11. $b=((\frac{1}{a})\cdot a)\cdot b$ (Symmetry of eq on 10) 12. $b=\frac{1}{a}$ (Transitivity of eq on 11, 7)

(b) 1. $a\cdot b=0$ (Given) 2. Let, $a\neq 0$. (Assume). $\therefore \exists (\frac{1}{a})\in R$ s.t. $(\frac{1}{a})\cdot a=1$ (M4)

3. $(\frac{1}{a})\cdot(a\cdot b)=(\frac{1}{a})\cdot 0$ (Substitution of eq on 1)

4. $(\frac{1}{a})\cdot 0=0$ (Thm 2.1.2(c)) 5. $(\frac{1}{a})\cdot(a\cdot b)=0$ (Transitivity of eq on 3,4)

6. $((\frac{1}{a})\cdot a)\cdot b=(\frac{1}{a})\cdot(a\cdot b)$ (M2) 7. $((\frac{1}{a})\cdot a)\cdot b=0$ (Transitivity of eq on 6,5)

8. $((\frac{1}{a})\cdot a)\cdot b=1\cdot b$ (Substitution of eq on 2) 9. $1\cdot b=b$ (M3)

10. $((\frac{1}{a})\cdot a)\cdot b=b$ (Transitivity of eq on 8,9)

11. $b=((\frac{1}{a})\cdot a)\cdot b$ (Symmetry of eq on 10) 12. $b=0$ (Transitivity of eq on 11,7)

13. If $a=0$, we are done

(Very small subset of properties we have proven)

- Subtraction: $\forall a,b\in R$, $a-b=a+(-b)$
- Division: $\forall a,b\in R$, and $b\neq 0$, $a/b=a\cdot(1/b)$