

9. $u \cdot (b \cdot (\frac{1}{b})) = u \cdot 1$ (Substitution of eq on 7) 10. $u \cdot 1 = u$ (M3)
 11. $u \cdot (b \cdot (\frac{1}{b})) = u$ (Transitivity of eq on 9, 10) 12. $u = u \cdot (b \cdot (\frac{1}{b}))$ (Symmetry of eq on 11)
 13. $u = 1$ (Transitivity of eq on 12, 8)

- (1)
 (c) Suppose, $\exists a \in R$ where $a \cdot 0 = b$, $b \neq 0$ and $b \in R$
 2) Let $a \neq 0 \therefore \exists (\frac{1}{a}) \in R$ s.t. $a \cdot (\frac{1}{a}) = 1$ (M4) and $(\frac{1}{a}) \cdot a = 1$
 3) $(\frac{1}{a}) \cdot (a \cdot 0) = (\frac{1}{a}) \cdot b$ (Substitution of eq on 1)
 4) $((\frac{1}{a}) \cdot a) \cdot 0 = (\frac{1}{a}) \cdot (a \cdot 0)$ (M2) 5) $((\frac{1}{a}) \cdot a) \cdot 0 = (\frac{1}{a}) \cdot b$ (Transitivity of eq on 4, 3)
 6) $((\frac{1}{a}) \cdot a) \cdot 0 = 1 \cdot 0$ (Substitution of eq on 2) 7) $1 \cdot 0 = 0$ (M3)
 8) $((\frac{1}{a}) \cdot a) \cdot 0 = 0$ (Transitivity of eq on 6, 7) \rightarrow It will not work

- (c) Very beautiful proof (From book)
 1) $a \cdot 1 = a \quad \forall a \in R$ (M3) 2) $a \cdot 1 + a \cdot 0 = a + a \cdot 0$ (Substitution of eq on 1)
 3) $a \cdot (1+0) = a \cdot 1 + a \cdot 0$ (D) 4) $a \cdot (1+0) = a + a \cdot 0$ (Transitivity of eq on 3, 2)
 5) $1+0 = 1$ (A3) 6) $a \cdot (1+0) = a \cdot 1$ (Substitution of eq on 5)
 7) $a \cdot (1+0) = a$ (Transitivity of eq on 6, 1) 8) $a + a \cdot 0 = a \cdot (1+0)$ (Symmetry of eq on 4)
 9) $a + a \cdot 0 = a$ (Transitivity of eq on 8, 7) 10) $a \cdot 0 = 0$ (Thm 2.1.2(a))

Theorem 2.1.3: (a) If $a \neq 0$ and b in R are such that $a \cdot b = 1$, then $b = 1/a$ (Uniqueness of multiplicative inverse)
 (b) If $a \cdot b = 0$, then either $a = 0$ or $b = 0$

Proof: (a) 1. $\because a \neq 0, \exists (\frac{1}{a}) \in R$ s.t. $(\frac{1}{a}) \cdot a = 1$ (M4)
 2. $a \cdot b = 1$ (Given) 3. $(\frac{1}{a}) \cdot (a \cdot b) = (\frac{1}{a}) \cdot 1$ (Substitution of eq on 2)
 4. $(\frac{1}{a}) \cdot 1 = \frac{1}{a}$ (M3) 5. $(\frac{1}{a}) \cdot (a \cdot b) = \frac{1}{a}$ (Transitivity of eq on 3, 4)
 6. $((\frac{1}{a}) \cdot a) \cdot b = (\frac{1}{a}) \cdot (a \cdot b)$ (M2) 7. $((\frac{1}{a}) \cdot a) \cdot b = \frac{1}{a}$ (Transitive prop of eq on 6, 5)
 8. $(\frac{1}{a}) \cdot a \cdot b = 1 \cdot b$ (Substitution of eq on 1) 9. $1 \cdot b = b$ (M3)
 10. $((\frac{1}{a}) \cdot a) \cdot b = b$ (Transitivity of eq on 9, 8)