

3. Suppose,  $a \in P$  holds.  $\therefore$  By defn,  $a > 0$
4. By Trichotomy Property, exactly one of the following hold:  $b \in P, b = 0, -b \in P$
5. Suppose,  $b \in P$  holds.  $\therefore$  By defn,  $b > 0$
6.  $b \cdot a > b \cdot 0$  (Thm 2.1.7(c)) 7.  $b \cdot a = a \cdot b$  (M1) 8.  $a \cdot b > b \cdot 0$   
(Substitute eq 7 in 6)
9.  $b \cdot 0 = 0$  (Thm 2.1.2(c)) 10.  $a \cdot b > 0$  (Substitute eq 9 in 8)
11.  $\therefore$  When  $a > 0, b > 0, ab > 0$ . ✓
12. Suppose,  $b = 0$  holds 13.  $a \cdot b = a \cdot 0$  (Substitution of eq 12)
14.  $a \cdot 0 = 0$  (Thm 2.1.2(c)) 15.  $a \cdot b = 0$  (Transitivity of eq on 13, 14)
16. By Trichotomy Property, exactly one of the following hold:  $ab \in P, ab = 0, -ab \in P$ .  
 $-ab \in P. \therefore b \neq 0$
17. Suppose,  ~~$a \in P$~~   $-b \in P$  holds.  $\therefore$  by defn,  $b < 0$ .
18.  $b \cdot a < b \cdot 0$  (Thm 2.1.7(c)) 19.  $a \cdot b < b \cdot 0$  (Substitute eq 7 in 18)
20.  $a \cdot b < 0$  (Substitute eq 9 in 19) 21.  ~~$ab \in P$~~   $-ab \in P$  (By defn.)
22. Applying 16, we conclude  $b < 0$  is False
23. Suppose,  $a = 0$  holds 24.  $a \cdot b = 0 \cdot b$  (Substitute eq 23)
25.  $0 \cdot b = b \cdot 0$  (M1) 26.  $0 \cdot b = 0$  (Transitivity of eq on 24, 25)
27.  $a \cdot b = 0$  (Transitivity of eq on 24, 26)
28. Applying 16 and 1, we conclude  $a \neq 0$
29. Suppose,  $-a \in P$  holds.  $\therefore$  By defn,  $a < 0$  or,  $0 > a$
30. Statement 4 holds 31. Suppose,  $b \in P$  holds.  $\therefore$  By defn,  $b > 0$
32.  $a \cdot b < a \cdot 0$  (Thm 2.1.7(c)) 33.  $a \cdot b < 0$  (Substitute eq 14 in 32)
34. Applying 16 and 1, we know  $b > 0$  is F
35. By 1, 15, 16 we know  $b \neq 0$
36. Suppose,  $-b \in P$  holds. By defn,  $b < 0$
37.  ~~$b \cdot 0 < b \cdot a$~~   $b \cdot 0 < b \cdot a$  (Thm 2.1.7(c))
38.  $b \cdot 0 < a \cdot b$  (Substitute eq 7 in 37) 39.  $0 < a \cdot b$  (Substitute eq 9 in 38)  
on  $ab > 0$
- $\therefore$  When  $a < 0$  and  $b < 0, ab > 0$