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Lecture 23: Recursion - 3

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Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein. Introduction to
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Tail Recursion

Tail Recursion

- In a tail-recursive method, the last action is a recursive call.
- This call performs a repetition that can be done by using iteration.
- Converting a tail-recursive method to an iterative one is usually a straightforward process. (Well..., sometimes, not always.)

Tail Recursion

```
public static void countDown(int n)
{
    System.out.println(n);
    if (n > 1)
        countDown(n - 1);
} // end countDown
```

When **the last action** performed by a recursive method is **a recursive call**.

Tail Recursion

- Converting a recursive method to an iterative one

```
public static void countDown(int integer)
{
    if (integer >= 1)
    {
        System.out.println(integer);
        countDown(integer - 1);
    } // end if
} // end countDown
```

- An interactive version

```
public static void countDown(int integer)
{
    while (integer >= 1)
    {
        System.out.println(integer);
        integer = integer - 1;
    } // end while
} // end countDown
```



Using a Stack Instead of Recursion

Using a Stack Instead of Recursion

```
public static void displayArray(int array[], int first, int last)
{
    if (first == last)
        System.out.print(array[first] + " ");
    else
    {
        int mid = first + (last - first) / 2;
        displayArray(array, first, mid);
        displayArray(array, mid + 1, last);
    } // end if
} // end displayArray
```

Recursive approach

Using a Stack Instead of Recursion

```
public class Record
{
    private int first, last;

    private Record(int firstIndex, int lastIndex)
    {
        first = firstIndex;
        last = lastIndex;

    } // end constructor
} // end Record
```

- An **iterative** `displayArray` to maintain its own stack

```
public void displayArray(int first, int last)
{
    boolean done = false;
    StackInterface<Record> programStack = new LinkedStack<>();
    programStack.push(new Record(first, last));

    while (!done && !programStack.isEmpty())
    {
        Record topRecord = programStack.pop();
        first = topRecord.first;
        last = topRecord.last;

        if (first == last){
            System.out.println(array[first] + " ");
            done = true;
        }else{
            int mid = first + (last - first) / 2;
            // Note the order of the records pushed onto the stack
            programStack.push(new Record(mid + 1, last));
            programStack.push(new Record(first, mid));
        } // end if
    } // end while

} // end displayArray
```

Exercise

- Write a method to return all subsets of a set

$$\begin{array}{c} \{ 3, 4 \} \\ \downarrow \\ \{ \{ \}, \{ 3 \}, \{ 4 \}, \{ 3, 4 \} \} \end{array}$$

i.e. the power set of $\{ 3, 4 \}$ is $\{ \{ \}, \{ 3 \}, \{ 4 \}, \{ 3, 4 \} \}$

Answer

```
public class Subsets {
    private static ArrayList<ArrayList<Integer>> getSubsets(ArrayList<Integer> set, int index) {

        ArrayList<ArrayList<Integer>> allsubsets;

        if(set.size() == index){
            allsubsets = new ArrayList<ArrayList<Integer>>();
            allsubsets.add(new ArrayList<Integer>());
        }else{
            allsubsets = getSubsets(set, index+1);
            int item = set.get(index);

            ArrayList<ArrayList<Integer>> moresubsets = new ArrayList<ArrayList<Integer>>();
            for(ArrayList<Integer> subset: allsubsets){
                ArrayList<Integer> newsubset = new ArrayList<Integer>();
                newsubset.addAll(subset);
                newsubset.add(item);
                moresubsets.add(newsubset);
            }
            allsubsets.addAll(moresubsets);
        }
        return allsubsets;
    }

    public static void main(String[] args){
        ArrayList<Integer> data = new ArrayList<Integer>();
        for(int i = 0; i < 5; i++){
            data.add(i+1);
        }
        ArrayList<ArrayList<Integer>> result = getSubsets(data, 0);
        System.out.println(result);
    }
}
```

Dynamic Programming

Definition

- Dynamic programming was developed by Richard Bellman in the 1950s. It is a math optimization method and an algorithmic paradigm.
- Dynamic programming, like the divide-and-conquer method, solves problems by combining solutions to subproblems.
- Programming in this context refers to a tabular method, not to writing computer code.

Definition (cont.)

- A dynamic-programming algorithm solves each subproblem just once and saves the answer in a table.
- It avoids the work of recomputing the answer every time when it solves each subproblem.
- Dynamic programming is an extension to the divide-and-conquer method. When you recursively divide the original problem into a smaller problem, you can apply it to save the answer.

Example: Fibonacci Numbers

- The Fibonacci series begins with 0 and 1.
- Each subsequent number is the sum of the previous two.

The series: 0 1 1 2 3 5 8 13 21 34 55 89 ...

Indexes: 0 1 2 3 4 5 6 7 8 9 10 11

- The series can be recursively defined as:

» `fib(0) = 0;`

`fib(1) = 1;`

`fib(index) = fib(index-2) + fib(index-1) ; index >= 2`

Fibonacci Numbers

$\text{fib}(0) = 0;$

$\text{fib}(1) = 1;$

$$\begin{aligned}\text{fib}(4) &= \text{fib}(3) + \text{fib}(2) \\&= (\text{fib}(2) + \text{fib}(1)) + \text{fib}(2) \\&= ((\text{fib}(1) + \text{fib}(0)) + \text{fib}(1)) + \text{fib}(1) + \text{fib}(0) \\&= ((1 + \text{fib}(0)) + \text{fib}(1)) + \text{fib}(1) + \text{fib}(0) \\&= \dots \\&= 1 + 0 + 1 + 1 + 0 \\&= 3\end{aligned}$$

Fibonacci Numbers (cont.) - recursion approach

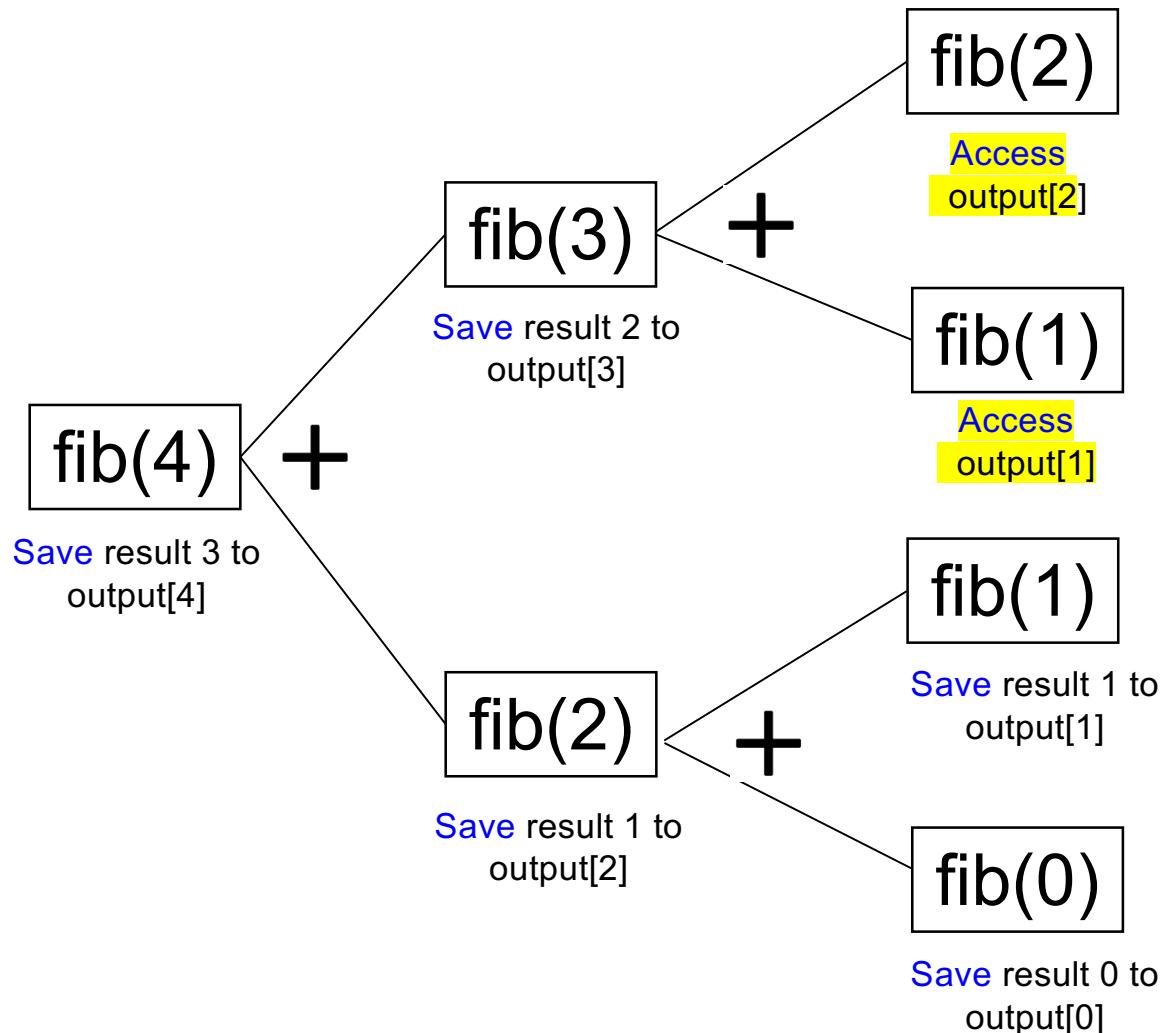
```
public class ComputeFibonacci {  
    /** Main method */  
    public static void main(String[] args) {  
        // Create a Scanner  
        Scanner input = new Scanner(System.in);  
        System.out.print("Enter an index for a Fibonacci number: ");  
        int index = input.nextInt();  
  
        // Find and display the Fibonacci number  
        System.out.println("The Fibonacci number at index " + index + " is " + fib(index));  
        input.close();  
    }  
  
    /** The method for finding the Fibonacci number */  
    public static int fib(int index) {  
        if (index == 0) // Base case  
            return 0;  
        else if (index == 1) // Base case  
            return 1;  
        else // Reduction and recursive calls  
            return fib(index - 1) + fib(index - 2);  
    }  
}
```

```
Enter an index for a Fibonacci number: 6  
The Fibonacci number at index 6 is 8
```

Fibonacci Numbers (cont.)

$\text{fib}(0) = 0;$

$\text{fib}(1) = 1;$



Exercise

- Please update `fib(int index)` method such that you can apply dynamic programming skills to calculate Fibonacci number.

```
/** The method for finding the Fibonacci number */
public static long fib(int index) {
    if (index == 0) // Base case
        return 0;
    else if (index == 1) // Base case
        return 1;
    else // Reduction and recursive calls
        return fib(index - 1) + fib(index - 2);
}
```

```
Enter an index for a Fibonacci number: 6
The Fibonacci number at index 6 is 8
```

Answer

```
public class ComputeFibonacci {  
    /** Main method */  
    public static void main(String[] args) {  
        // Create a Scanner  
        Scanner input = new Scanner(System.in);  
        System.out.print("Enter an index for a Fibonacci number: ");  
        int index = input.nextInt();  
  
        // Find and display the Fibonacci number  
        System.out.println("The Fibonacci number at index " + index + " is " + fib(index));  
        input.close();  
    }  
  
    public static int[] output = new int[1000];  
  
    /** The method for finding the Fibonacci number */  
    public static int fib(int index) {  
        int result = output[index];  
        if(result == 0) {  
            if (index == 0) // Base case  
                return 0;  
            else if (index == 1) // Base case  
                return 1;  
            else // Reduction and recursive calls  
                return fib(index - 1) + fib(index - 2);  
        }  
        output[index] = result;  
        return result;  
    }  
}
```