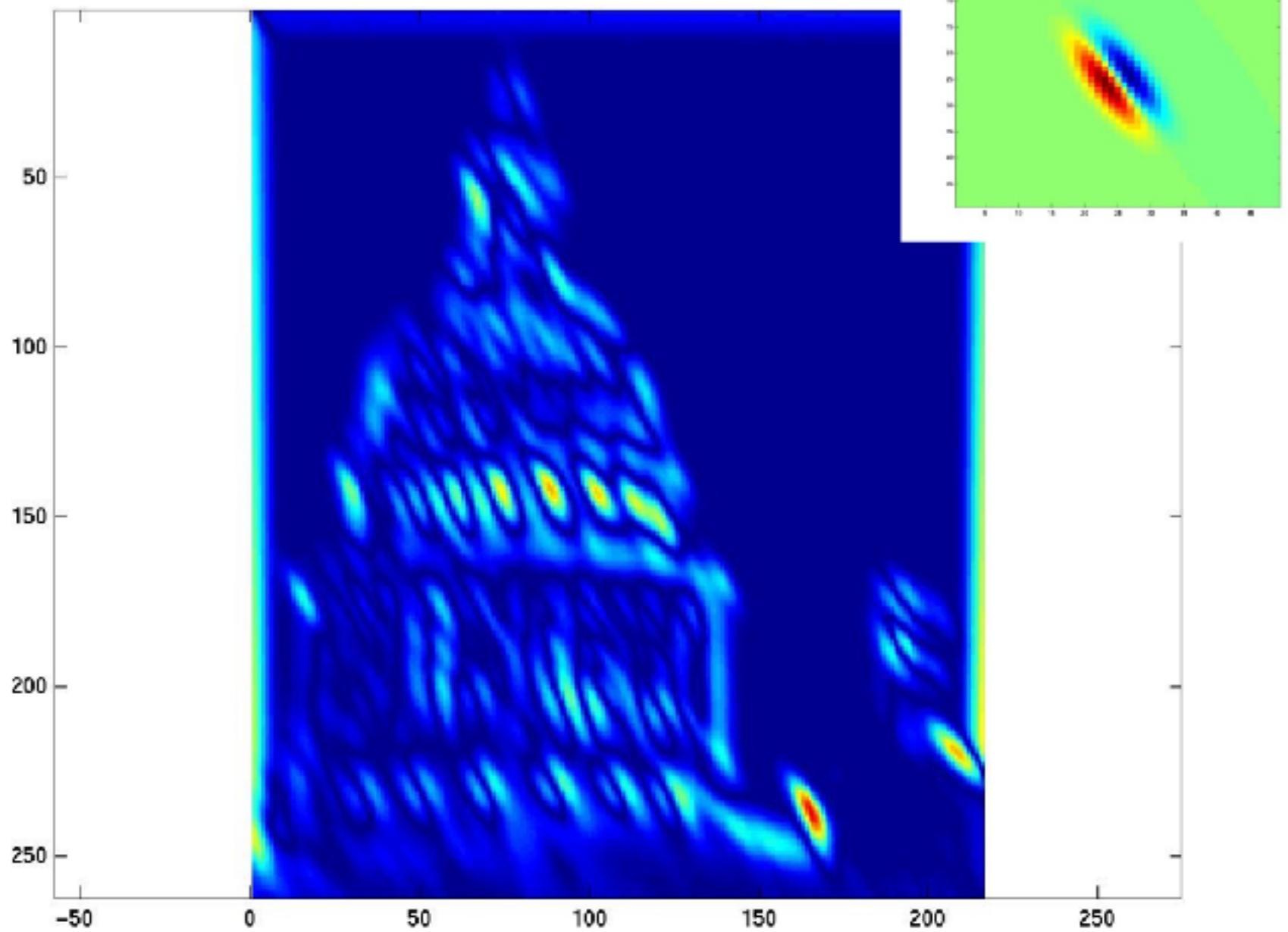


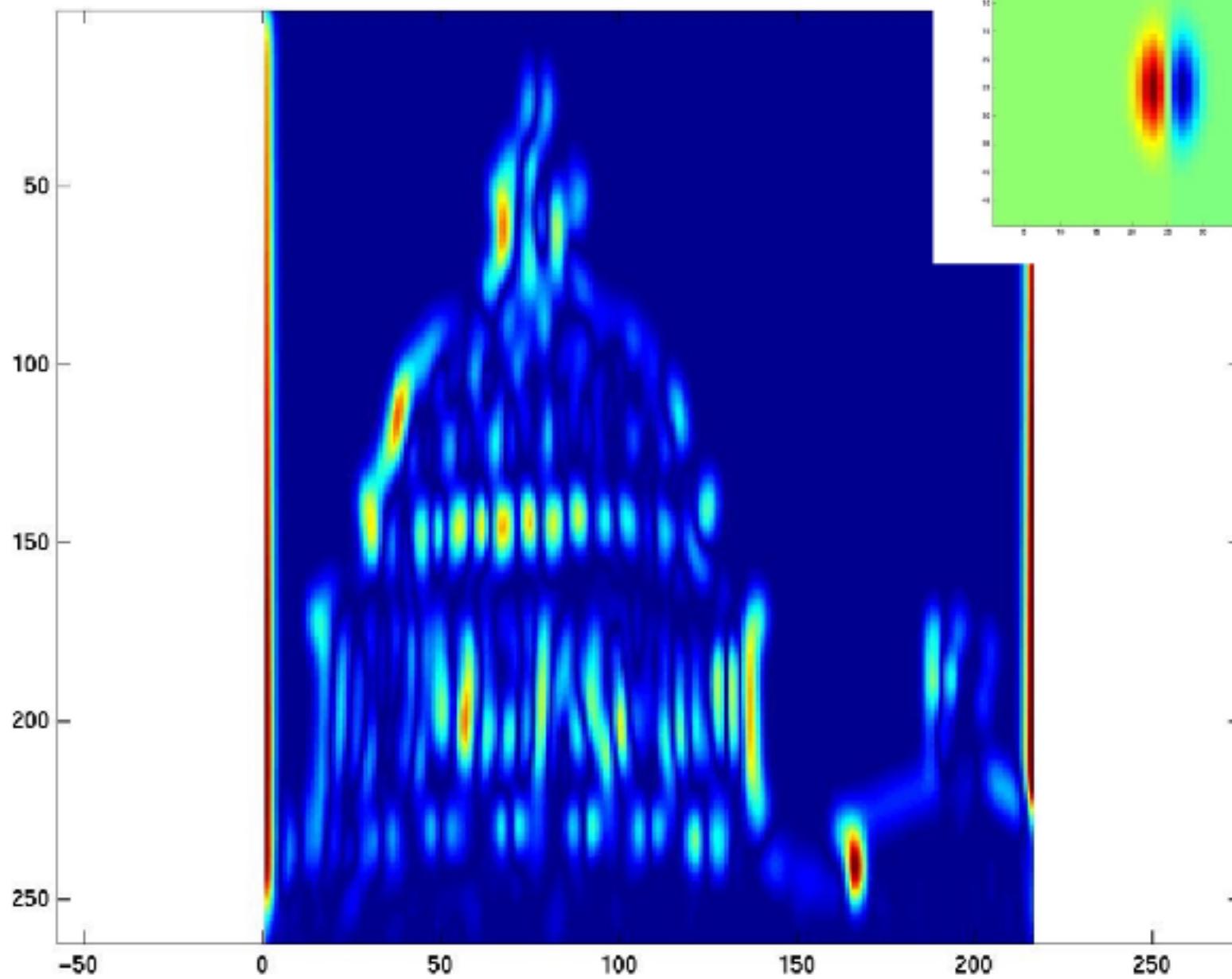


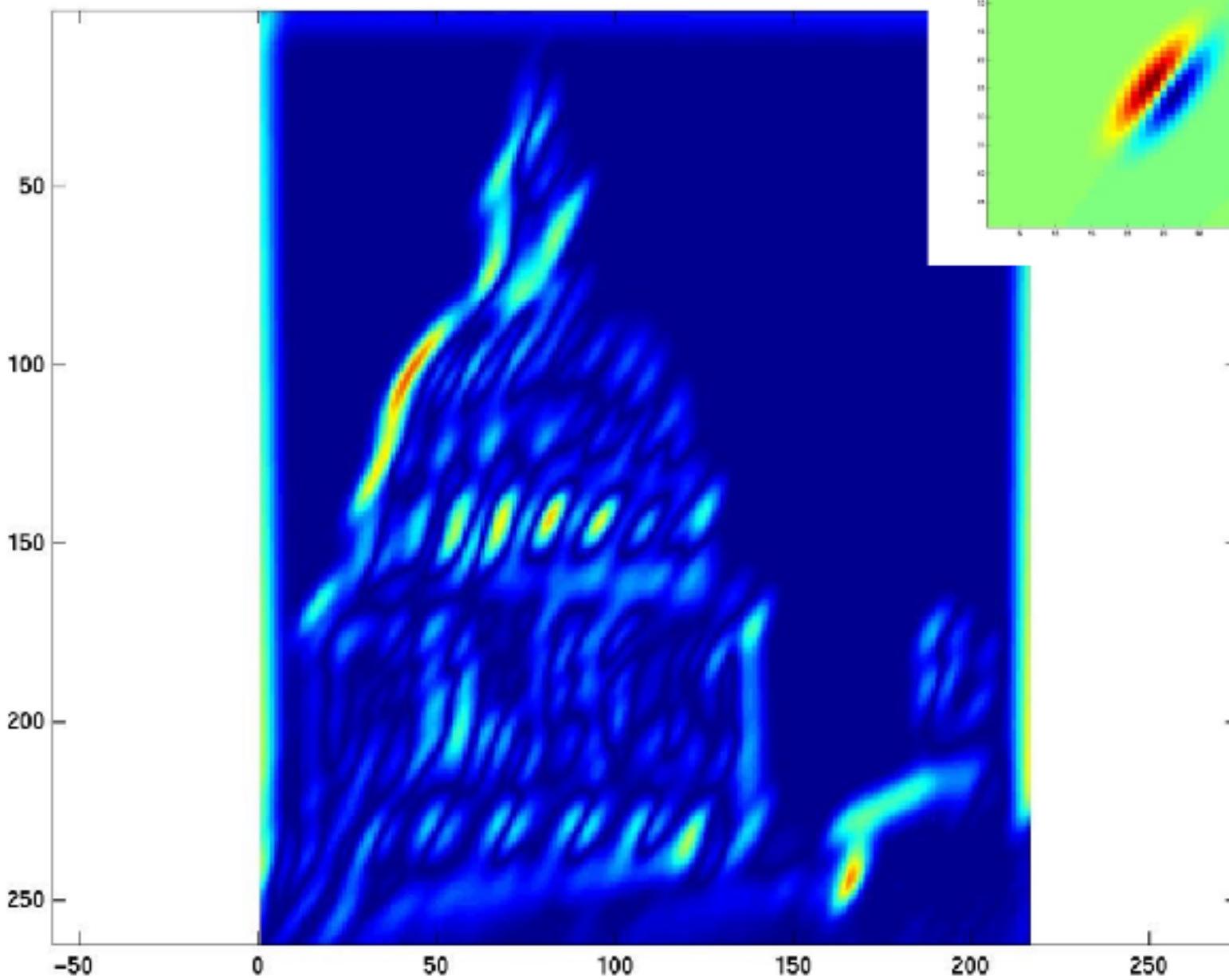


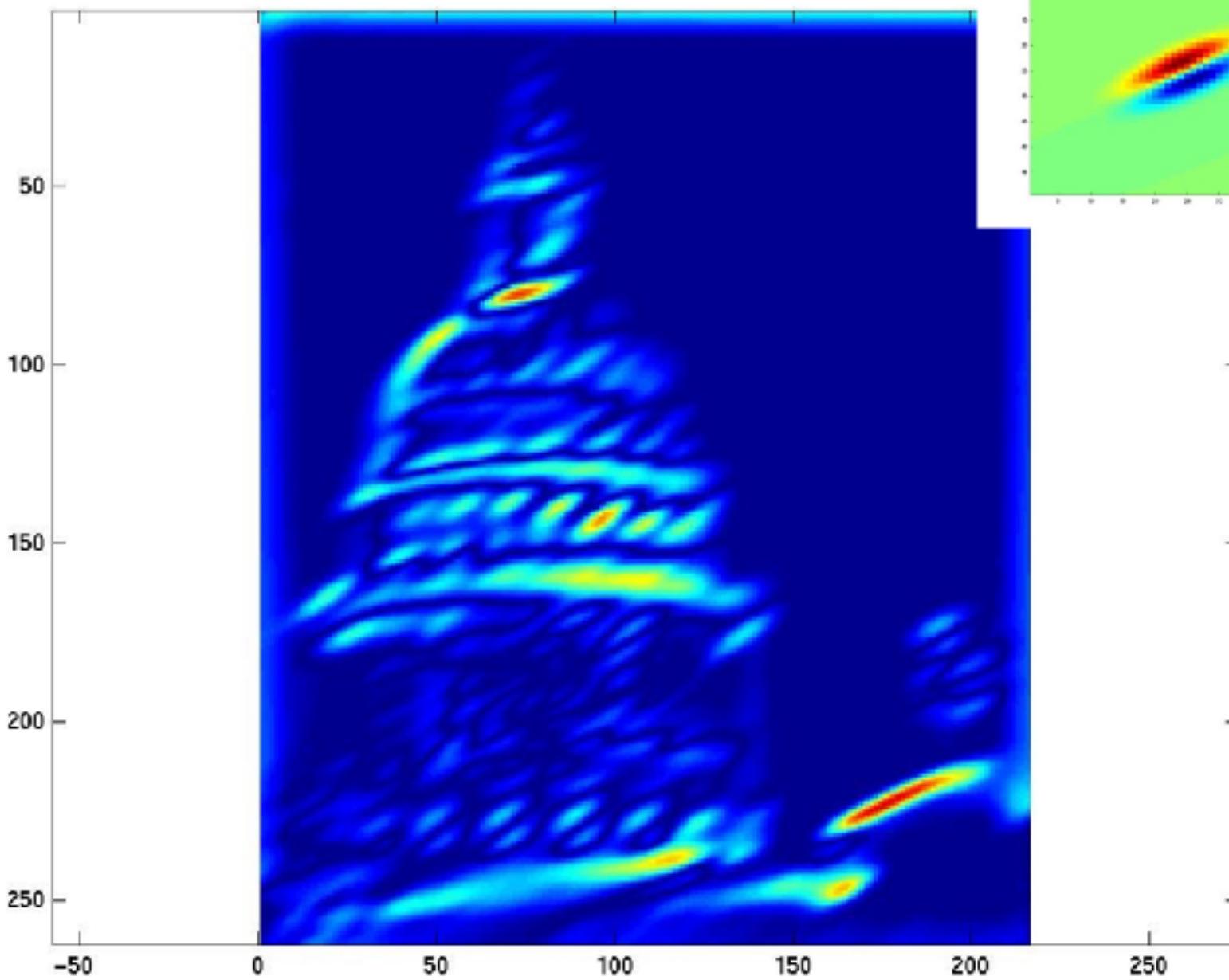


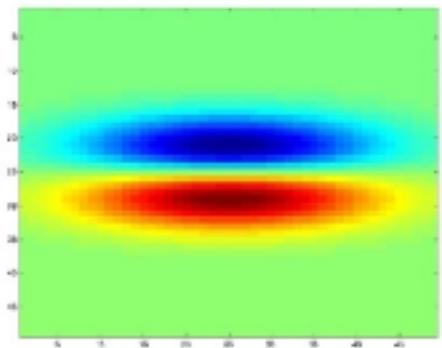
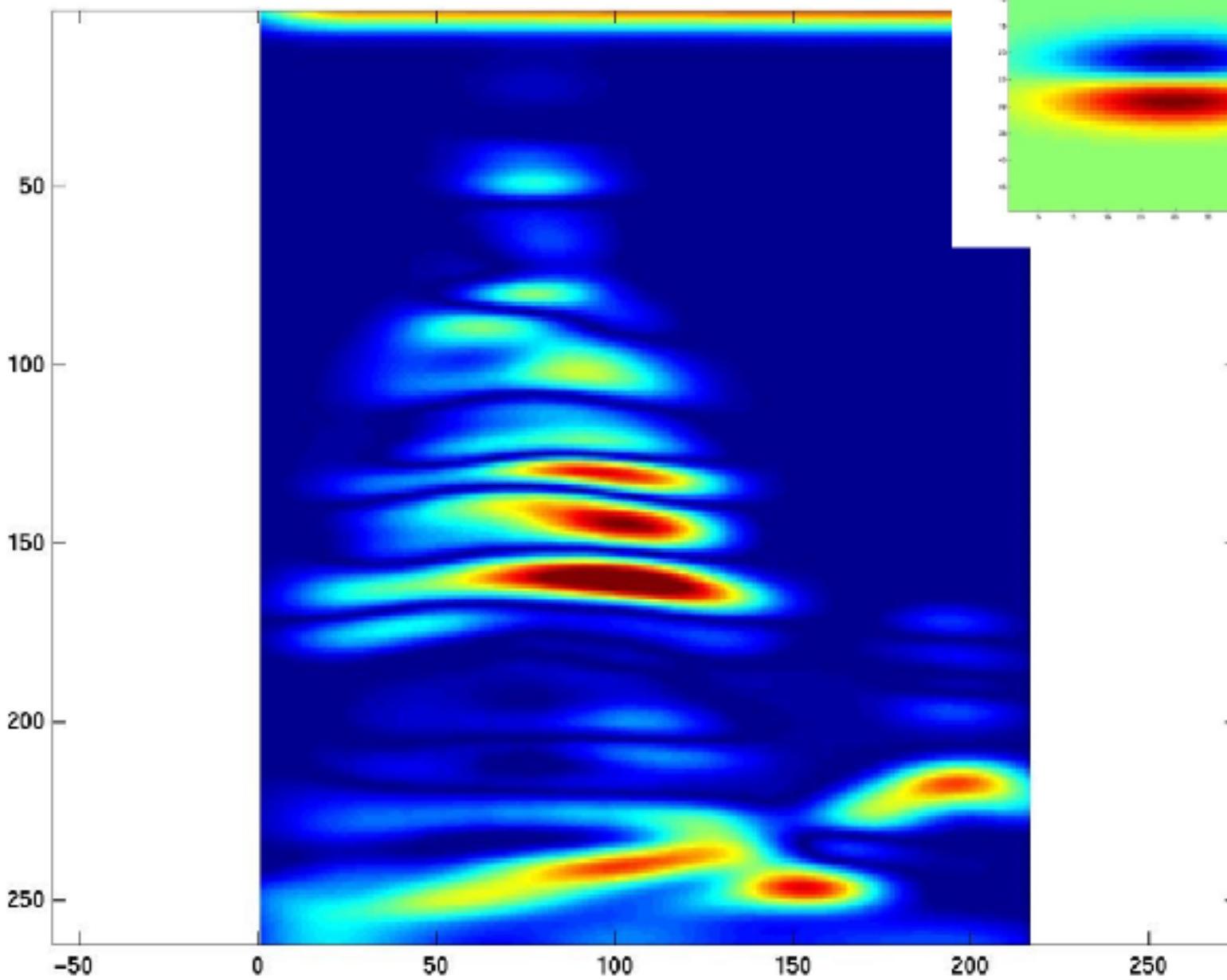


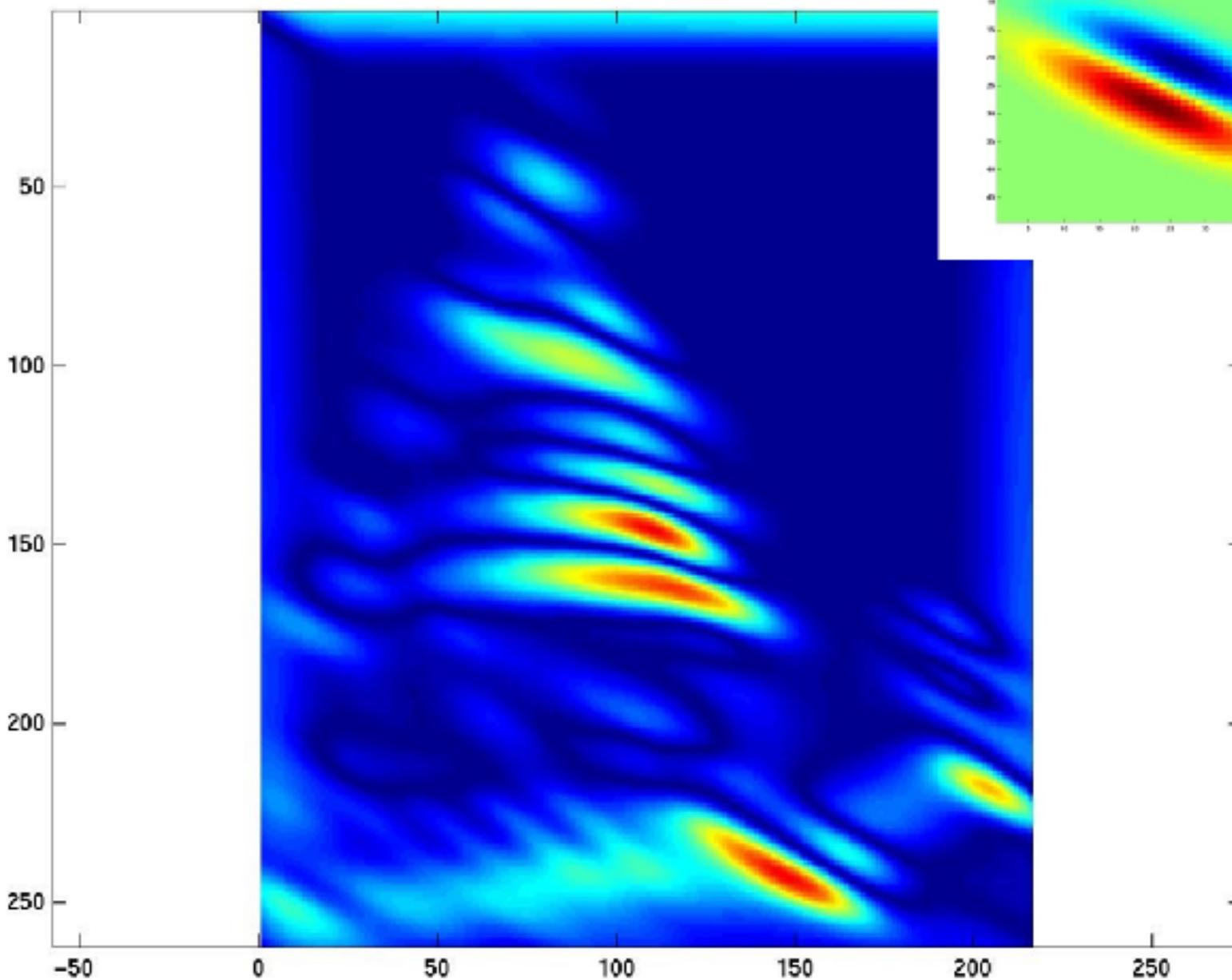


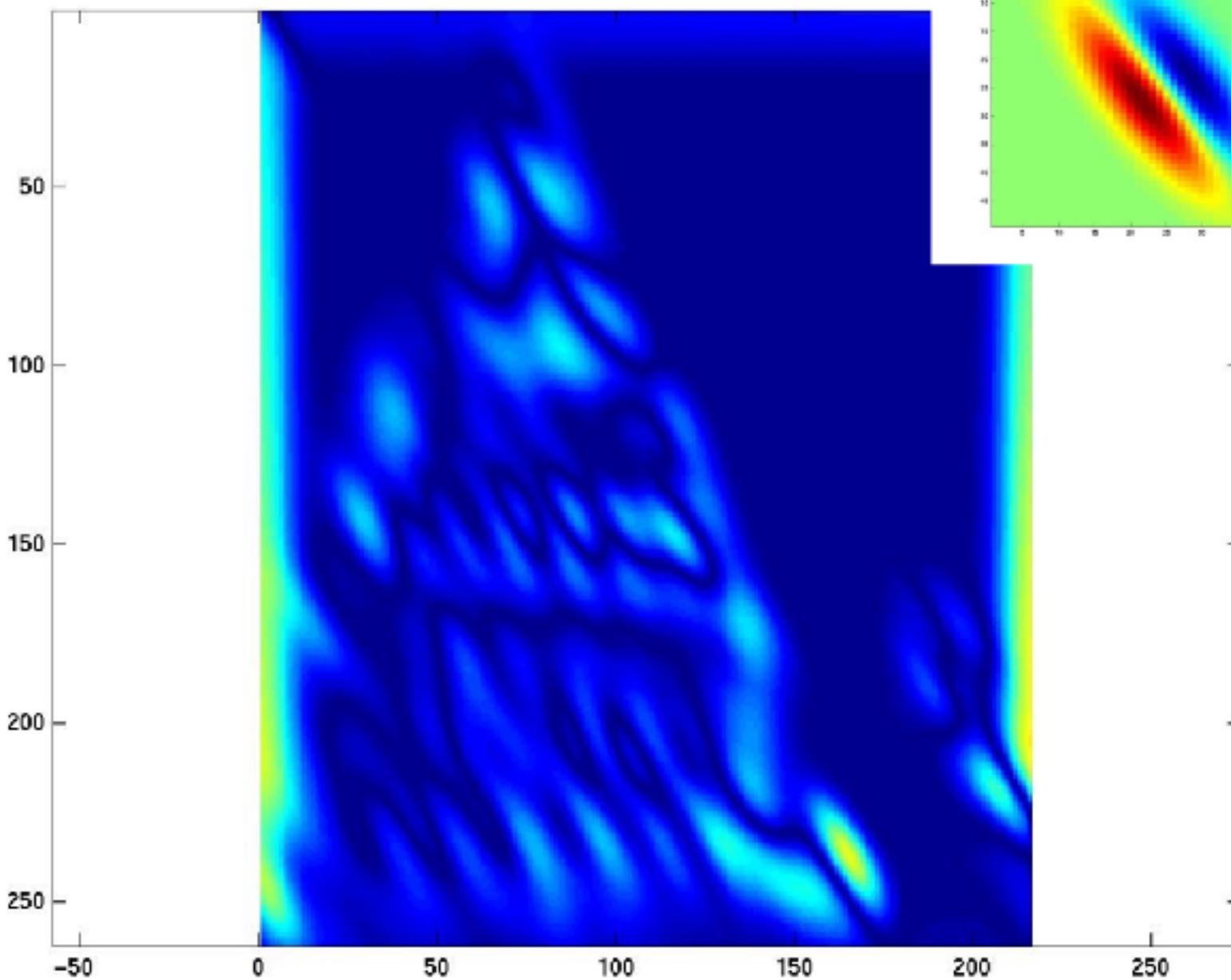


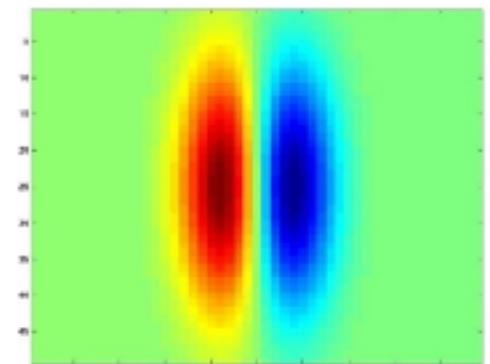
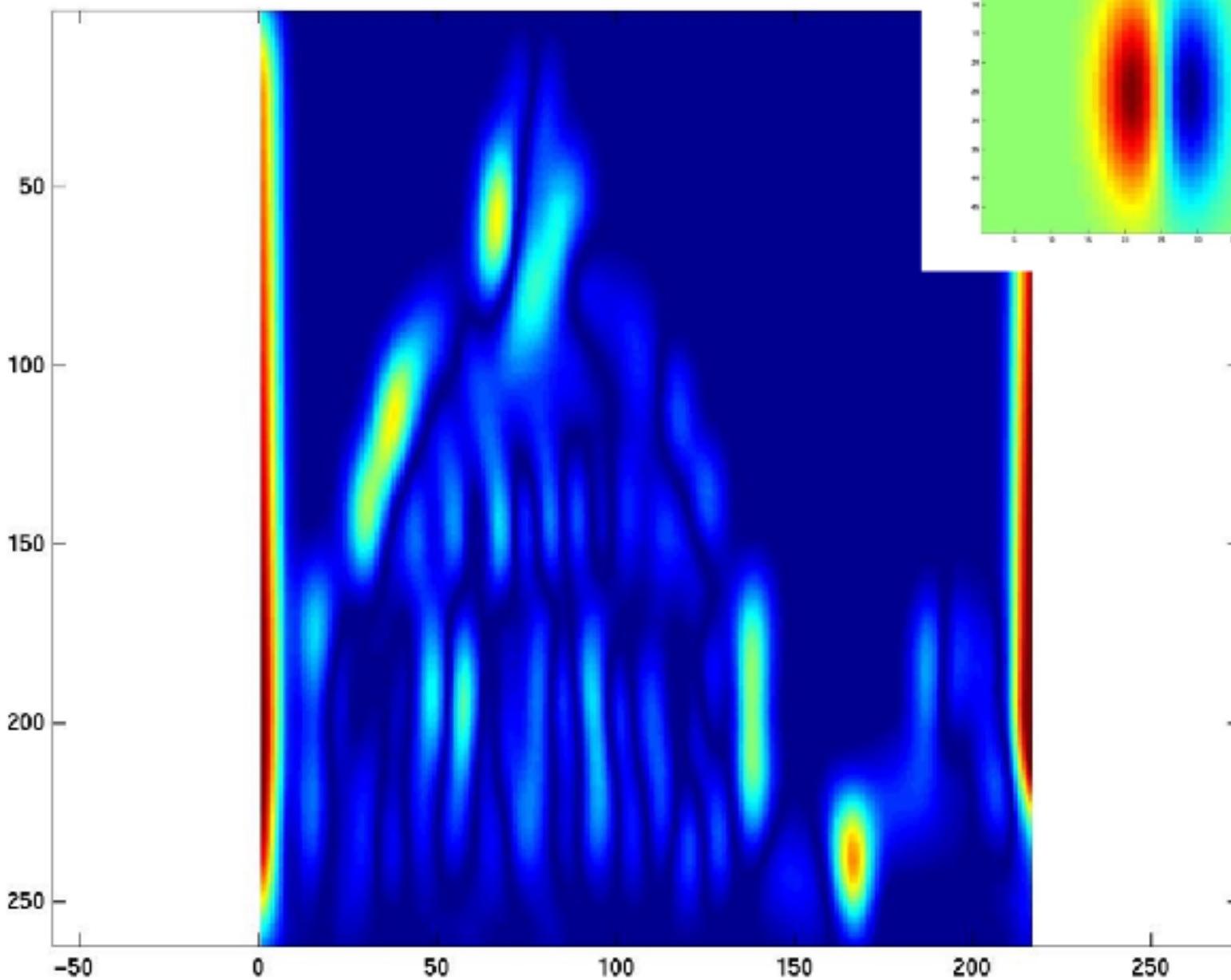


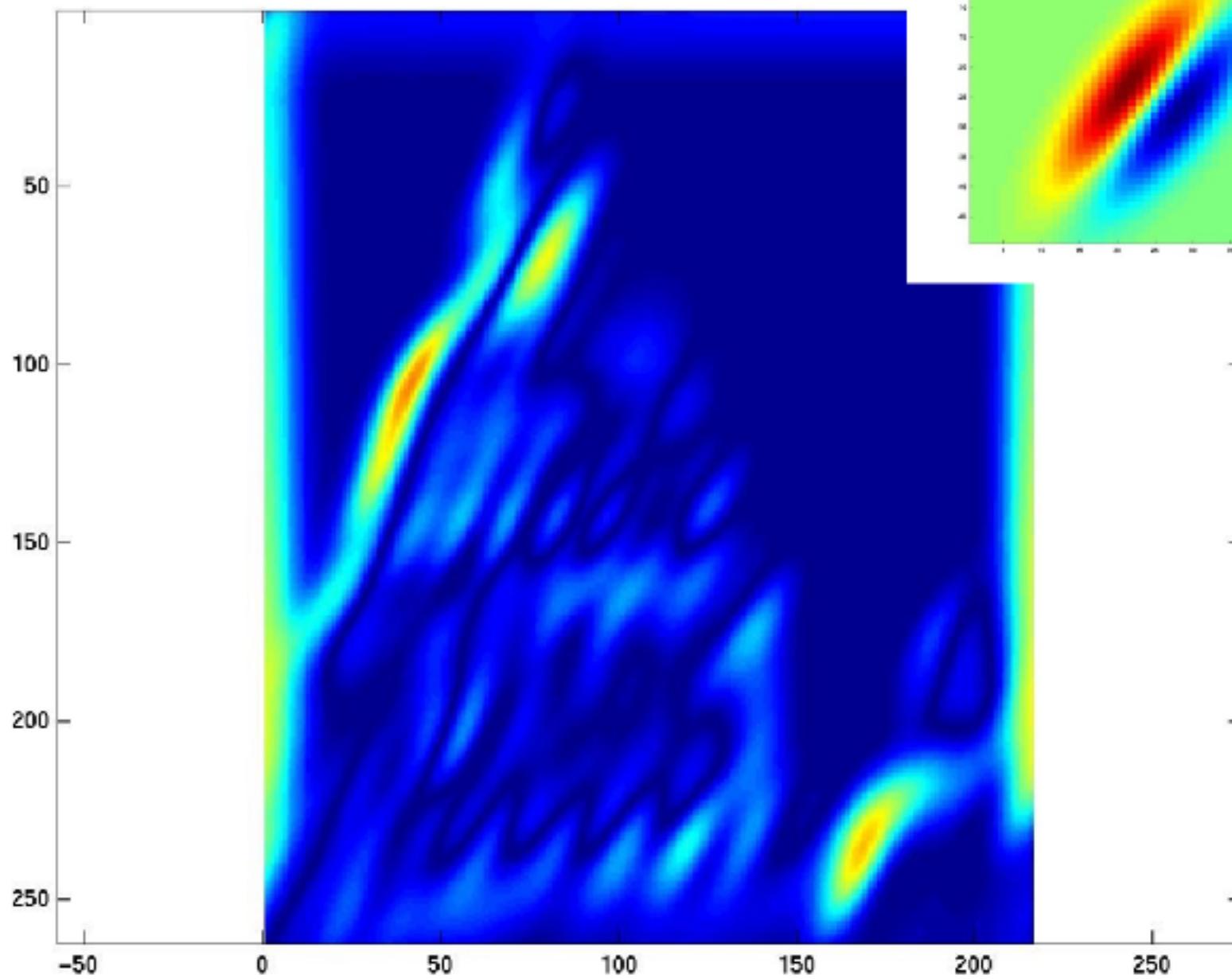


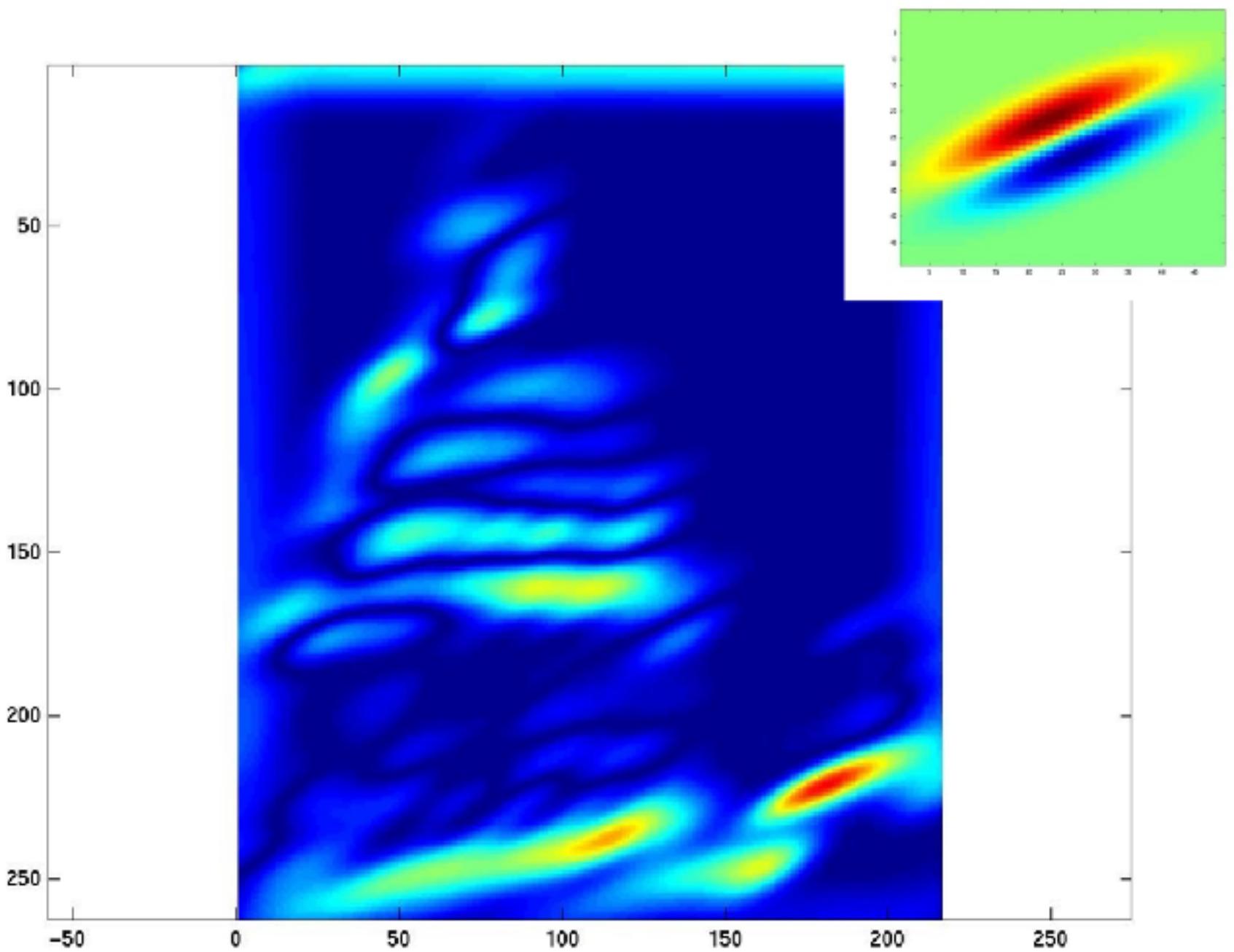


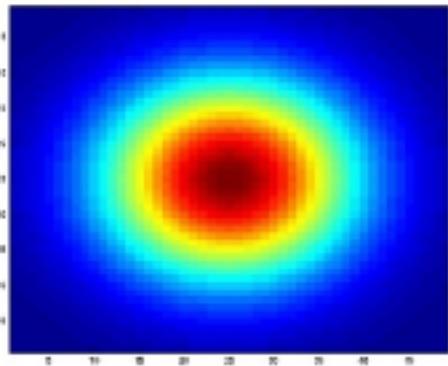
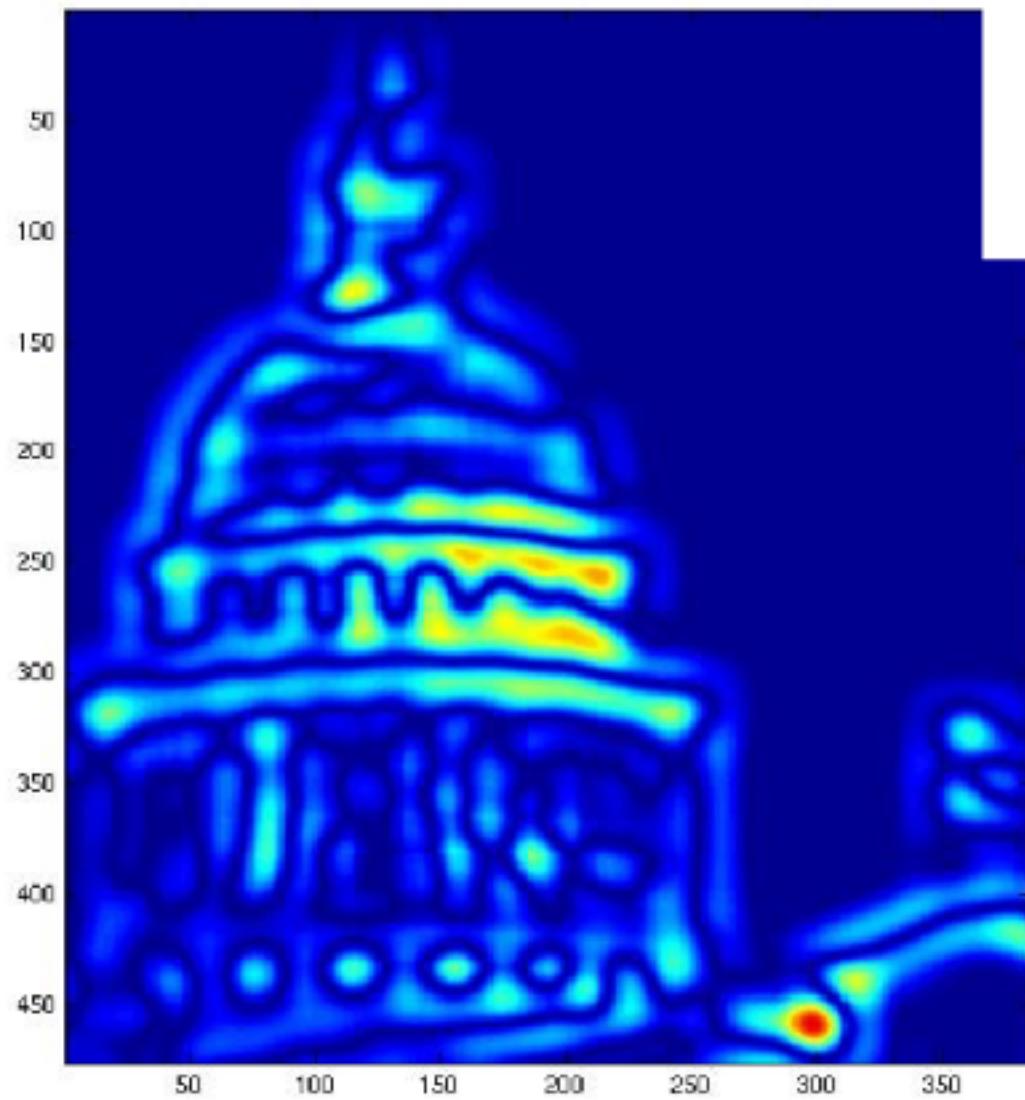


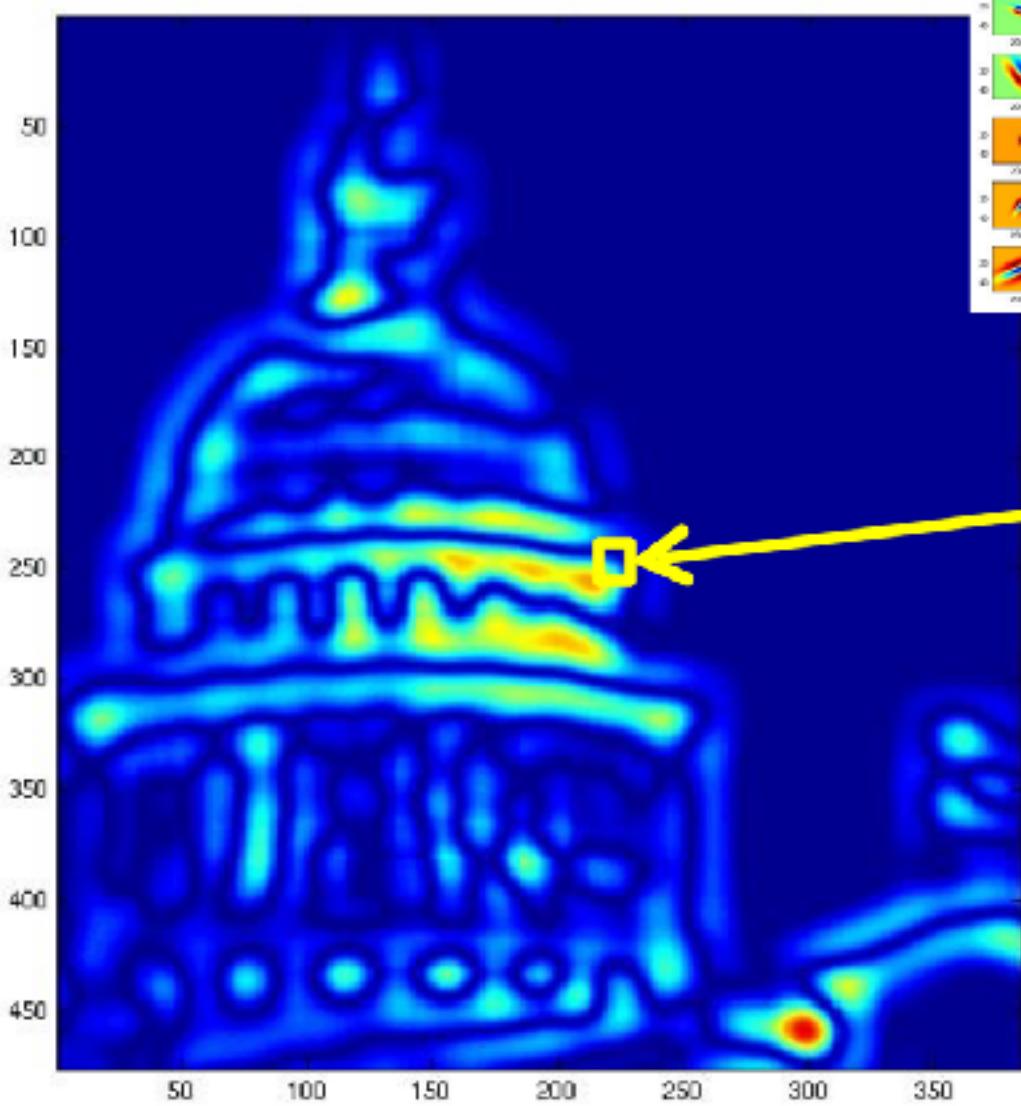




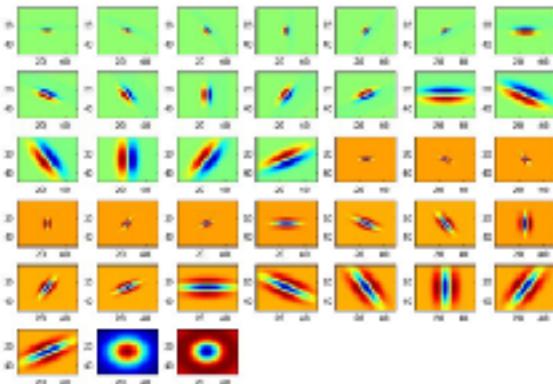






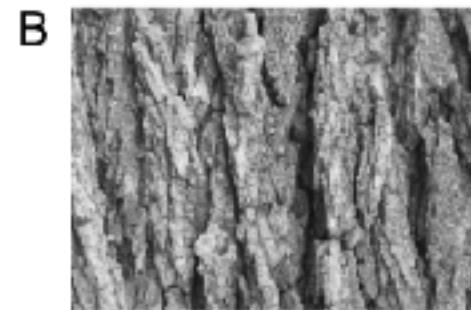
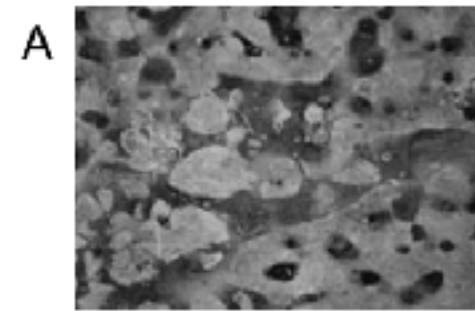
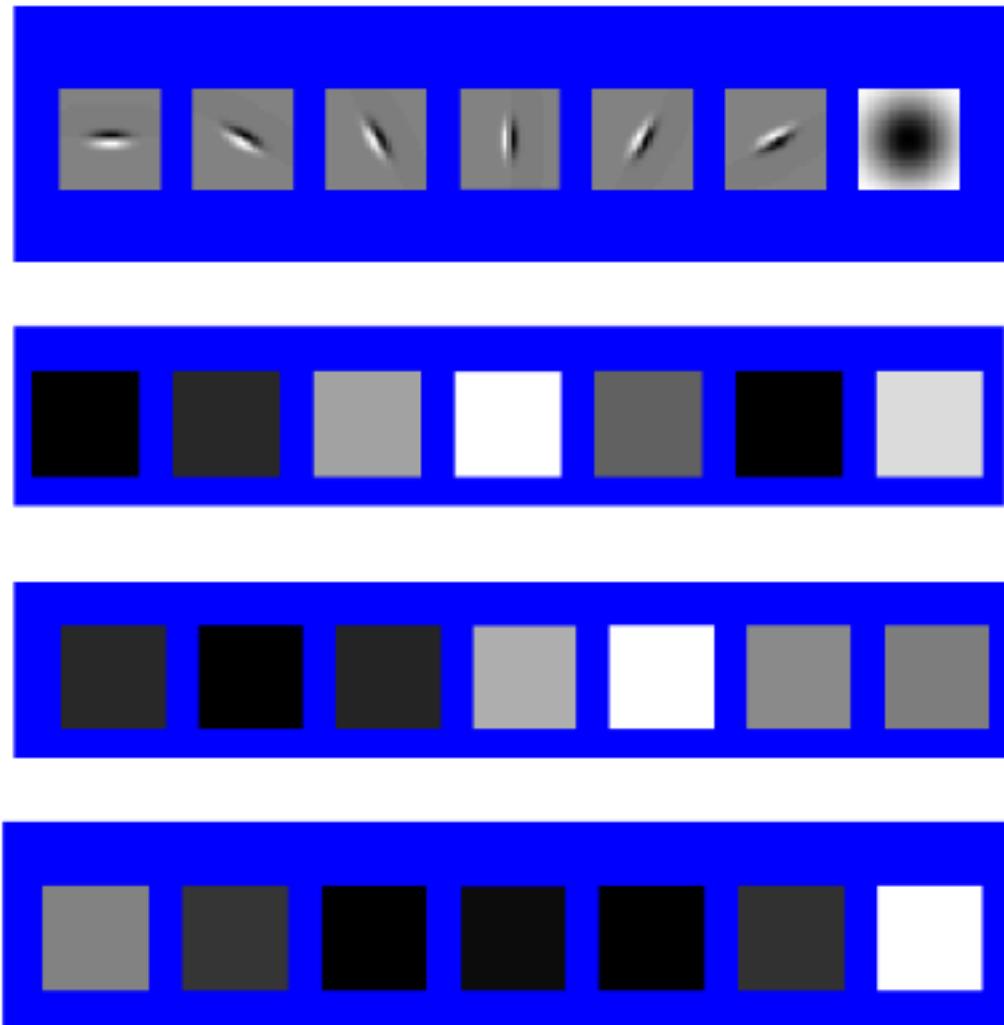


We can form a
feature vector
from the list of
responses at
each pixel.

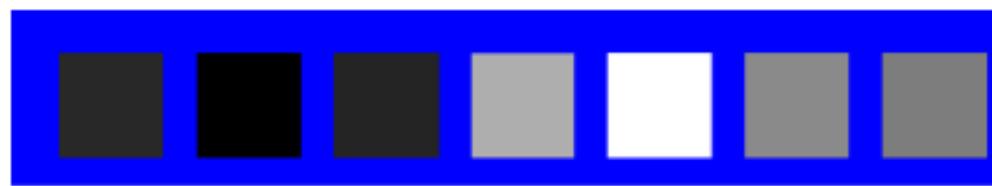
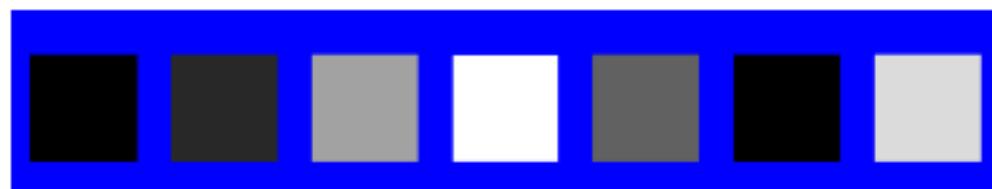
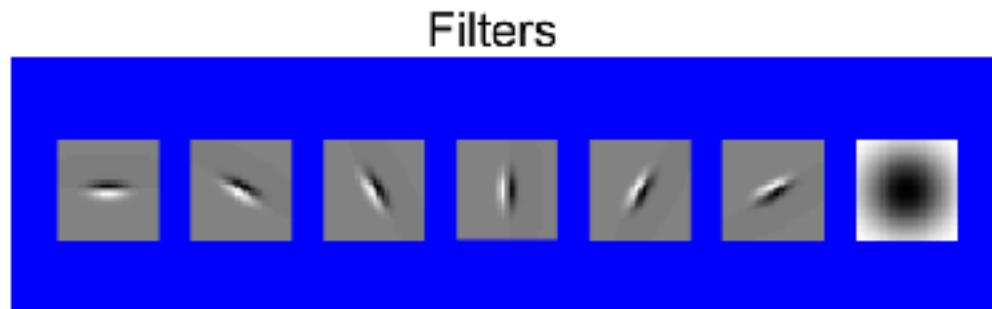
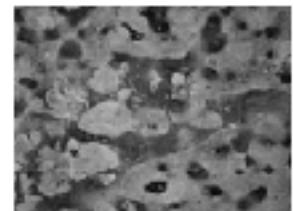


You try: Can you match the texture to the response?

Filters



Representing texture by mean abs response



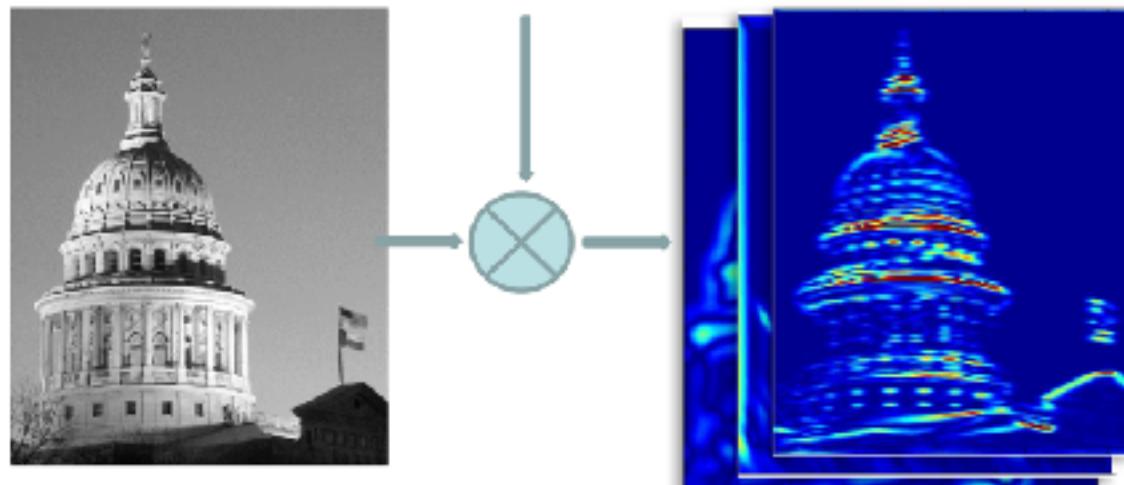
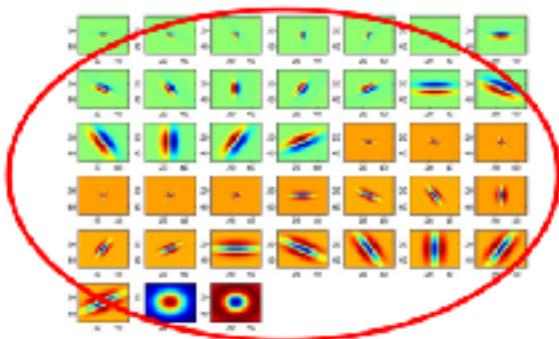
Mean abs responses



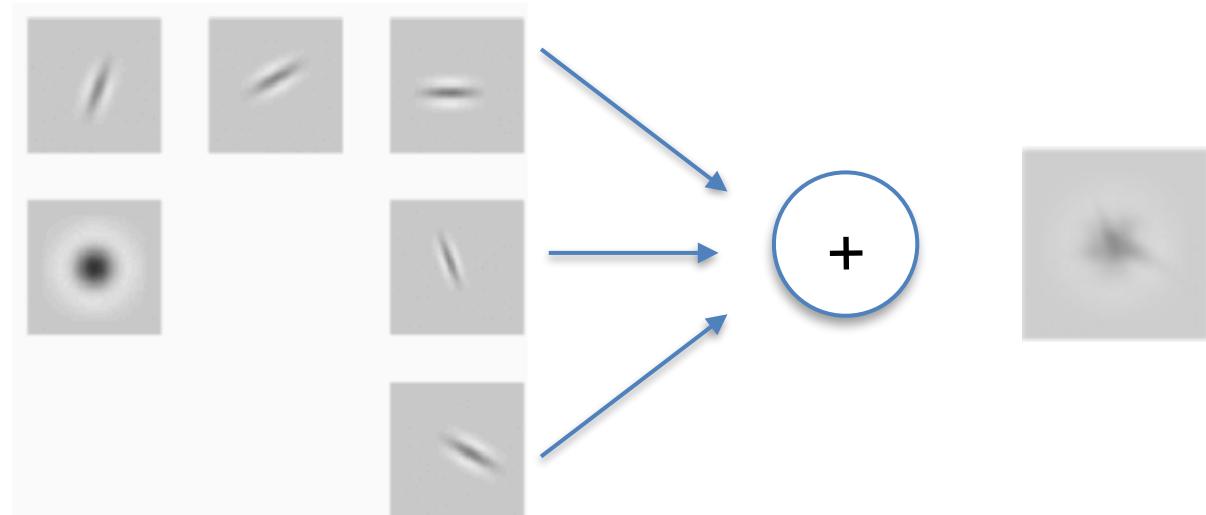
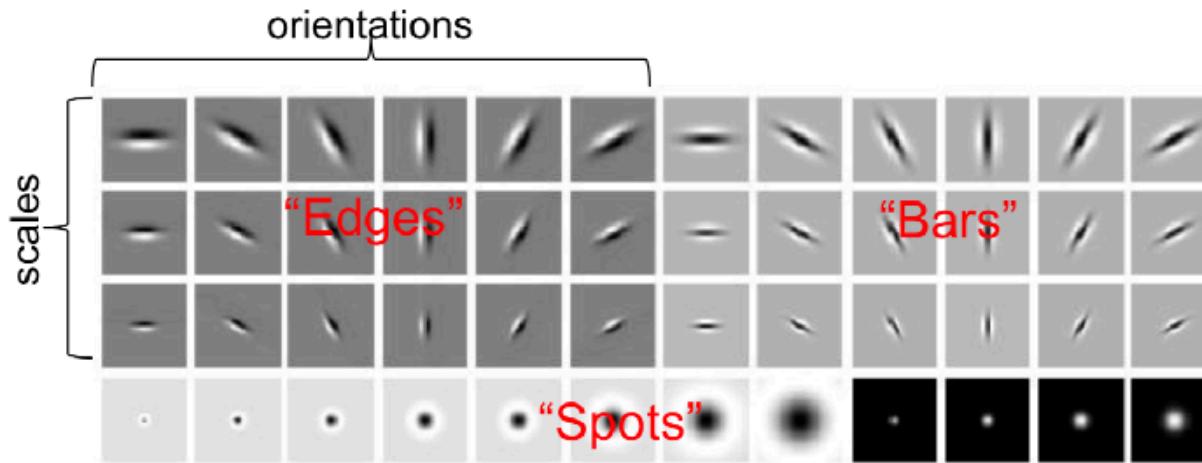
Derek Hoiem

Convolutional Neural Network

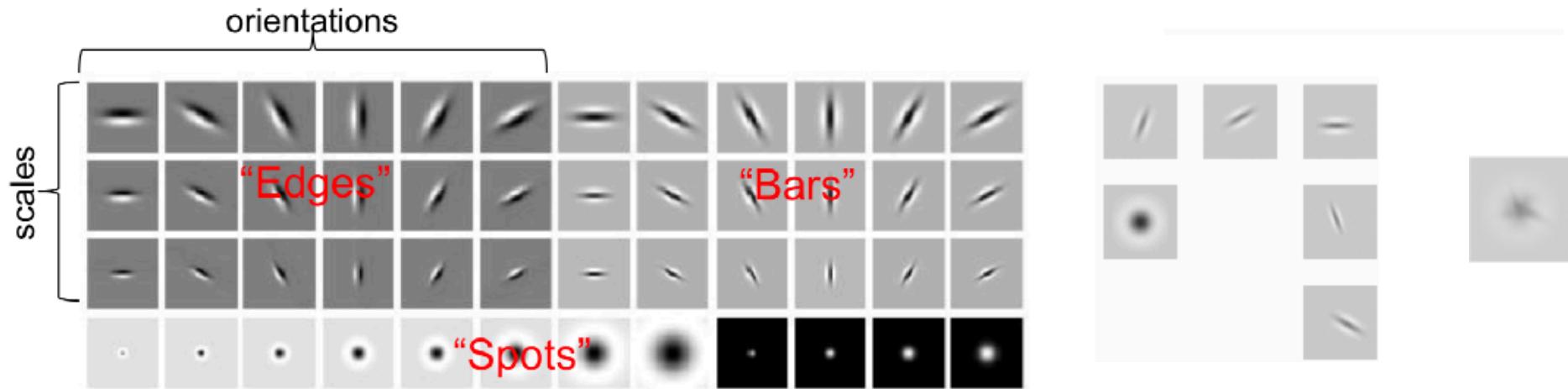
Learn from
Data what
these filters
should be



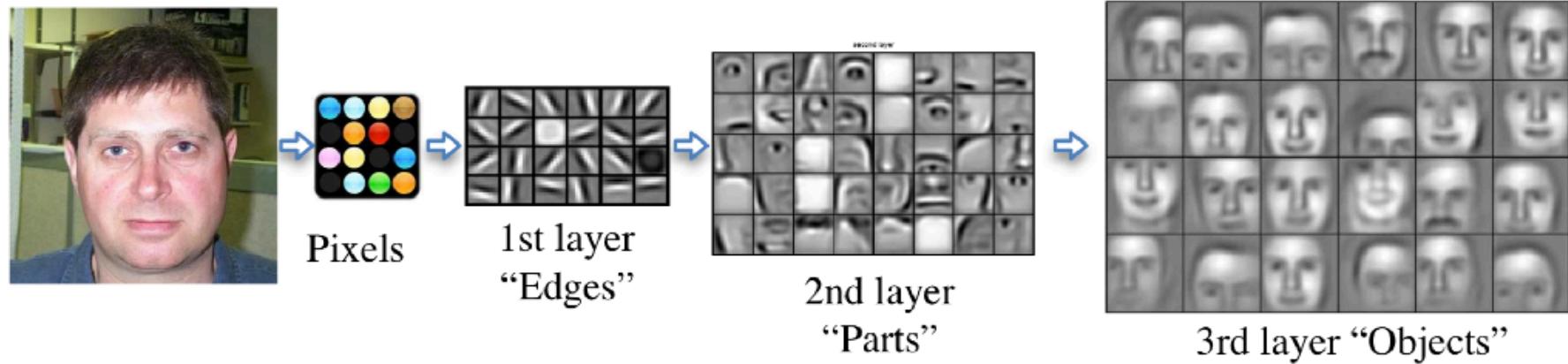
More on Edge Detection



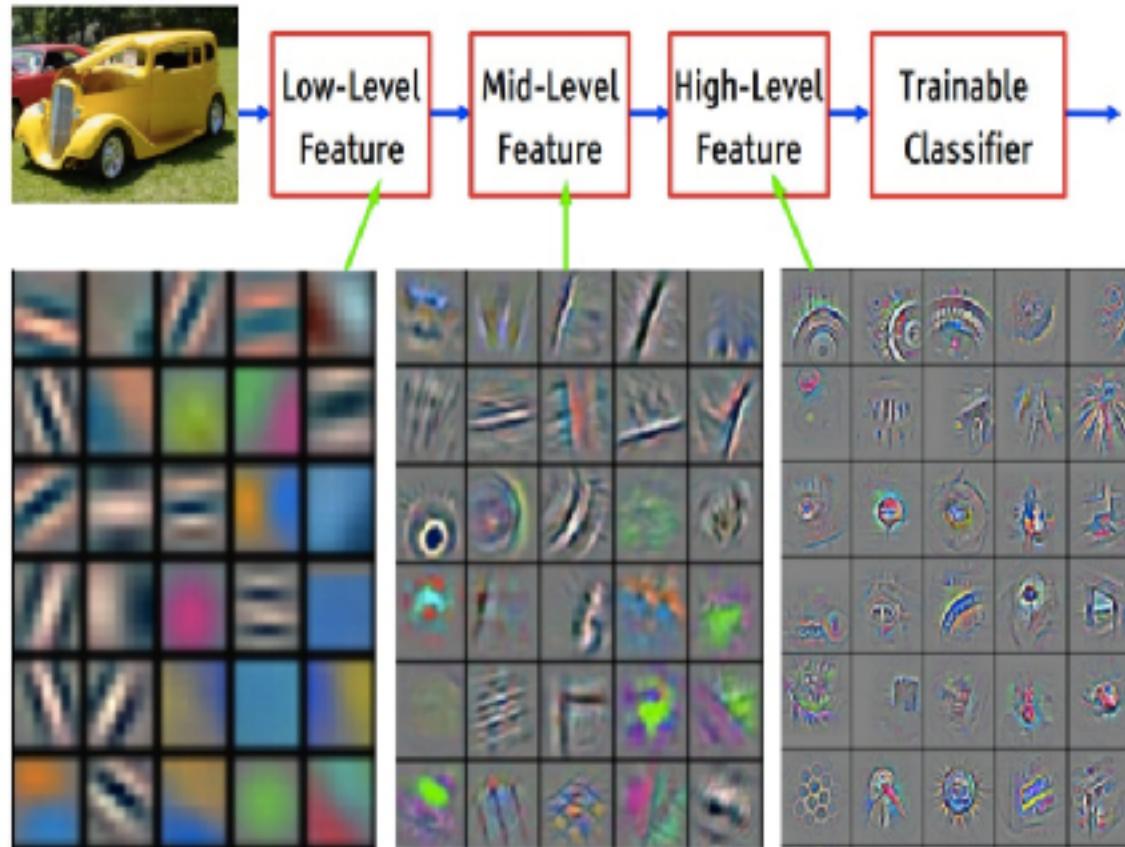
More on Edge Detection



Learned hierarchical feature representation: Sparse DBN



Convolutional Neural Network



Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

Convolutional Neural Network

► Le Cun 1998

