Northeastern University

Lecture 28: Faster Sorting Methods - 1

Prof. Chen-Hsiang (Jones) Yu, Ph.D.

College of Engineering

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Merge Sort

Merge Sort

- Divides an array into halves
- Sorts the two halves,
  - Then merges them into one sorted array.
- The algorithm for merge sort is usually stated recursively.
- Major programming effort is in the merge process.

Merging Arrays

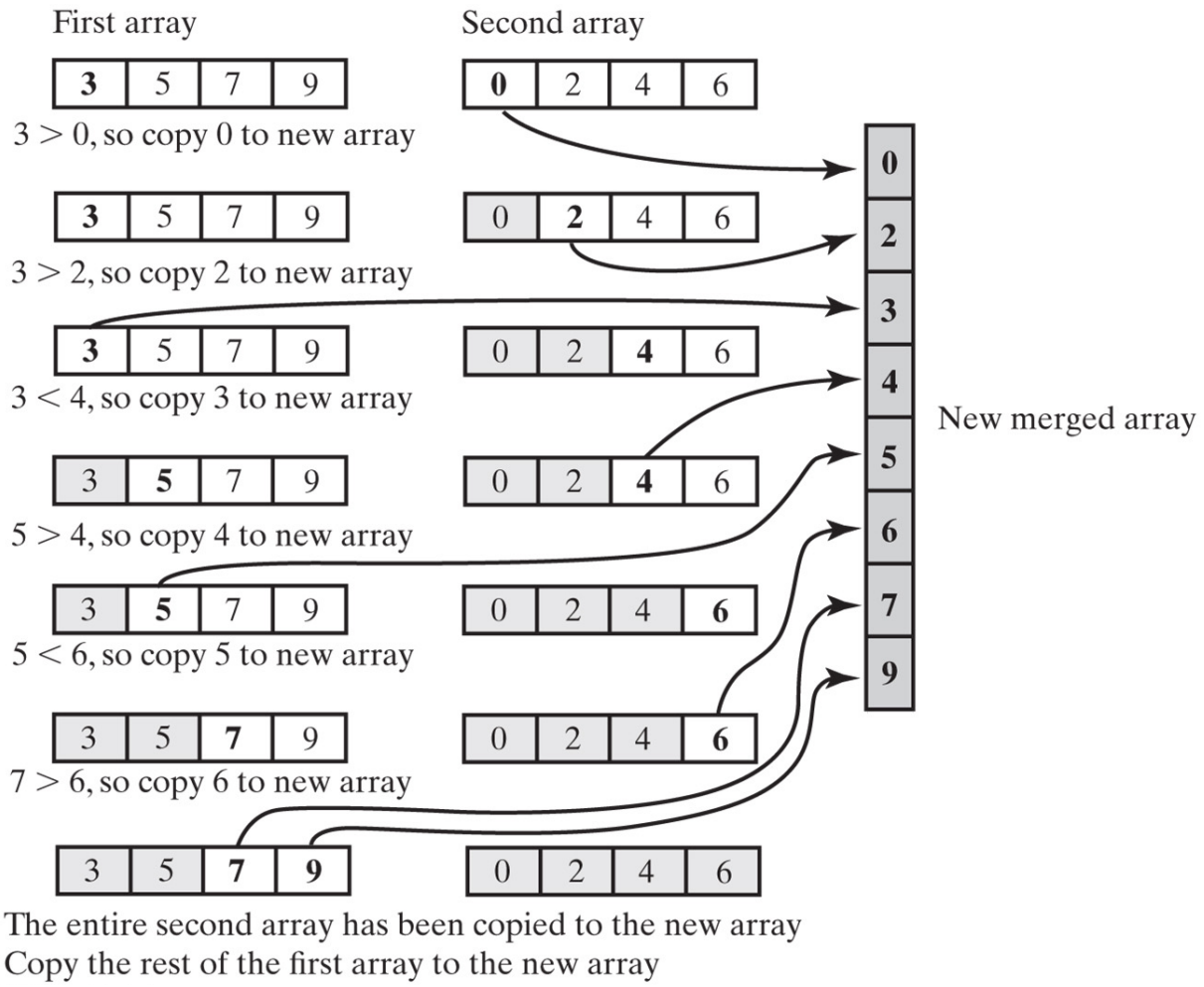


Figure 16-1: Merging two sorted arrays into one sorted array

Recursive Merge Sort

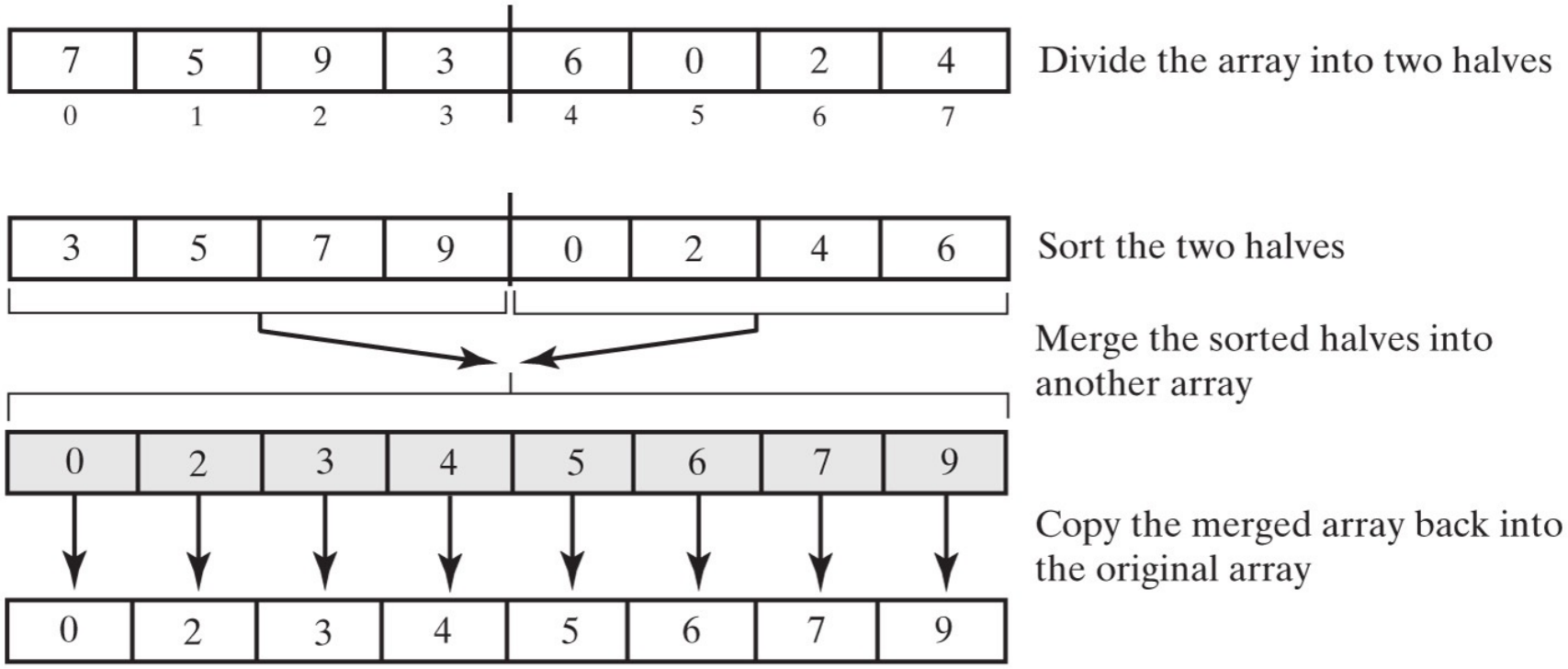


Figure 16-2: The major steps in a merge sort

Recursive Merge Sort

Algorithm mergeSort(a, tempArray, **first**, last)
// Sorts the array entries `a[first..last]` recursively.

```
if (first < last)
{
    mid = approximate midpoint between first and last
    mergeSort(a, tempArray, first, mid)
    mergeSort(a, tempArray, mid + 1, last)
    Merge the sorted halves a[first..mid] and a[mid+1..last] using the array tempArray
}
```

Recursive algorithm for merge sort

Algorithm merge(a, tempArray, **first, mid, last)**

// Merges the adjacent subarrays a[first..mid] and a[mid+1..last].
beginHalf1 = first, endHalf1 = mid, beginHalf2 = mid + 1, endHalf2 = last

// While both subarrays are not empty, compare an entry in one subarray with
// an entry in the other; then copy the smaller item into the temporary array

index = 0 //Next available location in `tempArray`

```
while ( (beginHalf1 <= endHalf1) and (beginHalf2 <= endHalf2) ){
        if (a[beginHalf1] <= a[beginHalf2]){
                tempArray[index]  =  a[beginHalf1]
                beginHalf1++
        } else{
                tempArray[index]  =  a[beginHalf2]
                beginHalf2++
        }
        index++
}
```

// Assertion: One subarray has been completely copied to `tempArray`.
Copy remaining entries from other subarray to `tempArray`
Copy entries from `tempArray` to array a

Pseudocode describes the merge step

Recursive Merge Sort

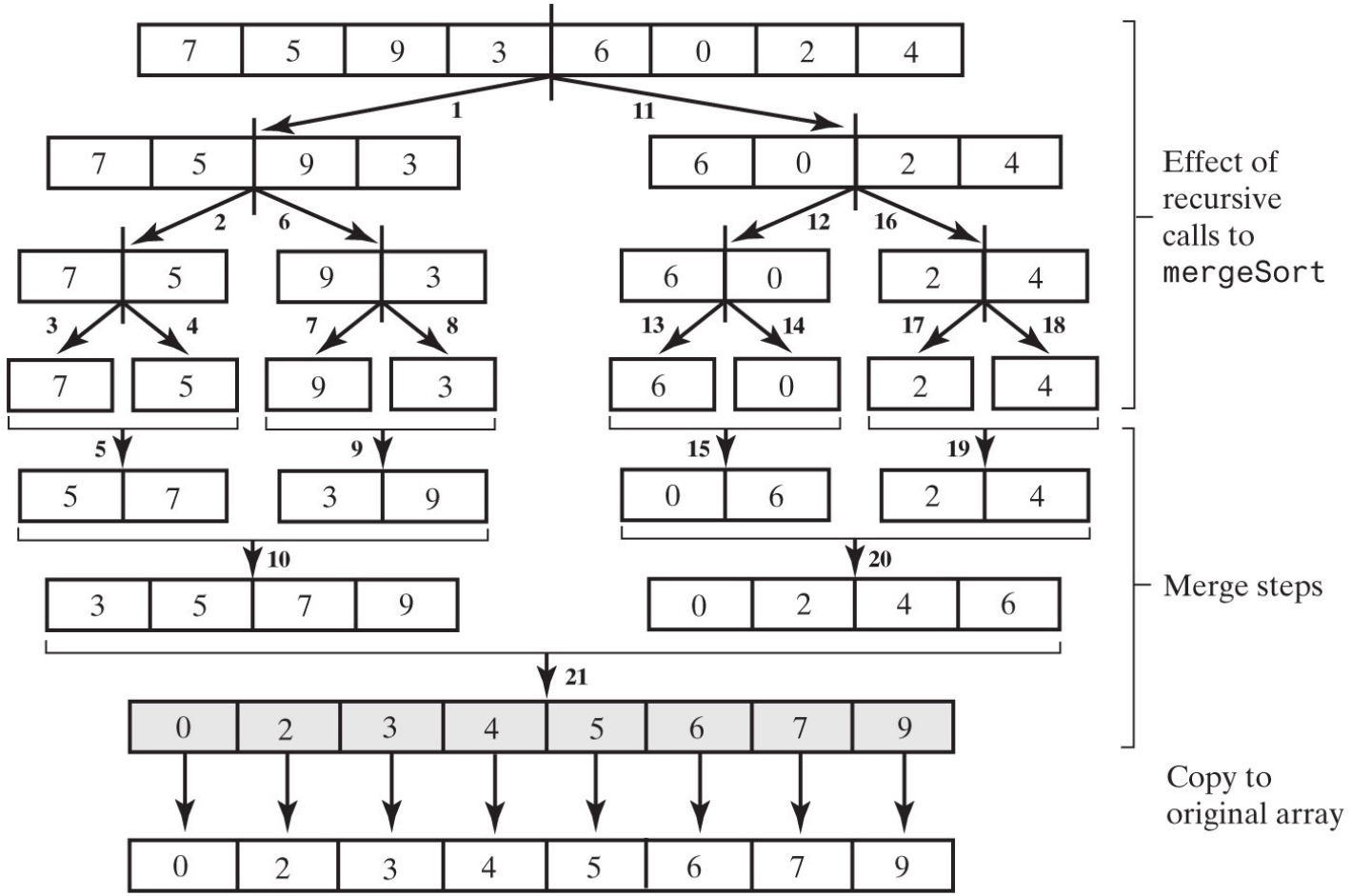


Figure 16-3: The effect of the recursive calls and the merges during a merge sort

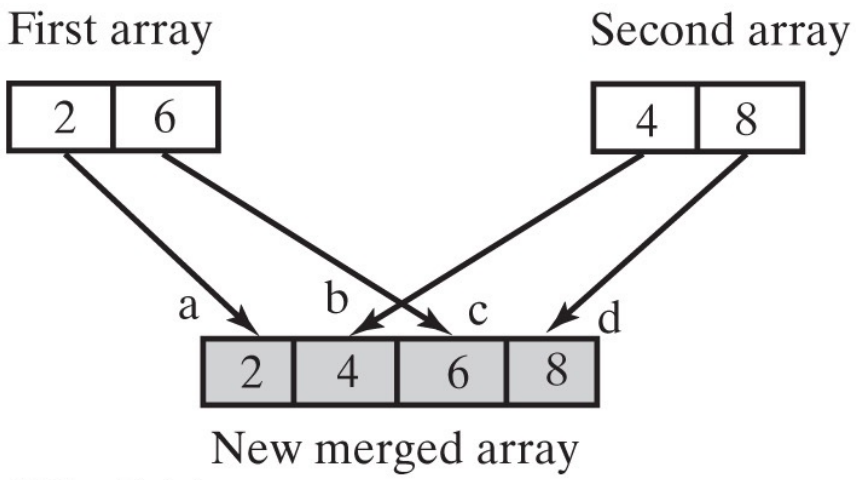
Recursive Merge Sort

```
public static <T extends Comparable<? super T>>
              void mergeSort(T[] a, int first, int last)
{
   // The cast is safe because the new array contains null entries
   @SuppressWarnings("unchecked")
   T[] tempArray = (T[])new Comparable<?>[a.length]; // Unchecked cast

   mergeSort(a, tempArray, first, last);
} // end mergeSort
```

Be careful to allocate the temporary array only once

Efficiency of Merge Sort



a. $2 < 4$, so copy 2 to new array

b. $6 > 4$, so copy 4 to new array

c. $6 < 8$, so copy 6 to new array

d. Copy 8 to new array

Figure 16-4: A worst-case merge of two sorted arrays. Efficiency is O(*n* log *n*).

Note that the proof is in section 16.6.

Iterative Merge Sort

- Less simple than recursive version
  - Need to control the merges
- Will be more efficient of both time and space
  - But, trickier to code without error

Iterative Merge Sort (cont.)

- Starts at beginning of array
  - Merges pairs of individual entries to form two-entry subarrays
- Returns to the beginning of array and merges pairs of the two-entry subarrays to form four-entry subarrays
  - And so on
- After merging all pairs of subarrays of a particular length, might have entries left over.

Exercise

- Suppose following list represents an array of Integer objects.
- Show the steps that a merge sort takes when sorting this array.

80 90 70 85 60 40 50 95

Answer

- Split the array in half until we reach single-element subarrays:

  80 90 70 85 60 40 50 95

  80 90 70 85 | 60 40 50 95

  80 90 | 70 85 | 60 40 | 50 95

  80 | 90 | 70 | 85 | 60 | 40 | 50 | 95

- Merge pairs of subarrays to produce larger sorted subarrays:

  80 90 | 70 85 | 40 60 | 50 95

  70 80 85 90 | 40 50 60 95

  40 50 60 70 80 85 90 95

Merge Sort in the Java Class Library

- Class `Arrays` in the package `java.util` defines versions of a static method `sort`

```
public static void sort(Object[] a)

public static void sort(Object[] a, int first, int after)
```

Exercise (10-15 mins)

- Please download "L28_E1" project from "In-class Exercises" on the Canvas.
- Please finish methods `mergeSort()` and `merge()`, and run Merge Sort to a given sequence of integers. (ArraySortRecursive.java)

Note: You should use `merge()` in `mergeSort()` to support the sorting.

Expected Result

```
Running Merge Sort:


Before sort:
11 2 5 7 9 21 8 18 24 10


After sort:
2 5 7 8 9 10 11 18 21 24
```

Answer

```
private static <T extends Comparable<? super T>>
void mergeSort(T[] a, T[] tempArray, int first, int last)
{

    if (first < last)
    {
        int mid = first + (last - first) / 2;  // Index of midpoint
        mergeSort(a, first, mid);              // Sort left half array
        mergeSort(a, mid + 1, last);           // Sort right half array

        merge(a, tempArray, first, mid, last);  // Merge the two halves
    }

}
```

Answer (cont.)

```
private static <T extends Comparable<? super T>>
   void merge(T[] a, T[] tempArray, int first, int mid, int last){

   int beginHalf1 = first;
   int endHalf1 = mid;
   int beginHalf2 = mid + 1;
   int endHalf2 = last;

   int index = beginHalf1;
   for (; (beginHalf1 <= endHalf1) && (beginHalf2 <= endHalf2); index++){
      if (a[beginHalf1].compareTo(a[beginHalf2]) < 0)
      {
         tempArray[index] = a[beginHalf1];
         beginHalf1++;
      } else {
         tempArray[index] = a[beginHalf2];
         beginHalf2++;
      } // end if
   } // end for

   for (; beginHalf1 <= endHalf1; beginHalf1++, index++)
      tempArray[index] = a[beginHalf1];

   for (; beginHalf2 <= endHalf2; beginHalf2++, index++)
      tempArray[index] = a[beginHalf2];

   for (index = first; index <= last; index++)
      a[index] = tempArray[index];

} // end merge
```