Equivariant homotopy theory and geometric Langlands

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Overview

1. Motivation
2. Equivariance
3. Proofs and generalizations

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But equivariance fixes the difficulties!

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The first line is the derived geometric Satake theorem/Fourier transform for tori. Note that the right-hand side can be viewed as the coadjoint quotient stack $\check{\mathfrak{t}}^*/\check{T}$ since $\check{T}$ is commutative.

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Question: What about KU-coefficients?

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In any case, the key point will be to compute $\mathrm{H}_*^G(\Omega G; k)$; exactly the kind of thing algebraic topologists love to do! As with all such calculations, one does this by first computing $\mathrm{H}_*^T(\Omega G; k)$ for a maximal torus $T \subseteq G$. I would like to illustrate this when $G = \mathrm{SU}_2$.

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