28. $(-1)\cdot b+(-1)\cdot c=(-b)+(-c)$ (Transitive prop of eq on 27, 25)

29. $(-1)\cdot(b+c)=(-1)\cdot b+(-1)\cdot c$ (D) 30. $(-1)\cdot(b+c)=(-b)+(-c)$ (Transitive prop of eq on 29, 28)

31. $(-1)\cdot(b+c)=-(b+c)$ (Ex 2.1, 1(c))

32. $-(b+c)=(-1)\cdot(b+c)$ (Symmetry of eq on 31) 33. $-(b+c)=(-b)+(-c)$ (Transitive prop of eq on 32, 31)

34. $(a+c)+(-(b+c))=(a+c)+((-b)+(-c))$ (Substitution of eq on 33)

35. $(a+c)+(-(b+c))=(a-b)$ (Transitive prop of eq on 34, 23)

36. $(a+c)-(b+c)=(a+c)+(-(b+c))$ (Defn. of subtraction)

37. $(a+c)-(b+c)=(a-b)$ (Transitivity of eq on 36, 35)

38. $(a+c)-(b+c)=c_1$ (Transitivity of eq on 37, 2)

39. $\because c_1>0$, $\therefore (a+c)-(b+c)\in P$, and by Defn. 2.1.6(a), $\boxed{a+c>b+c}$ (Proved)

(c) 1. Given, $a>b$ $\therefore$ By defn 2.1.6(a), $a-b\in P$

2. $\because$ P is a non-empty subset of R, $\exists c_1\in P$ s.t. $(a-b)=c_1$

3. $c_1=(a-b)$ (Symmetry of eq on 2) 4. $a-b=a+(-b)$ (Defn. of subtraction)

5. $c_1=a+(-b)$ (Transitivity of eq on 3, 4)

6. $(-1)\cdot b=(-b)$ (Ex-2.1 1(c)) 7. $c>0$ (Given) $\therefore$ By defn 2.1.6(a), $c-0\in P$

8. $\because$ P is a non-empty subset of R, $\exists c_2\in P$ s.t. $c-0=c_2$

9. $c_2=c-0$ (Symmetry of eq on 8) 10. $c-0=c+(-0)$ (Defn. of subtract)

11. $c_2=c+(-0)$ (Symmetry of eq on 9, 10) 12. $(-1)\cdot 0=(-0)$ (Ex 2.1 1(c))

13. $(-1)\cdot 0=0$ (Thm 2.1.2(c)) 14. $0=(-1)\cdot 0$ (Symmetry of eq on 13)

15. $0=(-0)$ (Transitivity of eq on 14, 12) 16. $c+0=c+(-0)$ (Substitution of eq on 15)

17. $c+(-0)=c+0$ (Symmetry of eq on 16)

18. $c_2=c+0$ (Transitivity of eq on 11, 17) 19. $c+0=c$ (A3)

20. $c_2=c$ (Transitivity of eq on 18, 19) 21. $c\in P$ (From 8, 20)

~~22. $c_1 c=(a+(-b))\cdot c$ (Substitution of eq on 5)~~