

* register - similarity : $A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \underset{A}{\operatorname{argmin}} \sum_{i=1}^{68} \|Ax'_i - x_i\|^2$

$$\sum_{i=1}^{68} \left\| \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} x'_i \\ y'_i \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \end{bmatrix} \right\|^2 = \sum_{i=1}^{68} \left\| \begin{bmatrix} ax'_i - by'_i - x_i \\ bx'_i + ay'_i - y_i \end{bmatrix} \right\|^2$$

$$= \sum_{i=1}^{68} (ax'_i - by'_i - x_i)^2 + (bx'_i + ay'_i - y_i)^2 = M$$

$$\frac{\partial M}{\partial a} = \sum_{i=1}^{68} 2(ax'_i - by'_i - x_i)x'_i + 2(bx'_i + ay'_i - y_i)y'_i = 0$$

$$\Rightarrow \sum_{i=1}^{68} ax_i'^2 - \cancel{by'_i x'_i} - x_i x'_i + \cancel{bx'_i y'_i} + ay_i'^2 - y_i y'_i = 0$$

$$\Rightarrow a \sum_{i=1}^{68} x_i'^2 + y_i'^2 = \sum_{i=1}^{68} x_i x'_i + y_i y'_i \Rightarrow a = \frac{\sum_{i=1}^{68} x_i x'_i + y_i y'_i}{\sum_{i=1}^{68} x_i'^2 + y_i'^2}$$

$$\frac{\partial M}{\partial b} = \sum_{i=1}^{68} -2(ax'_i - by'_i - x_i)y'_i + 2(bx'_i + ay'_i - y_i)x'_i = 0$$

$$\Rightarrow \sum_{i=1}^{68} -\cancel{ax'_i y'_i} + by_i'^2 + x_i y'_i + \cancel{bx'_i y'_i} + ay'_i x'_i - x'_i y_i = 0$$

$$\Rightarrow b \sum_{i=1}^{68} x_i'^2 + y_i'^2 = \sum_{i=1}^{68} x'_i y_i - x_i y'_i \Rightarrow b = \frac{\sum_{i=1}^{68} x'_i y_i - x_i y'_i}{\sum_{i=1}^{68} x_i'^2 + y_i'^2}$$

* register - affine : $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \underset{A}{\operatorname{argmin}} \sum_{i=1}^{68} \|Ax'_i - x_i\|^2$

$$M = \sum_{i=1}^{68} \left\| \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x'_i \\ y'_i \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \end{bmatrix} \right\|^2 = \sum_{i=1}^{68} (ax'_i + by'_i - x_i)^2 + (cx'_i + dy'_i - y_i)^2$$

$$\frac{\partial M}{\partial a} = \sum_{i=1}^{68} 2(ax'_i + by'_i - x_i)x'_i = 0 \Rightarrow a = \frac{\sum_{i=1}^{68} (x_i - by'_i)x'_i}{\sum_{i=1}^{68} x_i'^2}$$

$$\frac{\partial M}{\partial b} = \sum_{i=1}^{68} 2(ax'_i + by'_i - x_i)y'_i = 0 \Rightarrow b = \frac{\sum_{i=1}^{68} (x_i - ax'_i)y'_i}{\sum_{i=1}^{68} y_i'^2}$$

$$\frac{\partial M}{\partial c} = \sum_{i=1}^{68} 2(c n_i' + d y_i' - y_i) n_i' = 0 \Rightarrow c = \frac{\sum_{i=1}^{68} (y_i - d y_i') n_i'}{\sum_{i=1}^{68} n_i'^2}$$

$$\frac{\partial M}{\partial d} = \sum_{i=1}^{68} 2((n_i' + d y_i' - y_i) y_i') = 0 \Rightarrow d = \frac{\sum_{i=1}^{68} (y_i - c n_i') y_i'}{\sum_{i=1}^{68} y_i'^2}$$

$$a = \frac{\sum n_i n_i'}{\sum n_i'^2} - b \left(\frac{\sum n_i' y_i'}{\sum n_i'^2} \right)^{m_2} \quad (1)$$

$$b = \frac{\sum n_i y_i'}{\sum y_i'^2} - a \frac{\sum n_i' y_i'}{\sum y_i'^2} \xRightarrow{(1)} b = \frac{\sum n_i y_i'}{\sum y_i'^2} - \left(\frac{\sum n_i n_i'}{\sum n_i'^2} - b \frac{\sum n_i' y_i'}{\sum n_i'^2} \right) \frac{\sum n_i' y_i'}{\sum y_i'^2}$$

$$\Rightarrow b = \frac{\sum n_i y_i'}{\sum y_i'^2} - \frac{\sum n_i n_i'}{\sum n_i'^2} \times \frac{\sum n_i' y_i'}{\sum y_i'^2} + b \times \frac{\sum n_i' y_i'}{\sum n_i'^2} \times \frac{\sum n_i' y_i'}{\sum y_i'^2}$$

$$\Rightarrow b = \frac{\frac{\sum n_i y_i'}{\sum y_i'^2} - \frac{\sum n_i n_i'}{\sum n_i'^2} \times \left(\frac{\sum n_i' y_i'}{\sum y_i'^2} \right)^{m_1}}{1 - \frac{\sum n_i' y_i'}{\sum n_i'^2} \times \frac{\sum n_i' y_i'}{\sum y_i'^2}} \rightarrow \text{divisor}$$

$$c = \frac{\sum y_i n_i'}{\sum n_i'^2} - d \left(\frac{\sum n_i' y_i'}{\sum n_i'^2} \right)^{m_2} \quad (2)$$

$$d = \frac{\sum y_i y_i'}{\sum y_i'^2} - c \frac{\sum n_i' y_i'}{\sum y_i'^2} \xRightarrow{(2)} d = \frac{\sum y_i y_i'}{\sum y_i'^2} - \left(\frac{\sum y_i n_i'}{\sum n_i'^2} - d \frac{\sum n_i' y_i'}{\sum n_i'^2} \right) \frac{\sum n_i' y_i'}{\sum y_i'^2}$$

$$= \frac{\sum y_i y_i'}{\sum y_i'^2} - \frac{\sum y_i n_i'}{\sum n_i'^2} \times \frac{\sum n_i' y_i'}{\sum y_i'^2} + d \frac{\sum n_i' y_i'}{\sum n_i'^2} \times \frac{\sum n_i' y_i'}{\sum y_i'^2}$$

$$\Rightarrow d = \frac{\frac{\sum y_i y_i'}{\sum y_i'^2} - \frac{\sum y_i n_i'}{\sum n_i'^2} \times \left(\frac{\sum n_i' y_i'}{\sum y_i'^2} \right)^{m_1}}{1 - \frac{\sum n_i' y_i'}{\sum n_i'^2} \times \left(\frac{\sum n_i' y_i'}{\sum y_i'^2} \right)^{m_1}} \rightarrow \text{divisor}$$

- 1) neutral - middle
- 2) neutral - turn left
- 3) neutral - turn right
- 4) laugh - eyes open - middle
- 5) laugh - eyes open - left
- 6) laugh - eyes open - right
- 7) laugh - eyes closed - middle
- 8) laugh - eyes closed - left
- 9) laugh - eyes closed - right
- 10) surprised - eyes normal - middle
- 11) surprised - eyes normal - left
- 12) surprised - eyes normal - right
- 13) surprised - eyes wide - middle
- 14) surprised - eyes wide - middle 2
- 15) wink - middle

- 16) sad - middle
- 17) wink - middle 2
- 18) surprised - eyebrow up - middle
- 19) surprised - eyebrow up - open mouth - middle
- 20) 19 - right
- 21) 19 - left
- 22) sad - eyes closed - middle
- 23) 22 - 2
- 24) face up
- 25) angry long face - middle
- 26) 25 - right
- 27) 25 - left
- 28) creepy laugh middle
- 29) 28 - right
- 30) 28 - left
- ~~31) 28 - middle~~