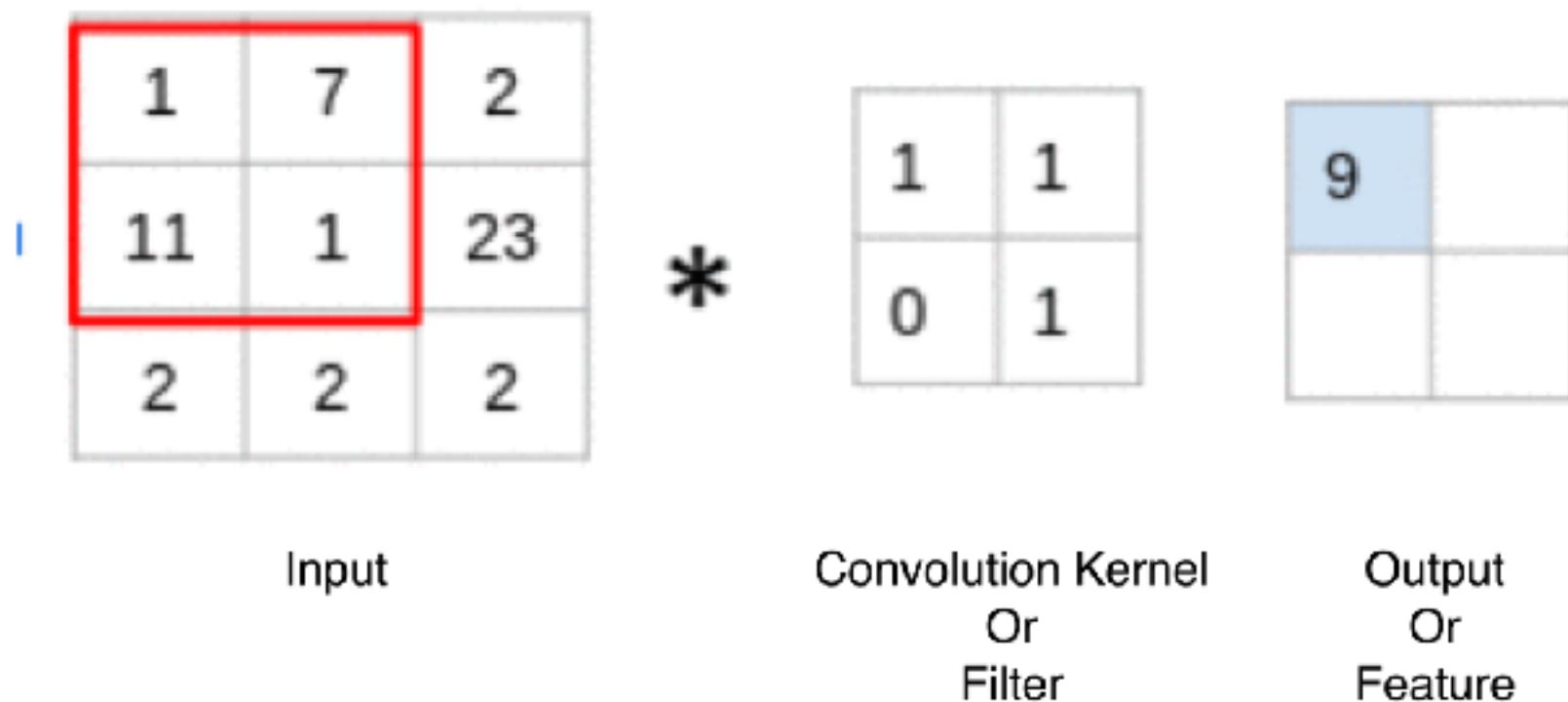
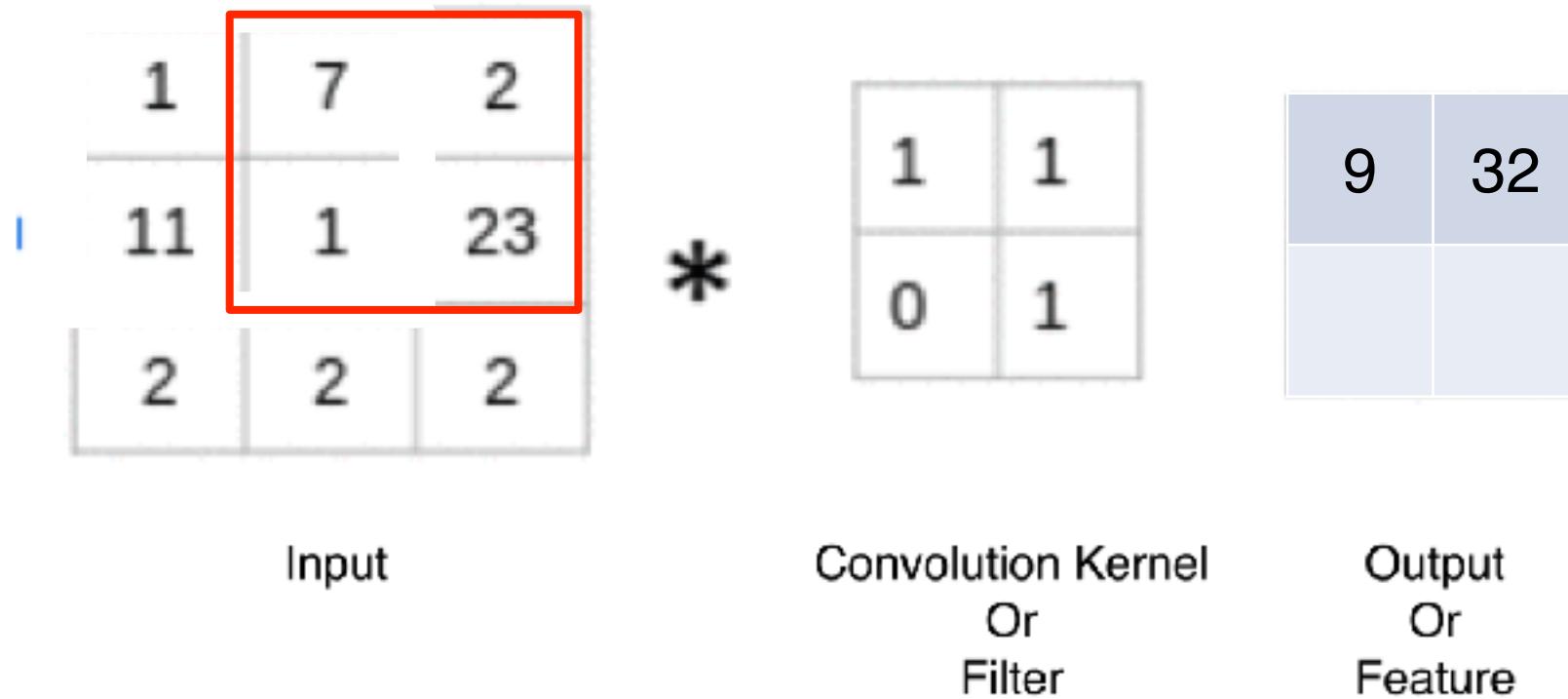


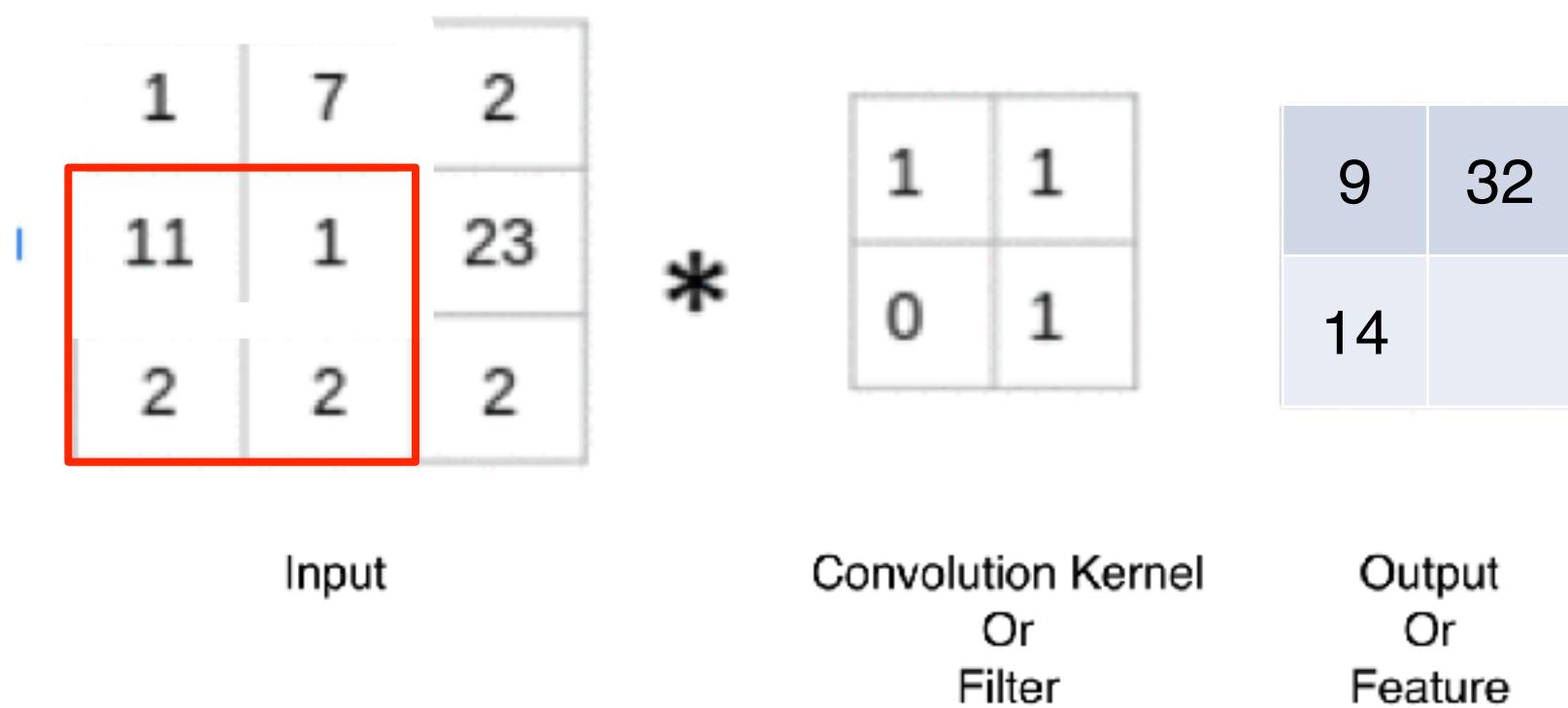
Recap: Convolution



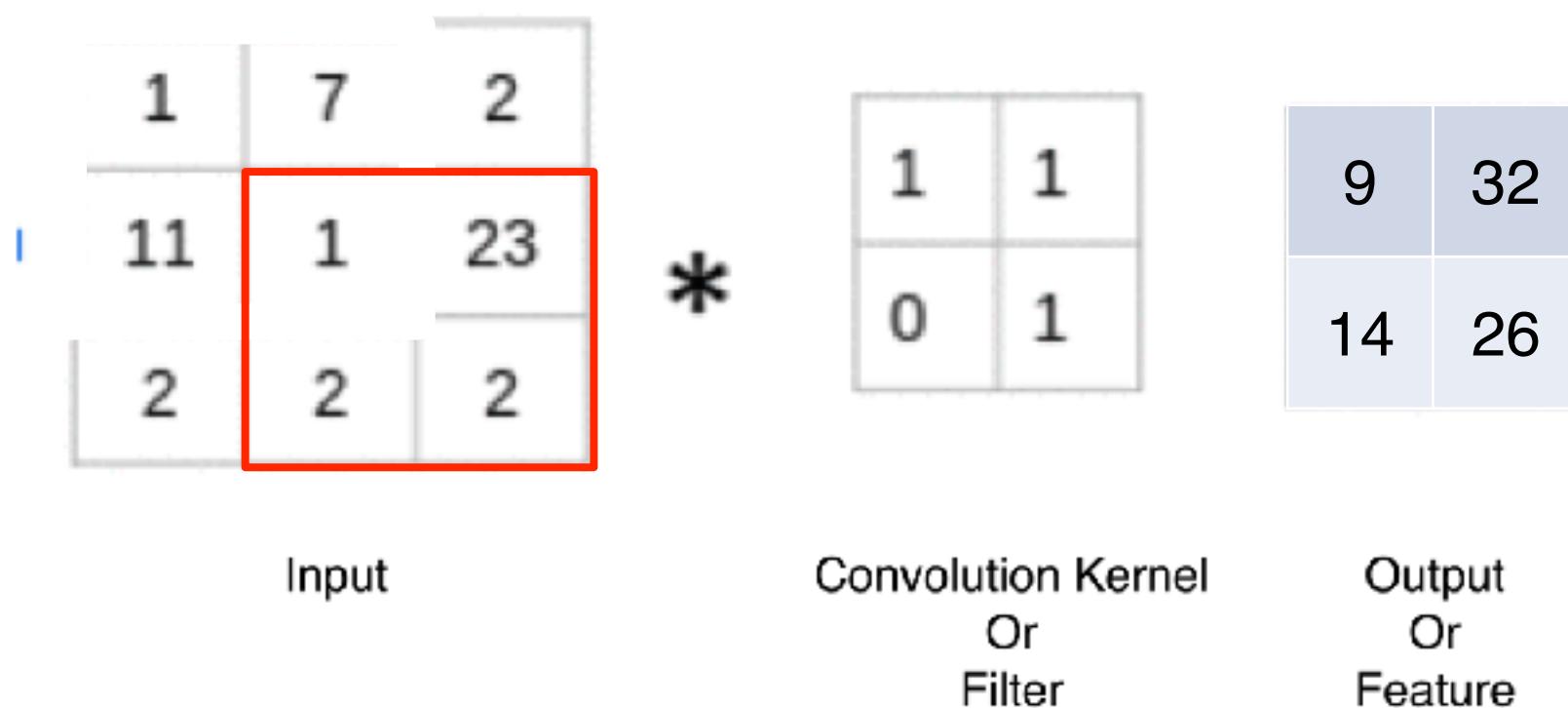
Recap: Convolution



Recap: Convolution



Recap: Convolution

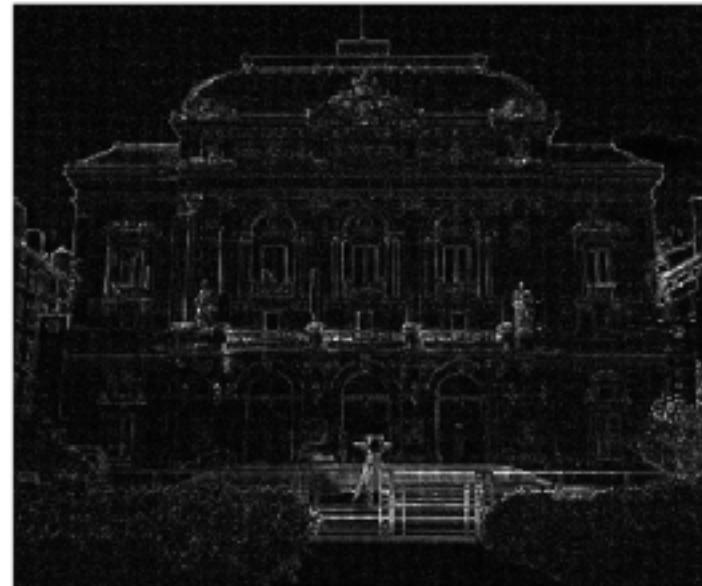


Laplacian Filter



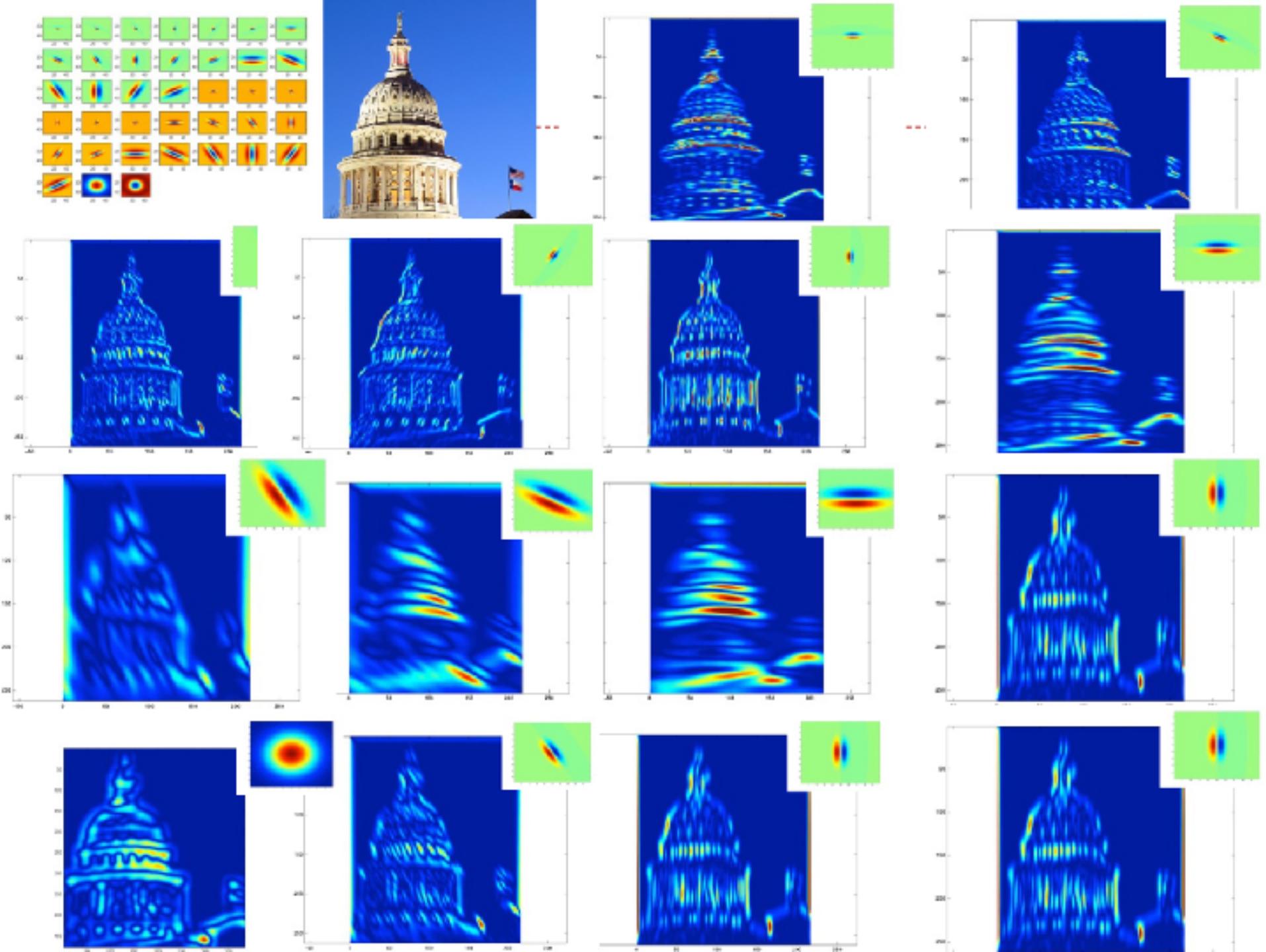
Input

$$* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} =$$



Convolution Kernel
Or
Filter

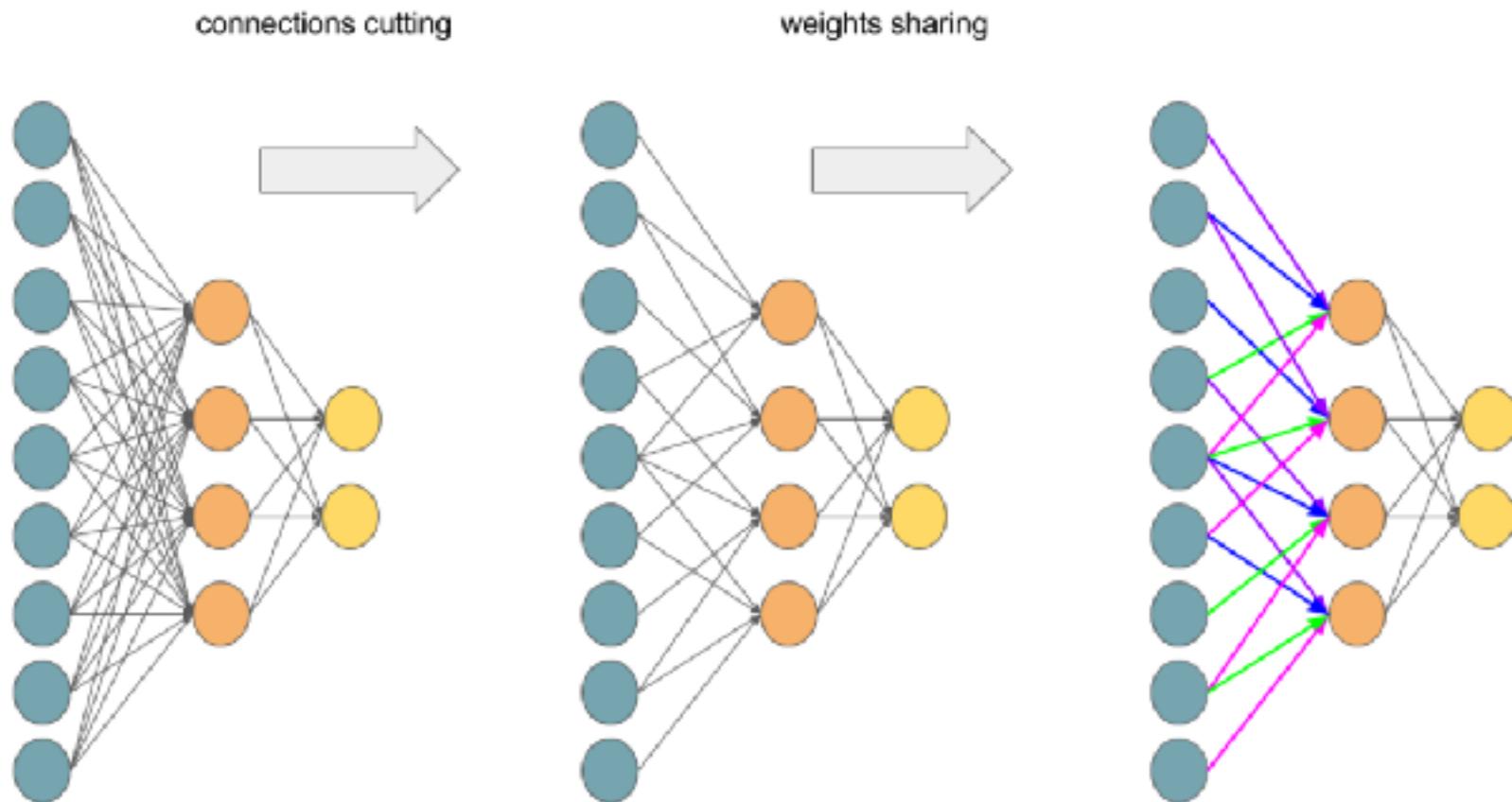
Output
Or
Feature



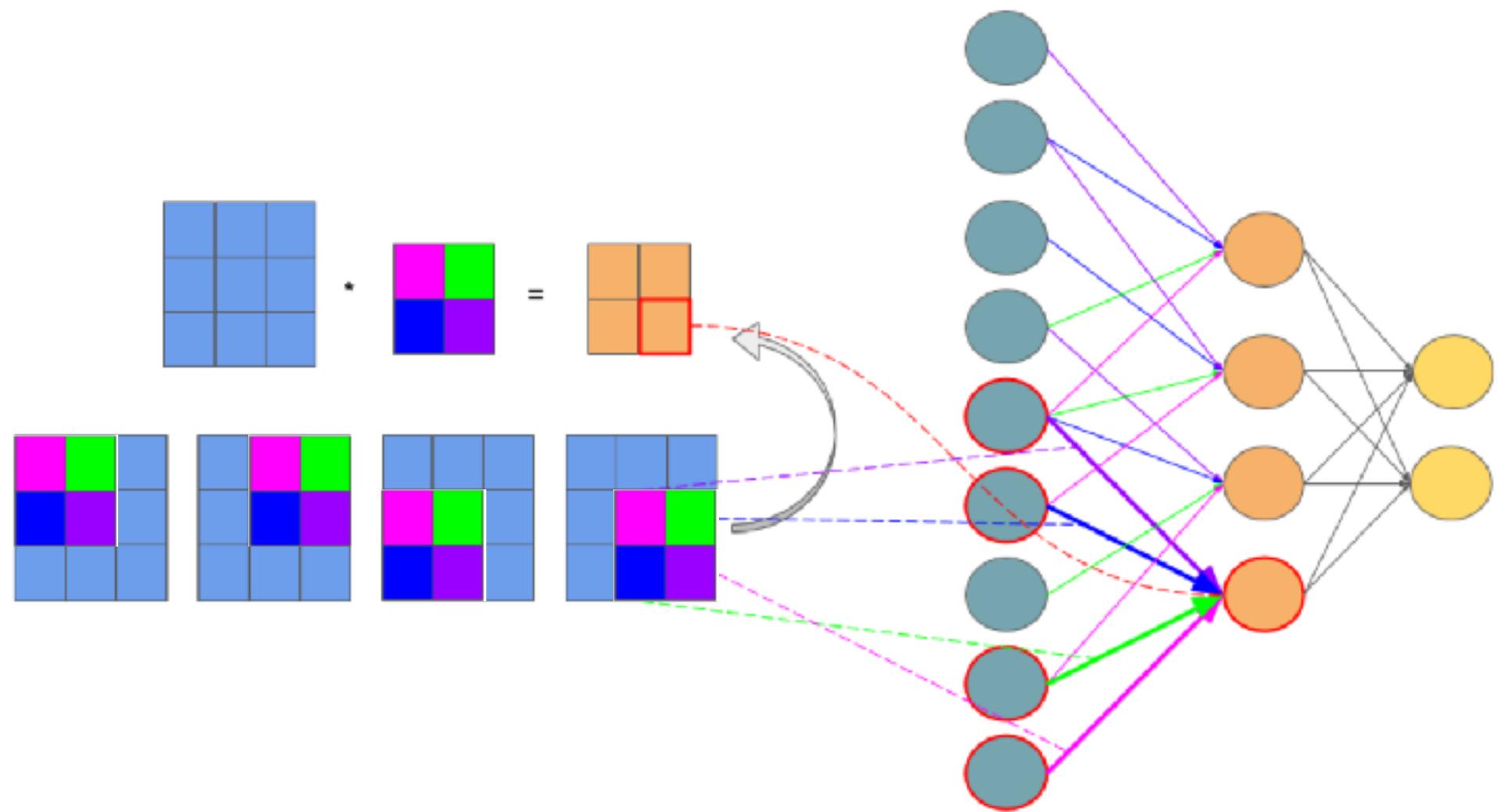
Any Questions?

Transforming Multilayer Perceptron to Convolutional Neural Network

Transforming Multilayer Perceptron to Convolutional Neural Network



Feedforward in CNN is identical with convolution operation



Forward and Backward Propagation using Convolution operation

- ▶ Forward Pass

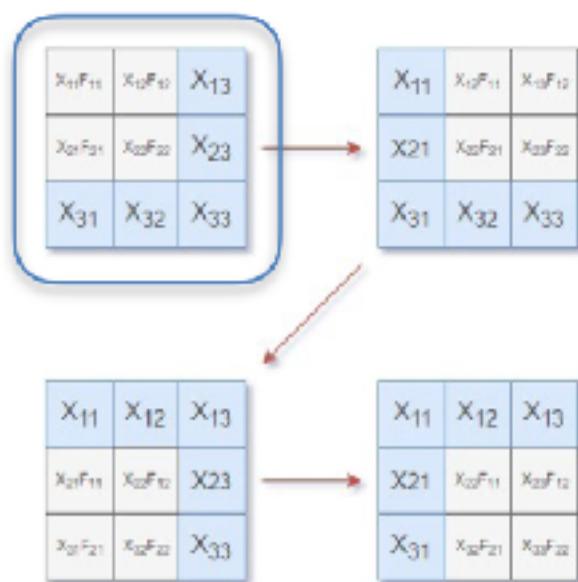
- ▶ Ignoring 180° rotation

$$\begin{matrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{matrix} = \text{Convolution} \left(\begin{matrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{matrix}, \begin{matrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{matrix} \right)$$

- ▶ Note: F instead of w

$$O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

- ▶ It can be visualized as:



Forward and Backward Propagation using Convolution operation

▶ Forward Pass

- ▶ Ignoring 180° rotation

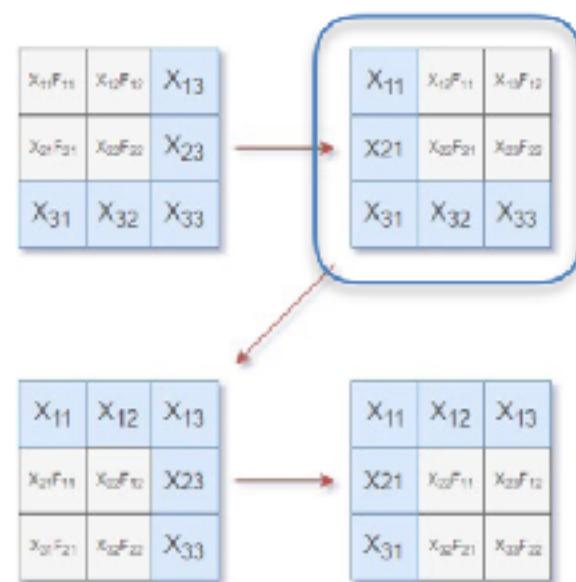
$$\begin{matrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{matrix} = \text{Convolution} \left(\begin{matrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{matrix}, \begin{matrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{matrix} \right)$$

- ▶ Note: F instead of w

$$O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

▶ It can be visualized as:



Forward and Backward Propagation using Convolution operation

- ▶ Forward Pass

- ▶ Ignoring 180° rotation

$$\begin{matrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{matrix} = \text{Convolution} \left(\begin{matrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{matrix}, \begin{matrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{matrix} \right)$$

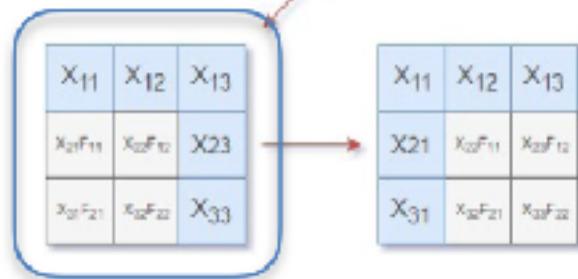
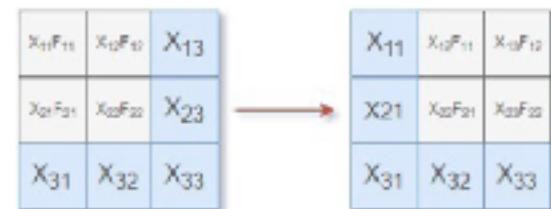
- ▶ Note: F instead of w

$$O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

$$O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

- ▶ It can be visualized as:



Forward and Backward Propagation using Convolution operation

▶ Forward Pass

- ▶ Ignoring 180° rotation

$$\begin{matrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{matrix} = \text{Convolution} \left(\begin{matrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{matrix}, \begin{matrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{matrix} \right)$$

- ▶ Note: F instead of w

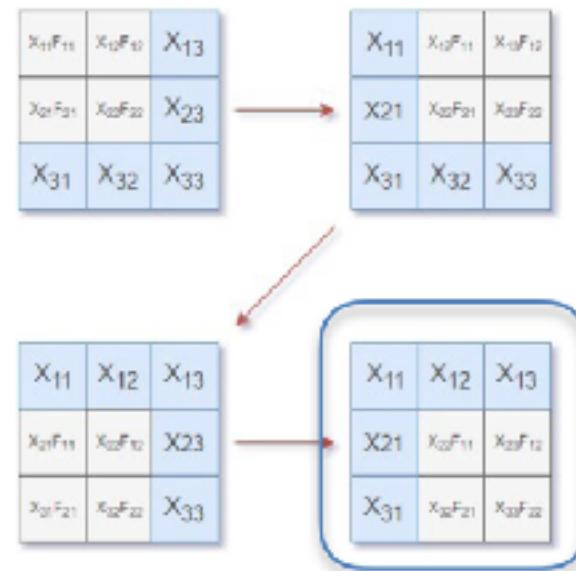
$$O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

$$O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

$$O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33}$$

- ▶ It can be visualized as:



Feedforward in CNN is identical with convolution operation

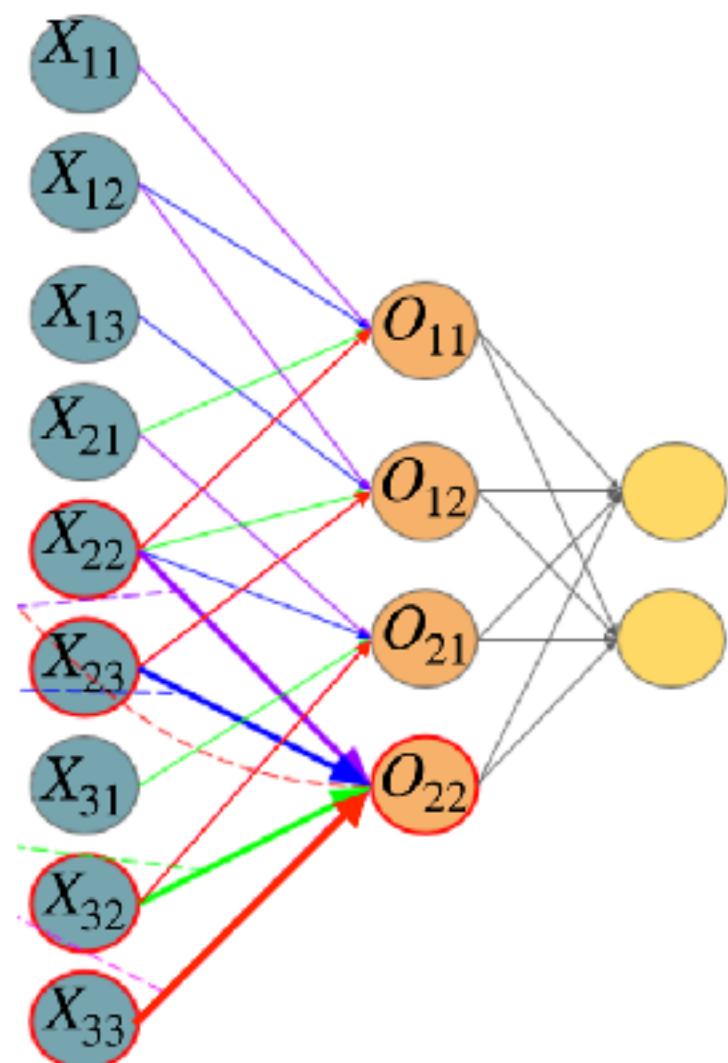
$$\begin{bmatrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{bmatrix} = \text{Convolution} \left(\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}, \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \right)$$

$$O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

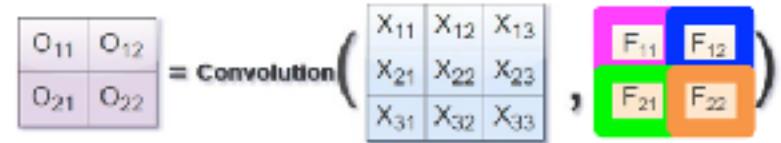
$$O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

$$O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33}$$



Forward and Backward Propagation Using Convolution Operation

- ▶ Gradient of error ‘E’ with respect to the filer ‘F’



$$O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

$$O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

$$O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33}$$

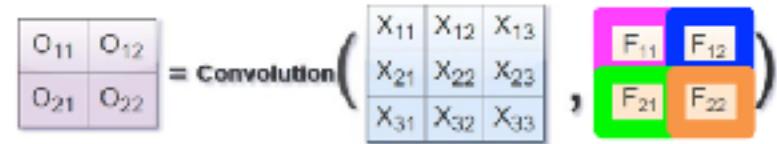
Forward and Backward Propagation Using Convolution Operation

► Gradient of error 'E' with respect to the filer 'F'

$$E = E_{O_{11}} + E_{O_{12}} + E_{O_{21}} + E_{O_{22}}$$

$$\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial F_{11}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial F_{11}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial F_{11}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial F_{11}}$$

→ $\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} X_{11} + \frac{\partial E}{\partial O_{12}} X_{12} + \frac{\partial E}{\partial O_{21}} X_{21} + \frac{\partial E}{\partial O_{22}} X_{22}$

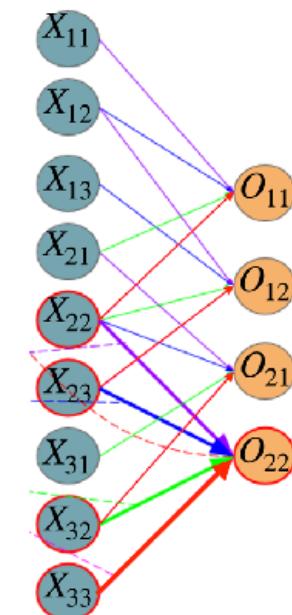


$$O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

$$O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

$$O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33}$$



Forward and Backward Propagation Using Convolution Operation

► Gradient of error 'E' with respect to the filer 'F'

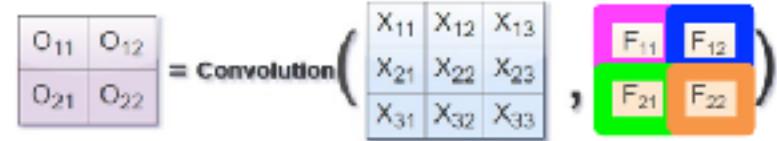
$$E = E_{O_{11}} + E_{O_{12}} + E_{O_{21}} + E_{O_{22}}$$

$$\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial F_{11}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial F_{11}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial F_{11}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial F_{11}}$$

→ $\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} X_{11} + \frac{\partial E}{\partial O_{12}} X_{12} + \frac{\partial E}{\partial O_{21}} X_{21} + \frac{\partial E}{\partial O_{22}} X_{22}$

→ $\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial F_{12}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial F_{12}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial F_{12}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial F_{12}}$

→ $\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}} X_{12} + \frac{\partial E}{\partial O_{12}} X_{13} + \frac{\partial E}{\partial O_{21}} X_{22} + \frac{\partial E}{\partial O_{22}} X_{23}$



$$O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

$$O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

$$O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33}$$

Forward and Backward Propagation Using Convolution Operation

► Gradient of error 'E' with respect to the filer 'F'

$$E = E_{O_{11}} + E_{O_{12}} + E_{O_{21}} + E_{O_{22}}$$

$$\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial F_{11}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial F_{11}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial F_{11}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial F_{11}}$$

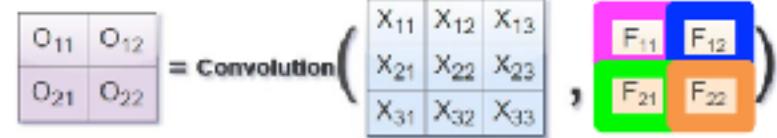
→ $\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} X_{11} + \frac{\partial E}{\partial O_{12}} X_{12} + \frac{\partial E}{\partial O_{21}} X_{21} + \frac{\partial E}{\partial O_{22}} X_{22}$

→ $\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial F_{12}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial F_{12}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial F_{12}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial F_{12}}$

$\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}} X_{12} + \frac{\partial E}{\partial O_{12}} X_{13} + \frac{\partial E}{\partial O_{21}} X_{22} + \frac{\partial E}{\partial O_{22}} X_{23}$

→ $\frac{\partial E}{\partial F_{21}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial F_{21}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial F_{21}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial F_{21}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial F_{21}}$

$\frac{\partial E}{\partial F_{21}} = \frac{\partial E}{\partial O_{11}} X_{21} + \frac{\partial E}{\partial O_{12}} X_{22} + \frac{\partial E}{\partial O_{21}} X_{31} + \frac{\partial E}{\partial O_{22}} X_{32}$



$$O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

$$O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

$$O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33}$$

Forward and Backward Propagation Using Convolution Operation

► Gradient of error 'E' with respect to the filer 'F'

$$E = E_{O_{11}} + E_{O_{12}} + E_{O_{21}} + E_{O_{22}}$$

$$\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial F_{11}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial F_{11}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial F_{11}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial F_{11}}$$

→ $\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} X_{11} + \frac{\partial E}{\partial O_{12}} X_{12} + \frac{\partial E}{\partial O_{21}} X_{21} + \frac{\partial E}{\partial O_{22}} X_{22}$

$$\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial F_{12}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial F_{12}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial F_{12}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial F_{12}}$$

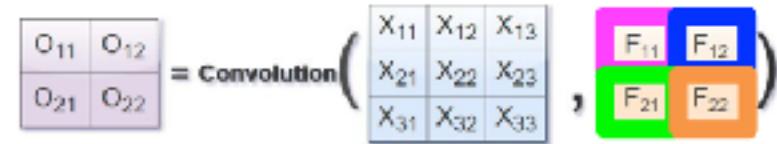
→ $\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}} X_{12} + \frac{\partial E}{\partial O_{12}} X_{13} + \frac{\partial E}{\partial O_{21}} X_{22} + \frac{\partial E}{\partial O_{22}} X_{23}$

→ $\frac{\partial E}{\partial F_{21}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial F_{21}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial F_{21}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial F_{21}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial F_{21}}$

→ $\frac{\partial E}{\partial F_{21}} = \frac{\partial E}{\partial O_{11}} X_{21} + \frac{\partial E}{\partial O_{12}} X_{22} + \frac{\partial E}{\partial O_{21}} X_{31} + \frac{\partial E}{\partial O_{22}} X_{32}$

→ $\frac{\partial E}{\partial F_{22}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial F_{22}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial F_{22}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial F_{22}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial F_{22}}$

→ $\frac{\partial E}{\partial F_{22}} = \frac{\partial E}{\partial O_{11}} X_{22} + \frac{\partial E}{\partial O_{12}} X_{23} + \frac{\partial E}{\partial O_{21}} X_{32} + \frac{\partial E}{\partial O_{22}} X_{33}$

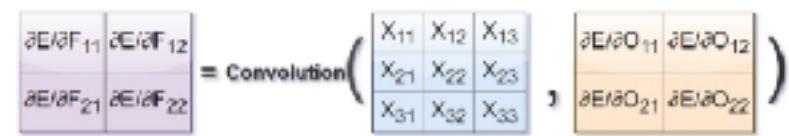


$$O_{11} = F_{11} X_{11} + F_{12} X_{12} + F_{21} X_{21} + F_{22} X_{22}$$

$$O_{12} = F_{11} X_{12} + F_{12} X_{13} + F_{21} X_{22} + F_{22} X_{23}$$

$$O_{21} = F_{11} X_{21} + F_{12} X_{22} + F_{21} X_{31} + F_{22} X_{32}$$

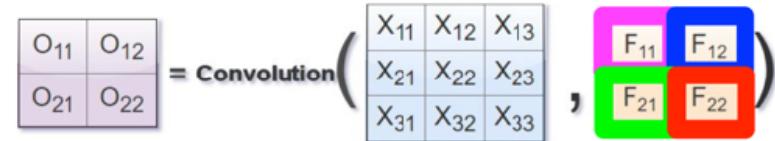
$$O_{22} = F_{11} X_{22} + F_{12} X_{23} + F_{21} X_{32} + F_{22} X_{33}$$



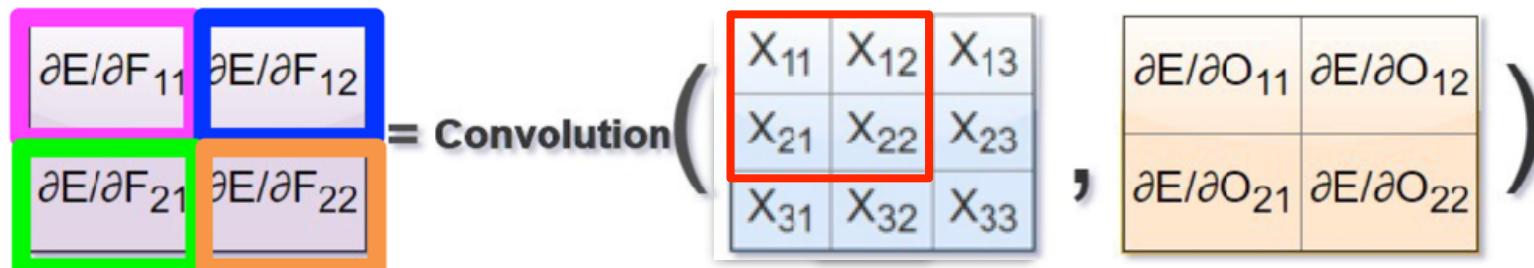
Forward and Backward Propagation using Convolution operation

► Gradients of error 'E' with respect to the filter 'F'

$$E = E_{O_{11}} + E_{O_{12}} + E_{O_{21}} + E_{O_{22}}$$



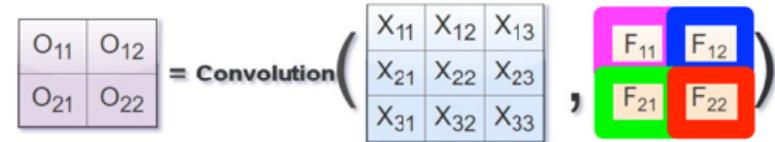
$$\rightarrow \frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} X_{11} + \frac{\partial E}{\partial O_{12}} X_{12} + \frac{\partial E}{\partial O_{21}} X_{21} + \frac{\partial E}{\partial O_{22}} X_{22}$$



Forward and Backward Propagation using Convolution operation

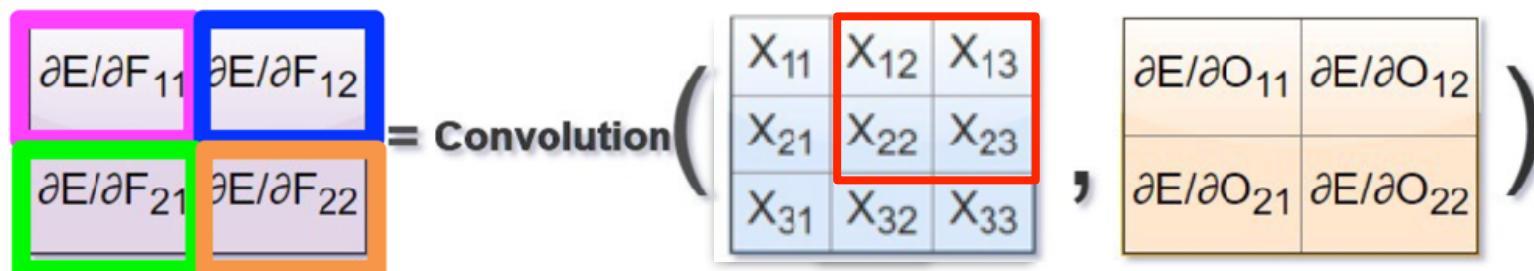
► Gradients of error 'E' with respect to the filter 'F'

$$E = E_{O_{11}} + E_{O_{12}} + E_{O_{21}} + E_{O_{22}}$$



$$\rightarrow \frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} X_{11} + \frac{\partial E}{\partial O_{12}} X_{12} + \frac{\partial E}{\partial O_{21}} X_{21} + \frac{\partial E}{\partial O_{22}} X_{22}$$

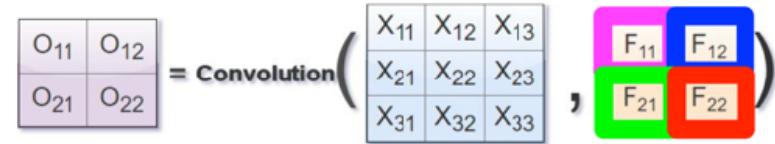
$$\rightarrow \frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}} X_{12} + \frac{\partial E}{\partial O_{12}} X_{13} + \frac{\partial E}{\partial O_{21}} X_{22} + \frac{\partial E}{\partial O_{22}} X_{23}$$



Forward and Backward Propagation using Convolution operation

► Gradients of error 'E' with respect to the filter 'F'

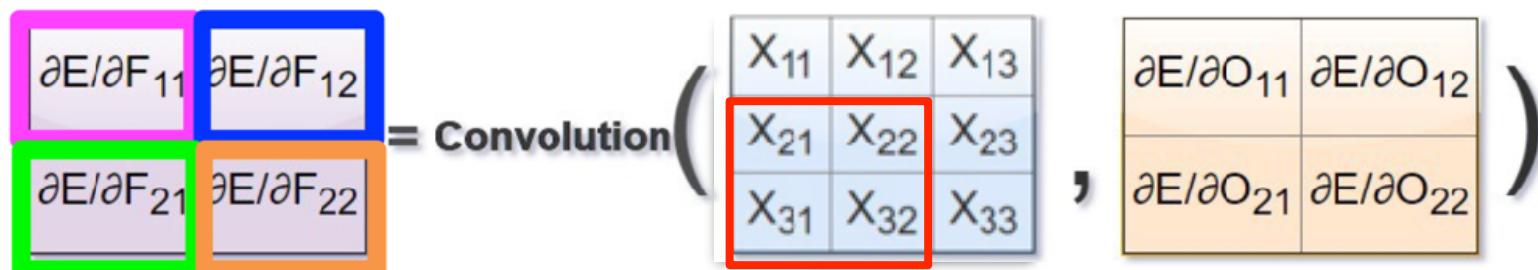
$$E = E_{O_{11}} + E_{O_{12}} + E_{O_{21}} + E_{O_{22}}$$



► $\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} X_{11} + \frac{\partial E}{\partial O_{12}} X_{12} + \frac{\partial E}{\partial O_{21}} X_{21} + \frac{\partial E}{\partial O_{22}} X_{22}$

► $\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}} X_{12} + \frac{\partial E}{\partial O_{12}} X_{13} + \frac{\partial E}{\partial O_{21}} X_{22} + \frac{\partial E}{\partial O_{22}} X_{23}$

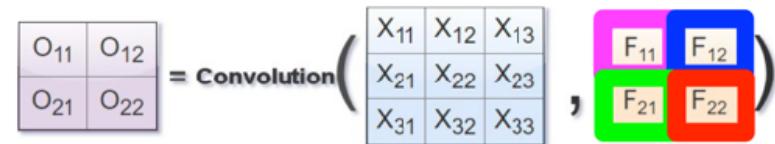
► $\frac{\partial E}{\partial F_{21}} = \frac{\partial E}{\partial O_{11}} X_{21} + \frac{\partial E}{\partial O_{12}} X_{22} + \frac{\partial E}{\partial O_{21}} X_{31} + \frac{\partial E}{\partial O_{22}} X_{32}$



Forward and Backward Propagation using Convolution operation

► Gradients of error 'E' with respect to the filter 'F'

$$E = E_{O_{11}} + E_{O_{12}} + E_{O_{21}} + E_{O_{22}}$$

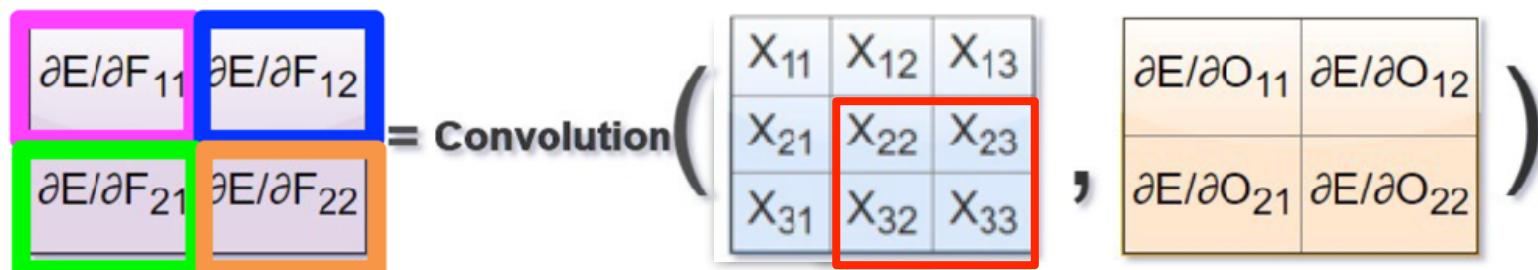


► $\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} X_{11} + \frac{\partial E}{\partial O_{12}} X_{12} + \frac{\partial E}{\partial O_{21}} X_{21} + \frac{\partial E}{\partial O_{22}} X_{22}$

► $\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}} X_{12} + \frac{\partial E}{\partial O_{12}} X_{13} + \frac{\partial E}{\partial O_{21}} X_{22} + \frac{\partial E}{\partial O_{22}} X_{23}$

► $\frac{\partial E}{\partial F_{21}} = \frac{\partial E}{\partial O_{11}} X_{21} + \frac{\partial E}{\partial O_{12}} X_{22} + \frac{\partial E}{\partial O_{21}} X_{31} + \frac{\partial E}{\partial O_{22}} X_{32}$

► $\frac{\partial E}{\partial F_{22}} = \frac{\partial E}{\partial O_{11}} X_{22} + \frac{\partial E}{\partial O_{12}} X_{23} + \frac{\partial E}{\partial O_{21}} X_{32} + \frac{\partial E}{\partial O_{22}} X_{33}$



Any Questions?

Forward and Backward Propagation using Convolution operation

► Gradients of the error 'E' with respect to the input matrix 'X'

$$E = E_{O_{11}} + E_{O_{12}} + E_{O_{21}} + E_{O_{22}}$$

$$\frac{\partial E}{\partial X_{11}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial X_{11}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial X_{11}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial X_{11}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial X_{11}}$$

$$\frac{\partial E}{\partial X_{11}} = \frac{\partial E}{\partial O_{11}} F_{11} + \frac{\partial E}{\partial O_{12}} 0 + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} 0$$

$$\begin{array}{|c|c|} \hline O_{11} & O_{12} \\ \hline O_{21} & O_{22} \\ \hline \end{array} = \text{Convolution} \left(\begin{array}{|c|c|c|} \hline X_{11} & X_{12} & X_{13} \\ \hline X_{21} & X_{22} & X_{23} \\ \hline X_{31} & X_{32} & X_{33} \\ \hline \end{array}, \begin{array}{|c|c|} \hline F_{11} & F_{12} \\ \hline F_{21} & F_{22} \\ \hline \end{array} \right)$$

$$O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

$$O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

$$O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33}$$

Forward and Backward Propagation using Convolution operation

► Gradients of the error ‘E’ with respect to the input matrix ‘X’

$$E = E_{O_{11}} + E_{O_{12}} + E_{O_{21}} + E_{O_{22}}$$

$$\begin{matrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{matrix} = \text{Convolution} \left(\begin{matrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{matrix}, \begin{matrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{matrix} \right)$$

$$O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

$$O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

$$O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33}$$

$$\frac{\partial E}{\partial X_{11}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial X_{11}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial X_{11}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial X_{11}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial X_{11}}$$

$$\frac{\partial E}{\partial X_{11}} = \frac{\partial E}{\partial O_{11}} F_{11} + \frac{\partial E}{\partial O_{12}} 0 + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} 0$$

$$\frac{\partial E}{\partial X_{23}} = \frac{\partial E}{\partial O_{11}} \frac{\partial O_{11}}{\partial X_{23}} + \frac{\partial E}{\partial O_{12}} \frac{\partial O_{12}}{\partial X_{23}} + \frac{\partial E}{\partial O_{21}} \frac{\partial O_{21}}{\partial X_{23}} + \frac{\partial E}{\partial O_{22}} \frac{\partial O_{22}}{\partial X_{23}}$$

$$\frac{\partial E}{\partial X_{23}} = \frac{\partial E}{\partial O_{11}} 0 + \frac{\partial E}{\partial O_{12}} F_{22} + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} F_{12}$$

Forward and Backward Propagation using Convolution operation

► Gradients of the error 'E' with respect to the input matrix 'X'

$$\frac{\partial E}{\partial X_{11}} = \frac{\partial E}{\partial O_{11}} F_{11} + \frac{\partial E}{\partial O_{12}} 0 + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} 0$$

$$\frac{\partial E}{\partial X_{12}} = \frac{\partial E}{\partial O_{11}} F_{12} + \frac{\partial E}{\partial O_{12}} F_{11} + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} 0$$

$$\frac{\partial E}{\partial X_{13}} = \frac{\partial E}{\partial O_{11}} 0 + \frac{\partial E}{\partial O_{12}} F_{12} + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} 0$$

$$\frac{\partial E}{\partial X_{21}} = \frac{\partial E}{\partial O_{11}} F_{21} + \frac{\partial E}{\partial O_{12}} 0 + \frac{\partial E}{\partial O_{21}} F_{11} + \frac{\partial E}{\partial O_{22}} 0$$

$$\frac{\partial E}{\partial X_{22}} = \frac{\partial E}{\partial O_{11}} F_{22} + \frac{\partial E}{\partial O_{12}} F_{21} + \frac{\partial E}{\partial O_{21}} f_{12} + \frac{\partial E}{\partial O_{22}} F_{11}$$

$$\frac{\partial E}{\partial X_{23}} = \frac{\partial E}{\partial O_{11}} 0 + \frac{\partial E}{\partial O_{12}} F_{22} + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} F_{12}$$

$$\frac{\partial E}{\partial X_{31}} = \frac{\partial E}{\partial O_{11}} 0 + \frac{\partial E}{\partial O_{12}} 0 + \frac{\partial E}{\partial O_{21}} F_{21} + \frac{\partial E}{\partial O_{22}} 0$$

$$\frac{\partial E}{\partial X_{32}} = \frac{\partial E}{\partial O_{11}} 0 + \frac{\partial E}{\partial O_{12}} 0 + \frac{\partial E}{\partial O_{21}} F_{22} + \frac{\partial E}{\partial O_{22}} F_{21}$$

$$\frac{\partial E}{\partial X_{33}} = \frac{\partial E}{\partial O_{11}} 0 + \frac{\partial E}{\partial O_{12}} 0 + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} F_{22}$$

$$\begin{array}{|c|c|} \hline O_{11} & O_{12} \\ \hline O_{21} & O_{22} \\ \hline \end{array} = \text{Convolution} \left(\begin{array}{|c|c|c|} \hline X_{11} & X_{12} & X_{13} \\ \hline X_{21} & X_{22} & X_{23} \\ \hline X_{31} & X_{32} & X_{33} \\ \hline \end{array}, \begin{array}{|c|c|} \hline F_{11} & F_{12} \\ \hline F_{21} & F_{22} \\ \hline \end{array} \right)$$

$$O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

$$O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

$$O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33}$$

$$\begin{array}{|c|c|c|} \hline \partial E / \partial X_{11} & \partial E / \partial X_{12} & \partial E / \partial X_{13} \\ \hline \partial E / \partial X_{21} & \partial E / \partial X_{22} & \partial E / \partial X_{23} \\ \hline \partial E / \partial X_{31} & \partial E / \partial X_{32} & \partial E / \partial X_{33} \\ \hline \end{array} = \text{Full_Convolution} \left(\begin{array}{|c|c|} \hline \partial E / \partial O_{11} & \partial E / \partial O_{12} \\ \hline \partial E / \partial O_{21} & \partial E / \partial O_{22} \\ \hline \end{array}, \begin{array}{|c|c|} \hline F_{22} & F_{21} \\ \hline F_{12} & r_{12}O_{11} + S_{12}O_{12} \\ \hline \end{array} \right)$$

$$\begin{array}{|c|c|} \hline F_{22} & F_{21} \\ \hline F_{12} & r_{12}O_{11} + S_{12}O_{12} \\ \hline \end{array}$$

$r_{12}O_{11} + S_{12}O_{12}$

$$\begin{array}{|c|c|} \hline F_{22} & F_{21} \\ \hline r_{12}O_{11} & r_{12}O_{12} \\ \hline \end{array}$$

$r_{12}O_{11} + S_{12}O_{12}$

$$\begin{array}{|c|c|} \hline F_{22} & F_{21} \\ \hline S_{12}O_{11} & F_{11} \\ \hline \end{array}$$

$r_{12}O_{11} + S_{12}O_{12}$

$$\begin{array}{|c|c|} \hline F_{22} & r_{12}O_{11} + S_{12}O_{12} \\ \hline F_{12} & r_{12}O_{11} + S_{12}O_{12} \\ \hline \end{array}$$

$r_{12}O_{11} + S_{12}O_{12}$

$$\begin{array}{|c|c|} \hline r_{12}O_{11} & r_{12}O_{12} \\ \hline r_{12}O_{11} & r_{12}O_{12} \\ \hline \end{array}$$

$r_{12}O_{11} + S_{12}O_{12}$

$$\begin{array}{|c|c|} \hline S_{12}O_{11} & F_{11} \\ \hline S_{12}O_{11} & F_{11} \\ \hline \end{array}$$

$r_{12}O_{11} + S_{12}O_{12}$

$$\begin{array}{|c|c|} \hline S_{12}O_{11} & S_{12}O_{12} \\ \hline F_{22} & r_{12}O_{11} + S_{12}O_{12} \\ \hline \end{array}$$

$r_{12}O_{11} + S_{12}O_{12}$

$$\begin{array}{|c|c|} \hline r_{12}O_{11} & r_{12}O_{12} \\ \hline F_{12} & F_{11} \\ \hline \end{array}$$

$r_{12}O_{11} + S_{12}O_{12}$

$$\begin{array}{|c|c|} \hline S_{12}O_{11} & F_{21} \\ \hline S_{12}O_{11} & F_{21} \\ \hline \end{array}$$

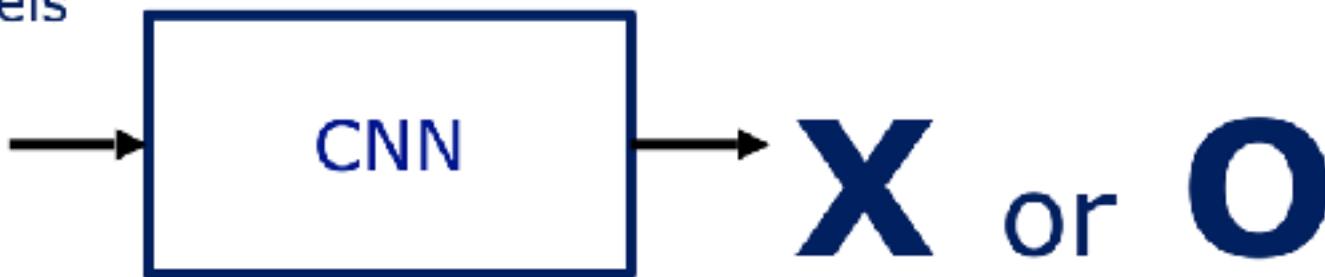
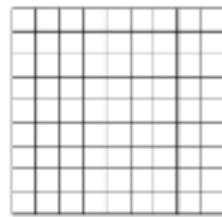
$r_{12}O_{11} + S_{12}O_{12}$

A Toy ConvNet X's and O's

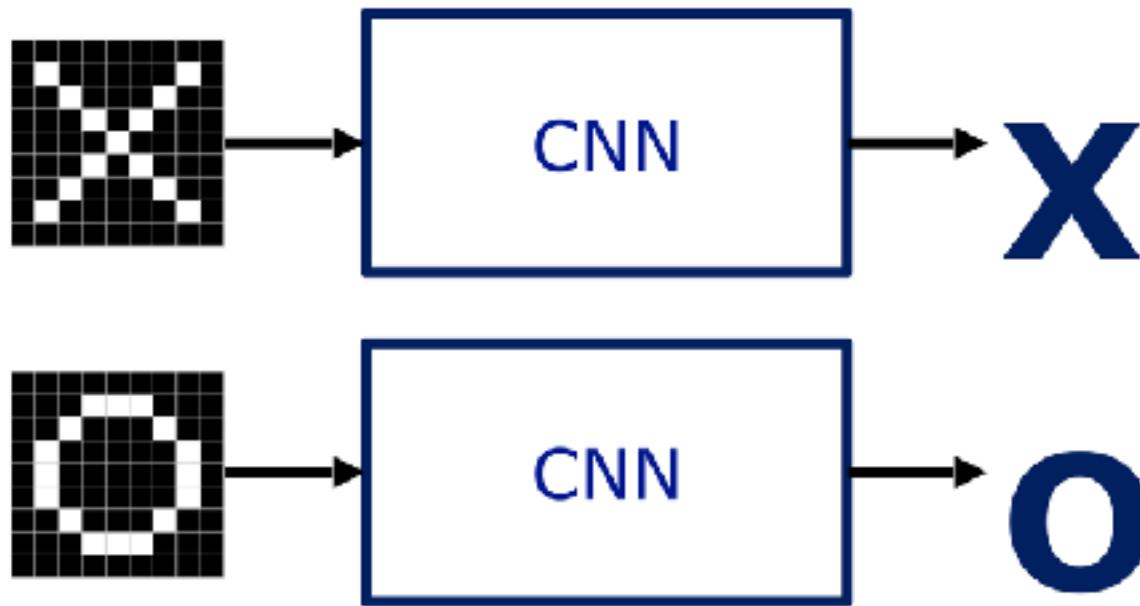
A toy ConvNet: X's and O's

Says whether a picture is of an X or an O

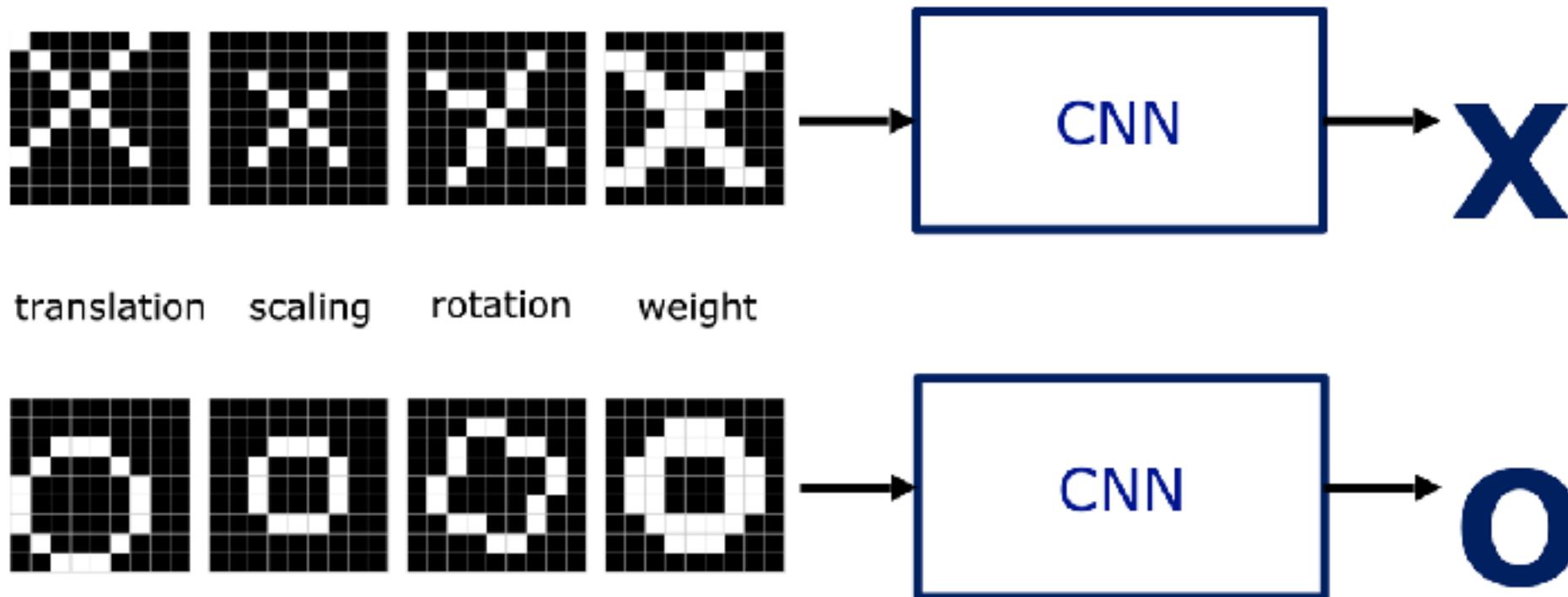
A two-dimensional
array of pixels



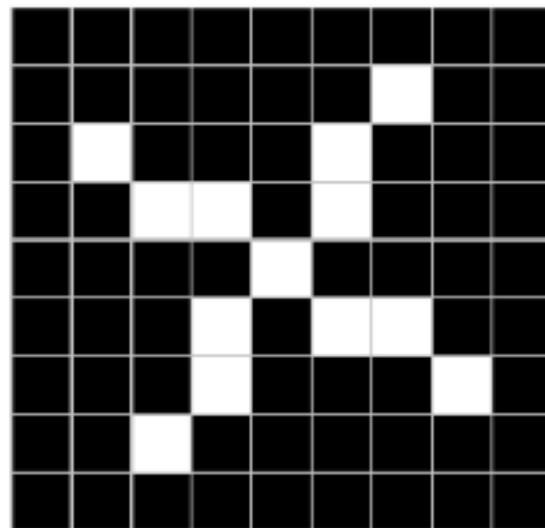
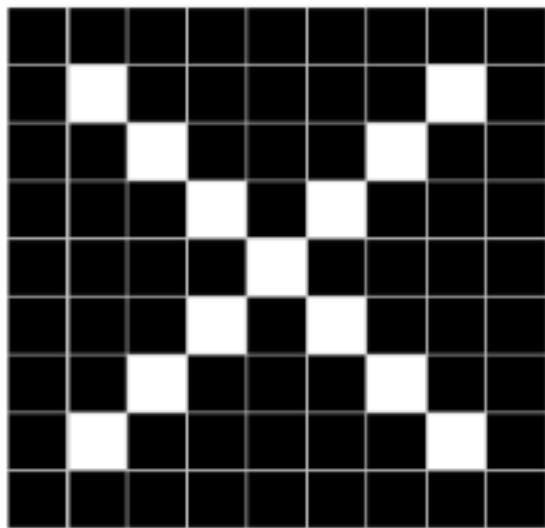
For example



Trickier cases



Deciding is hard



What computers see

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1



-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	1	-1
-1	1	-1	-1	-1	1	-1	-1	-1
-1	-1	1	1	-1	1	-1	-1	-1
-1	-1	1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	1	-1	-1
-1	-1	-1	1	-1	1	1	-1	-1
-1	-1	-1	1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

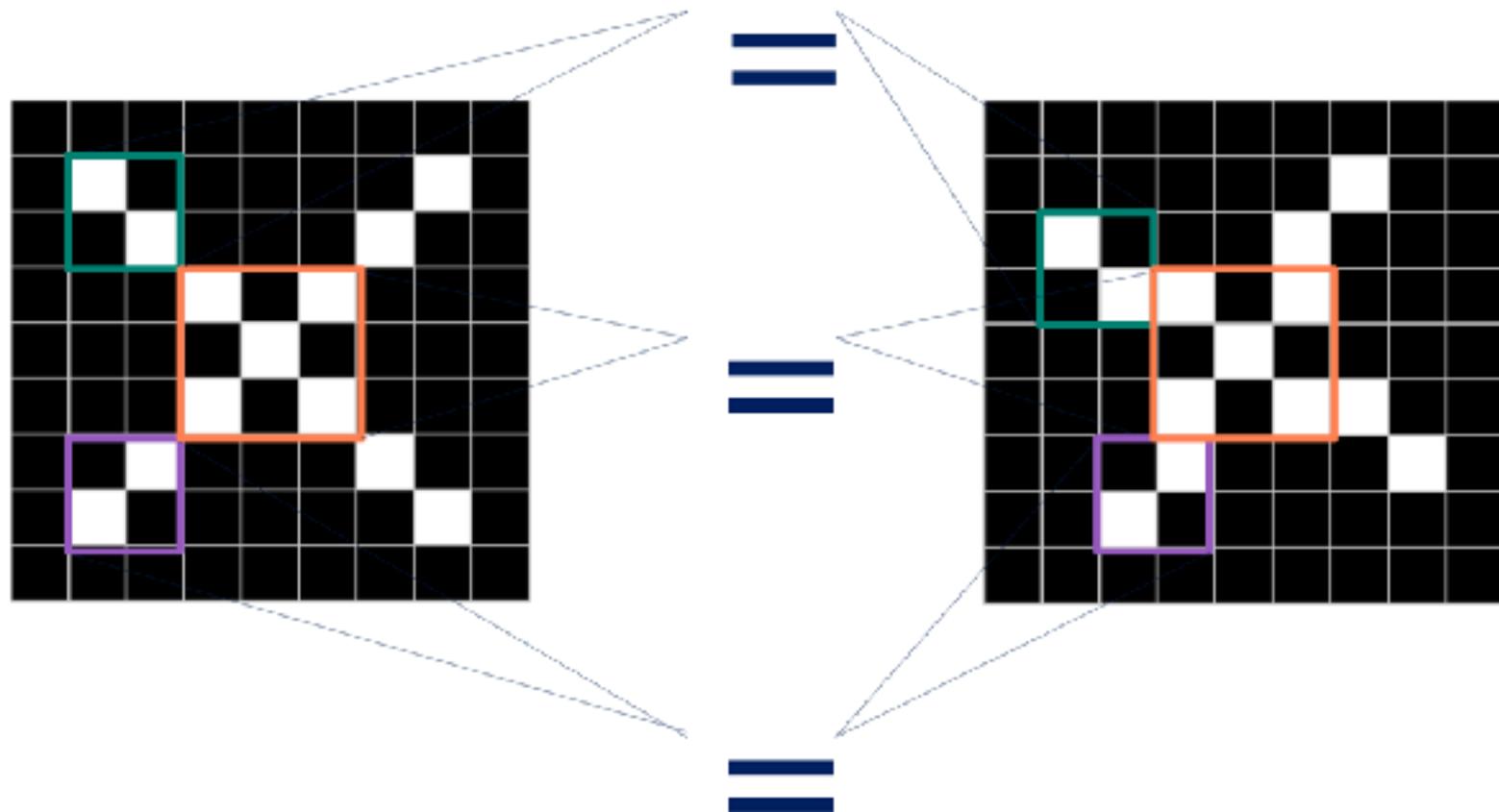
Computers are literal

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1



-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	1	-1
-1	1	-1	-1	-1	1	-1	-1	-1
-1	-1	1	1	-1	1	-1	-1	-1
-1	-1	-1	1	1	-1	1	-1	-1
-1	-1	-1	-1	1	-1	1	-1	-1
-1	-1	-1	1	-1	1	1	-1	-1
-1	-1	-1	1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

ConvNets match pieces of the image



Features match pieces of the image

1	-1	-1
-1	1	-1
-1	-1	1

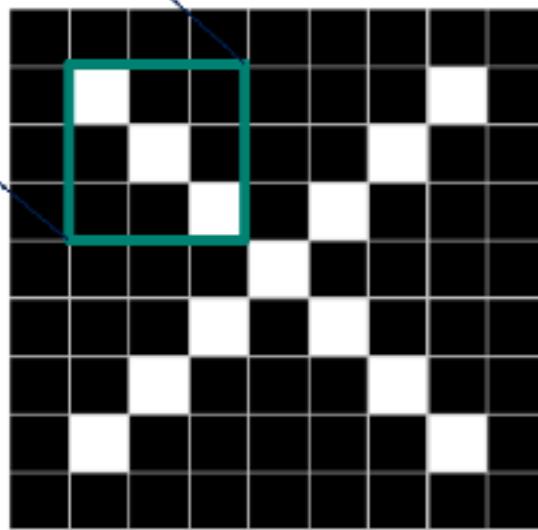
1	-1	1
-1	1	-1
1	-1	1

-1	-1	1
-1	1	-1
1	-1	-1

1	-1	-1
-1	1	-1
-1	-1	1

1	-1	1
-1	1	-1
1	-1	1

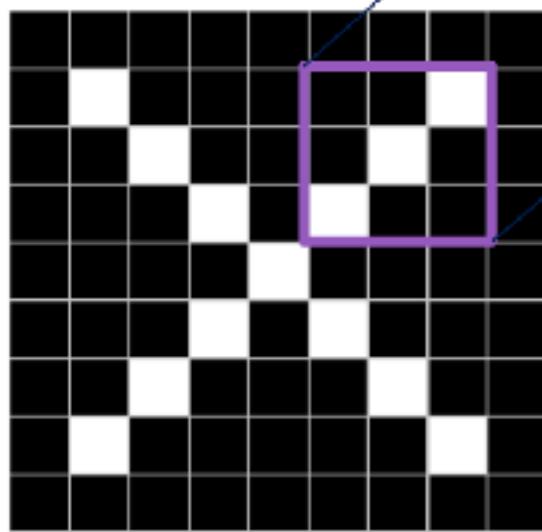
-1	-1	1
-1	1	-1
1	-1	-1



1	-1	-1
-1	1	-1
-1	-1	1

1	-1	1
-1	1	-1
1	-1	1

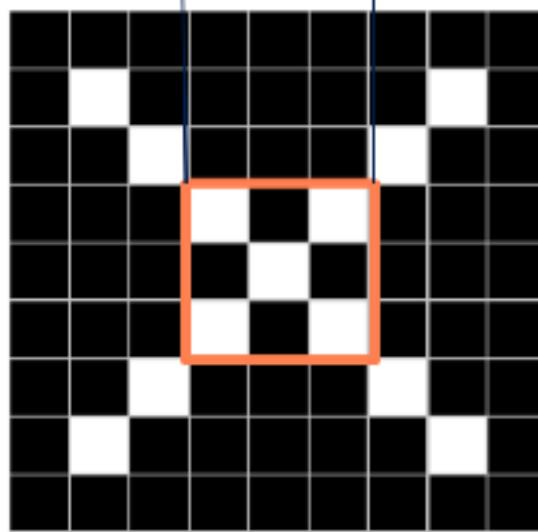
-1	-1	1
-1	1	-1
1	-1	-1



1	-1	-1
-1	1	-1
-1	-1	1

1	-1	1
-1	1	-1
1	-1	1

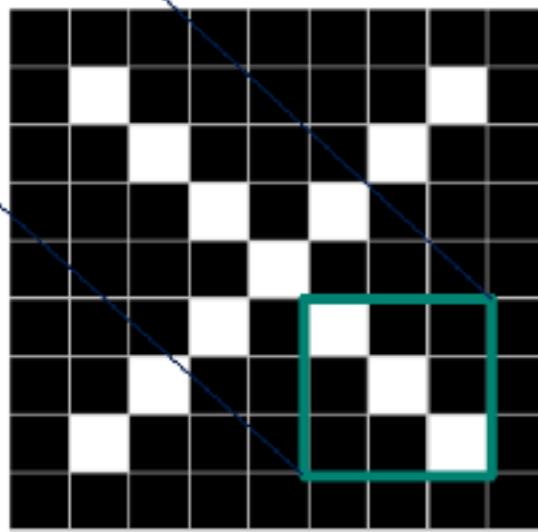
-1	-1	1
-1	1	-1
1	-1	-1



1	-1	-1
-1	1	-1
-1	-1	1

1	-1	1
-1	1	-1
1	-1	1

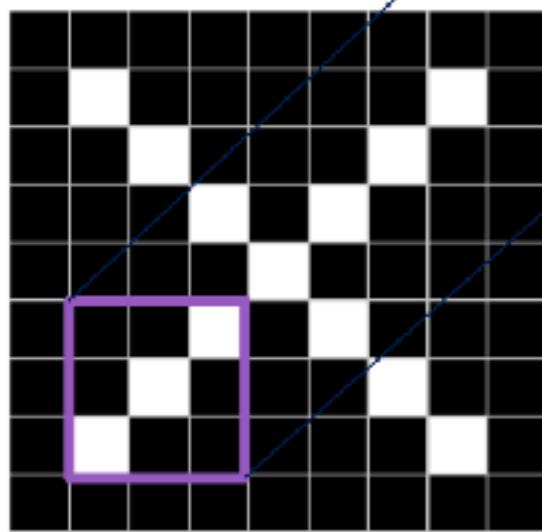
-1	-1	1
-1	1	-1
1	-1	-1



1	-1	-1
-1	1	-1
-1	-1	1

1	-1	1
-1	1	-1
1	-1	1

-1	-1	1
-1	1	-1
1	-1	-1



Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

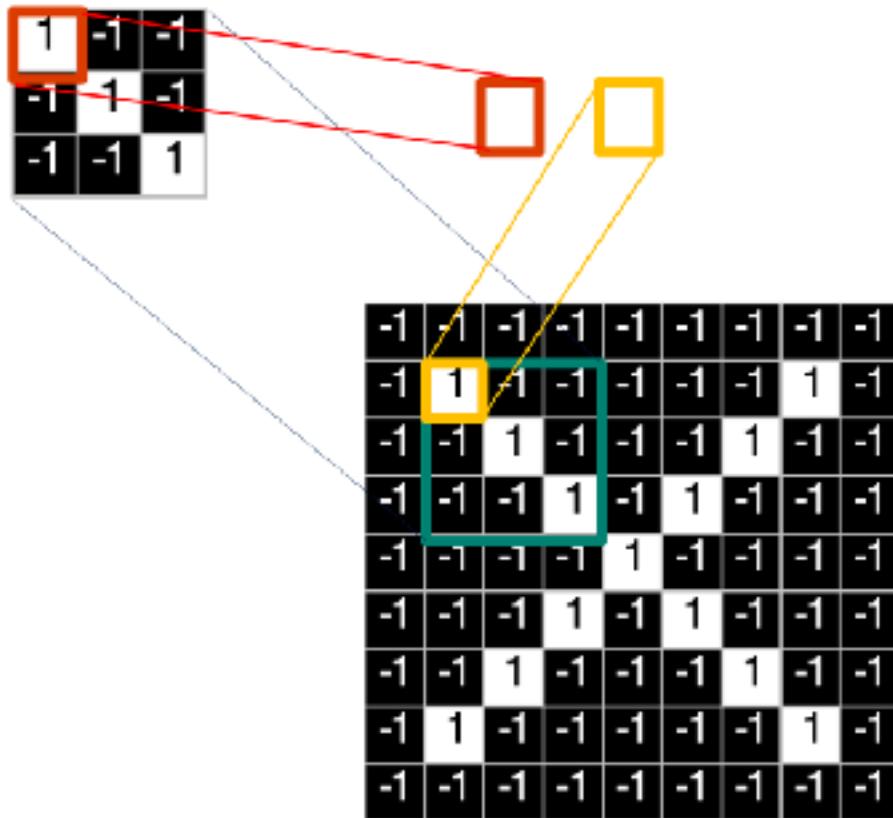
The diagram illustrates the application of a 3x3 filter to a larger 3x3 input matrix. The filter, shown in the top-left corner, has weights: 1, -1, -1; -1, 1, -1; and -1, -1, 1. It is applied to the input matrix at its top-left position, where the input values are: -1, -1, -1; -1, 1, -1; and -1, -1, 1. The result of this convolution step is highlighted with a green box.

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

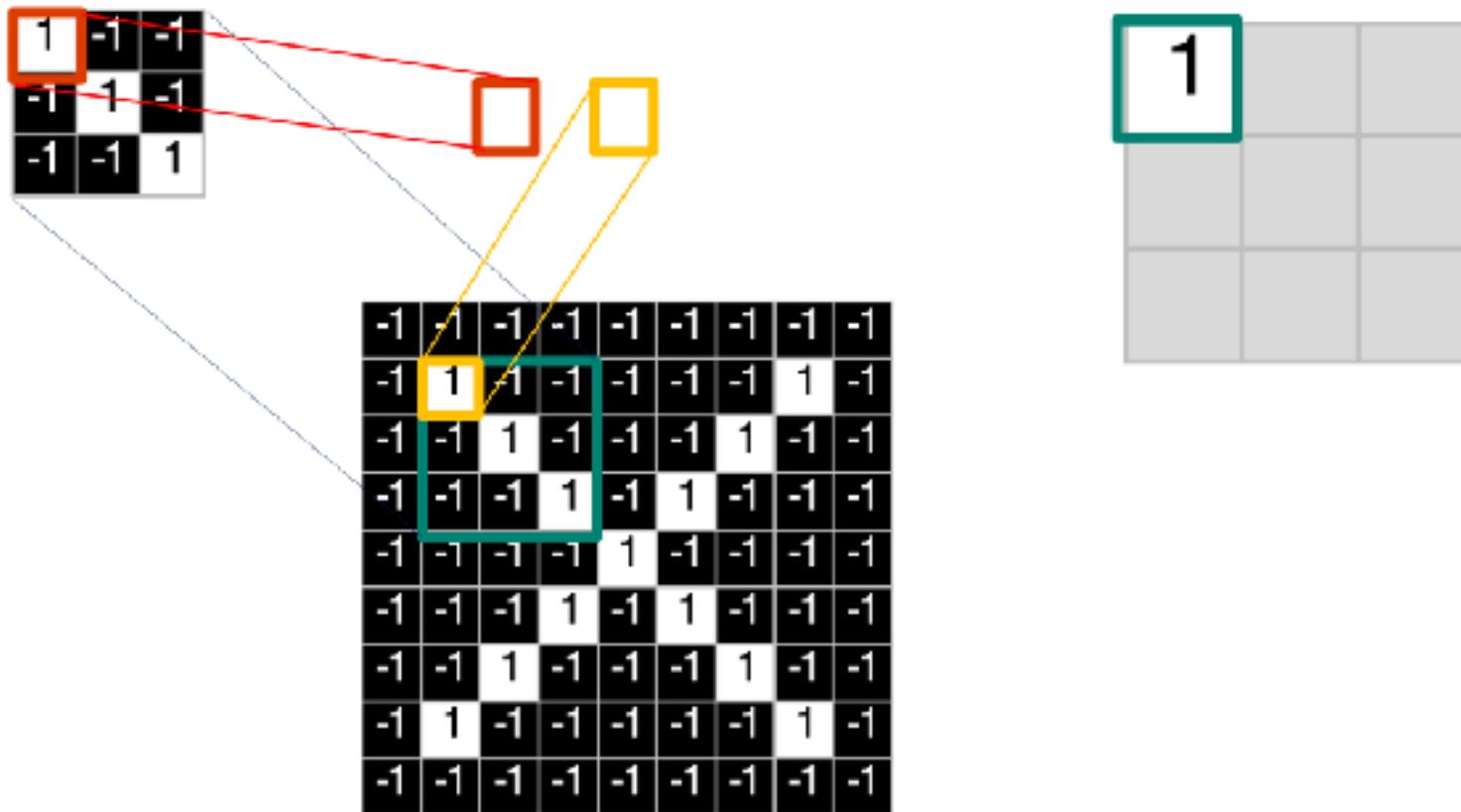
Filtering: The math behind the match

1. Line up the feature and the image patch.
2. Multiply each image pixel by the corresponding feature pixel.
3. Add them up.
4. Divide by the total number of pixels in the feature.

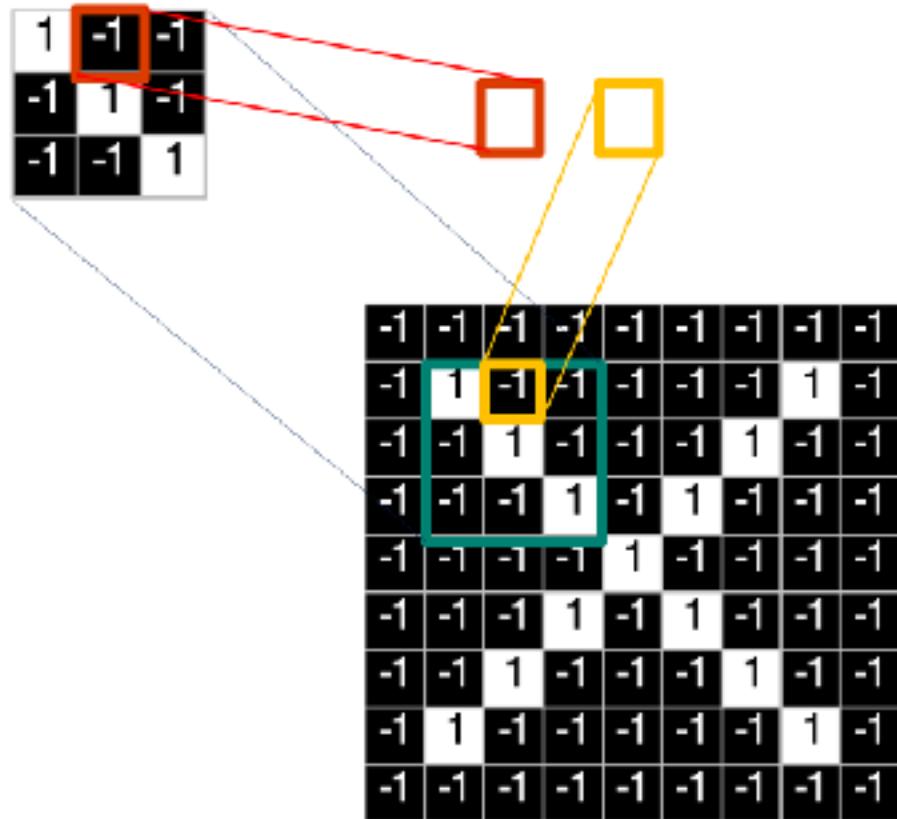
Filtering: The math behind the match



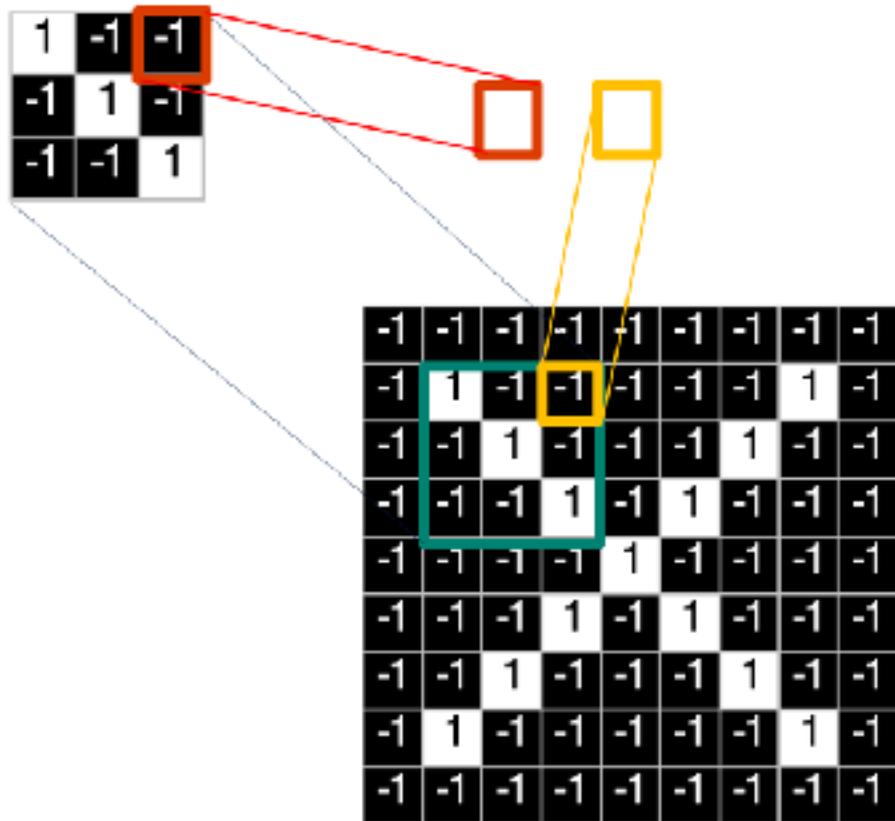
Filtering: The math behind the match



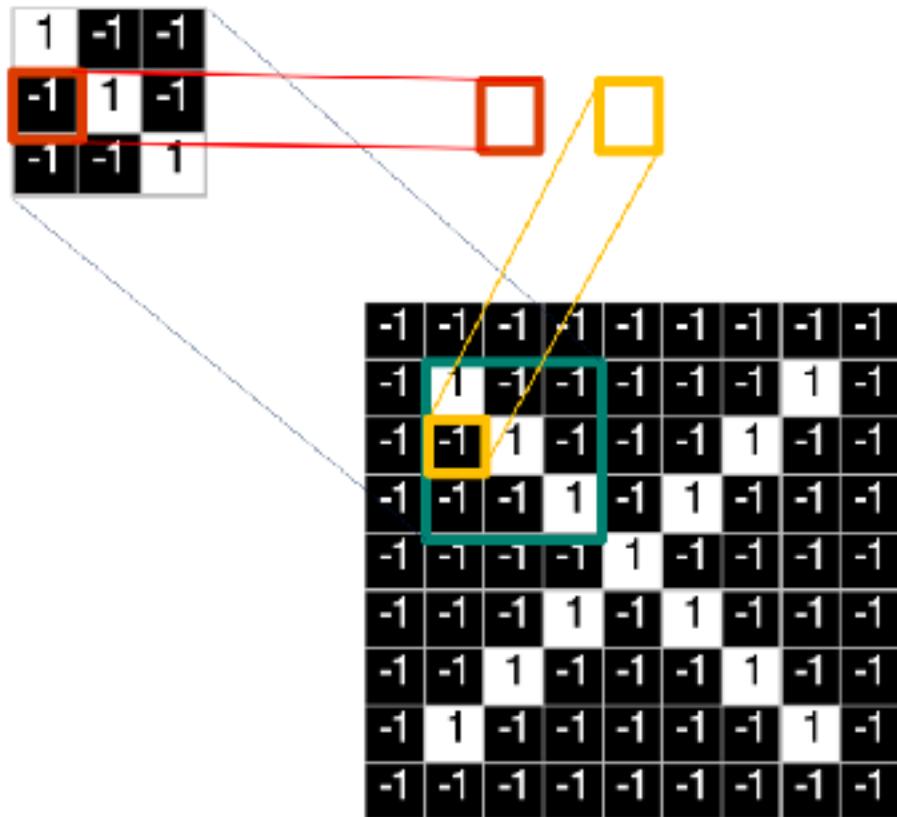
Filtering: The math behind the match



Filtering: The math behind the match

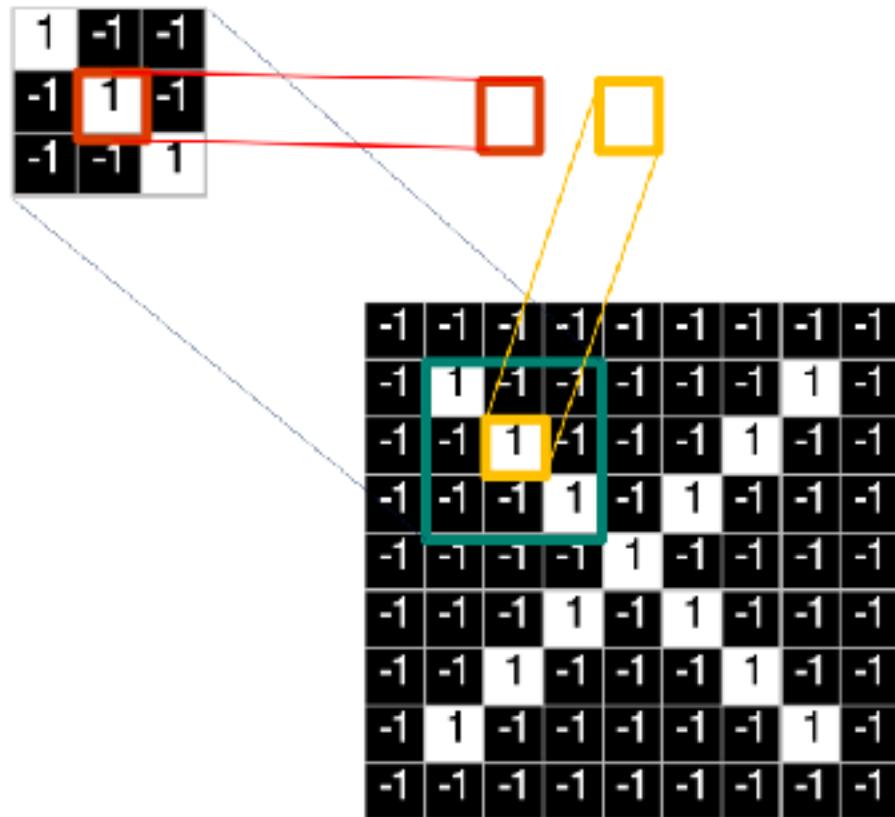


Filtering: The math behind the match

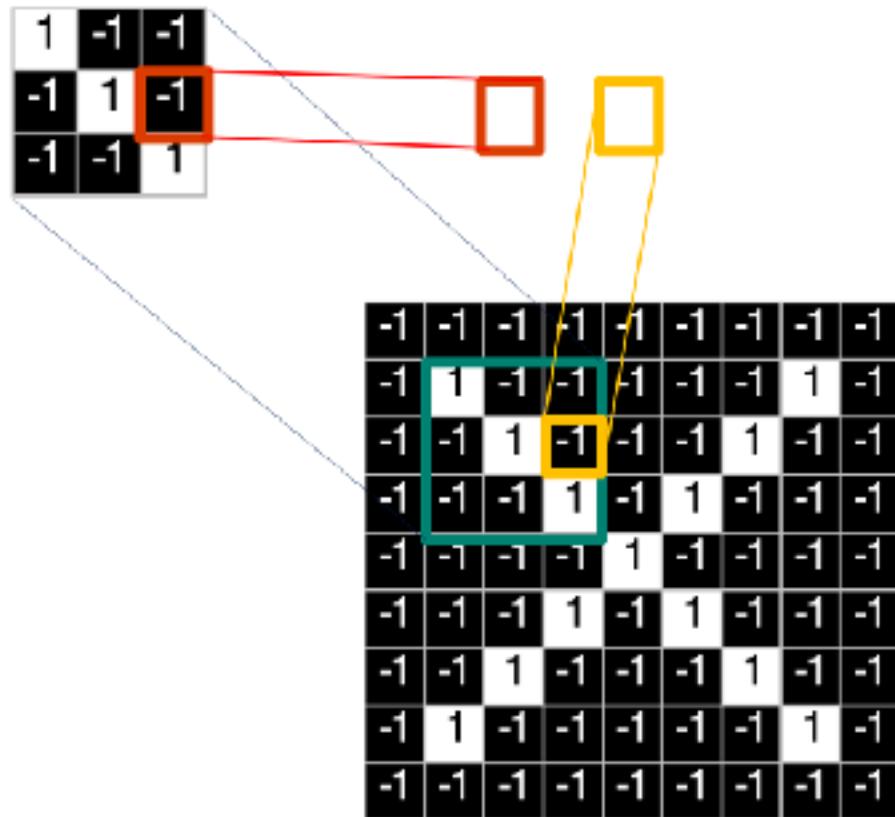


1	1	1
1		

Filtering: The math behind the match

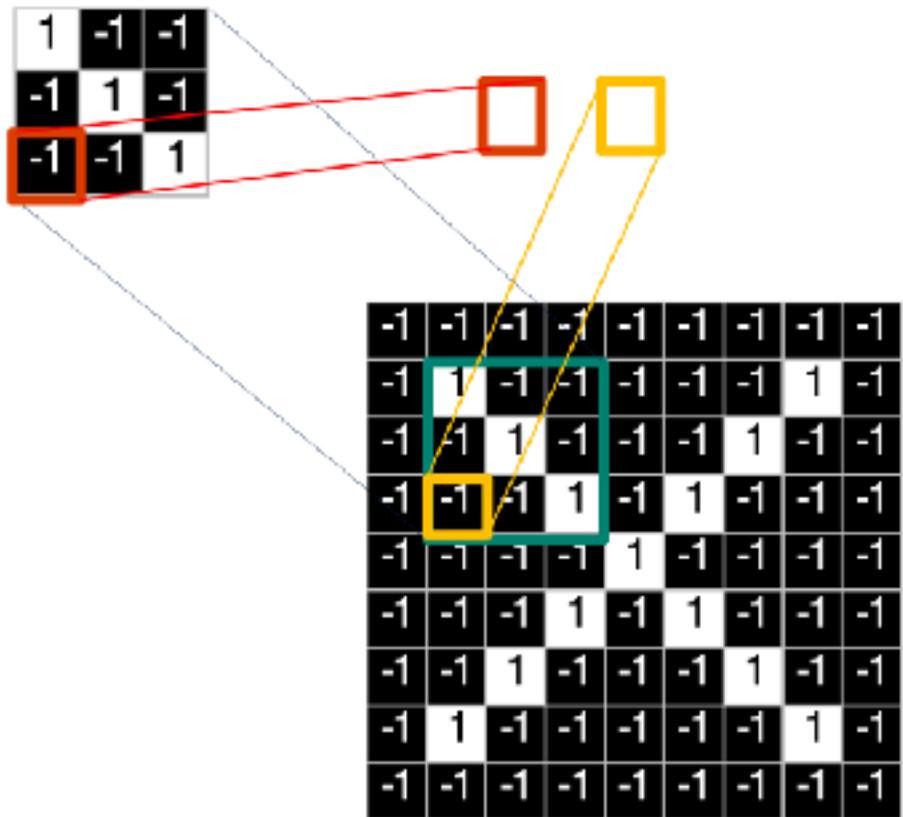


Filtering: The math behind the match



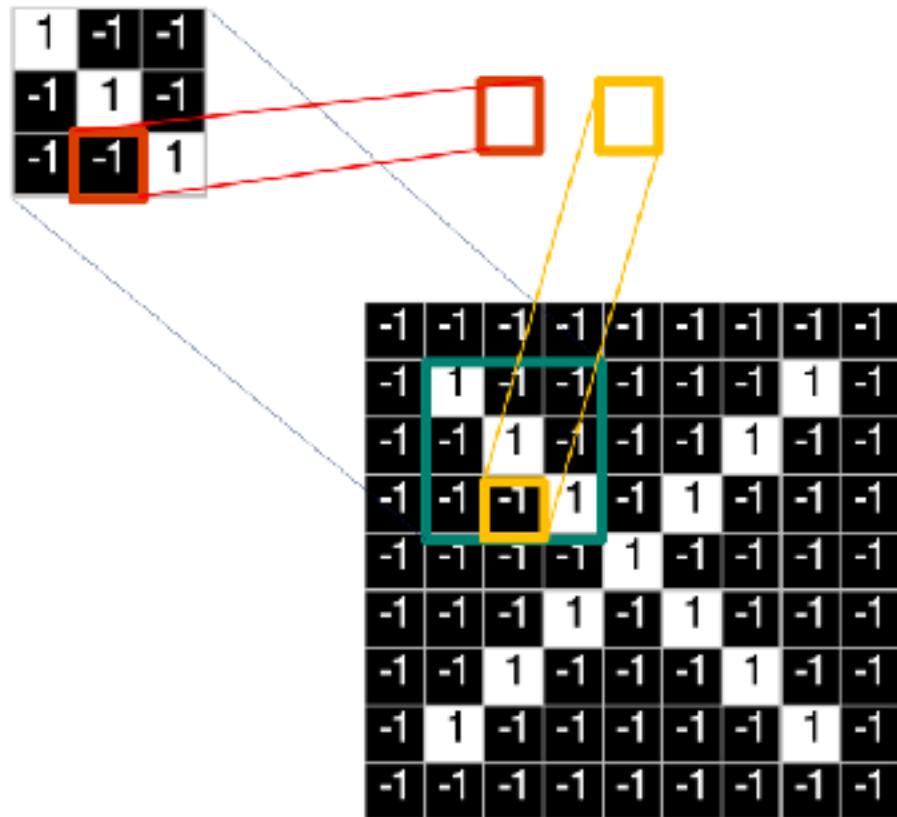
1	1	1
1	1	1
1	1	1

Filtering: The math behind the match



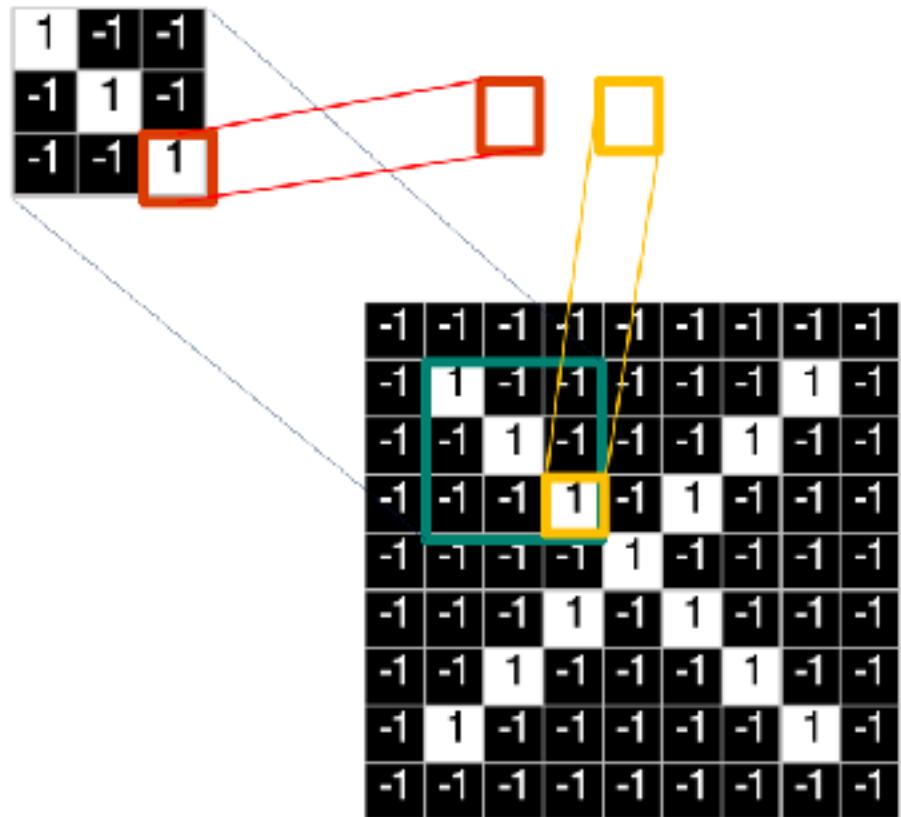
1	1	1
1	1	1
1		

Filtering: The math behind the match



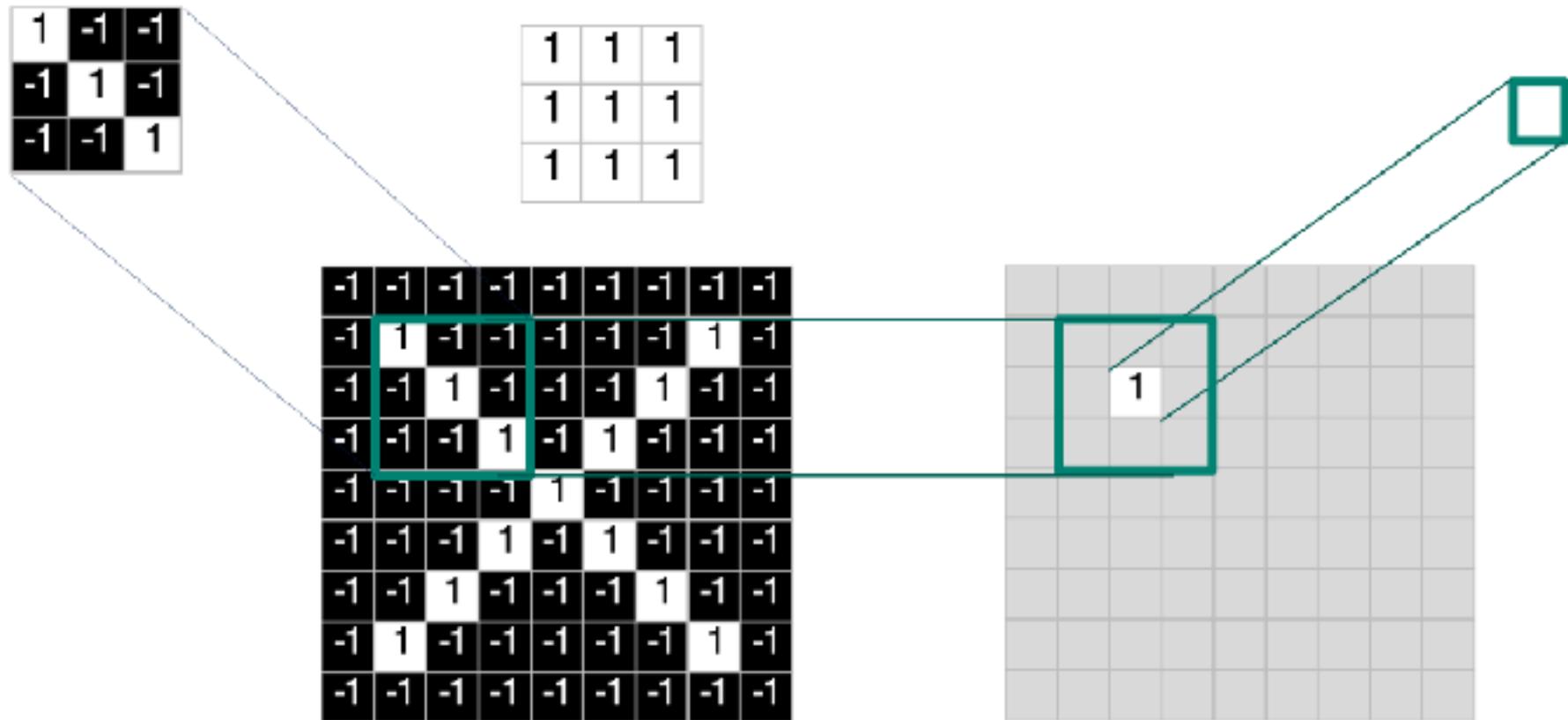
1	1	1
1	1	1
1	1	1

Filtering: The math behind the match

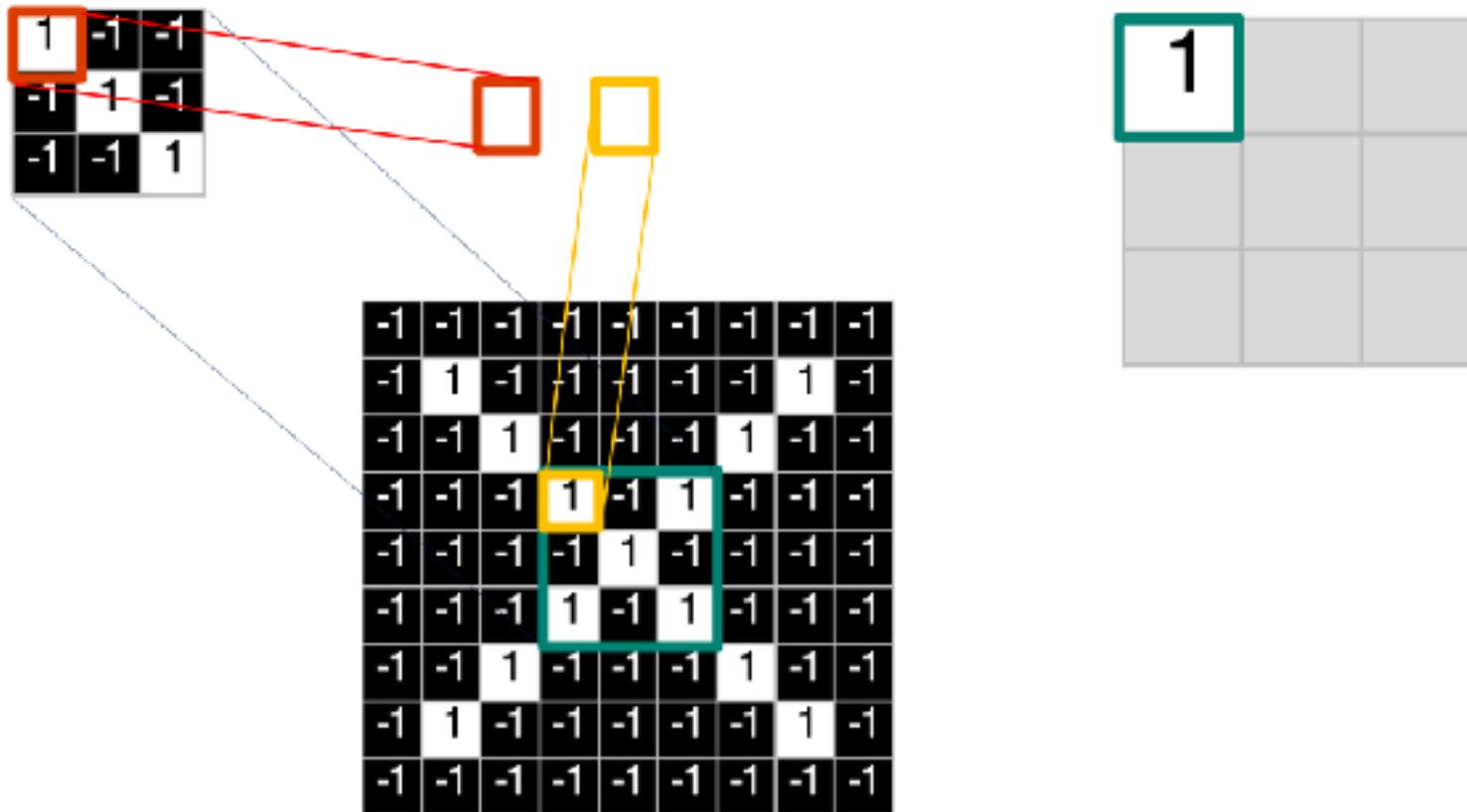


1	1	1
1	1	1
1	1	1

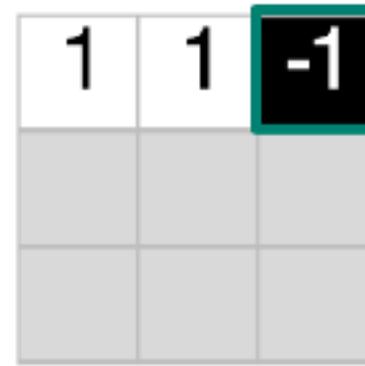
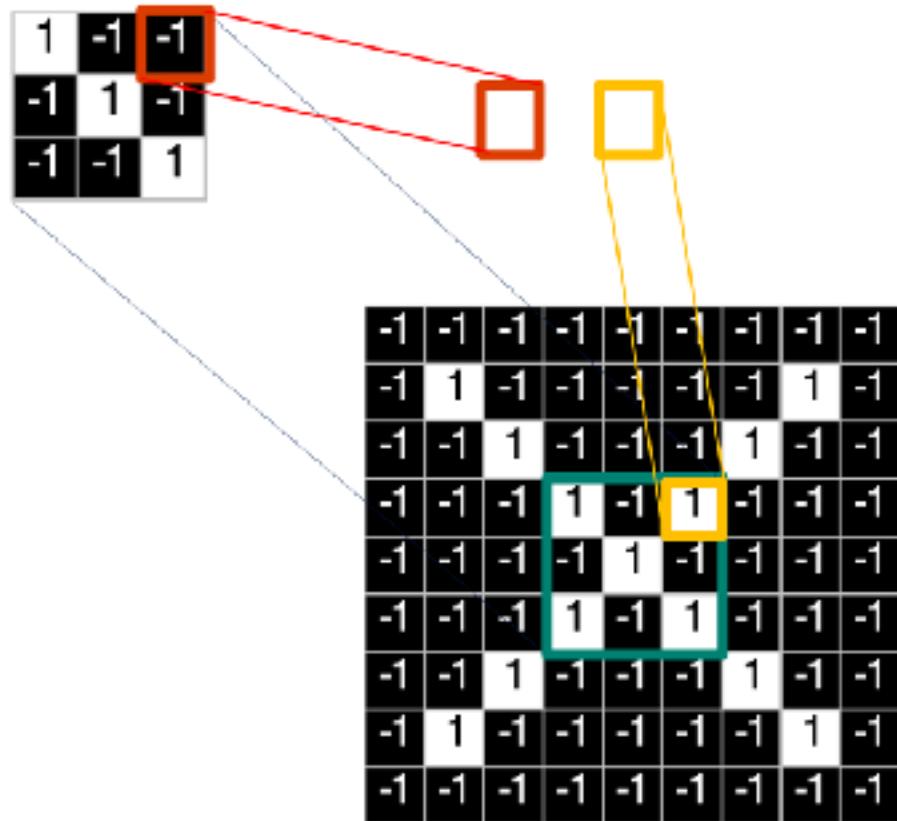
Filtering: The math behind the match



Filtering: The math behind the match



Filtering: The math behind the match



Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

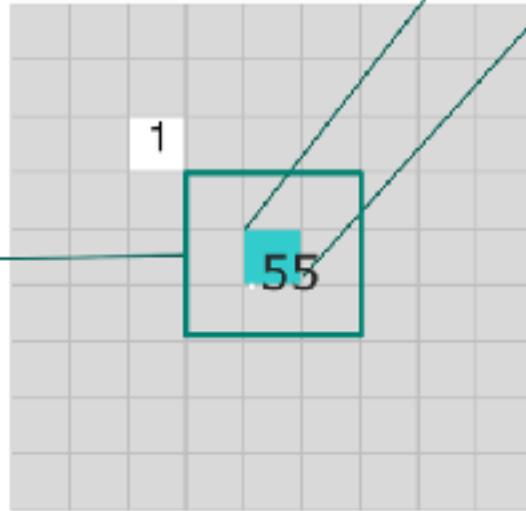
1	1	-1
1	1	1
-1	1	1

Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

1	1	-1
1	1	1
-1	1	1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1



Convolution: Trying every possible match

1	-1	-1
-1	1	-1
-1	-1	1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	-1	1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1



0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

Convolution: Trying every possible match

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1



1	-1	-1
-1	1	-1
-1	-1	1

=

0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	1	-1	-1



1	-1	-1
-1	1	-1
-1	-1	1



0.27	-0.11	0.11	0.28	0.86	0.81	0.88
-0.11	0.00	-0.11	0.88	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.22	0.11	-0.11	0.22
0.28	0.88	-0.22	0.88	-0.22	0.22	0.88
0.86	-0.11	0.11	-0.33	1.00	-0.11	0.11
0.81	0.11	-0.11	0.22	-0.11	1.00	-0.11
0.88	-0.11	0.88	0.22	0.11	-0.11	0.77

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	1	-1	-1	-1	-1	1	-1	-1
-1	-1	-1	-1	-1	-1	1	-1	-1



1	-1	1
-1	1	-1
1	-1	1



0.20	-0.25	0.11	-0.35	0.11	0.20	0.20
-0.25	0.25	-0.55	0.00	-0.55	0.55	-0.55
0.11	0.88	0.22	0.22	0.88	0.11	0.11
-0.35	0.39	-0.19	1.00	-0.27	0.00	-0.11
0.11	-0.25	0.25	-0.27	0.25	-0.25	0.11
0.20	0.99	-0.55	0.00	-0.33	0.00	-0.55
0.20	-0.55	0.11	-0.11	0.11	-0.55	0.38

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	1	-1	-1	-1	-1	1	-1	-1
-1	-1	-1	-1	-1	-1	1	-1	-1



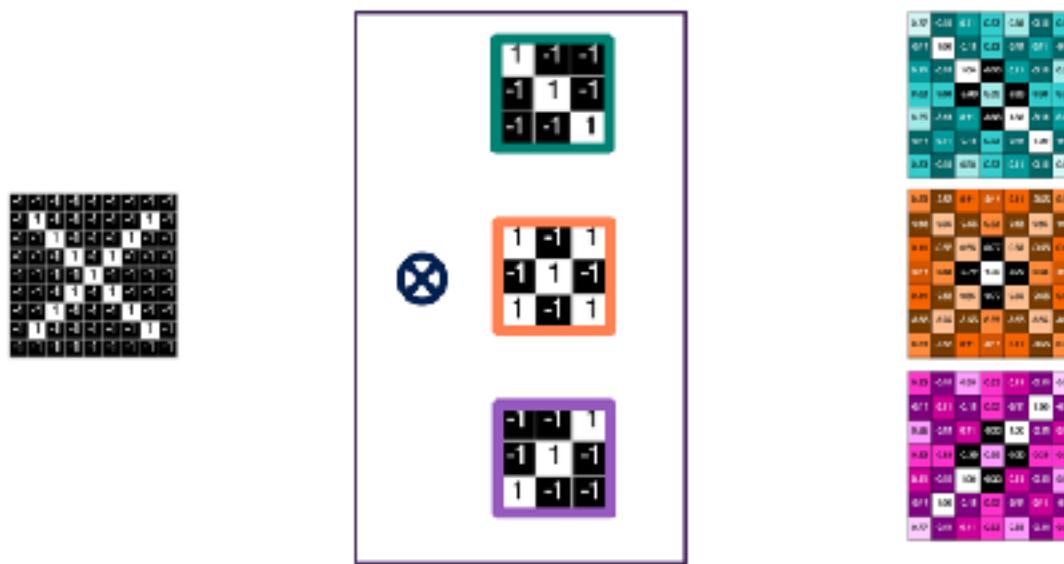
-1	-1	1
-1	1	-1
1	-1	-1



0.20	-0.11	0.66	0.22	0.11	-0.11	0.77
-0.11	0.88	-0.11	0.22	-0.11	1.00	-0.11
0.66	-0.11	0.11	-0.22	1.00	-0.11	0.11
0.22	0.39	-0.55	0.00	-0.33	0.00	0.39
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.11
-0.11	0.00	-0.11	0.22	-0.11	0.11	-0.11
0.77	-0.11	0.11	0.00	0.55	-0.11	0.39

Convolution layer

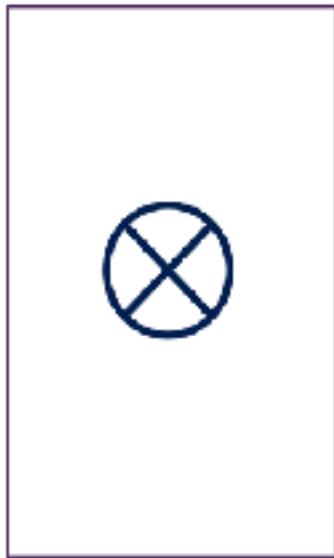
One image becomes a stack of filtered images



Convolution layer

One image becomes a stack of filtered images

1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	1



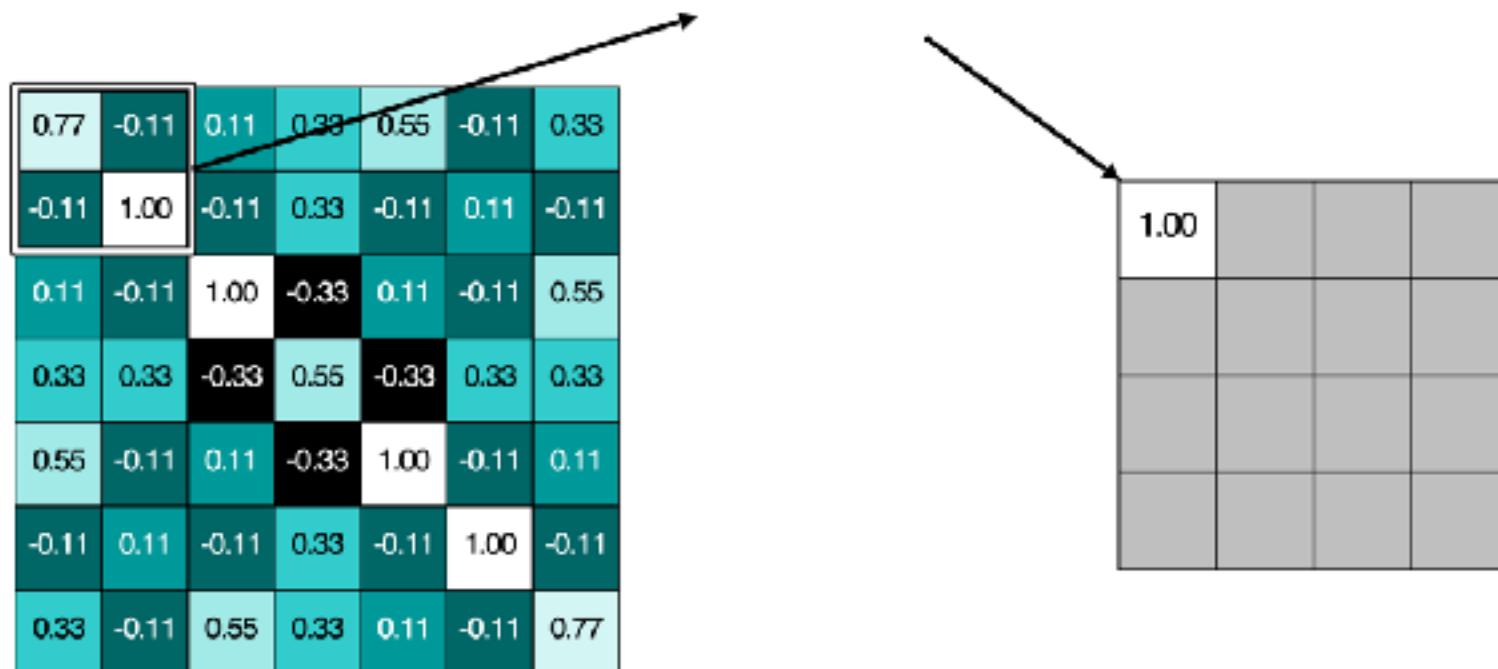
0.7	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

-0.05	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

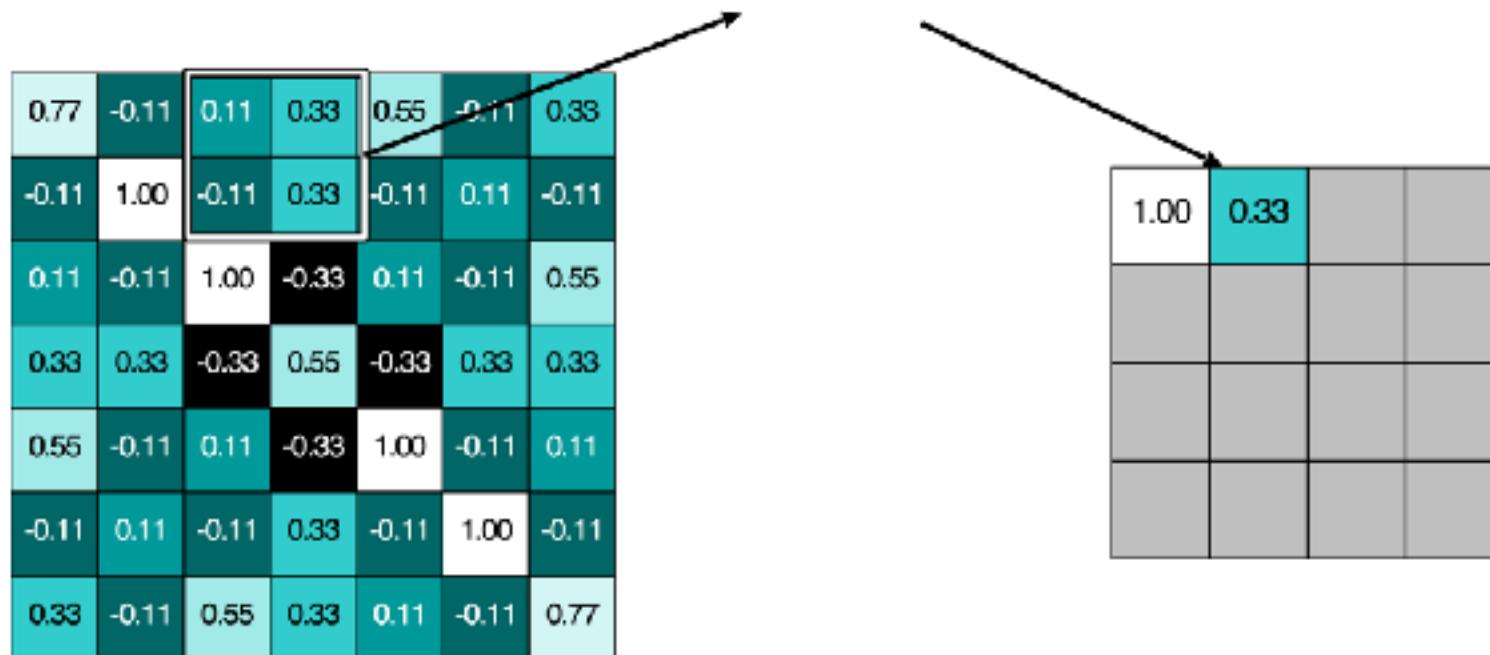
Pooling: Shrinking the image stack

1. Pick a window size (usually 2 or 3).
2. Pick a stride (usually 2).
3. Walk your window across your filtered images.
4. From each window, take the maximum value.

Pooling



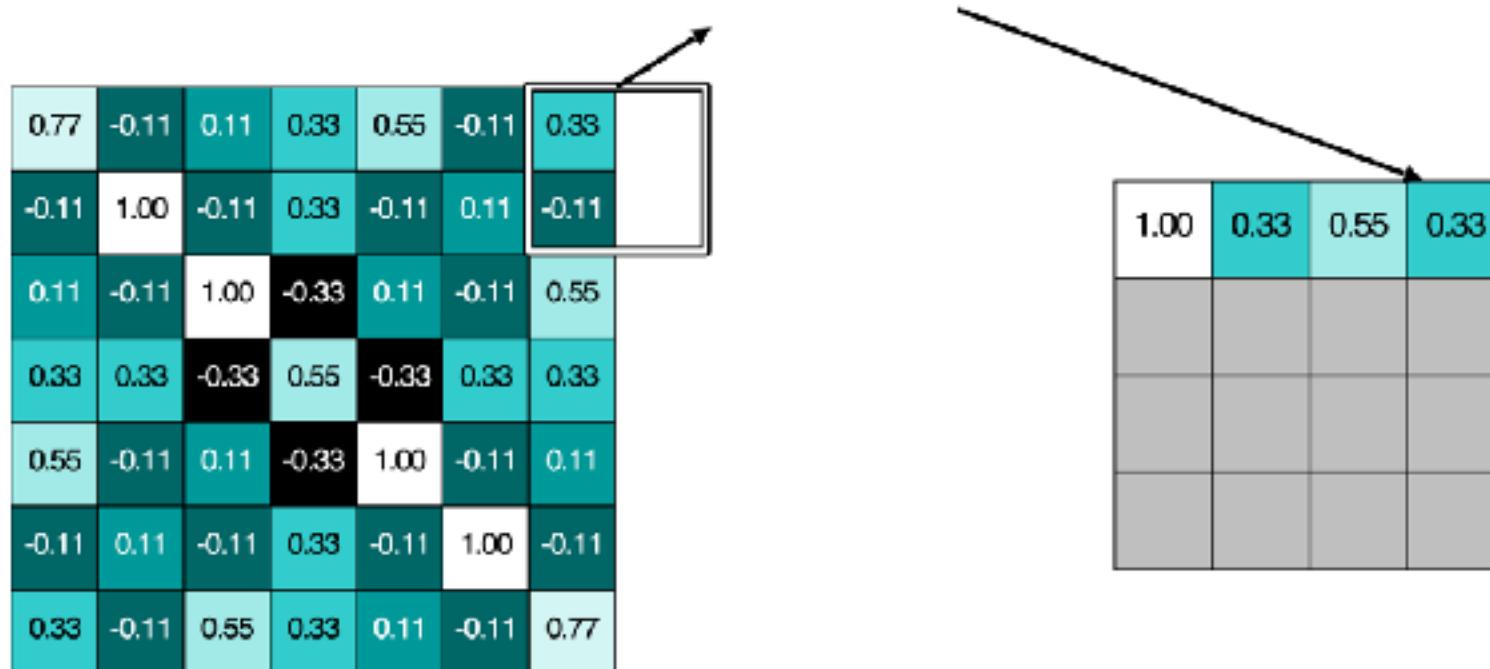
Pooling



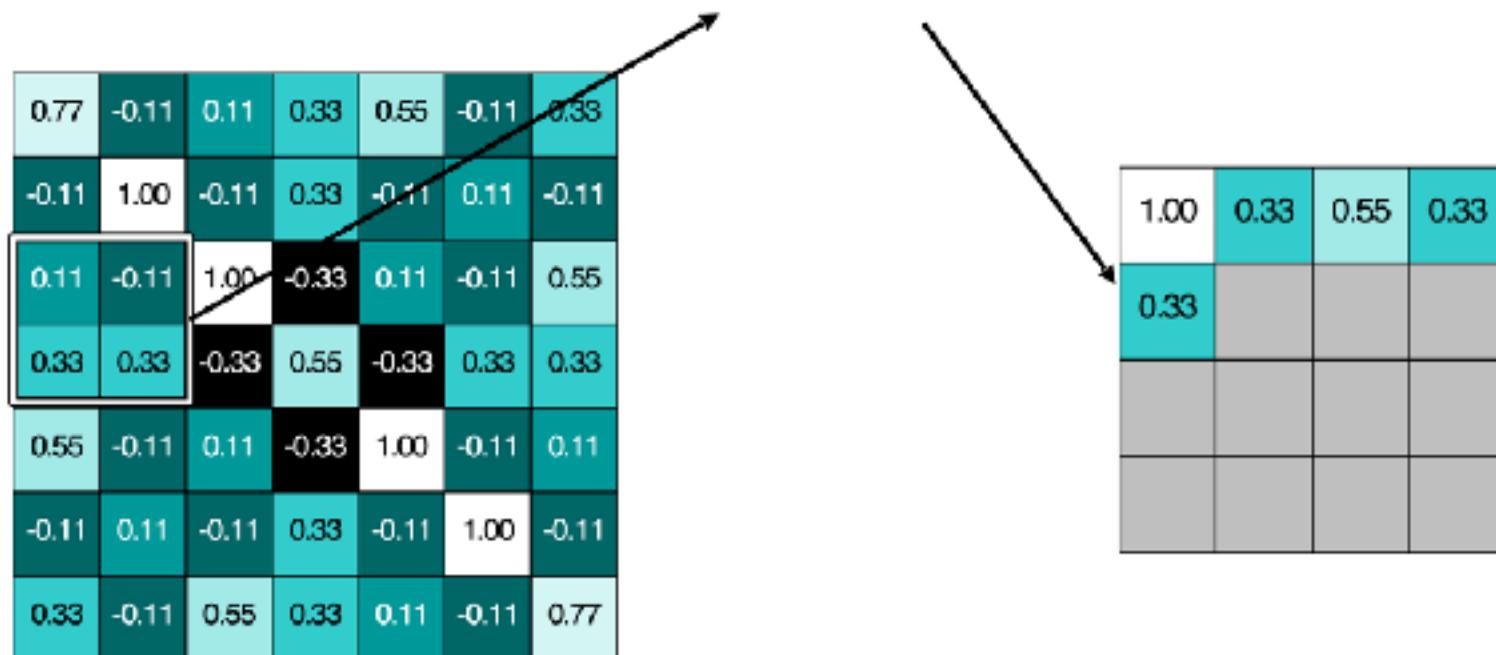
Pooling



Pooling



Pooling



Pooling

0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77



1.00	0.33	0.55	0.33
0.33	1.00	0.33	0.55
0.55	0.33	1.00	0.11
0.33	0.55	0.11	0.77

0.77	-0.11	0.11	0.33	0.66	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.33
0.33	0.33	-0.33	0.66	-0.33	0.33	1.00
0.66	-0.11	0.11	-0.33	1.00	-0.11	0.33
-0.11	0.33	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.33	0.33	0.11	-0.11	0.33



1.00	0.33	0.55	0.33
0.33	1.00	0.33	0.55
0.55	0.33	1.00	0.11
0.33	0.55	0.11	0.77

0.33	-0.55	0.11	-0.11	0.11	-0.55	1.00
-0.55	0.55	-0.55	0.33	-0.55	0.55	-0.55
0.11	-0.55	0.55	0.33	0.55	-0.55	1.00
-0.11	0.33	-0.77	1.00	-0.77	0.33	-0.11
0.11	-0.55	0.55	-0.77	0.55	-0.55	1.00
-0.55	0.55	-0.55	0.33	-0.55	0.55	-0.33
0.33	-0.55	0.11	-0.11	0.11	-0.55	1.00



0.55	0.33	0.55	0.33
0.33	1.00	0.55	0.11
0.55	0.55	0.55	0.11
0.33	0.11	0.11	0.33

0.99	-0.11	0.11	0.33	0.11	-0.11	0.77
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.11	-0.11	0.11	-0.33	1.00	-0.11	0.11
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.66
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.77	-0.11	0.11	0.33	0.55	-0.11	0.33



0.33	0.55	1.00	0.77
0.55	0.55	1.00	0.33
1.00	1.00	0.11	0.55
0.77	0.33	0.55	0.33

Pooling layer

A stack of images becomes a stack of smaller images.

0.0	-0.1	0.1	0.0	0.1	-0.1	0.0
0.1	0.0	-0.1	0.0	0.0	0.1	-0.1
0.1	-0.1	-0.1	0.0	0.1	0.1	0.1
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	-0.1	0.1	0.0	0.0	0.1	0.1
0.1	0.1	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0

0.0	-0.1	0.1	0.0	0.1	-0.1	0.0
0.1	0.0	-0.1	0.0	0.0	0.1	-0.1
0.1	-0.1	-0.1	0.0	0.1	0.1	0.1
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	-0.1	0.1	0.0	0.0	0.1	0.1
0.1	0.1	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0

0.0	-0.1	0.1	0.0	0.1	-0.1	0.0
0.1	0.0	-0.1	0.0	0.0	0.1	-0.1
0.1	-0.1	-0.1	0.0	0.1	0.1	0.1
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	-0.1	0.1	0.0	0.0	0.1	0.1
0.1	0.1	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0



1.00	0.39	0.66	0.38
0.39	1.00	0.39	0.36
0.66	0.39	1.00	0.11
0.38	0.36	0.11	0.77

0.66	0.39	0.66	0.39
0.39	1.00	0.39	0.11
0.66	0.39	0.66	0.11
0.38	0.11	0.11	0.38

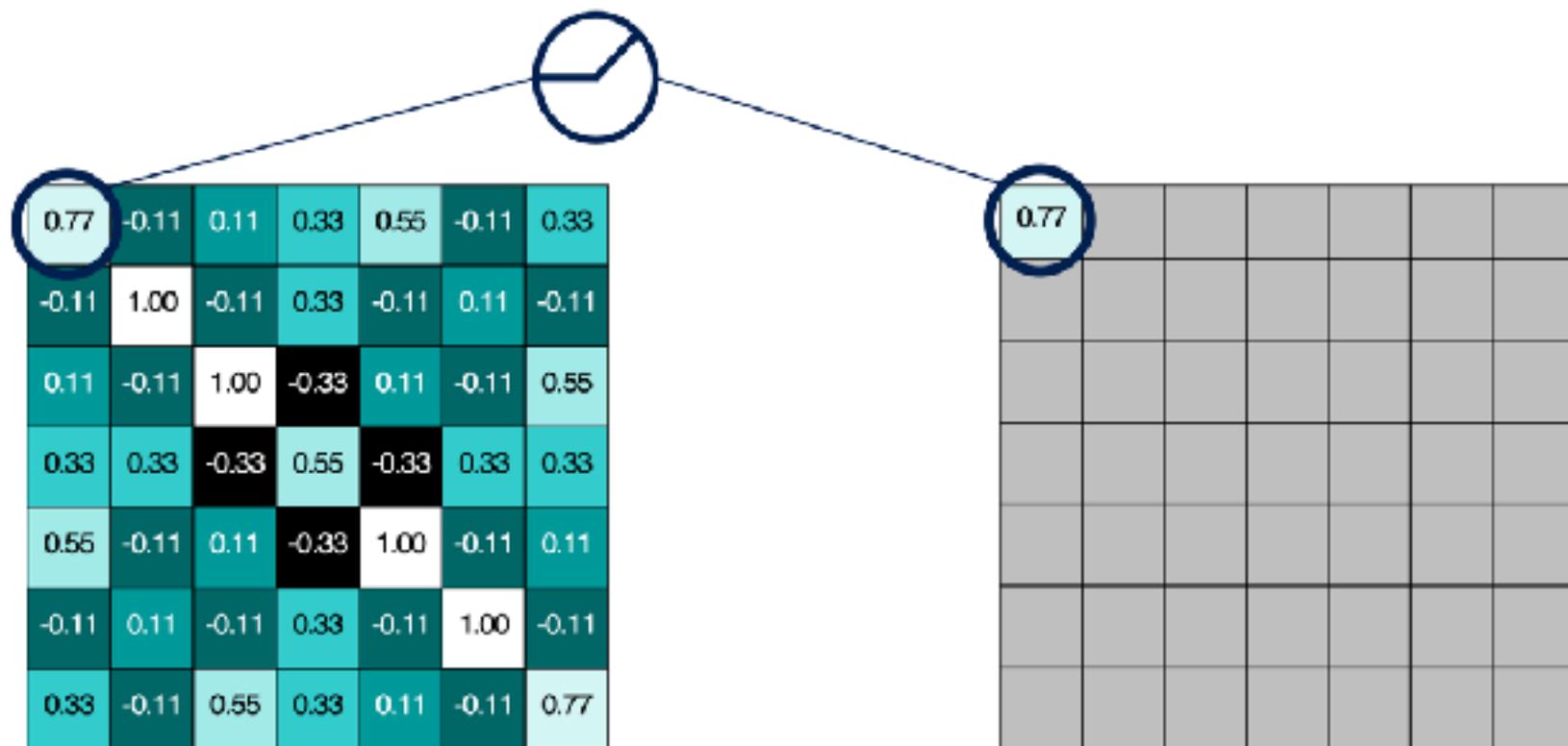
0.38	0.39	1.00	0.77
0.38	0.39	1.00	0.38
1.00	0.39	0.11	0.38
0.77	0.38	0.38	0.38

Normalization

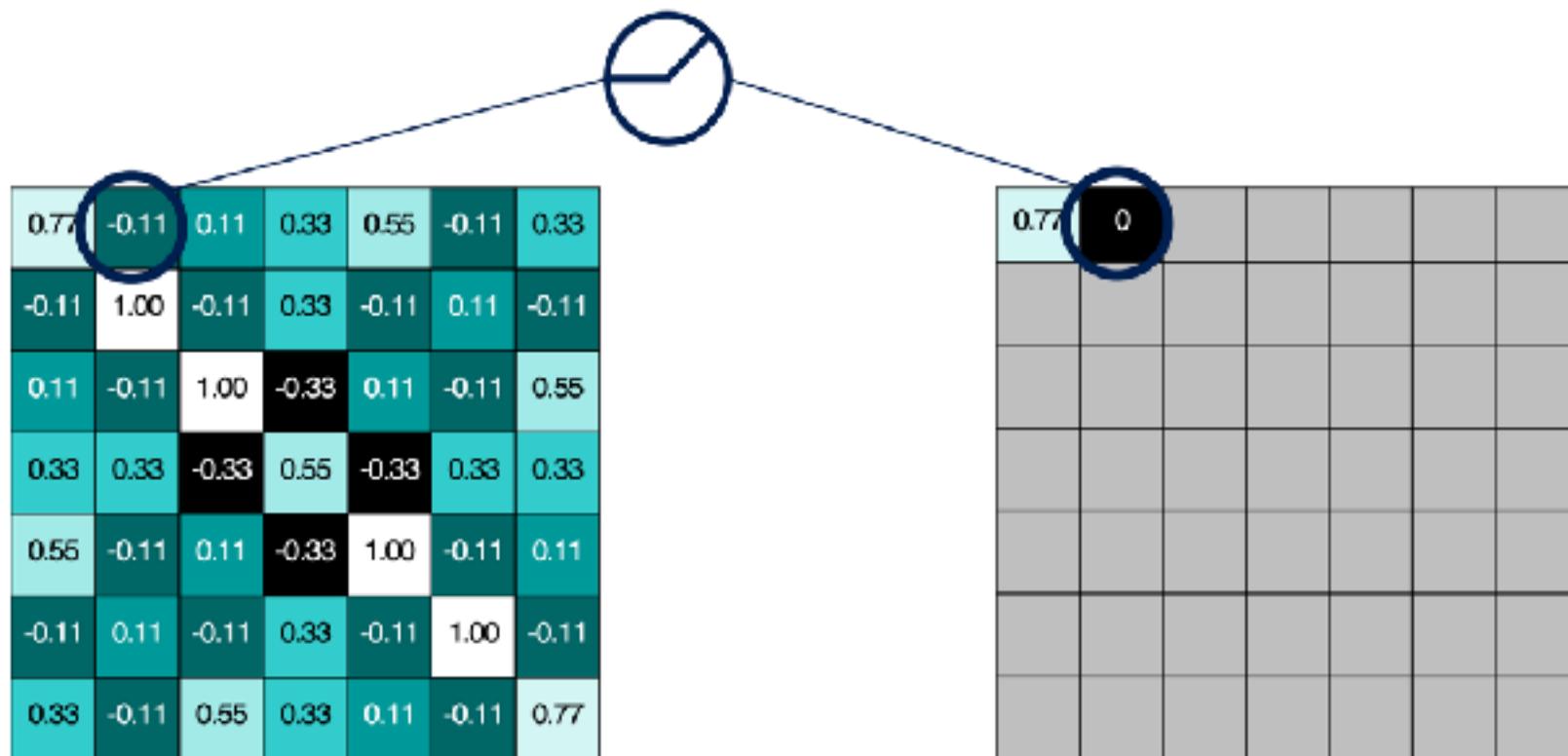
Keep the math from breaking by tweaking each of the values just a bit.

Change everything negative to zero.

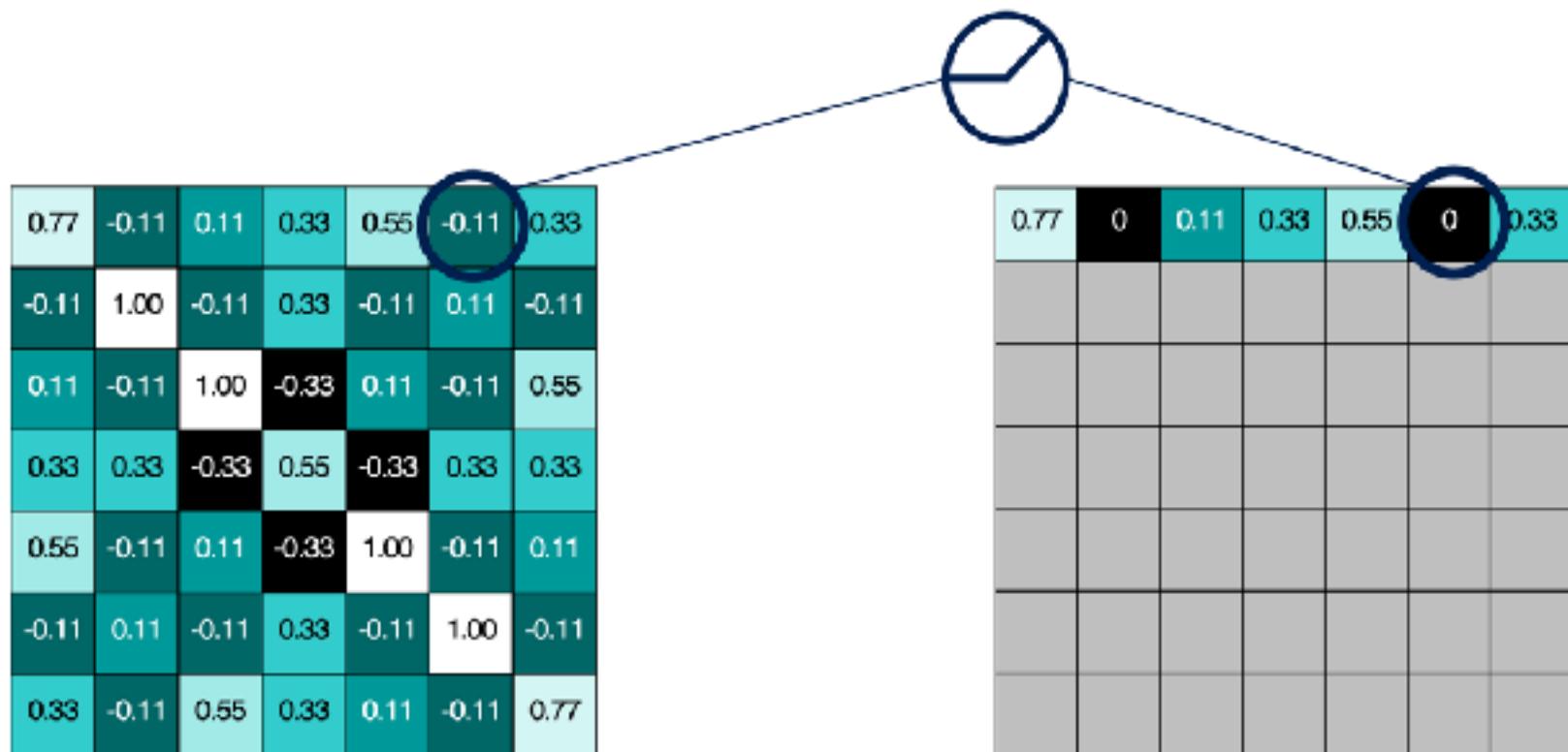
Rectified Linear Units (ReLUs)



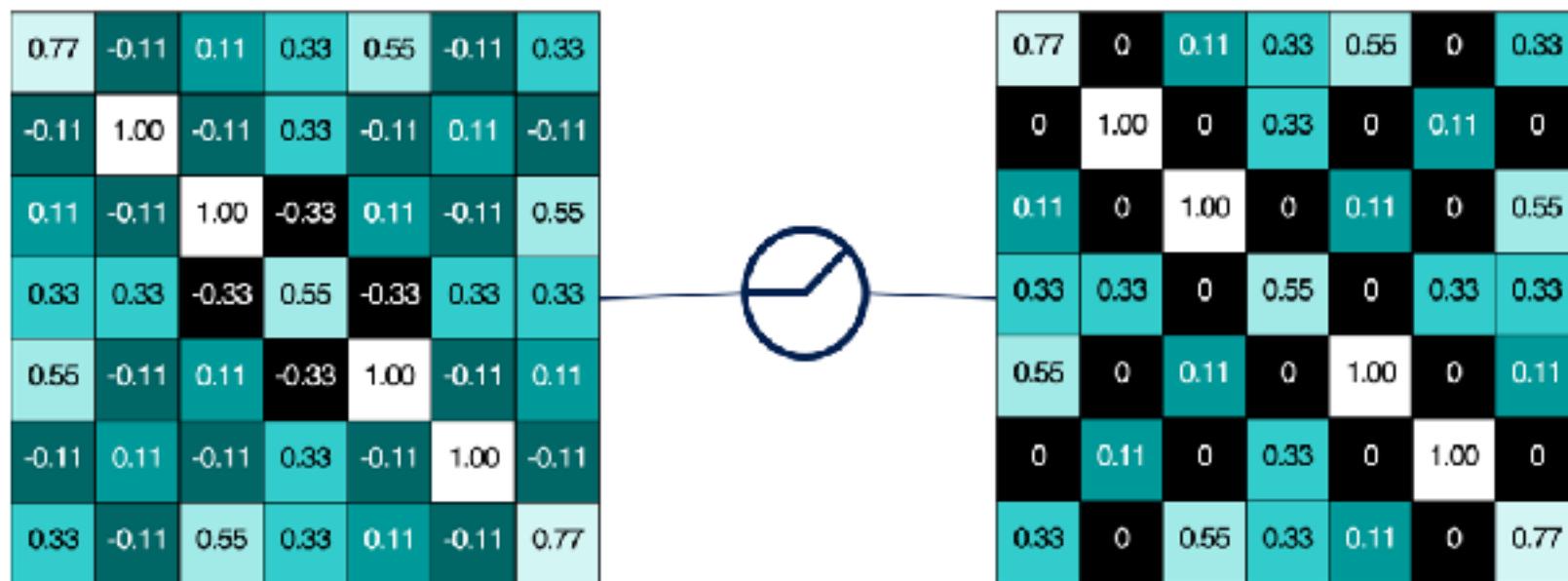
Rectified Linear Units (ReLUs)



Rectified Linear Units (ReLUs)



Rectified Linear Units (ReLUs)



ReLU layer

A stack of images becomes a stack of images with no negative values.

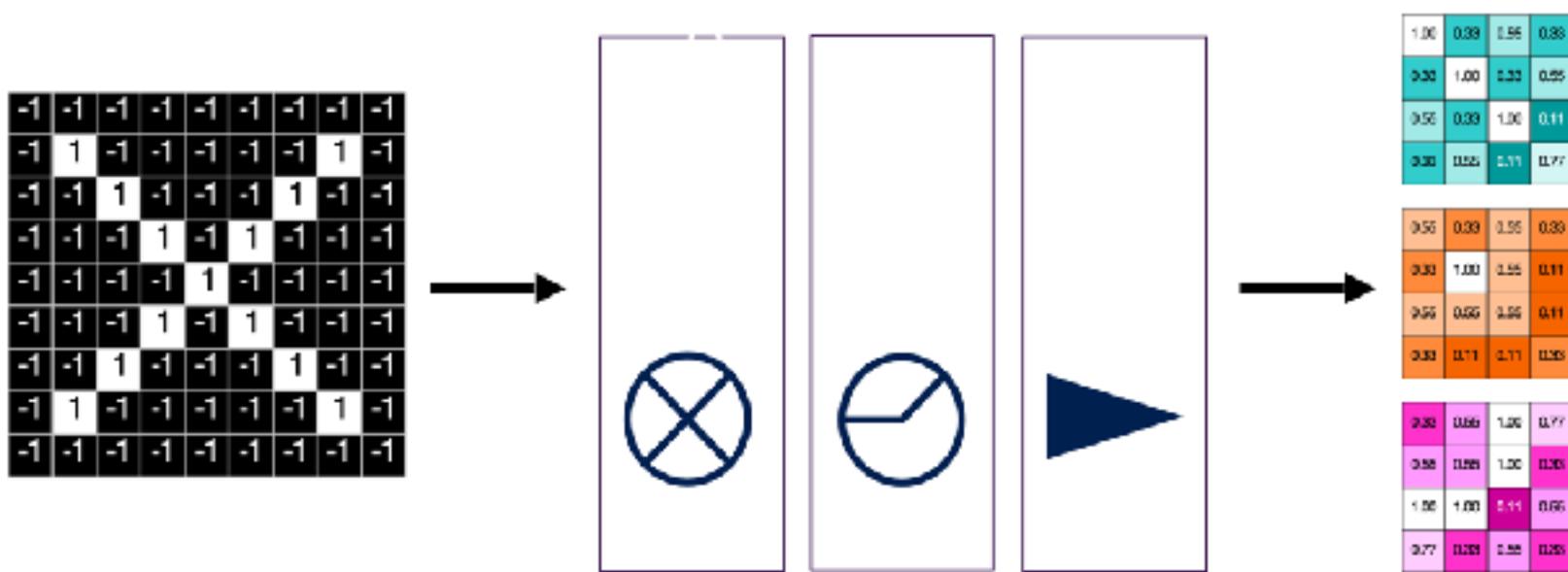
0.7	-0.1	0.1	0.3	0.5	-0.1	0.2
0.1	0.6	0.0	0.4	0.1	0.0	0.1
0.3	0.0	1.0	0.3	0.1	0.1	0.5
0.5	0.8	0.6	0.2	0.8	0.0	0.0
0.6	0.1	0.1	0.5	0.0	-0.1	0.1
0.1	0.1	0.1	0.0	0.1	1.0	0.1
0.2	0.0	0.5	0.0	0.1	-0.1	0.1



0.7	0	0.8	0.0	0.6	0	0.2
0.1	0	0.6	0.0	0.1	0	0.0
0.3	0.2	0	0.6	0	0.2	0.0
0.5	0	0.8	0	0.8	0	0.0
0.6	0.1	0.1	0	0.1	0	0.0
0.1	0	0.5	0	0.1	0	0.1
0.2	0	0.0	0	0.2	0	0.0

Layers get stacked

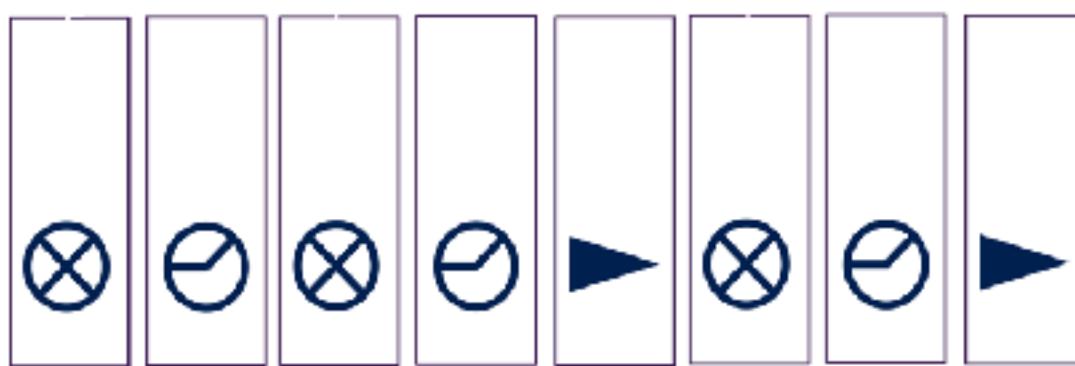
The output of one becomes the input of the next.



Deep stacking

Layers can be repeated several (or many) times.

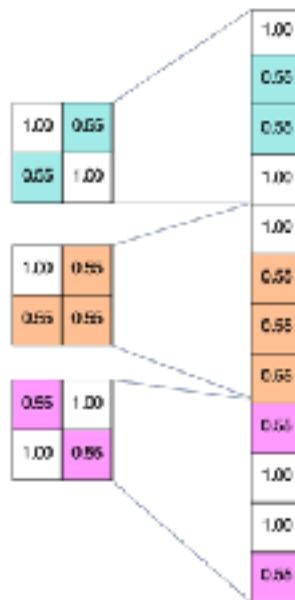
-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	-1	-1	-1	1	-1	-1
-1	-1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	1



1.00	0.85
0.85	1.00
1.00	0.85
0.85	0.85
0.85	1.00
1.00	0.85

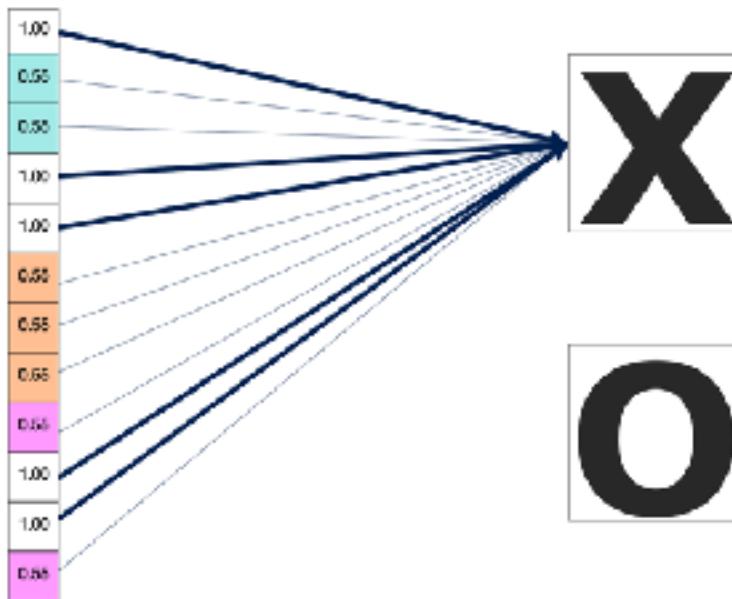
Fully connected layer

Every value gets a vote



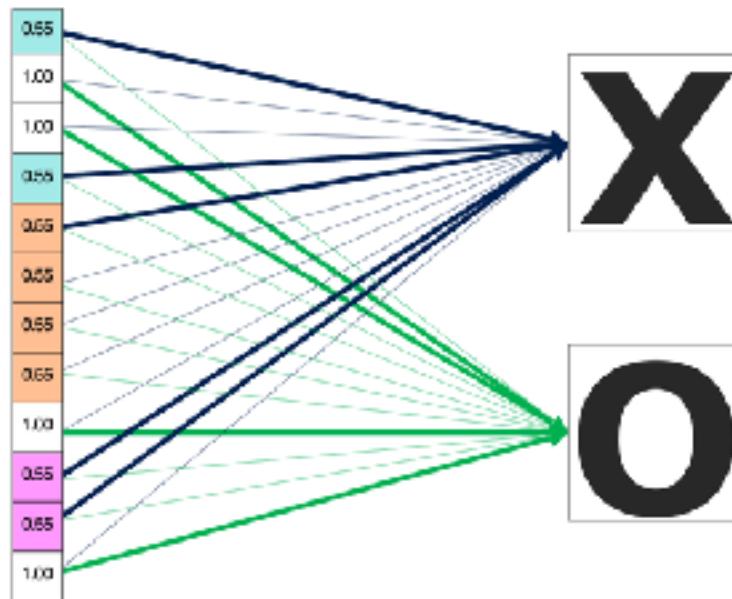
Fully connected layer

Vote depends on how strongly a value predicts X or O



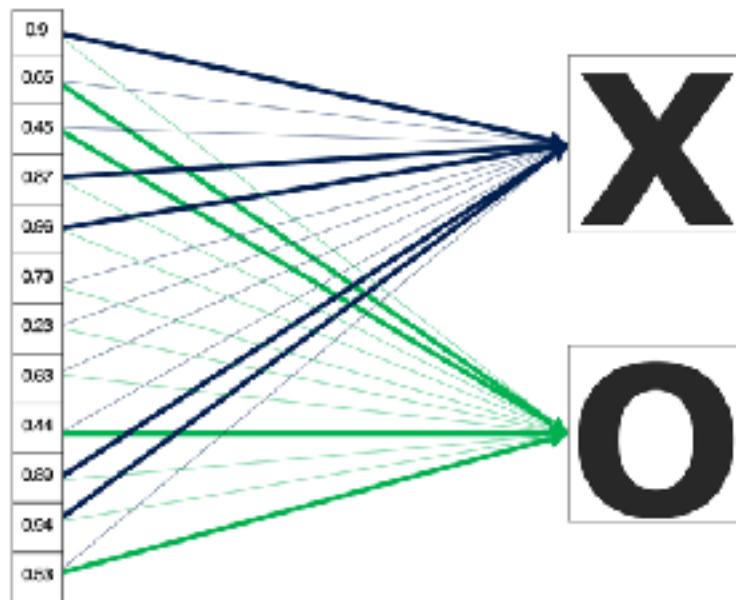
Fully connected layer

Vote depends on how strongly a value predicts X or O



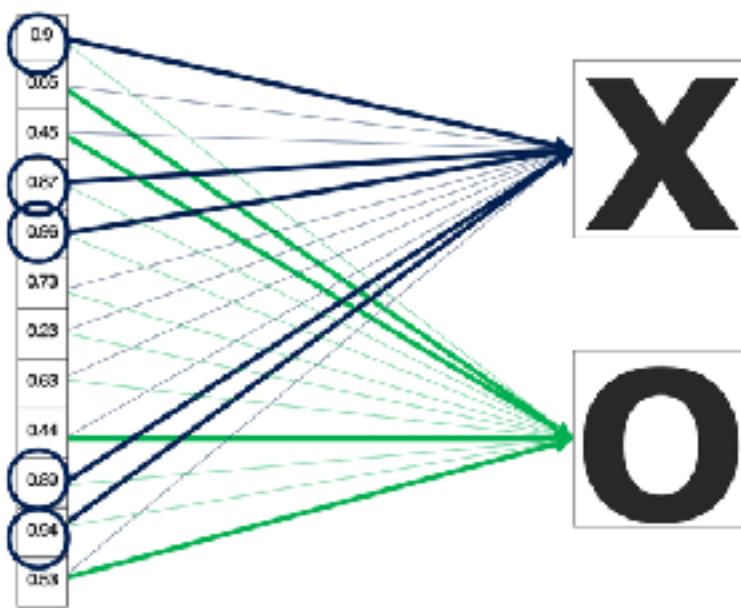
Fully connected layer

Future values vote on X or O



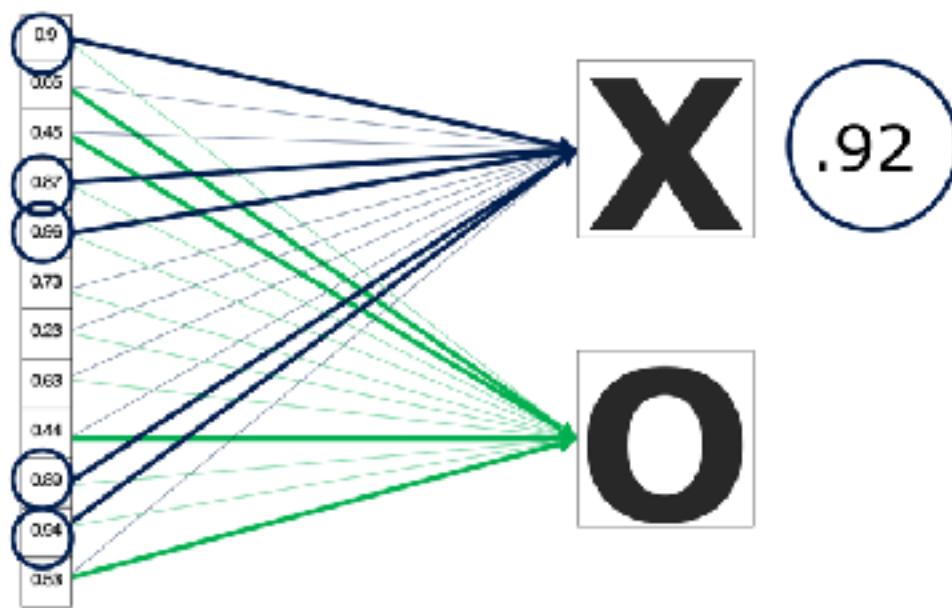
Fully connected layer

Future values vote on X or O



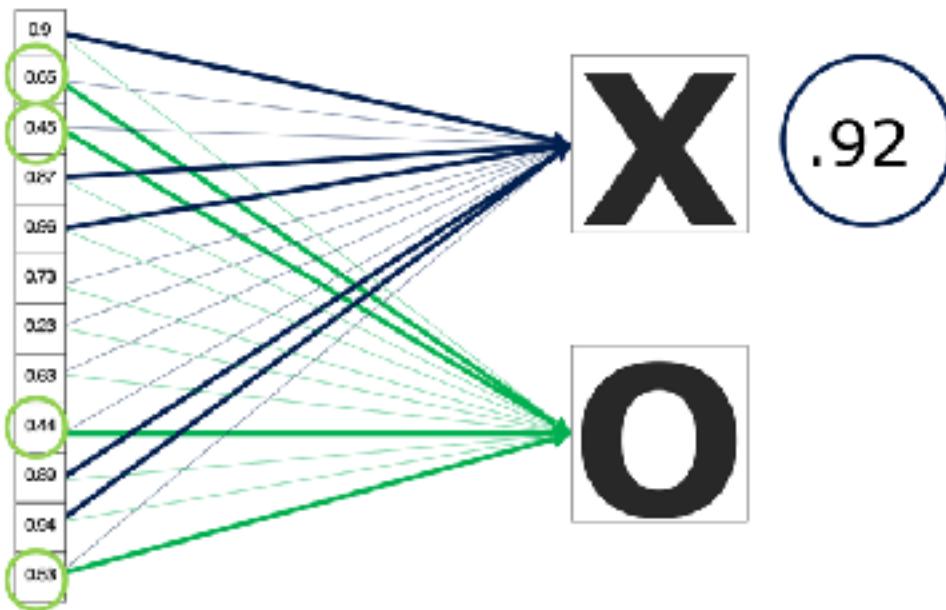
Fully connected layer

Future values vote on X or O



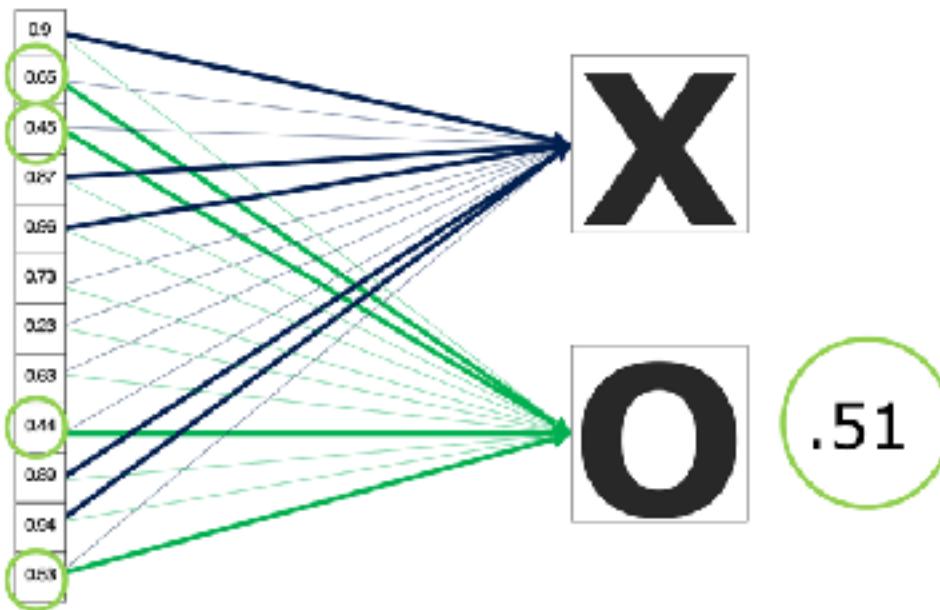
Fully connected layer

Future values vote on X or O



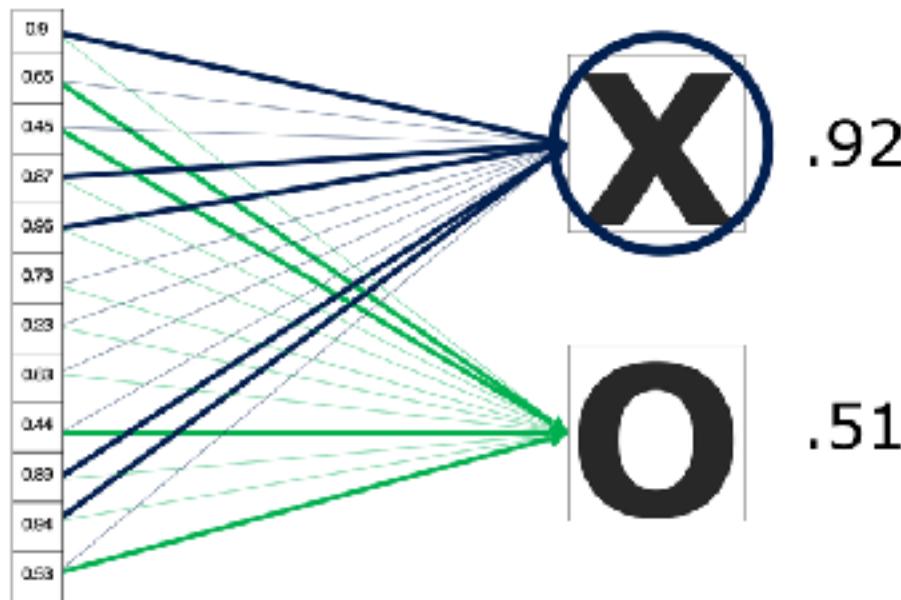
Fully connected layer

Future values vote on X or O



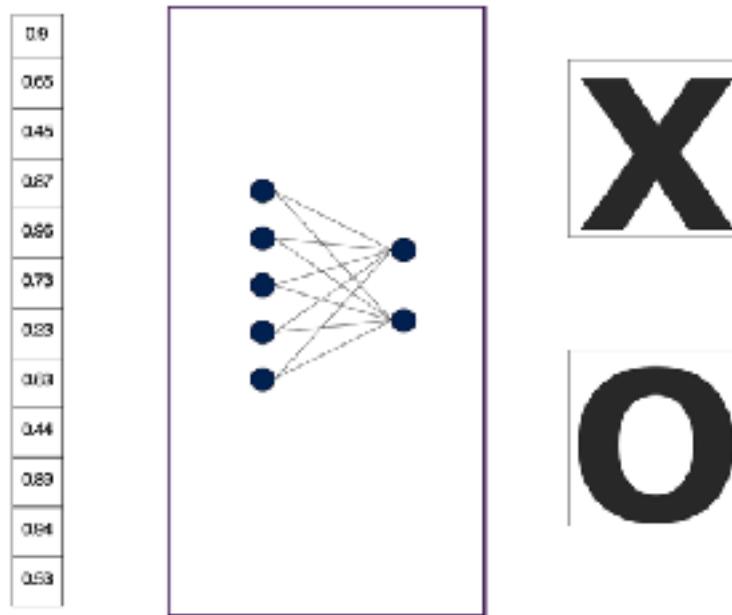
Fully connected layer

Future values vote on X or O



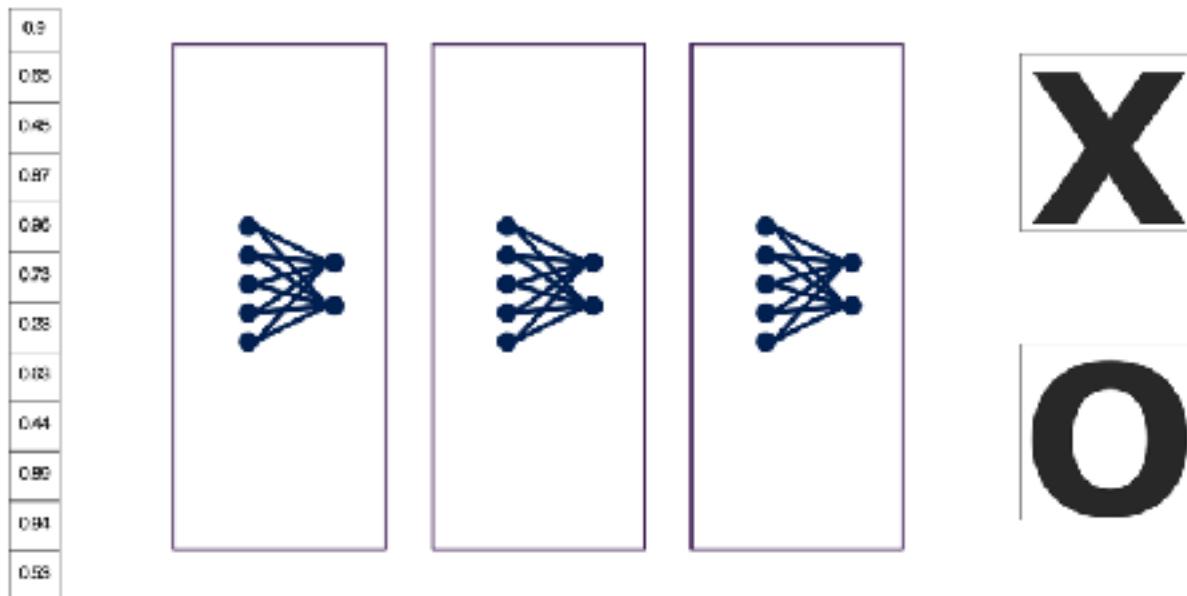
Fully connected layer

A list of feature values becomes a list of votes.



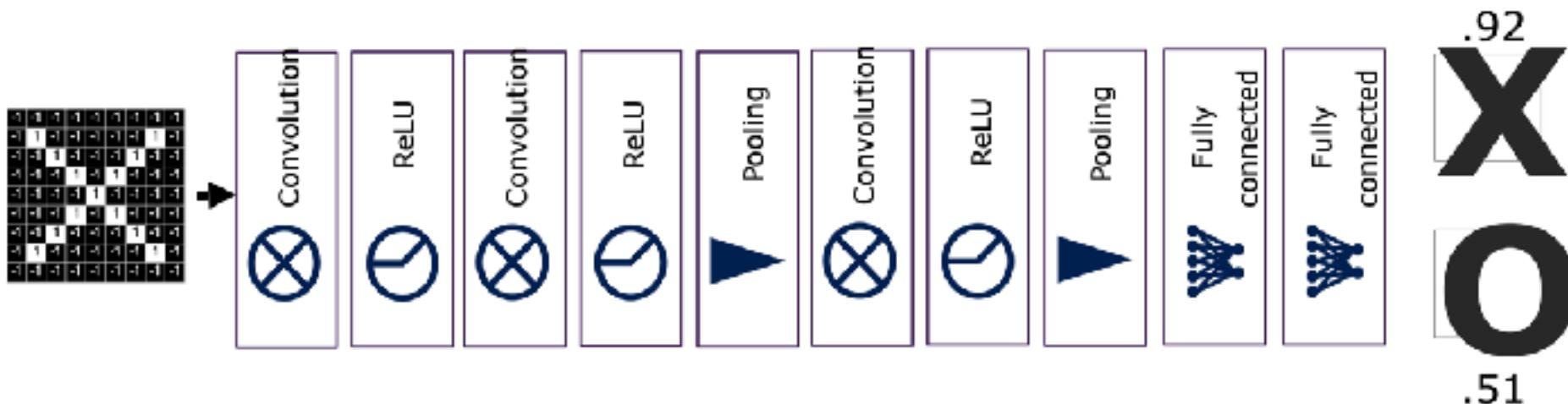
Fully connected layer

These can also be stacked.



Putting it all together

A set of pixels becomes a set of votes.



Learning

Q: Where do all the magic numbers come from?

Features in convolutional layers

Voting weights in fully connected layers

A: Backpropagation

Hyperparameters (knobs)

Convolution

- Number of features

- Size of features

Pooling

- Window size

- Window stride

Fully Connected

- Number of neurons

Architecture

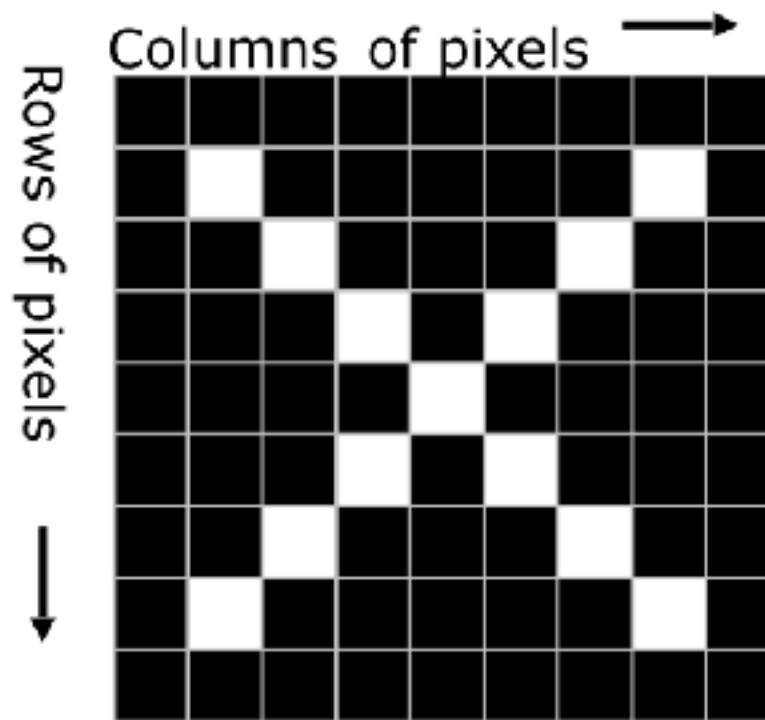
How many of each type of layer?
In what order?

Not just images

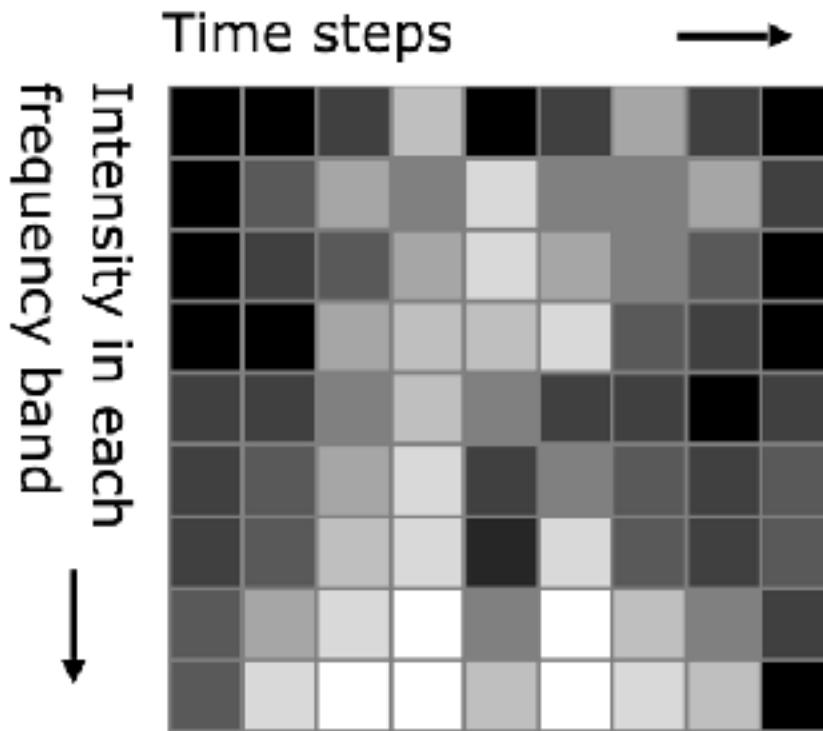
Any 2D (or 3D) data.

Things closer together are more closely related than things far away.

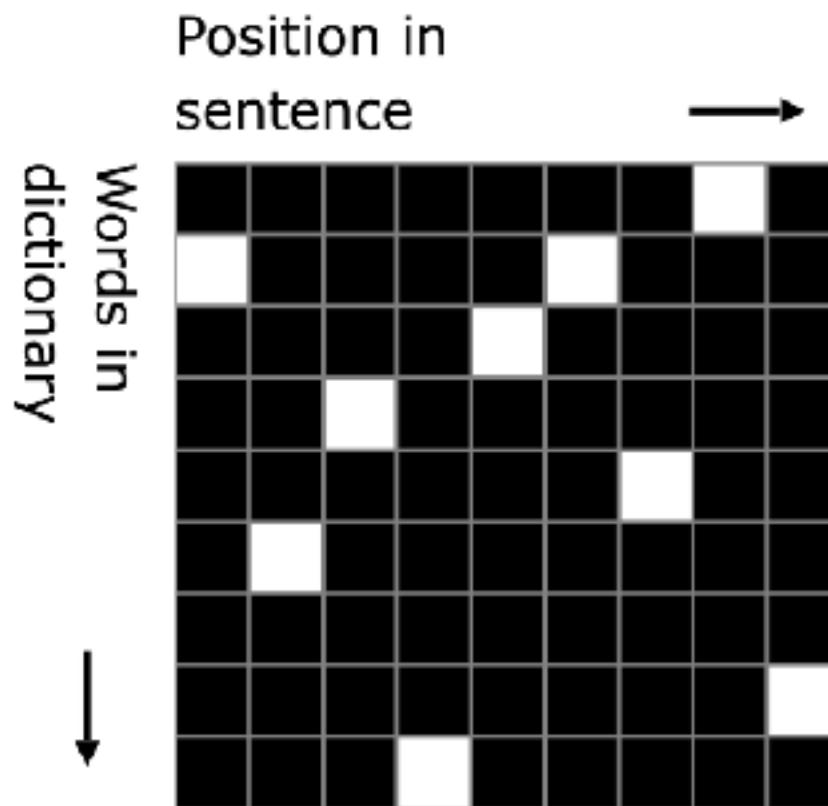
Images



Sound



Text



Limitations

ConvNets only capture local “spatial” patterns in data.
If the data can’t be made to look like an image, ConvNets
are less useful.

Customer data

Name, age,
address, email,
purchases, →
browsing activity,...

Customers



A	22	1A	a@a	1	aa	a1.a	123	aa1
B	33	2B	b@b	2	bb	b2.b	234	bb2
C	44	3C	c@c	3	cc	c3.c	345	cc3
D	55	4D	d@d	4	dd	d4.d	456	dd4
E	66	5E	e@e	5	ee	e5.e	567	ee5
F	77	6F	f@f	6	ff	f6.f	678	ff6
G	88	7G	g@g	7	gg	g7.g	789	gg7
H	99	8H	h@h	8	hh	h8.h	890	hh8
I	111	9I	i@i	9	ii	i9.i	901	ii9

Readings

- ▶ DL Book: Convolutional Networks
 - ▶ <https://www.deeplearningbook.org/contents/convnets.html>