Theorem 2.1.7: Let a, b, c be any elements of $\mathbb{R}$

(a) If $a > b$ and $b > c$, then $a > c$. (b) If $a > b$, then $a+c > b+c$

(c) If $a > b$ and $c > 0$, then $ca > cb$. If $c < 0$, then $ca < cb$

Proof: (a) 1. $a > b$ is Given, So $a-b \in P$ (Defn 2.1.6(a))

2. $\because P$ is non-empty, let $\exists c_1 \in P$ s.t. $(a-b) = c_1$

3. $b > c$ is Given, So $b-c \in P$ (Defn 2.1.6(a))

4. $\because P$ is non-empty, let $\exists c_2 \in P$ s.t. $(b-c) = c_2$

5. $(a-b)+(b-c) = c_1+(b-c)$ (Substitution of eq on 2)

6. $c_1+(b-c) = c_1+c_2$ (Substitution of eq on 4)

7. $(a-b)+(b-c) = c_1+c_2$ (Transitivity of eq on 5, 6)

8. $(a-b) = a+(-b)$ (Defn. of subtraction)

9. $(b-c) = b+(-c)$ (Defn. of subtraction)

10. $(a-b)+(b-c) = (a+(-b))+(b-c)$ (Substitution of eq on 8)

11. $(a+(-b))+(b-c) = (a+(-b))+(b+(-c))$ (Substitution of eq on 9)

12. $(a-b)+(b-c) = (a+(-b))+(b+(-c))$ (Transitivity of eq on 10, 11)

13. $(a+(-b))+(b+(-c)) = (a-b)+(b-c)$ (Symmetry of eq on 12)

14. $(a+(-b))+(b+(-c)) = c_1+c_2$ (Transitivity of eq on 13, 7)

15. $((a+(-b))+b)+(-c) = (a+(-b))+(b+(-c))$ (A2)

16. $(a+(-b))+b = a+((-b)+b)$ (A2)

17. $(-b)+b = 0$ (A4) 18. $a+((-b)+b) = a+0$ (Substitution of eq on 17)

19. $a+0 = a$ (A3) 20. $a+((-b)+b) = a$ (Transitivity of eq on 18, 19)

21. $(a+(-b))+b = a$ (Transitivity of eq on 16, 20)

22. $((a+(-b))+b)+(-c) = a+(-c)$ (Substitution of eq on 21)

23. $((a+(-b))+b)+(-c) = c_1+c_2$ (Transitivity of eq on 15, 14)

24. $a+(-c) = ((a+(-b))+b)+(-c)$ (Symmetry of eq on 22)

25. $a+(-c) = c_1+c_2$ (Transitivity of eq on 24, 23)