31. $((-1)\cdot(-1))\cdot(c\cdot b)=(-1)\cdot((-1)\cdot(c\cdot b))$ (M2)

32. $(-1)\cdot(-1)=1$ (Ex 2.1, 1(d)) 33. $((-1)\cdot(-1))\cdot(c\cdot b)=1\cdot(c\cdot b)$ (Substitution of eq on 32)

34. $1\cdot(c\cdot b)=c\cdot b$ (M3)

35. $((-1)\cdot(-1))\cdot(c\cdot b)=c\cdot b$ (Transitivity of eq on 33, 34)

36. $c\cdot b=((-1)\cdot(-1))\cdot(c\cdot b)$ (Symmetry of eq on 35)

37. $c\cdot b=(-1)\cdot((-1)\cdot(c\cdot b))$ (Transitivity of eq on 36, 31)

38. $(-1)\cdot((-1)\cdot(c\cdot b))=c\cdot b$ (Symmetry of eq on 37)

39. $(-1)\cdot(c_1\cdot b)=c\cdot b$ (Transitivity of eq on 28, 38)

40. $-(c_1\cdot b)=c\cdot b$ (Transitivity of eq on 30, 39)

41. $(-c)\cdot a+(-(c_1\cdot b))=(-c)\cdot a+c\cdot b$ (Substitution of eq on 40)

42. $(-c)\cdot a+c\cdot b=(-c)\cdot a+(-(c_1\cdot b))$ (Symmetry of eq on 41)

43. $(-c)\cdot a+(c\cdot b)=c_2$ (Transitivity of eq 42, 19)

44. $(c\cdot b)+(-c)\cdot a=(-c)\cdot a+(c\cdot b)$ (A1) 45. $(c\cdot b)+(-c)\cdot a=c_2$ (Transitivity of eq on 44, 43)

46. $(-1)\cdot c=-(c)$ (Ex 2.1, 1(c)) 47. $((-1)\cdot c)\cdot a=(-c)\cdot a$ (Substitution of eq on 46)

48. $((-1)\cdot c)\cdot a=(-1)\cdot(c\cdot a)$ (M2)

49. $(-1)\cdot(c\cdot a)=((-1)\cdot c)\cdot a$ (Symmetry of eq on 48)

50. $(-1)\cdot(c\cdot a)=(-c)\cdot a$ (Symmetry of eq on 49, 47)

51. $c\cdot b+(-1)\cdot(c\cdot a)=c\cdot b+(-c)\cdot a$ (Substitution of eq on 50)

52. $c\cdot b+(-1)\cdot(c\cdot a)=c_2$ (Transitivity of eq on 51, 45)

53. $c\cdot b-c\cdot a=c\cdot b+(-1)\cdot(c\cdot a)$ (Defn. of subtraction)

54. $c\cdot b-c\cdot a=c_2$ (Transitivity of eq on 53, 52)

55. $c\cdot b-c\cdot a\in P$ (By 12) $\therefore ca<cb$ (Defn 2.1.6(a))

- It is natural to expect that the natural numbers are positive real numbers. This property is derived from the basic properties of order. The key observation is that the square of any nonzero real number is positive