10. $5+0 = 5$ (A3) 11. $5+(1+(-1)) = 5$ (Transitivity of eq on 9, 10)

12. $(5+1)+(-1) = 5$ (Transitivity of eq on 7, 11)

13. $6+(-1) = 5$ (Transitivity of eq on 6, 12)

14. $5 > (5x+1)+(-1)$ (Substitute eq 13 in 4)

15. $(5x+1)+(-1) = 5x+(1+(-1))$ (A2) 16. $5x+(1+(-1)) = 5x+0$ (Substitute eq 8)

17. $5x+0 = 5x$ (A3) 18. $5x+(1+(-1)) = 5x$ (Transitivity of eq on 16, 17)

19. $(5x+1)+(-1) = 5x$ (Transitivity of eq on 15, 18)

20. $5 > 5x$ (Substitute eq 19 in 14)

21. ~~~~ Claim: $1/5 > 0$

22. Proof: By Trichotomy property, exactly one of the following hold
$$\frac{1}{5} \varepsilon P, \quad \frac{1}{5} = 0, \quad -\left(\frac{1}{5}\right) \varepsilon P$$

23. Let $-(1/5) \varepsilon P$ hold $\therefore$ By defn, $0 > \frac{1}{5}$

24. $\because 5 \varepsilon N$, $\therefore 5 > 0$ (Thm 2.1.8(c)) 26. $5 \cdot 0 = 0$ (Thm 2.1.2(c))

25. ~~~~ $5 \cdot 0 > 5 \cdot \frac{1}{5}$ (Thm 2.1.7(c)) 28. $5 \cdot \frac{1}{5} = 1$ (Apply 24, order prop) and M4

27. $0 > 5 \cdot \frac{1}{5}$ (Substitute eq 26 in 25)

29. $0 > 1$ (Substitute eq 28 in 27)

30. $1 > 0$ (Thm 2.1.8(b)) 31. Statement 29 contradicts Statement 30

because by ~~~~ trichotomy prop of R, exactly 1 holds : $1 > 0$, $1 = 0$, $1 < 0$

$\therefore$ 29 is F, and $0 > 1/5$ is F

32. Let, $0 = \frac{1}{5}$ hold 33. ~~~~ $5 \cdot 0 = 5 \cdot \frac{1}{5}$ (Substitute eq 32)

34. $0 = 5 \cdot 0$ (Symmetry of eq 26) 35. $0 = 5 \cdot \frac{1}{5}$ (Transitivity of eq on 34, 33)

36. $0 = 1$ (Transitivity of eq on 35, 28)

37. $0 < 1$ (Thm 2.1.8(b)) 38. By Trichotomy prop, 36 is F, $\therefore 0 = \frac{1}{5}$ is F

39. To validate Stmt 22, $\therefore \frac{1}{5} \varepsilon P$ is T or $\frac{1}{5} > 0$ (By defn.)

40. $\frac{1}{5} \cdot 5 > \frac{1}{5} \cdot (5x \cdot x)$ (Thm 2.1.7(c))