

Thm 2.1.8: (a) If $a \in R$ and $a \neq 0$, then $a^2 > 0$

(b) $1 > 0$ (c) If $n \in N$, then $n > 0$

Proof: (a) 1. By Trichotomy property, if $a \in R$, then exactly one of $a \in P$, $a = 0$, $-a \in P$ holds.

2. Suppose, $a \in P$ holds 3. By defn, $a > 0$ 4. $a \cdot a > 0 \cdot a$ (Thm 2.1.7(c) on 3)
5. $0 \cdot a = a \cdot 0$ (M1) 6. $a \cdot 0 = 0$ (Thm 2.1.2(c)) 7. $0 \cdot a = 0$ (Transitivity of eq on 5, 6)
8. $a \cdot a > 0$ (Substitution of eq 7 on 4) 9. $a^2 > 0$ (We write a^2 as $a \cdot a$)
10. $a \in P$ (By defn.)

11. $\because a \neq 0$, $a = 0$ is F 12. Suppose, $-a \in P$ holds 13. By defn, $(-a) > 0$
14. $(-a)(-a) > 0 \cdot (-a)$ (Thm 2.1.7(c) on 13) 15. $0 \cdot (-a) = (-a) \cdot 0$ (M1)
16. $(-a) \cdot 0 = 0$ (Thm 2.1.2(c)) 17. $0 \cdot (-a) = 0$ (Transitive prop of eq on 15, 16)
18. $(-a)(-a) > 0$ (Substitution of eq 17 on 14)
19. $a(-1)a = (-a)$ (Ex 2.1, 1(c)) 20. $-a = (-1) \cdot a$ (Symmetric prop of eq on 19)
21. $(-a)(-a) = ((-1) \cdot a) \cdot ((-1) \cdot a)$ (Substitution prop of eq 20)
22. $(-1) \cdot a = a \cdot (-1)$ (M1) 23. $((-1) \cdot a) \cdot ((-1) \cdot a) = (a \cdot (-1)) \cdot ((-1) \cdot a)$ (Substitution prop of eq 22)

24. $(a \cdot (-1)) \cdot ((-1) \cdot a) = a \cdot ((-1) \cdot ((-1) \cdot a))$ (M2)
25. $((-1) \cdot a) \cdot ((-1) \cdot a) = a \cdot ((-1) \cdot ((-1) \cdot a))$ (Transitivity of eq on 23, 24)
26. $((-1) \cdot (-1)) \cdot a = (-1) \cdot ((-1) \cdot a)$ (M2) 27. $(-1) \cdot ((-1) \cdot a) = ((-1) \cdot (-1)) \cdot a$ (Symmetry of eq on 26)

28. $a \cdot ((-1) \cdot ((-1) \cdot a)) = a \cdot (((-1) \cdot (-1)) \cdot a)$ (Substitution of eq 27)
29. $((-1) \cdot a) \cdot ((-1) \cdot a) = a \cdot (((-1) \cdot (-1)) \cdot a)$ (Transitivity of eq on 25, 28)
30. $(-1) \cdot (-1) = 1$ (Ex 2.1, 1(d)) 31. $((-1) \cdot (-1)) \cdot a = 1 \cdot a$ (Substitution of eq 30)

32. $1 \cdot a = a$ (M3) 33. $((-1) \cdot (-1)) \cdot a = a$ (Transitivity of eq on 31, 32)
34. $a \cdot (((-1) \cdot (-1)) \cdot a) = a \cdot a$ (Substitution of eq 33)
35. $((-1) \cdot a) \cdot ((-1) \cdot a) = a \cdot a$ (Transitivity of eq on 29, 34)