Northeastern University

Lecture 29: Faster Sorting Methods - 2

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Quick Sort

Quick Sort

- Divides an array into two pieces
  - Pieces are not necessarily halves of the array
  - Chooses one entry in the array - called the pivot
- Partitions the array

Quick Sort

- When pivot is chosen, array rearranged such that:
  - Pivot is in position that it will occupy in final sorted array
  - Entries in positions before pivot are less than or equal to pivot
  - Entries in positions after pivot are greater than or equal to pivot

Quick Sort

Algorithm quicksort(a, first, last)

// Sorts the array entries a[first..last] recursively.

```
if (first < last)
{
    Choose a pivot
    Partition the array about the pivot
    pivotIndex = index of pivot
    quickSort(a, first, pivotIndex - 1)  // Sort Smaller
    quickSort(a, pivotIndex + 1, last)  //Sort Larger
}
```

Algorithm that describes our sorting strategy

Quick Sort

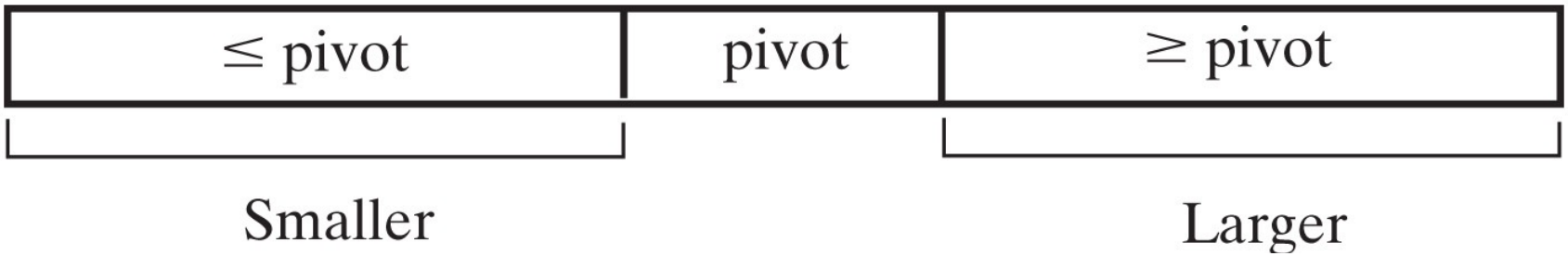


Figure 16-5: A partition of an array during a quick sort

- Quick sort: O(n log n) in average case, O(n^2) in worst case.
- Choice of pivots affects behavior

Creating the Partition

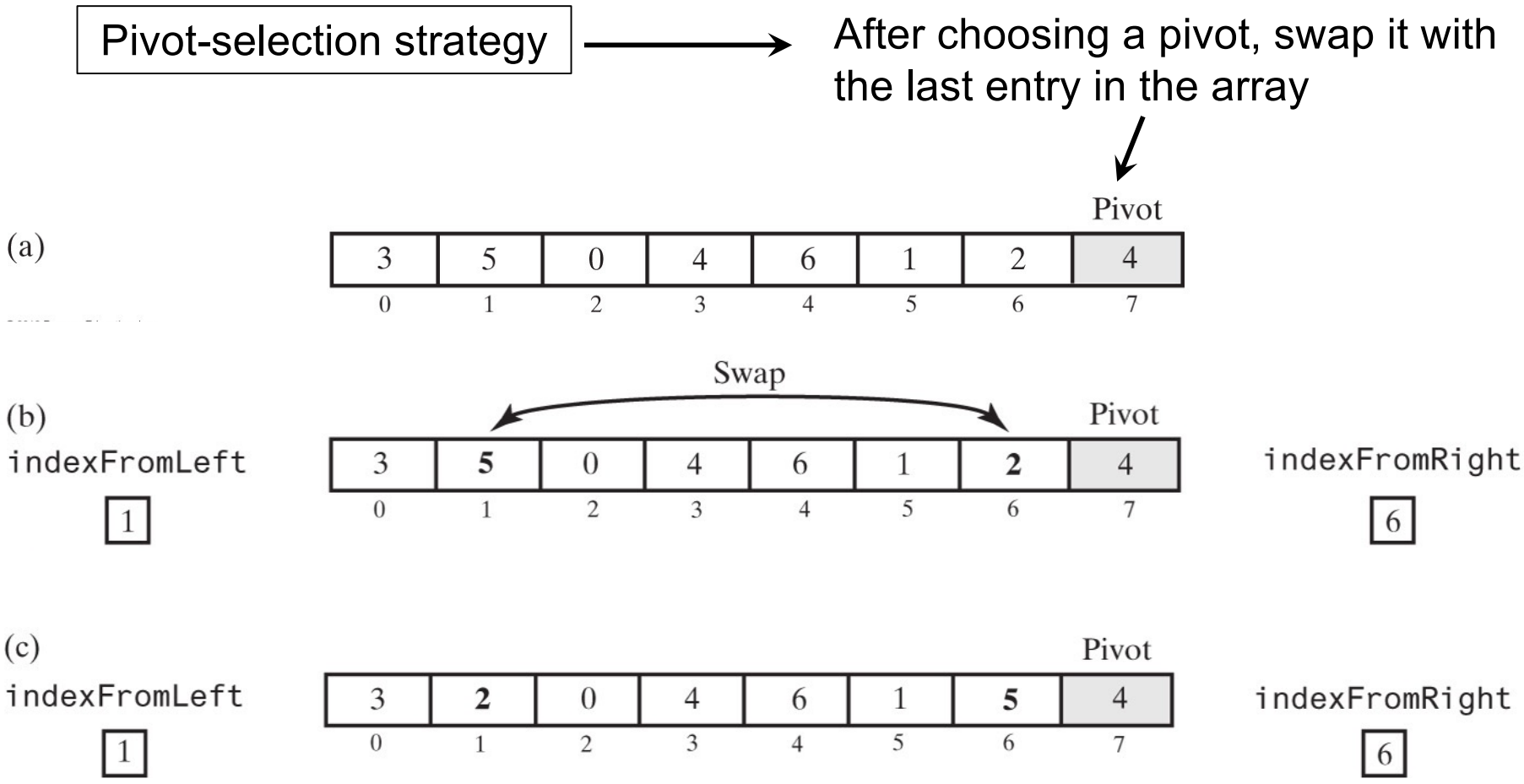


Figure 16-6: A partitioning strategy for quick sort

Creating the Partition

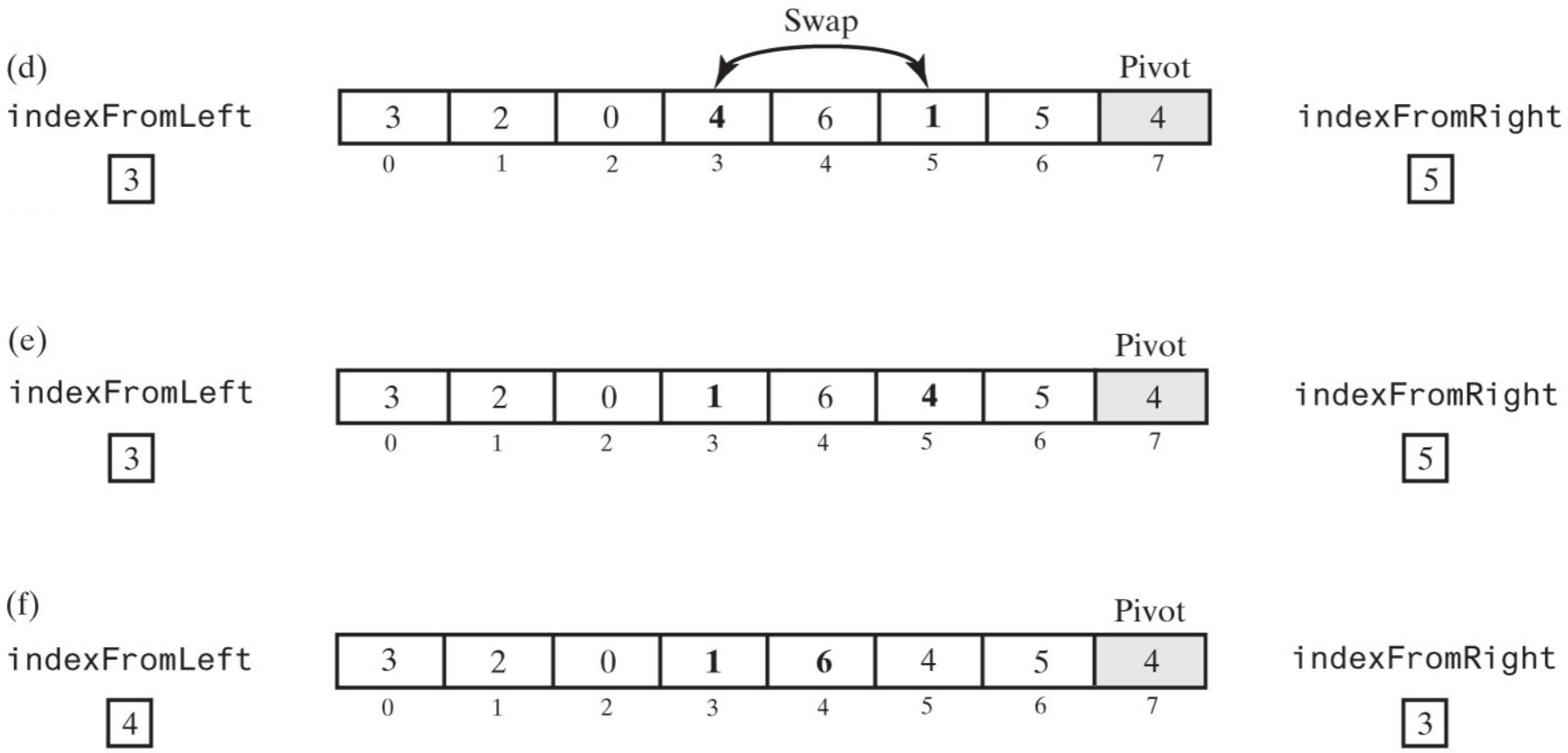


Figure 16-6: A partitioning strategy for quick sort

Creating the Partition

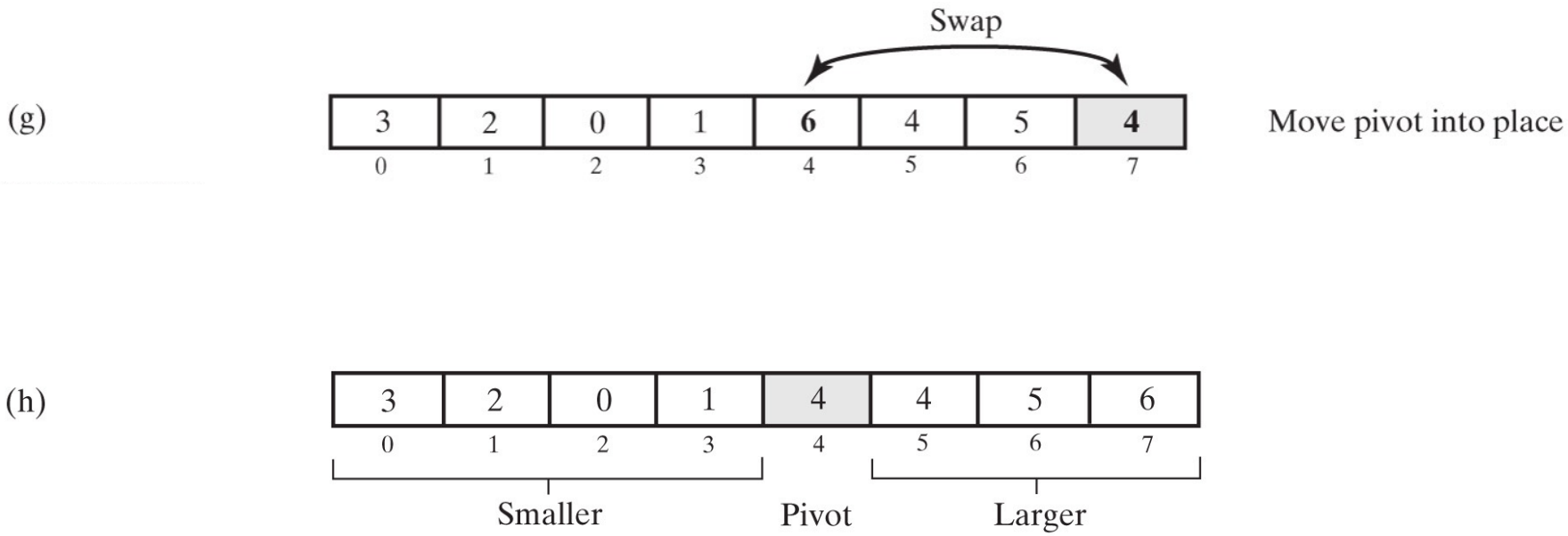


Figure 16-6: A partitioning strategy for quick sort

Pivot Selection

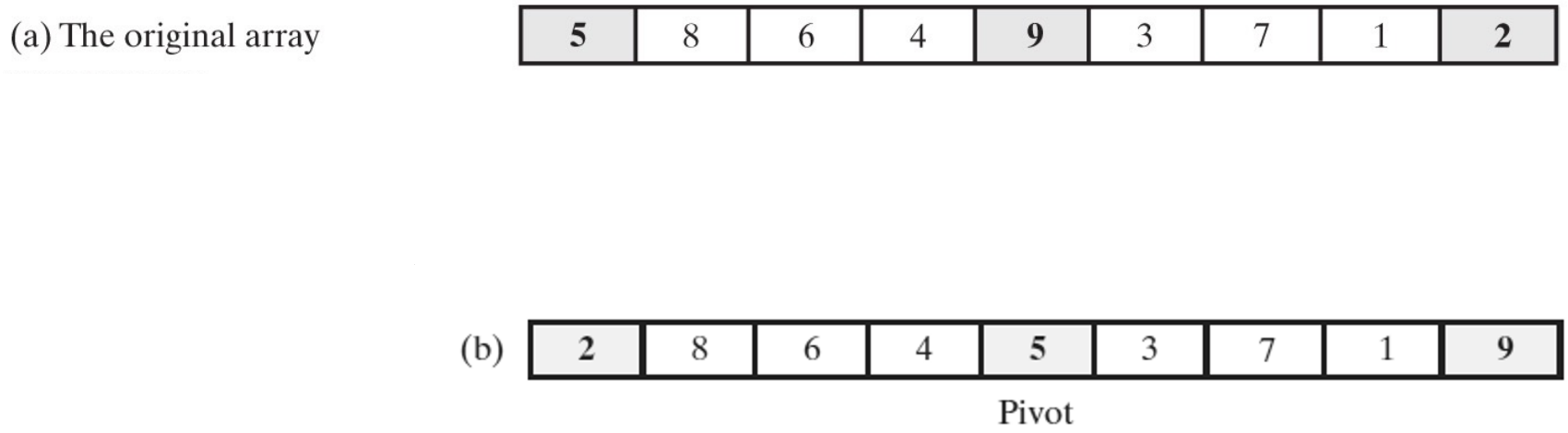
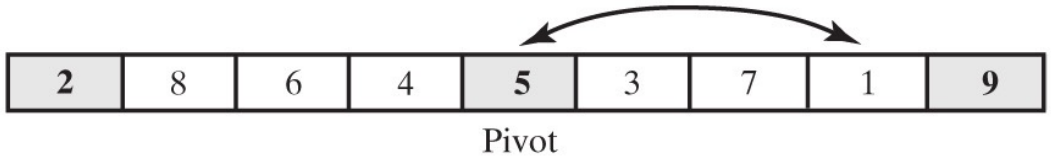


Figure 16-7: Median-of-three pivot selection: (a) The original array; (b) the array with its first, middle, and last entries sorted

Adjusting the Partition Algorithm

(a) The array after median-of-three pivot selection, as shown in Figure 16-7b



(b) The array before partitioning and just after positioning the pivot

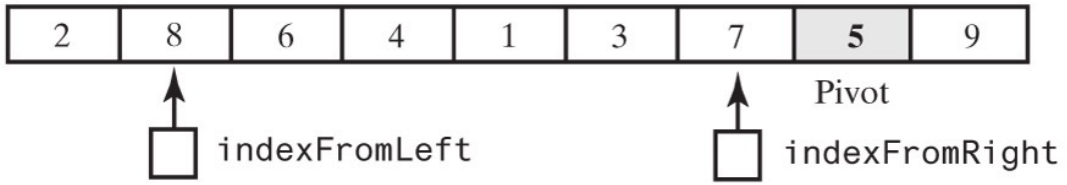


Figure 16-8: (a) The array with its first, middle, and last entries sorted; (b) the array after positioning the pivot and just before partitioning

Adjusting the Partition Algorithm

Algorithm partition(a, first, last)

// Partitions an array a[first..last] as part of quick sort into two subarrays named
// Smaller and Larger that are separated by a single entry -- the pivot -- named pivotValue.
// Entries in Smaller are <= pivotValue and appear before pivotValue in the array.
// Entries in Larger are >= pivotValue and appear after pivotValue in the array.
// Precondition: first >= 0; first < a.length; last – first >= 3; last < a.length.
// Returns the index of the pivot.

mid = index of the array's middle entry
sortFirstMiddleLast(a, first, mid, last)

// Assertion: a[mid] is the pivot, that is, pivotValue;
// a[first] <= pivotValue and a[last] >= pivotValue, so do not compare these two
// array entries with pivotValue.
// Move pivotValue to next-to-last position in array

Exchange a[mid] and a[last – 1]
pivotIndex = last – 1
pivotValue = a[pivotIndex]

```
// Determine two subarrays:
//   Smaller = a[first..endSmaller] and
//   Larger = a[endSmaller+1..last–1]
// such that entries in Smaller are <= pivotValue and entries in Larger are >= pivotValue.
// Initially, these subarrays are empty.

indexFromLeft = first + 1
indexFromRight = last − 2

done = false

while (!done)
{
      // Starting at the beginning of the array, leave entries that are < pivotValue and
      // locate the first entry that is >= pivotValue. You will find one, since the last
      // entry is >= pivotValue.
      while (a[indexFromLeft]  <  pivotValue)
            indexFromLeft++

      // Starting at the end of the array, leave entries that are > pivotValue and
      // locate the first entry that is <= pivotValue. You will find one, since the first
      // entry is <= pivotValue.
      while (a[indexFromRight]  >  pivotValue)
            indexFromRight−−
```

```
      // Assertion: a[indexFromLeft] >= pivotValue and a[indexFromRight] <= pivotValue

      if (indexFromLeft < indexFromRight)
      {
          Exchange a[indexFromLeft] and a[indexFromRight]
          indexFromLeft++
          indexFromRight−−
      } else {
          done = true
       }

}  // end of while

// Place pivotValue between the subarrays Smaller and Larger
Exchange a[pivotIndex] and a[indexFromLeft]
pivotIndex = indexFromLeft

// Assertion: Smaller = a[first..pivotIndex–1], pivotValue = a[pivotIndex], Larger = a[pivotIndex+1..last]

return pivotIndex
```

The Quick Sort Method

```
public static <T extends Comparable<? super T>>
       void quickSort(T[] a, int first, int last)
{
   if (last - first + 1 < MIN_SIZE)
   {
      insertionSort(a, first, last);
   }
   else
   {
      // Create the partition: Smaller | Pivot | Larger
      int pivotIndex = partition(a, first, last);

      // Sort subarrays Smaller and Larger
      quickSort(a, first, pivotIndex - 1);
      quickSort(a, pivotIndex + 1, last);
   } // end if
} // end quickSort
```

Above method implements quick sort

Quick Sort in the Java Class Library

```
public static void sort(type[] a)

public static void sort(type[] a, int first, int after)
```

Class **`Arrays`** in the package **`java.util`** uses a quick sort to sort arrays of primitive types into ascending order

Exercise

- Trace the steps that the method quickSort takes when sorting the following array into ascending order. Assume that MIN_SIZE = 4

9 6 2 4 8 7 5 3

Answer

```
quickSort(array, 0, 7)
partition(array, 0, 7)

 9 6 2 4 8 7 5 3
 3 6 2 4 8 7 5 9
 3 6 2 5 8 7 4 9
 3 2 6 5 8 7 4 9
 3 2 4 5 8 7 6 9

quickSort(array, 0, 1)
insertionSort(array, 0, 1)

 2 3 4 5 8 7 6 9

quickSort(array, 3, 7)
partition(array, 3, 7)

 2 3 4 5 8 7 6 9
 2 3 4 5 8 6 7 9
 2 3 4 5 6 8 7 9
 2 3 4 5 6 7 8 9

quickSort(array, 3, 4)
insertionSort(array, 3, 4)

 2 3 4 5 6 7 8 9

quickSort(array, 6, 7)
insertionSort(array, 6, 7)

 2 3 4 5 6 7 8 9
```

Radix Sort

Radix Sort

- Does not use comparison
- Treats array entries as if they were strings that have the same length.
  - Group integers according to their rightmost character (digit) into “buckets”
  - Repeat with next character (digit), etc.
- It is not suitable as a general-purpose sorting algorithm.

Radix Sort

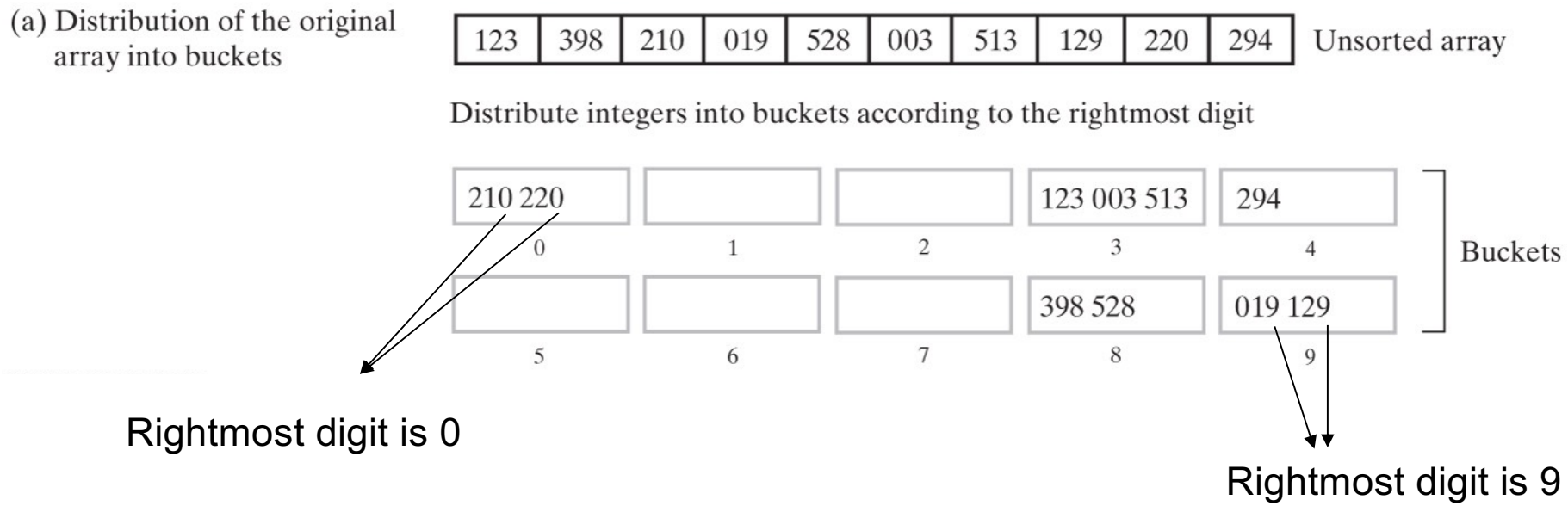


Figure 16-9: Radix sort: (a) Original array and buckets after first distribution;

Radix Sort

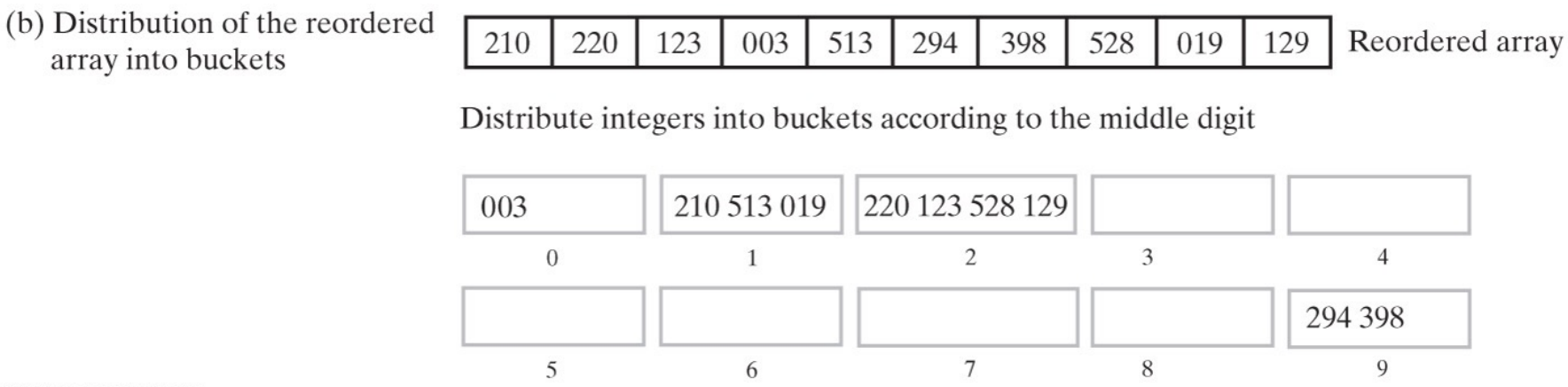


Figure 16-9: Radix sort: (b) reordered array and buckets after second distribution

Radix Sort

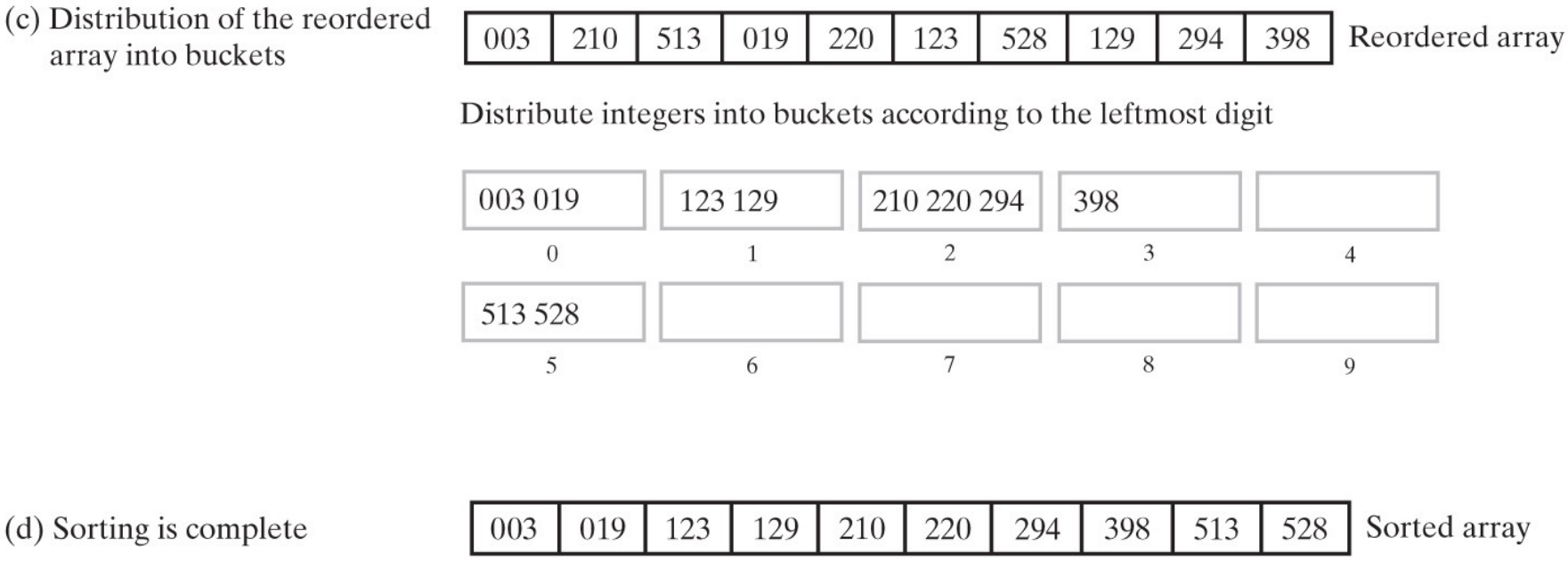


Figure 16-9: Radix sort: (c) reordered array and buckets after third distribution; (d) sorted array

Pseudocode for Radix Sort

```
Algorithm radixSort(a, first, last, maxDigits)
// Sorts the array of positive decimal integers a[first..last] into ascending order;
// maxDigits is the number of digits in the longest integer.

for (i = 0 to maxDigits - 1)
{
    Clear bucket[0], bucket[1], . . . , bucket[9]
    for (index = first to last)
    {
        digit = digit i of a[index]
        Place a[index] at end of bucket[digit]
    }
    Place contents of bucket[0], bucket[1], . . . , bucket[9] into the array a
}
```

Radix sort is an O(n) algorithm for certain data, it is not appropriate for all data

Comparing the Algorithms

	Best Case	Average Case	Worst Case
Radix Sort	$O(n)$	$O(n)$	$O(n)$
Merge Sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Quick Sort	$O(n \log n)$	$O(n \log n)$	$O(n^2)$
Shell Sort	$O(n)$	$O(n^{1.5})$	$O(n^{1.5})$
Insertion Sort	$O(n)$	$O(n^2)$	$O(n^2)$
Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$

Figure 16-10: The time efficiency of various sorting algorithms, expressed in Big Oh notation

Exercise

- You are given two sorted arrays, A and B, where A has a large enough buffer at the end to hold B.
- Write a method to merge B into A in sorted order.

Answer

```
public static void mergeArrays(int[] a, int[] b, int numA, int numB) {

      int newLastIndex = numA + numB - 1;
      int lastIndexA = numA - 1;
      int lastIndexB = numB - 1;

      int loop = numA >= numB? numB : numA;

      for(int i = loop; i >= 0; i--) {
            if(a[lastIndexA] > b[lastIndexB]) {
                  a[newLastIndex] = a[lastIndexA];
                  lastIndexA--;
            } else {
                  a[newLastIndex] = b[lastIndexB];
                  lastIndexB--;
            }
            newLastIndex--;
      }

      //Copy remaining elements of a or b to array a
      if(loop == numA) {
            for(int j = lastIndexB; j >= 0; j--) {
                  a[newLastIndex] = b[j];
                  newLastIndex--;
            }
      }else {
            for(int j = lastIndexA; j >= 0; j--) {
                  a[newLastIndex] = a[j];
                  newLastIndex--;
            }
      }
}
```