$\boxed{\text{Theorem 2.1.2:}}$ (a) If z and a are elements in R with $z+a=a$, then $z=0$
(b) If u and $b \neq 0$ are elements in R with $u \cdot b = b$, then $u=1$

$\Big\}$ → uniqueness of 0 and 1

(c) If $a \in R$, then $a \cdot 0 = 0$ (Multiplication by zero is always zero)

$\boxed{\text{Proof:}}$ (a) ① Suppose, $\exists\, z, a \in R$ s.t. $z+a=a$.

② . By A4, for each a, there exists $(-a) \in R$.

③ . $(z+a)+(-a) = \overset{\in R}{a}+(-a)$ (Substitution of equality on 1)

④ . $(z+a)+(-a) = z+(a+(-a))$ (A2)

⑤ . $z+(a+(-a)) = (z+a)+(-a)$ (Symmetry of equality on 4)

⑥ . $z+(a+(-a)) = a+(-a)$ (Transitivity of equality on 5,3)

⑦ . $a+(-a)=0$ (A4)

⑧ $z+(a+(-a)) = 0$ (Transitivity of equality on 6,7)

⑨ $z+(a+(-a)) = z+0$ (Substitution of equality on 7)

⑩ $z+0=z$ (A3) ⑪ $z+(a+(-a)) = z$ (Transitivity of equality on 9,10)

⑫ $z = z+(a+(-a))$ (Symmetry of eq on 11)

⑬ $z=0$ (Transitivity of eq on 12,8)

(b) 1. Suppose, $\exists\, u, b \in R$ and $b \neq 0$ s.t. $u \cdot b = b$

2. By M4, for each $b \in R$, $b \neq 0$, there exists $\left(\frac{1}{b}\right) \in R$

3. $(u \cdot b) \cdot \left(\frac{1}{b}\right) = b \cdot \left(\frac{1}{b}\right)$ (Substitution of equality on 1)

4. $(u \cdot b) \cdot \left(\frac{1}{b}\right) = u \cdot \left(b \cdot \left(\frac{1}{b}\right)\right)$ (M2)

5. $u \cdot \left(b \cdot \left(\frac{1}{b}\right)\right) = (u \cdot b) \cdot \left(\frac{1}{b}\right)$ (Symmetry of eq on 4)

6. $u \cdot \left(b \cdot \left(\frac{1}{b}\right)\right) = b \cdot \left(\frac{1}{b}\right)$ (Transitivity of eq on 5,3)

7. $b \cdot \left(\frac{1}{b}\right) = 1$ (M4)

8. $u \cdot \left(b \cdot \left(\frac{1}{b}\right)\right) = 1$ (Transitivity of eq on 6,7)