Theorem 2.1.7(c) Contd.

1. $c<0$ (Given) 2. $0-c\in P$ (by defn. 2.1.6(a)) 3. $\because P$ is a non-empty subset of $\mathbb{R}$, $\exists c_1\in P$ s.t. $0-c=c_1$ 4. $0-c=0+(-c)$ (Defn. of subtraction)

5. $c_1=0-c$ (Symmetry of eq on 3) 6. $c_1=0+(-c)$ (Transitivity of eq on 5,4)

7. $0+(-c)=(-c)$ (A3) 8. $c_1=(-c)$ (Transitivity of eq on 6,7)

9. $a>b$ (Given)

10. $c_1>0$ (By 3, Order Prop) 11. $c_1a>c_1b$ (1st part)

12. $c_1a-c_1b\in P$ (By defn. 2.1.6(a)) 13. $\because P$ is a non-empty subset of $\mathbb{R}$, $\exists c_2\in P$ s.t. $c_1a-c_1b=c_2$ (on 12)

14. $c_1\cdot a=(-c)\cdot a$ (Substitution of eq on 8) 15. $c_1\cdot a-c_1\cdot b=(-c)\cdot a-c_1\cdot b$ (Substitution of eq on 14)

16. $(-c)\cdot a-c_1\cdot b=c_1\cdot a-c_1\cdot b$ (Symmetry of eq on 15)

17. $(-c)\cdot a-c_1\cdot b=c_2$ (Transitivity of eq on 16,12)

15. $c_1\cdot a+(-(c_1\cdot b))=(-c)\cdot a+(-(c_1\cdot b))$ (Substitution of eq on 14)

16. $c_1\cdot a-c_1\cdot b=c_1\cdot a+(-(c_1\cdot b))$ (Defn. of subtraction)

17. $c_1\cdot a-c_1\cdot b=(-c)\cdot a+(-(c_1\cdot b))$ (Transitivity of eq on 16,15)

18. $(-c)\cdot a+(-(c_1\cdot b))=c_1\cdot a-c_1\cdot b$ (Symmetry of eq on 17)

19. $(-c)\cdot a+(-(c_1\cdot b))=c_2$ (Transitivity of eq on 18,12)

20. $c_1\cdot b=(-c)\cdot b$ (Substitution of eq on 8)

21. $(-1)\cdot c=(-c)$ (Ex-2.1.1(c)) 22. $((-1)\cdot c)\cdot b=(-c)\cdot b$ (Substitution of eq on 21)

23. $((-1)\cdot c)\cdot b=(-1)\cdot(c\cdot b)$ (M2) 24. $(-1)\cdot(c\cdot b)=((-1)\cdot c)\cdot b$ (Symmetry of eq on 23)

25. $(-1)\cdot(c\cdot b)=(-c)\cdot b$ (Transitivity of eq on 24,22)

26. $(-c)\cdot b=(-1)\cdot(c\cdot b)$ (Symmetry of eq on 25)

27. $c_1\cdot b=(-1)\cdot(c\cdot b)$ (Transitivity of eq on 20, 26)

28. $(-1)\cdot(c_1\cdot b)=(-1)\cdot((-1)\cdot(c\cdot b))$ (Substitution of eq on 27)

29. $(-1)\cdot(c_1\cdot b)=-(c_1\cdot b)$ (Ex 2.1 1(c)) 30. $-(c_1\cdot b)=(-1)\cdot(c_1\cdot b)$ (Symmetry of eq on 29)