The Upwind Scheme :-

$$u_n^{m+1} = u_n^m - \frac{6\Delta t}{\Delta x} u_n^m \left(h_{n+\frac{1}{2}}^m - h_{n-\frac{1}{2}}^m \right) - \frac{\Delta t}{2(\Delta x)^3} \left(u_{n+2}^M - 2u_{n+1}^m + 2u_{n-1}^m - u_{n-2}^m \right)$$

$\boxed{f(u) = \frac{u^2}{2}}$

where

$$h_{n+\frac{1}{2}} = \begin{cases} u_n , & \varphi_{n+\frac{1}{2}} > 0 \\ u_{n+1} , & \varphi_{n+\frac{1}{2}} < 0 \end{cases}$$

$$\varphi_{n+\frac{1}{2}} = \frac{\Delta t}{\Delta x} \frac{f_{n+1} - f_n}{u_{n+1} - u_n}$$

$$\boxed{\varphi_{n+\frac{1}{2}} = \frac{\Delta t}{\Delta x} \left(\frac{u_{n+1} + u_n}{2} \right)}$$

$$h_{n-\frac{1}{2}} = \begin{cases} u_{n-1} , & \varphi_{n-\frac{1}{2}} > 0 \\ u_n , & \varphi_{n-\frac{1}{2}} < 0 \end{cases}$$

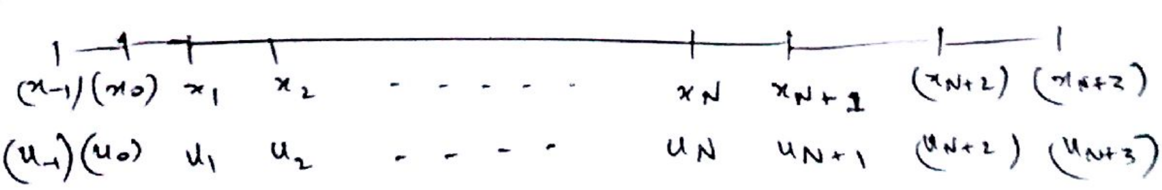
$$\boxed{\varphi_{n-\frac{1}{2}} = \frac{\Delta t}{\Delta x} \left(\frac{u_n + u_{n-1}}{2} \right)}$$

for the PDE :-

$$u_t + 6uu_x + u_{xxx} = 0 \qquad -20 < x < 20, \quad t > 0$$

IC: $u(x,0) = A \,\mathrm{sech}^2(kx)$, $k = \sqrt{\frac{A}{2}}$

BC (Periodic) $u(-20,t) = u(20,t)$



$$\boxed{\begin{array}{ll} u_0 = u_N & u_{-1} = u_{N-1} \\ u_{N+2} = u_2 & u_{N+3} = u_3 \end{array}}$$