Northeastern University

Lecture 30: Trees

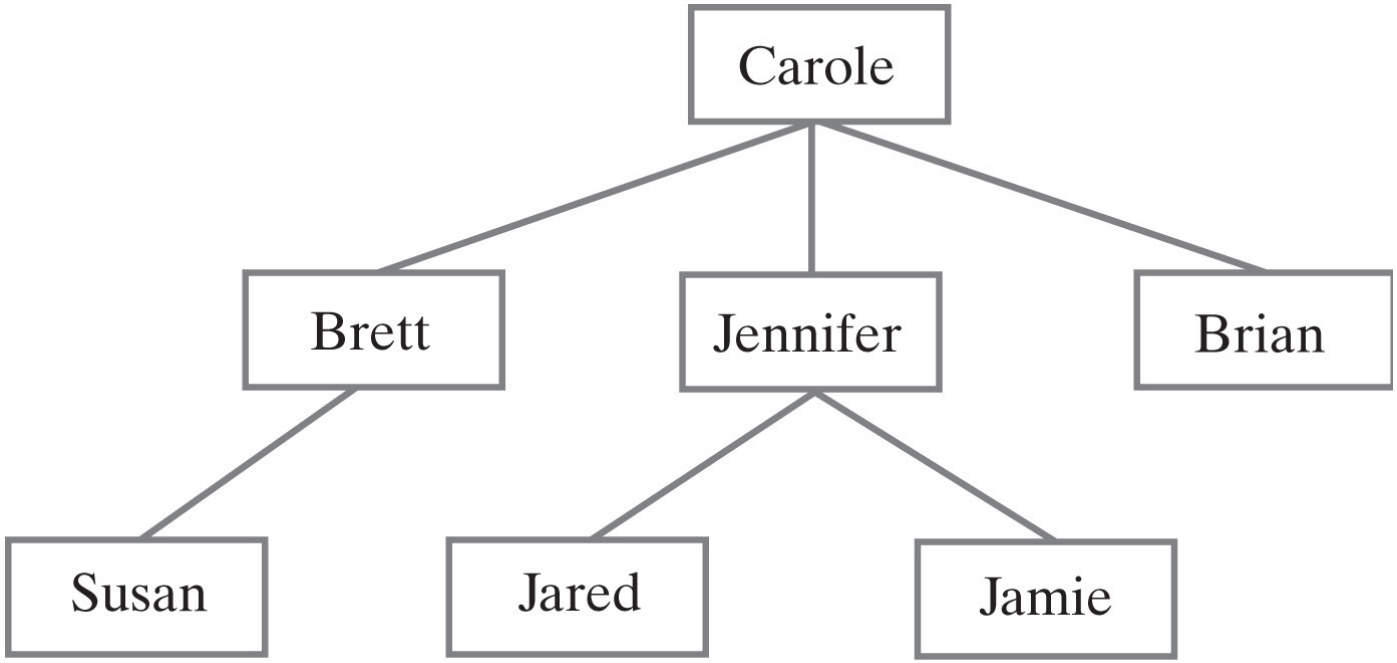
Prof. Chen-Hsiang (Jones) Yu, Ph.D.

College of Engineering

Materials are edited by Prof. Jones Yu from

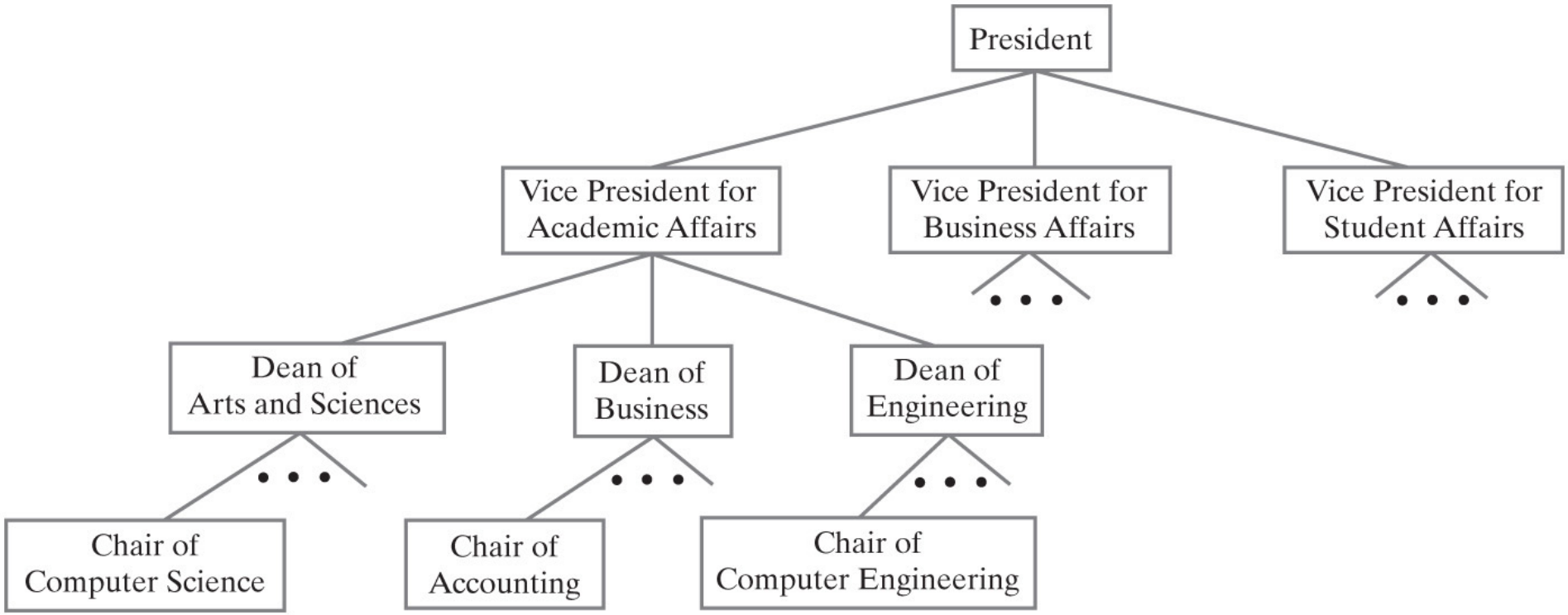
Data Structures and Abstractions with Java, 5th edition. By Frank M. Carrano and Timothy M. Henry.
ISBN-13 978-0-13-483169-5

Hierarchical Organizations



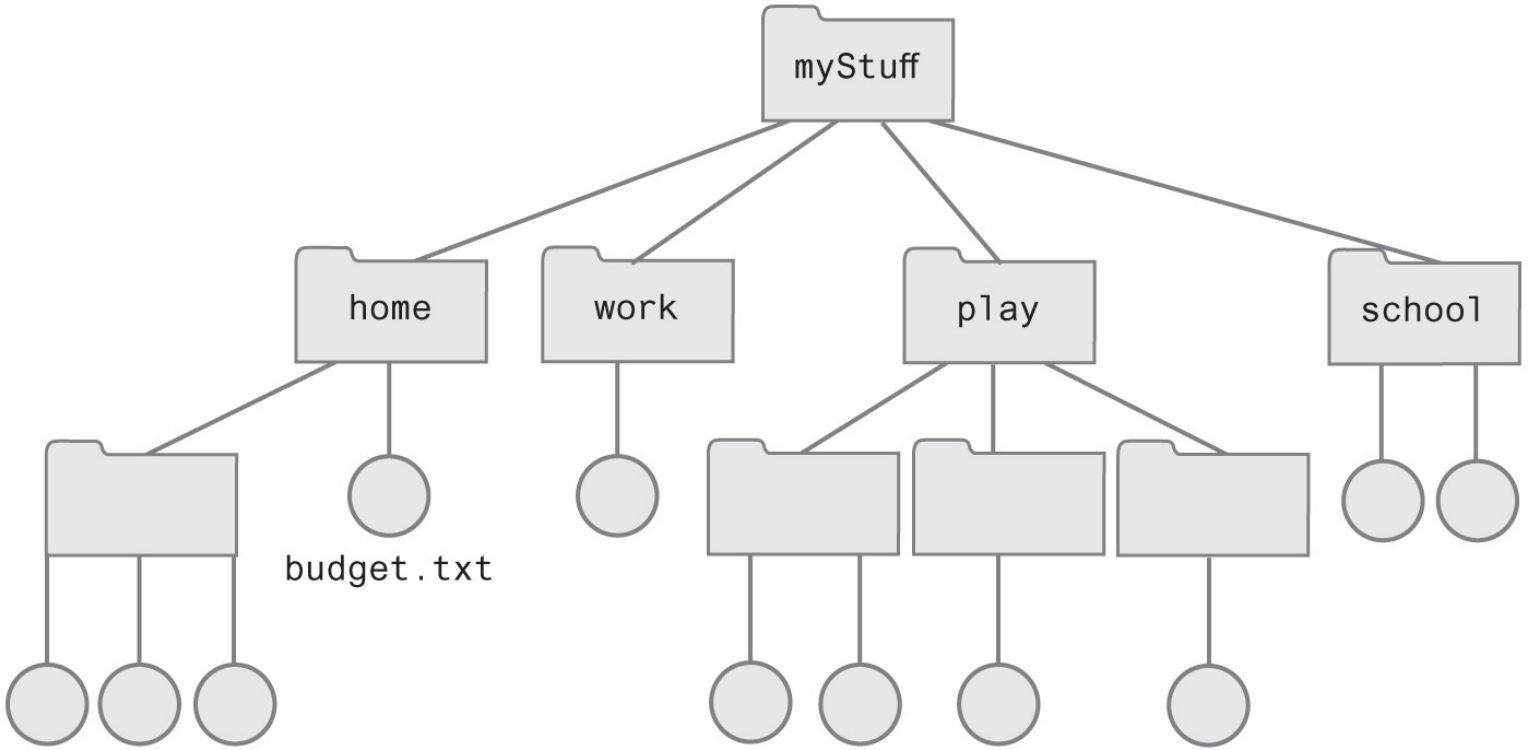
Carole's children and grandchildren

Hierarchical Organizations (cont.)



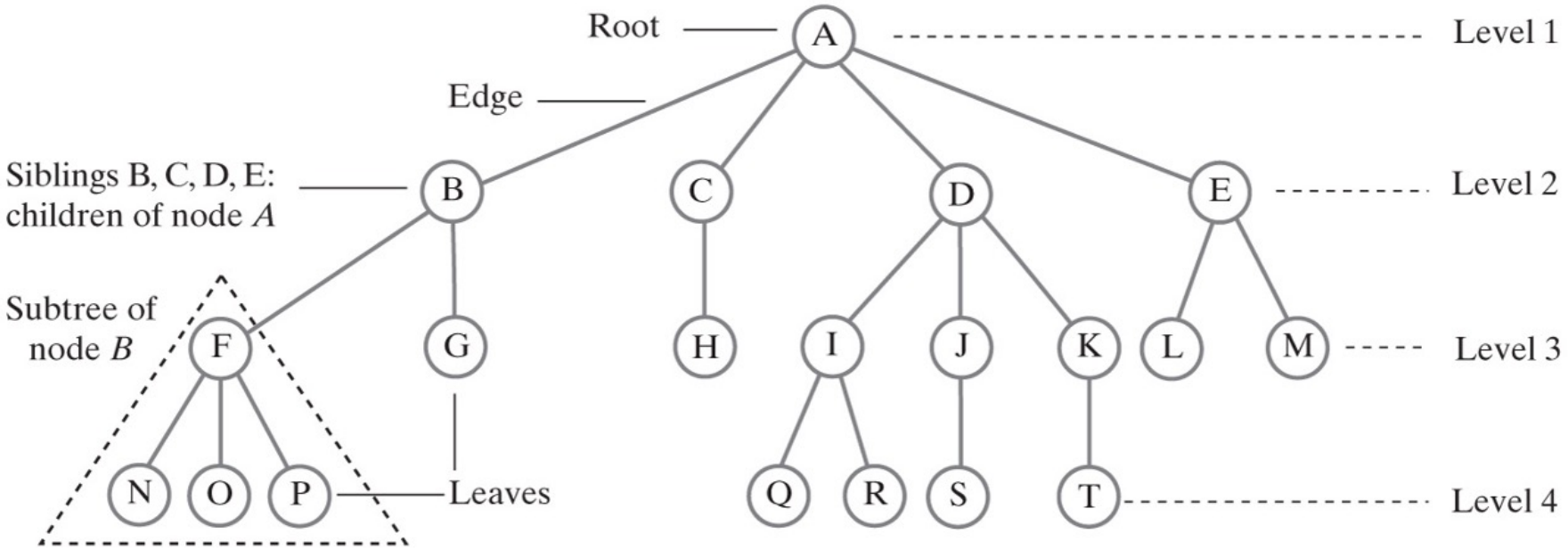
A portion of a university's administrative structure

Hierarchical Organizations (cont.)



Computer files organized into folders

Tree Terminology

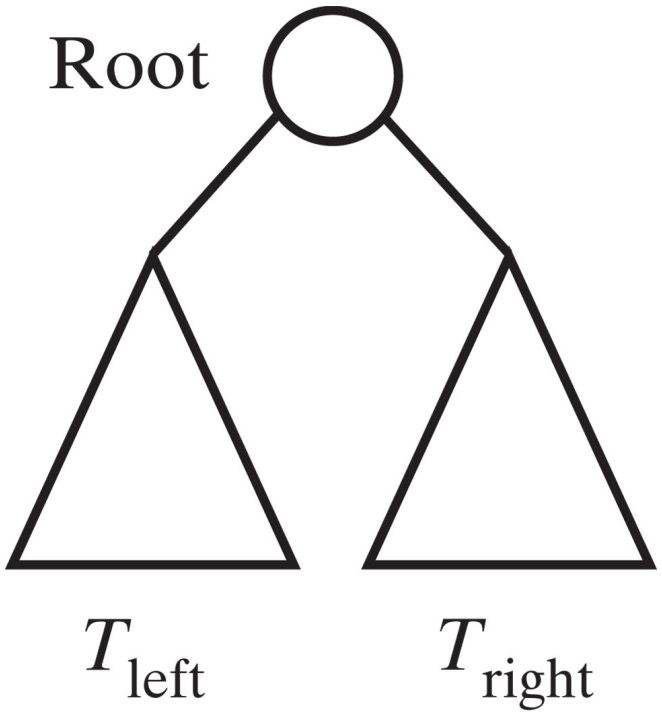


A tree equivalent to the tree in previous slide

Tree Terminology (cont.)

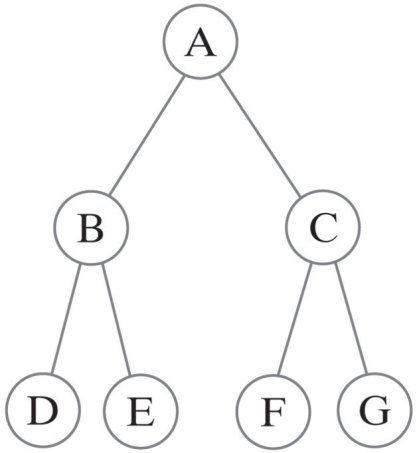
- Contrast plants with root at bottom
  - ADT tree with root at top
  - Root is only node with no parent
- A tree can be empty.
- Any node and its descendants form a subtree of the original tree.
- The height of a tree is the number of levels in the tree.

Binary Trees



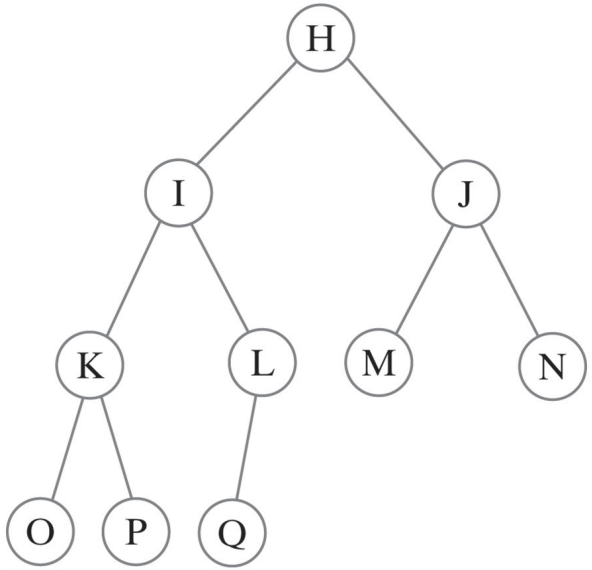
Binary Trees (cont.)

(a) Full tree

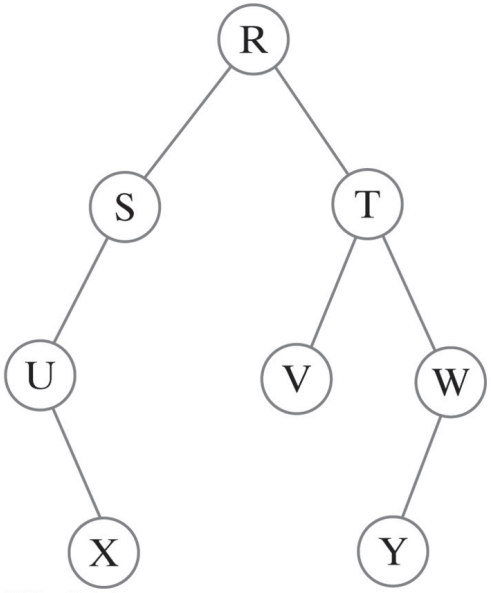


Left children: B, D, F
Right children: C, E, G

(b) Complete tree

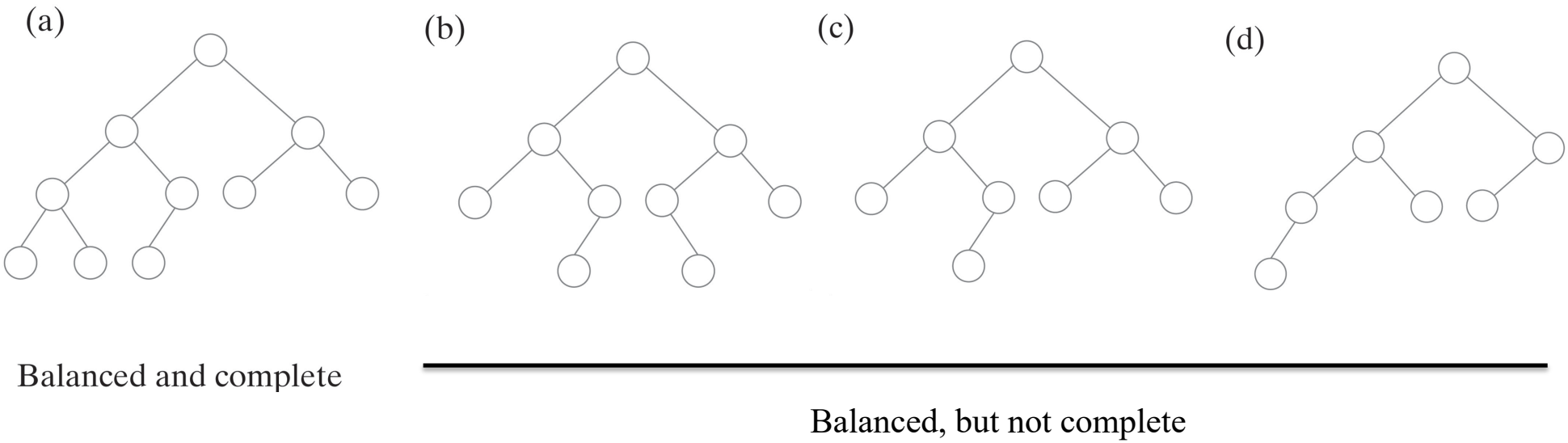


(c) Tree that is not full and not complete



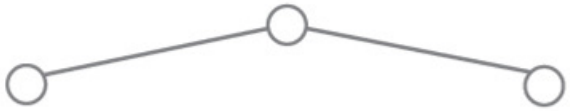
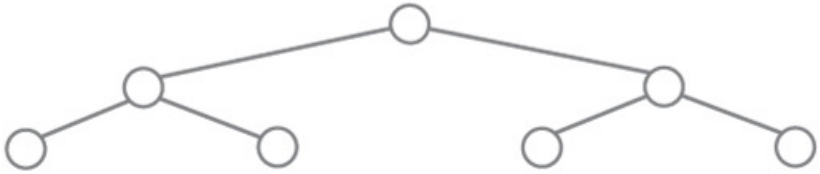
Three Binary Trees

Binary Trees (cont.)



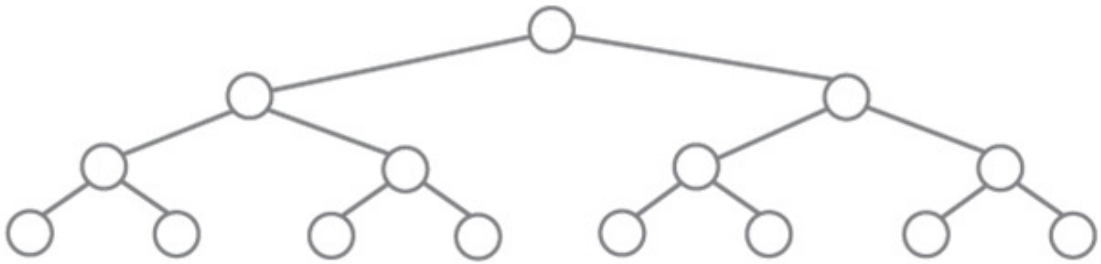
Some binary trees that are height balanced

Binary Trees Height (Part 1)

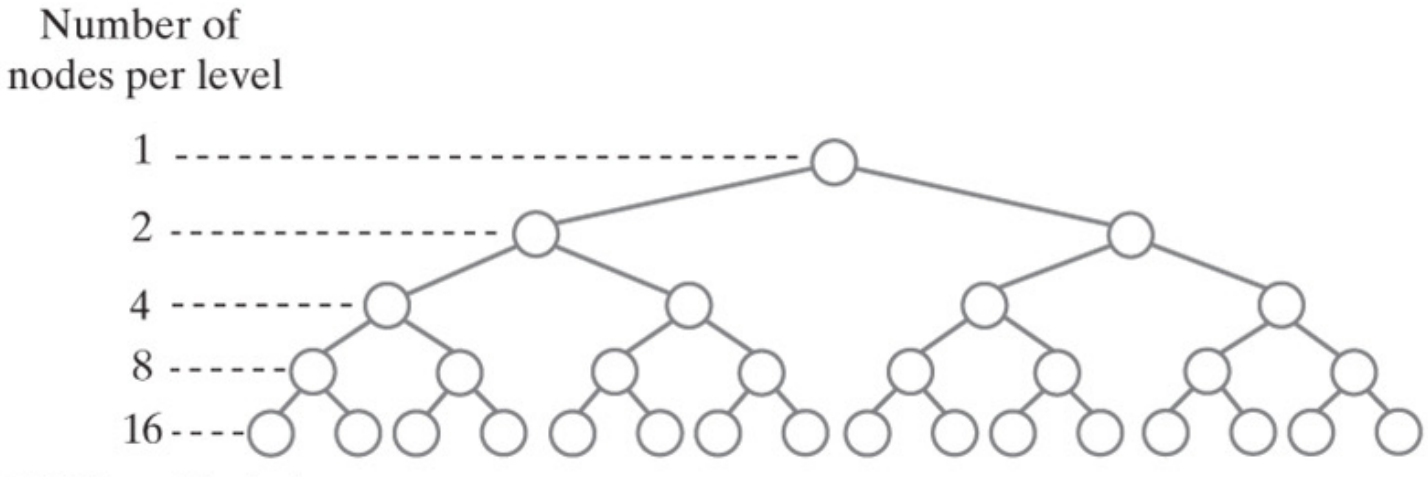
Full Tree	Height	Number of Nodes
	1	$1 = 2^1 - 1$
	2	$3 = 2^2 - 1$
	3	$7 = 2^3 - 1$

The number of nodes in a full binary tree as a function of the tree's height

Binary Trees Height (Part 2)



4 $15 = 2^4 - 1$



5 $31 = 2^5 - 1$

The number of nodes in a full binary tree as a function of the tree's height

Traversals of A Tree

- Traversal:
  - Visit, or process, each data item exactly once
- We will say that traversal can pass through a node without visiting it at that moment.
- Order in which we visit items is not unique.
- Traversals of a binary tree are somewhat easy to understand.

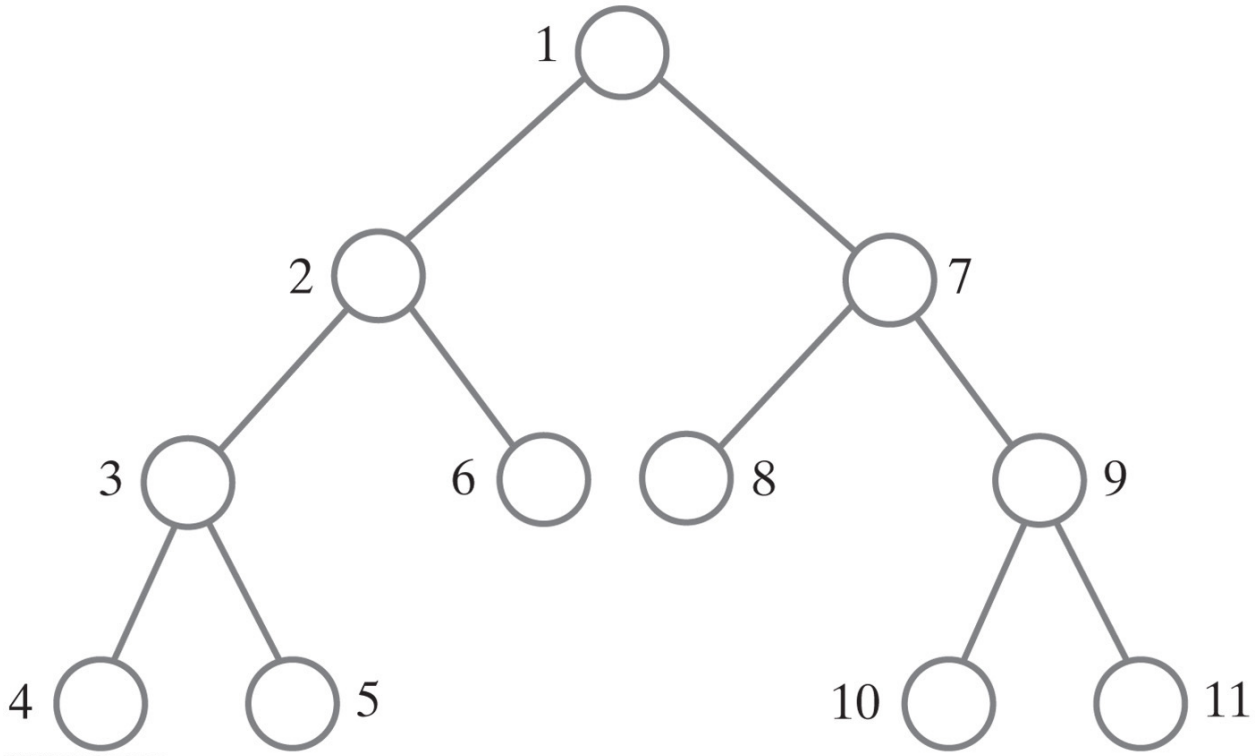
Traversals of a Binary Tree

- We use recursion.
- To visit all the nodes in a binary tree, we must
  - Visit the root
  - Visit all the nodes in the root's left subtree
  - Visit all the nodes in the root's right subtree

Traversals of a Binary Tree (cont.)

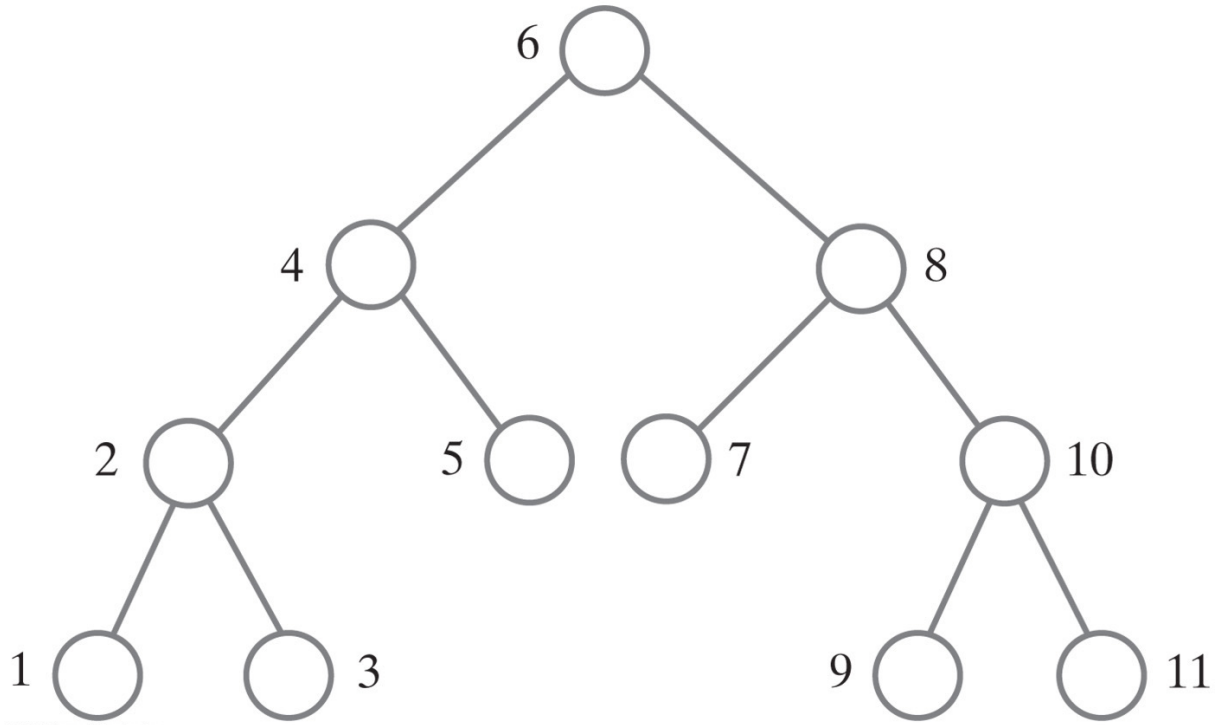
- Preorder traversal
  - Visit root before we visit root's subtrees
- Inorder traversal
  - Visit root of a binary tree between visiting nodes in root's subtrees.
- Postorder traversal
  - Visit root of a binary tree after visiting nodes in root's subtrees
- Level-order traversal
  - Begin at root and visit nodes one level at a time

Traversals of a Binary Tree (cont.)



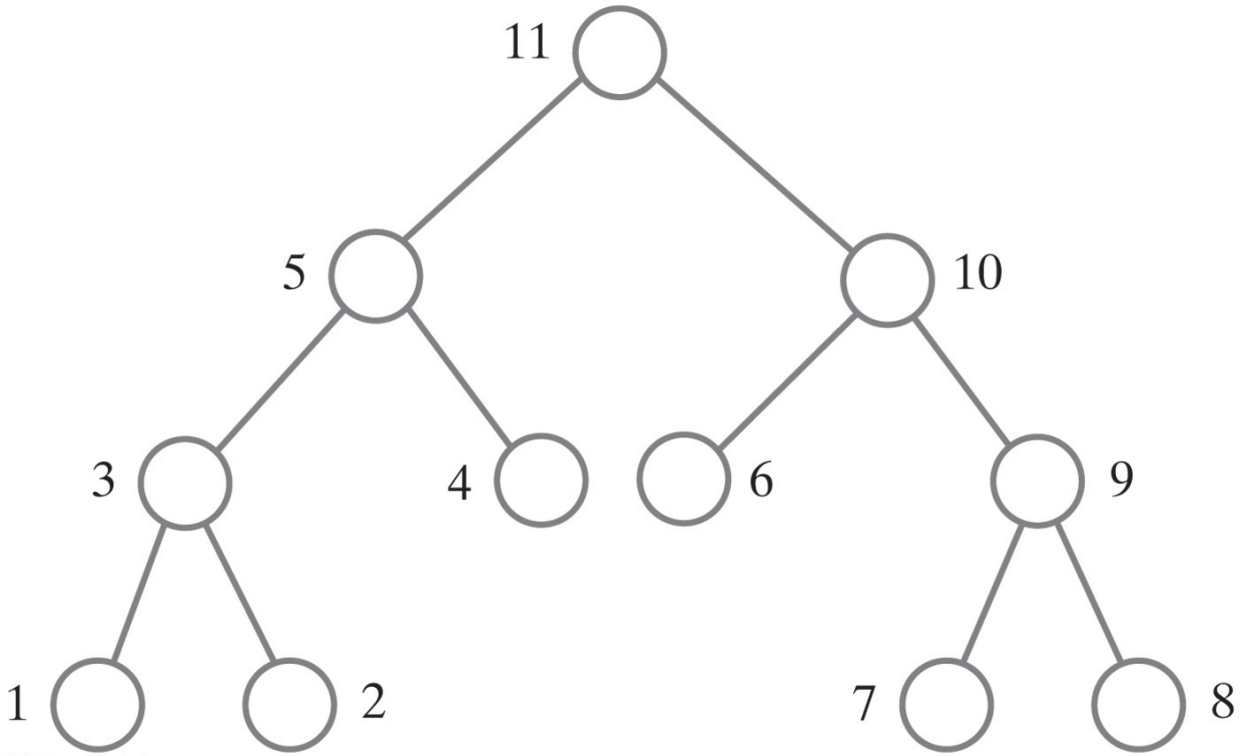
The visitation order of a preorder traversal

Traversals of a Binary Tree (cont.)



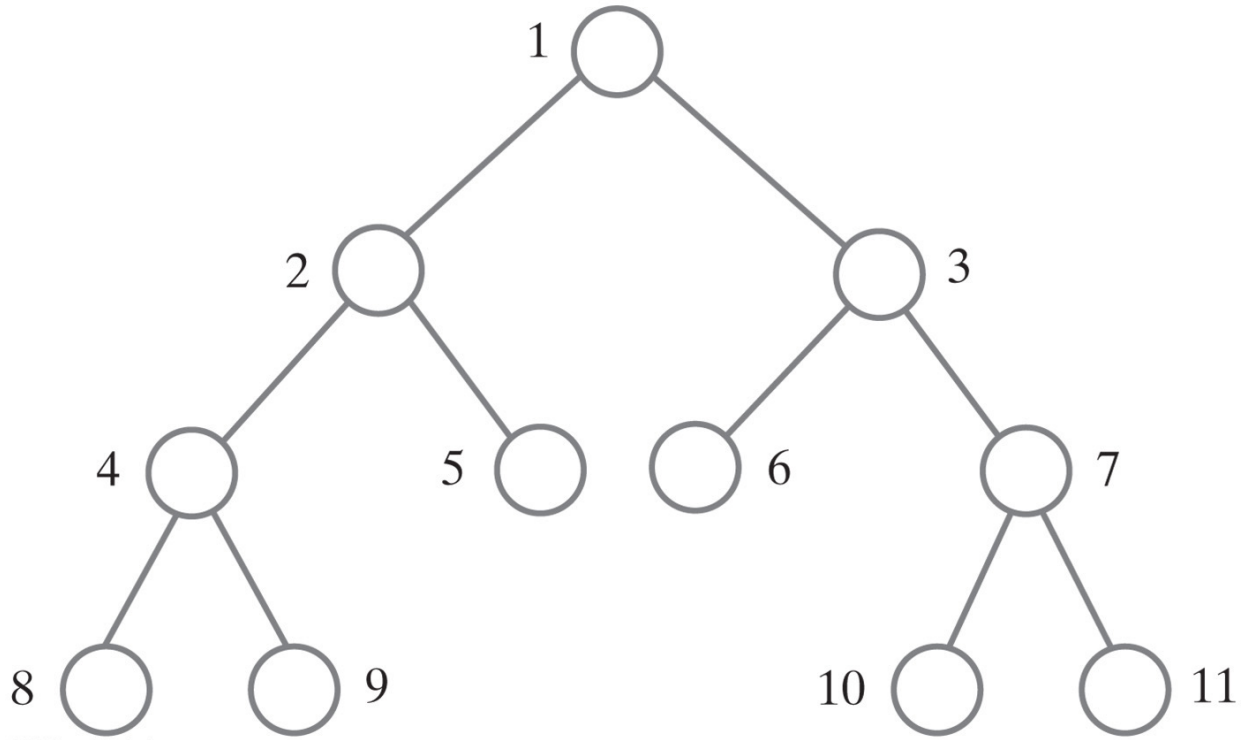
The visitation order of a inorder traversal

Traversals of a Binary Tree (cont.)



The visitation order of a postorder traversal

Traversals of a Binary Tree (cont.)

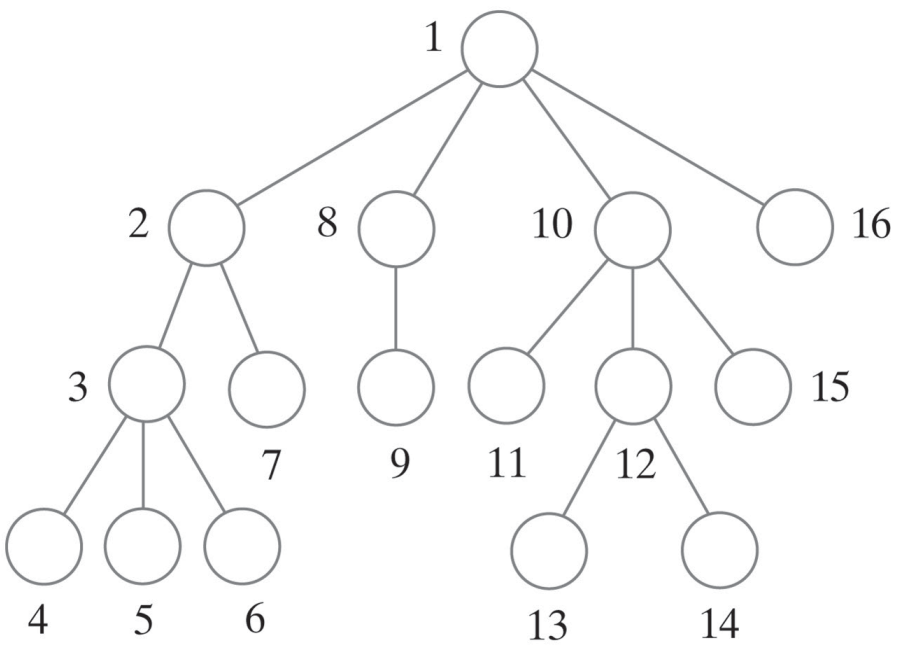


The visitation order of a level-order traversal

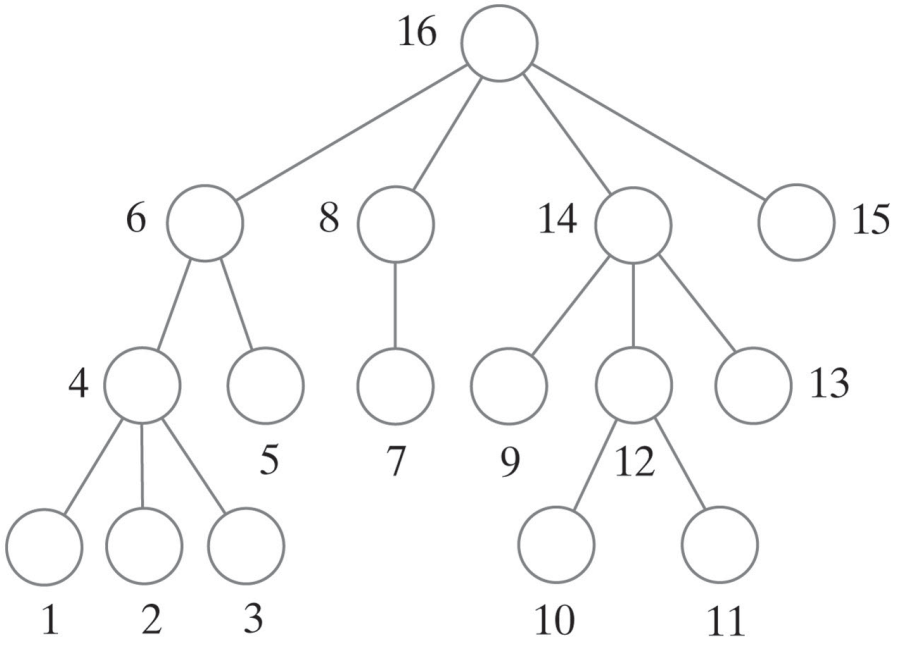
Traversals of a General Tree

- Types of traversals for general tree
  - Level order
  - Preorder
  - Postorder
- Not suited for general tree traversal
  - Inorder

Traversals of a General Tree (cont.)



(a) Preorder traversal



(b) Postorder traversal

The visitation order of two traversals of a general tree

Interfaces for All Trees

```
package TreePackage;
/** An interface of basic methods for the ADT tree. */

public interface TreeInterface<T>
{
    public T getRootData();
    public int getHeight();
    public int getNumberOfNodes();
    public boolean isEmpty();
    public void clear();
} // end TreeInterface
```

An interface of methods common to all trees

Traversals

```
package TreePackage;

import java.util.Iterator;
/** An interface of iterators for the ADT tree. */

public interface TreeIteratorInterface<T>
{
   public Iterator<T> getPreorderIterator();
   public Iterator<T> getPostorderIterator();
   public Iterator<T> getInorderIterator();
   public Iterator<T> getLevelOrderIterator();
} // end TreeIteratorInterface
```

An interface of traversal methods for a tree

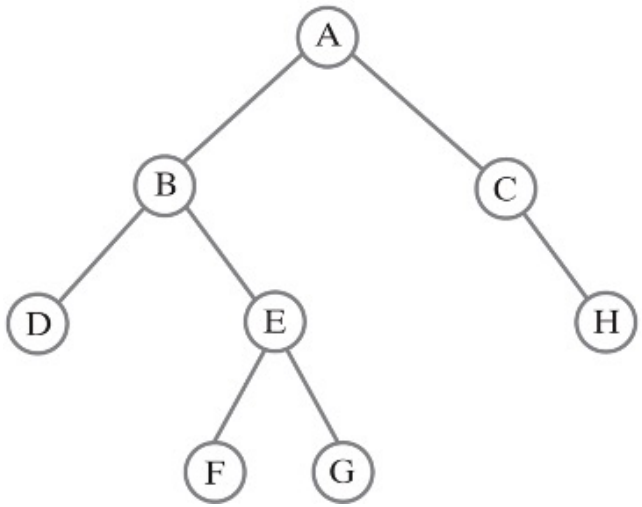
Interface for Binary Trees

```
package TreePackage;
/*   An interface for the ADT binary tree. */

public interface BinaryTreeInterface<T> extends TreeInterface<T>,
                                                TreeIteratorInterface<T>
{
   /** Sets the data in the root of this binary tree.
       @param rootData  The object that is the data for the tree's root.
   */
   public void setRootData(T rootData);

   /** Sets this binary tree to a new binary tree.
      @param rootData   The object that is the data for the new tree's root.
      @param leftTree   The left subtree of the new tree.
      @param rightTree  The right subtree of the new tree. */
   public void setTree(T rootData, BinaryTreeInterface<T> leftTree,
                                   BinaryTreeInterface<T> rightTree);
} // end BinaryTreeInterface
```

An interface for a binary tree

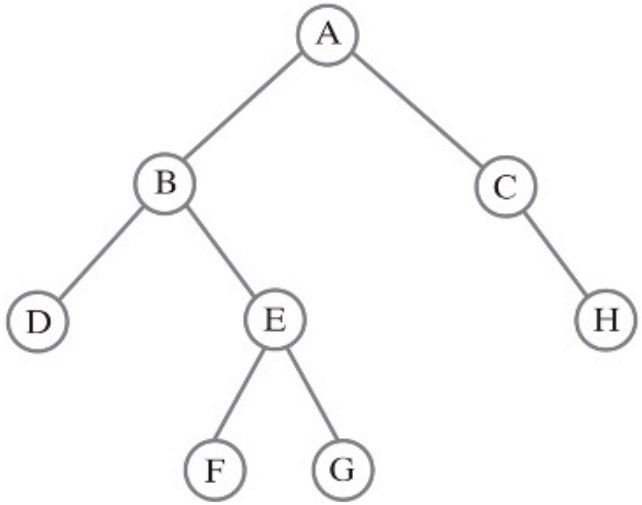
Building a Binary Tree

```
BinaryTreeInterface<String> dTree = new BinaryTree<>();
dTree.setTree("D", null, null);

BinaryTreeInterface<String> fTree = new BinaryTree<>();
fTree.setTree("F", null, null);

BinaryTreeInterface<String> gTree = new BinaryTree<>();
gTree.setTree("G", null, null);

BinaryTreeInterface<String> hTree = new BinaryTree<>();
hTree.setTree("H", null, null);

BinaryTreeInterface<String> emptyTree = new BinaryTree<>();

// Form larger subtrees
BinaryTreeInterface<String> eTree = new BinaryTree<>();
eTree.setTree("E", fTree, gTree); // Subtree rooted at E

BinaryTreeInterface<String> bTree = new BinaryTree<>();
bTree.setTree("B", dTree, eTree); // Subtree rooted at B

BinaryTreeInterface<String> cTree = new BinaryTree<>();
cTree.setTree("C", emptyTree, hTree); // Subtree rooted at C

BinaryTreeInterface<String> aTree = new BinaryTree<>();
aTree.setTree("A", bTree, cTree); // Desired tree rooted at A
```

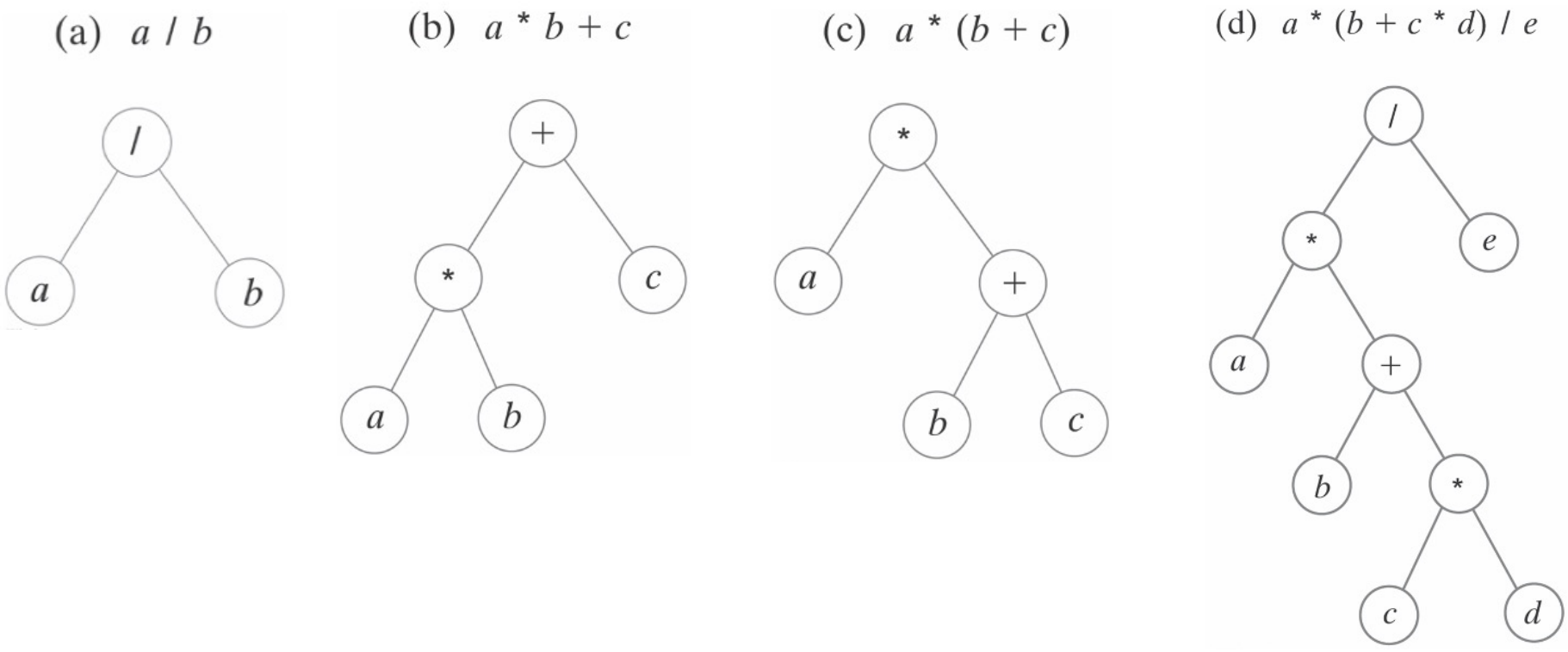


Building a Binary Tree (cont.)

```
// Display root, height, number of nodes
System.out.println("Root of tree contains " + aTree.getRootData());
System.out.println("Height of tree is " + aTree.getHeight());
System.out.println("Tree has " + aTree.getNumberOfNodes() + " nodes");

// Display nodes in preorder
System.out.println("A preorder traversal visits nodes in this order:");
Iterator<String> preorder = aTree.getPreorderIterator();
while (preorder.hasNext())
   System.out.print(preorder.next() + " ");
System.out.println();
```



Expression Trees



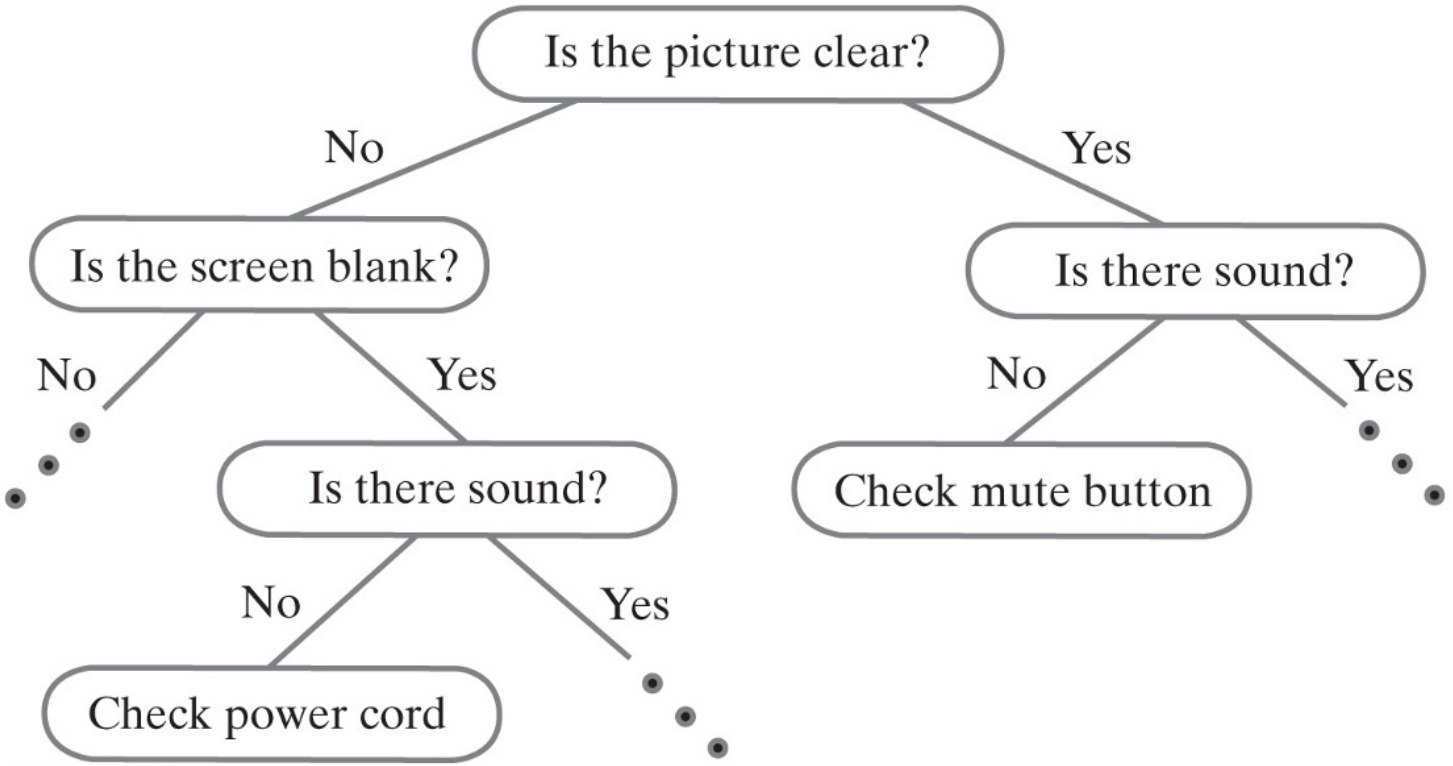
Expression trees for four algebraic expressions

Expression Trees (cont.)

```
Algorithm evaluate(expressionTree)

if (expressionTree is empty)
   return 0
else
{
   firstOperand = evaluate(left subtree of expressionTree)
   secondOperand = evaluate(right subtree of expressionTree)
   operator = the root of expressionTree

   return the result of the operation operator and its operands
          firstOperand and secondOperand
}
```

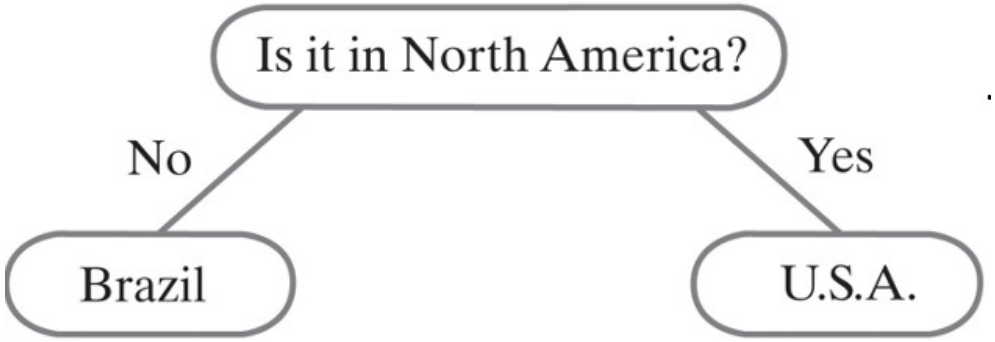
Algorithm for postorder traversal of an expression tree.

Expert System Using A Decision Tree

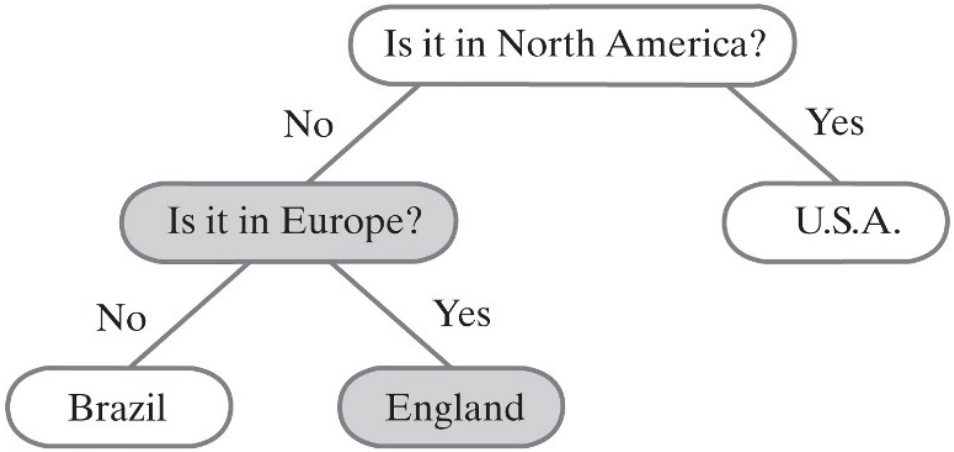


A portion of a binary decision tree

Expert System Using A Decision Tree (cont.)



An initial decision tree for a guessing game



The decision tree for a guessing game after acquiring another fact

Expert System Using A Decision Tree (algo.)

```
package TreePackage;
/** An interface for a decision tree. */
public interface DecisionTreeInterface<T> extends BinaryTreeInterface<T>
{
   /** Gets the data in the current node.
       @return  The data object in the current node, or
                null if the current node is null. */
   public T getCurrentData();

   /** Sets the data in the current node.
       Precondition: The current node is not null.
       @param newData  The new data object. */
   public void setCurrentData(T newData);

   /** Sets the data in the children of the current node,
       creating them if they do not exist.
       Precondition: The current node is not null.
       @param responseForNo   The new data object for the left child.
       @param responseForYes  The new data object for the right child. */
   public void setResponses(T responseForNo, T responseForYes);
```

An interface for a binary decision tree

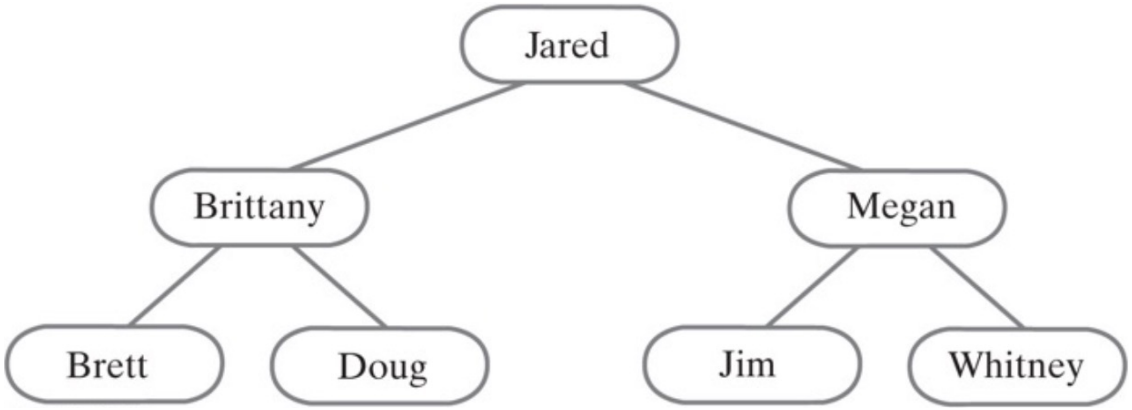
Expert System Using A Decision Tree (algo.)(cont.)

```
   /** Sees whether the current node contains an answer.
      @return  True if the current node is a leaf, or
               false if it is a nonleaf. */
   public boolean isAnswer();

   /** Sets the current node to its left child.
       If the child does not exist, sets the current node to null. */
   public void advanceToNo();

   /** Sets the current node to its right child.
       If the child does not exist, sets the current node to null. */
   public void advanceToYes();

   /** Sets the current node to the root of the tree. */
   public void resetCurrentNode();
} // end DecisionTreeInterface
```

An interface for a binary decision tree

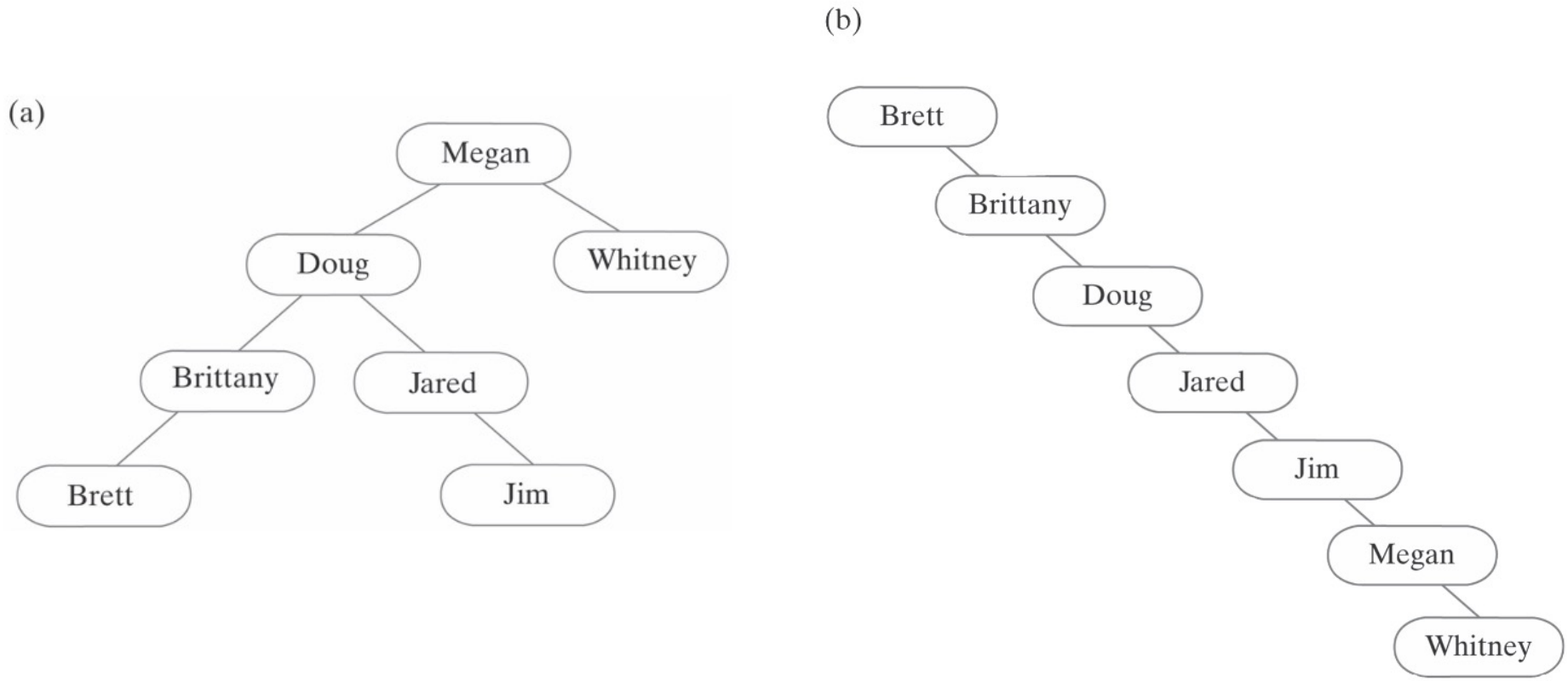
Binary Search Tree

- For each node in a binary search tree
  - Node's data is greater than all data in node's left subtree
  - Node's data is less than all data in node's right subtree
- Every node in a binary search tree is the root of a binary search tree.



A binary search tree of names

Binary Search Tree (cont.)



Two binary search trees containing the same data

Binary Search Tree (cont.)

```
Algorithm bstSearch(binarySearchTree, desiredObject)
// Searches a binary search tree for a given object.
// Returns true if the object is found.

if (binarySearchTree is empty)
   return false
else if (desiredObject == object in the root of binarySearchTree)
   return true
else if (desiredObject < object in the root of binarySearchTree)
   return bstSearch(left subtree of binarySearchTree, desiredObject)
else
   return bstSearch(right subtree of binarySearchTree, desiredObject)
```

Pseudocode for recursive search algorithm

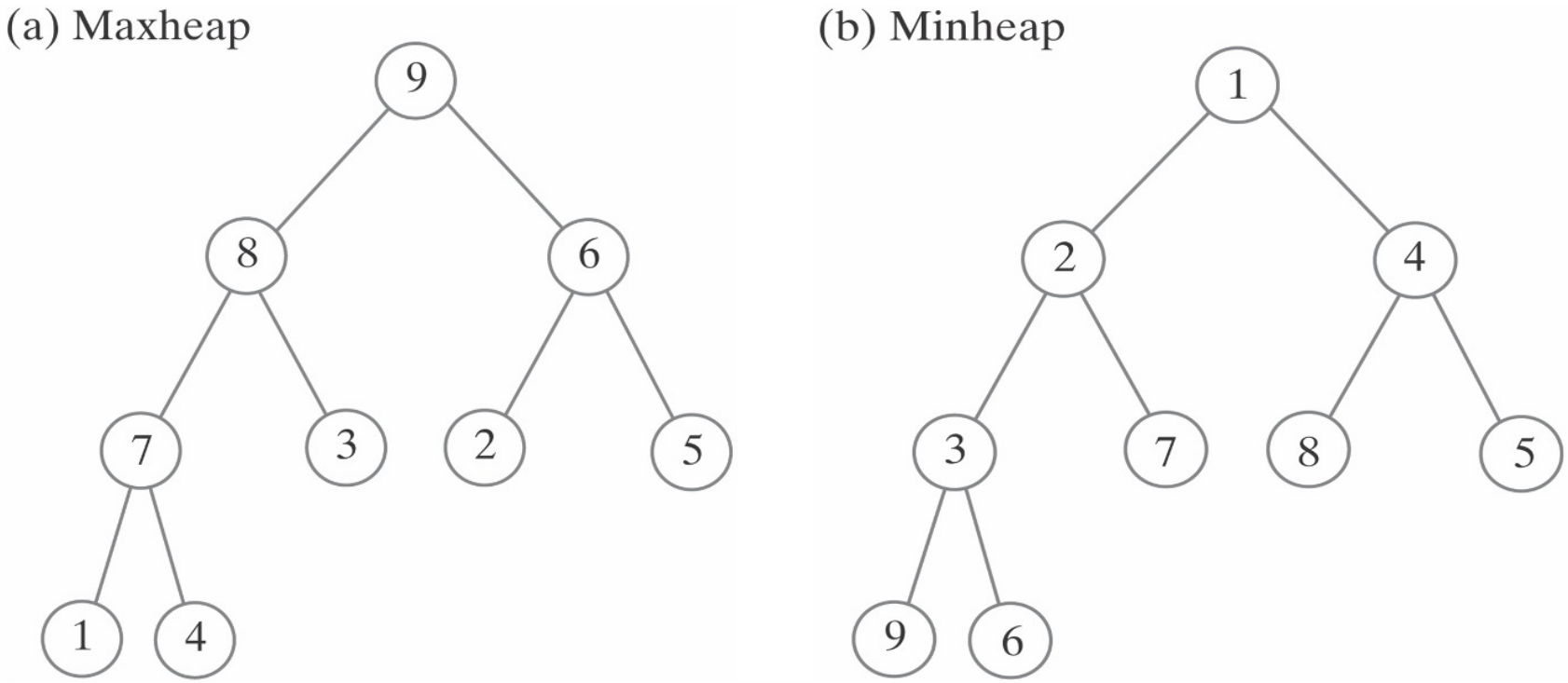
Binary Search Tree (cont.)

- Efficiency of a search
  - Searching a binary search tree of height *h* is O(h)
- To make searching a binary search tree efficient:
  - Tree must be as short as possible.

Heap

- Complete binary tree whose nodes contain Comparable objects and are organized as follows:
  - Each node contains an object no smaller/larger than objects in its descendants
  - Maxheap: object in node greater than or equal to its descendant objects
  - Minheap: object in node less than or equal to its descendant objects

Heap (cont.)



Two heaps that contain the same values

Heap (cont.)

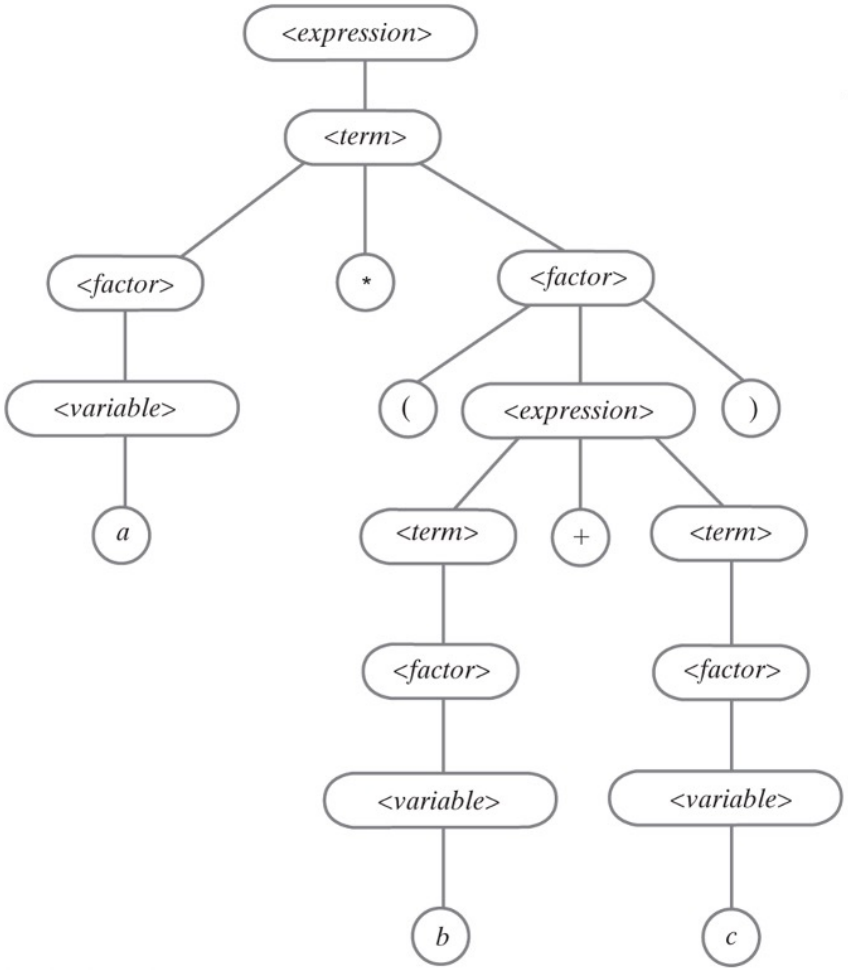
```java
/**  An interface for the ADT maxheap. */
public interface MaxHeapInterface<T extends Comparable<? super T>>
{
   /** Adds a new entry to this heap.
       @param newEntry  An object to be added. */
   public void add(T newEntry);

   /** Removes and returns the largest item in this heap.
       @return  Either the largest object in the heap or,
                if the heap is empty before the operation, null. */
   public T removeMax();

   /** Retrieves the largest item in this heap.
       @return  Either the largest object in the heap or,
                if the heap is empty, null. */
   public T getMax();

   /** Detects whether this heap is empty.
       @return  True if the heap is empty, or false otherwise. */
   public boolean isEmpty();

   /** Gets the size of this heap.
       @return  The number of entries currently in the heap. */
   public int getSize();

   /** Removes all entries from this heap. */
   public void clear();
} // end MaxHeapInterface
```

An interface for a maxheap

More on General Trees

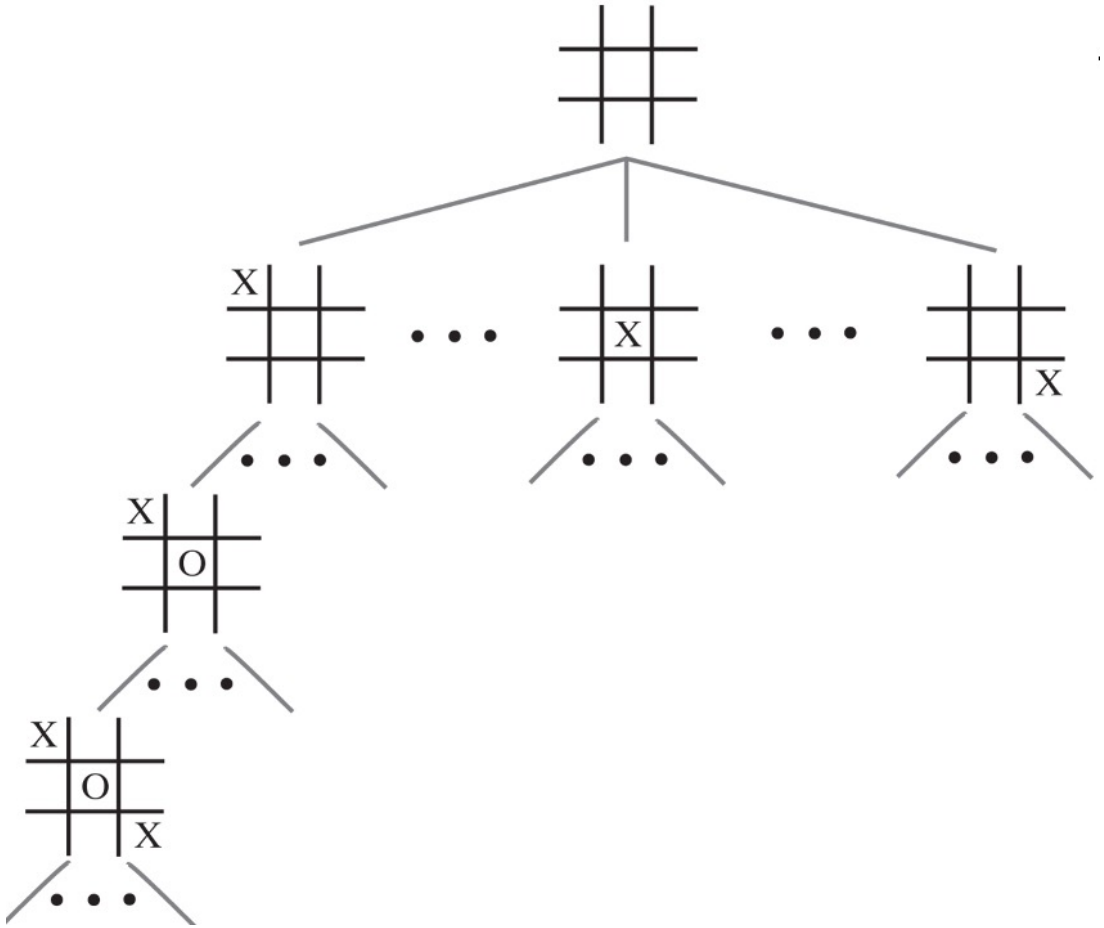
- Parse tree
  - Check syntax of a string for valid algebraic expression
  - If valid, it can be expressed as a parse tree.
- Parse tree must be a general tree.
  - So it can accommodate any expression.
- Compilers use parse trees.
  - Check syntax, produce code

Parse Tree for an Equation



A parse tree for the algebraic expression a * (b + c)

Parse Tree for a Game



A portion of a game tree for tic-tac-toe