

to zero, we see that it is a positive number.

• **Theorem 2.1.9:** If  $a \in \mathbb{R}$  is such that  $0 < a < \varepsilon$  for every  $\varepsilon > 0$ , then  $a = 0$ .

**Proof:** Suppose,  $a > 0$ . Let,  $a < \varepsilon$  for every  $\varepsilon > 0$ .  $\therefore$  If  $\varepsilon_0 = \frac{1}{2}a$ ,  
by our prev claim,  $\frac{1}{2}a = \varepsilon < a$ , which is a contradiction  
 $\therefore a = 0$

• **Theorem 2.1.10:** If  $ab > 0$ , then either i)  $a > 0$  and  $b > 0$ , or ii)  $a < 0$  and  $b < 0$

**Proof:** 1. Given,  $ab > 0$  2. By <sup>trichotomy</sup> order property, ~~either~~ exactly one of the following hold:  $a \in \mathbb{P}$ ,  $a = 0$ ,  $-a \in \mathbb{P}$