- l_g = length of the airgap
- l_c = length of the core
- A_g = Cross-sectional area of the airgap
- A_c = Cross-sectional area of the core

By using Ampere's law:

$$\oint \mathbf{H} \cdot d\mathbf{l} = H_c l_c + H_g l_g = NI \qquad (1)$$

$$B_c A_c = B_g A_g = \Phi_c \qquad (2)$$

$$B = \mu_0 \mu_r H$$

where $\mu_0 = 4\pi \times 10^{-7}$ and $\mu_r = 1.05$ for air.
For steel, μ_r is in the range of 2000 to 6000.

$$B = \mu_0 \mu_{rc} H$$
$$B = \mu_0 \mu_{rg} H$$

Now,

μ_{rc} = Relative permeability of the core Material

μ_{rg} = Relative permeability of the Air

$$\mu_0 \mu_{rc} H_c A_c = \mu_0 \mu_{rg} H_g A_g = \Phi_c \qquad (3)$$

$$H_c = \frac{\Phi_c}{\mu_0 \mu_{rc} A_c}, \quad H_g = \frac{\Phi_c}{\mu_0 \mu_{rg} A_g}$$

Now, put H_c and H_g values in eq. (1):

$$\frac{\Phi_c}{\mu_0 A_c}\left[\frac{l_c}{\mu_{rc} A_c} + \frac{l_g}{\mu_{rg} A_g}\right] = NI \qquad (1)$$

Now, $A_C = A_g$

$$\frac{l_g}{\mu_{rg}} > \frac{l_c}{\mu_{rc}}$$

Since $\mu_0 = 4\pi \times 10^{-7}$

while $\mu_r \approx 2000$ to 6000 for steel

So, $\Phi_c = \dfrac{N_i}{\frac{l_g}{\mu_0 A_g}}$

$$NI = \Phi_c \left[\frac{l_g}{\mu_0 A_g} \right]$$

$$\boxed{NI = \Phi_c \left[\frac{l_g}{\mu_0 A_g} \right]}$$