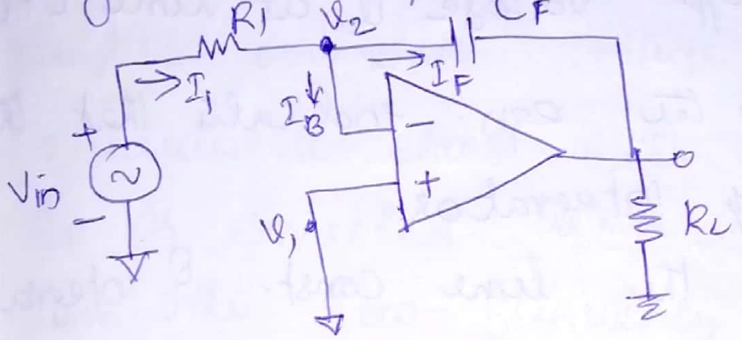
The Integrator

A ckt in which the o/p voltage is integral of i/p voltage is the integrator or the integration amper



In the inv amper the R_f is replaced with a capacitor C_f.

Write KCL at node v_2

$$i_1 = I_B + i_F$$

Since I_B is negligibly small

$$i_1 \cong i_F$$

The capacitor current $i_c = \frac{C\,dv_c}{dt}$

$$\therefore \quad \frac{v_{in} - v_2}{R_1} = C_F \frac{d}{dt}(v_2 - v_0)$$

However $v_1 = v_2 = 0$ (virtual gnd)

$$\frac{v_{in}}{R_1} = C_F \frac{d}{dt}(-V_0)$$

Integrating both sides w.r.to time

$$\int_0^t \frac{v_{in}}{R_1} = \int_0^t C_F \frac{d}{dt}(-V_0)$$

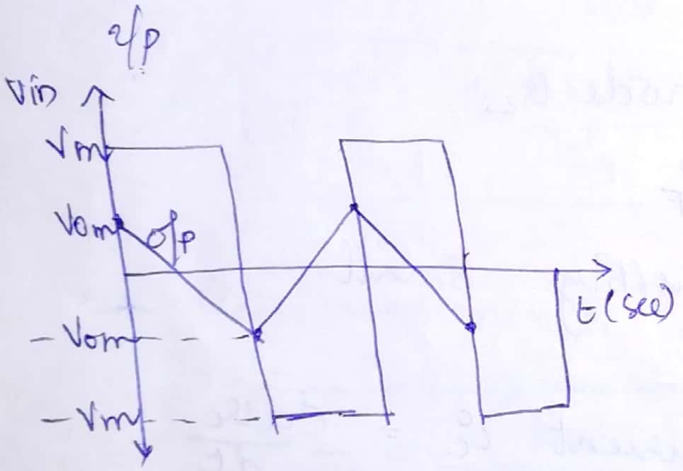
$$= C_F(V_0) + v_0 |_{t=0}.$$

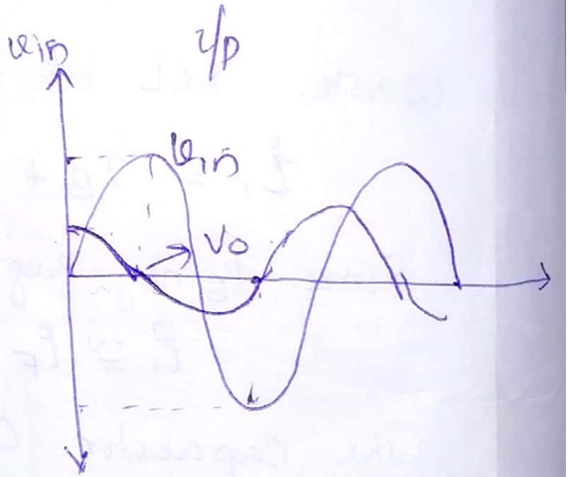
$$\therefore v_o = -\frac{1}{R_1 C_F} \int_0^t v_{in} \cdot dt + C$$

C → integration const and is proportional to the value of the o/p voltage v_o at time t=0 sec

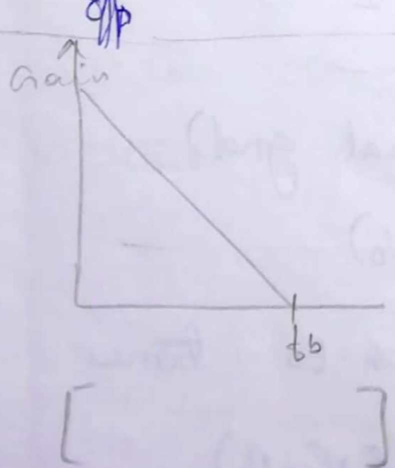
-ve sign in the eqn. indicates that the ckt as an inverting integrator

$R_1 C_F$ is the time const. of integrator.





o/p



o/p

$f_b = 10 f_a$

Generally $f_a < f_b$. If $f_a = f_b/10$ then $R_F = 10R_1$

The i/p signal will be integrated properly if the time period T of the signal is larger than or equal to $R_F C_F$.

$$T \geq R_F C_F$$

when $R_F C_F = \frac{1}{2\pi f_a}$.

If i/p freq < f_a → No integration

$f_{in} = f_a$ → 50% integration

$f_{in} = 10 f_a$ → 99% of accuracy

The integrator is most commonly used in analog computers and, ADC and signal waveshaping circuits

The Differentiator

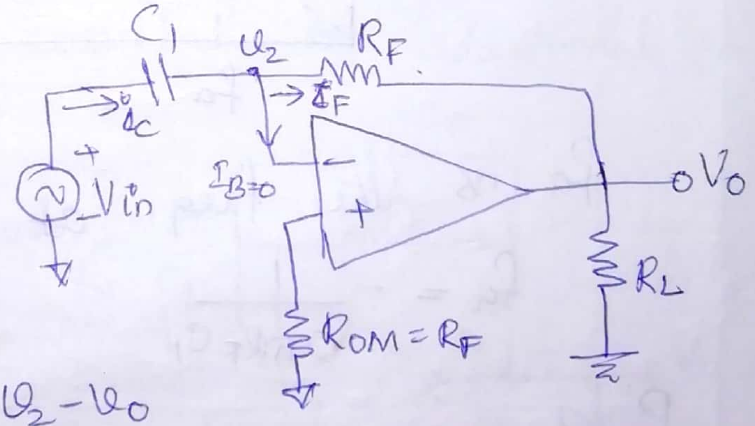
Differentiator or differentiation amplifier performs mathematical operation of differentiation. i.e., o/p waveform is the derivative of i/p waveform.

By KCL

$$i_C = I_B + i_F$$

Since $I_B = 0$

$$i_C = i_F$$



$$C_1 \frac{d}{dt}(V_{in} - v_2) = \frac{v_2 - v_o}{R_F}$$

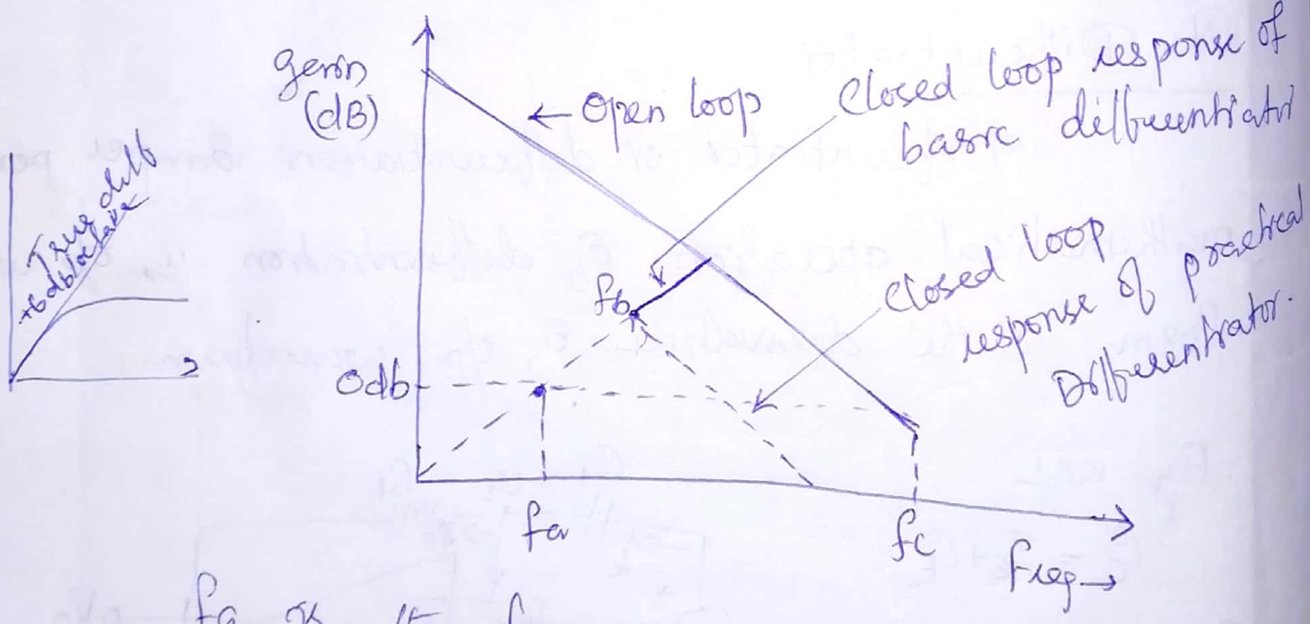
But $v_2 = 0$ (virtual gnd)

$$\frac{C_1 \, dv_{in}}{dt} = -\frac{v_o}{R_F}$$

$$\therefore \quad v_o = -R_F C_1 \frac{dv_{in}}{dt}$$

The gain of the circuit $\left(\frac{R_F}{X_{C_1}}\right)$ increases in freq. at a rate of 20dB/dec. This makes the ckt unstable. X_{C_1} decrease with increase in freq which makes the ckt very susceptible to high freq. noise.

Freq. response of differentiator



fa is the freq. at which gain is 0dB.

$$f_a = \frac{1}{2\pi R_F C_1}$$

Proof.
$$\frac{V_o(s)}{V_{in}(s)} = \frac{-Z_F(s)}{Z_i(s)}$$

where $Z_F(s) = R_F$ and $Z_i(s) = \frac{1}{sC_1}$