

$$35. 1+(-1)=0 \text{ (A4)} \quad 36. 2+(-1) < 0 \text{ (Substitution of eq 35 in 34)}$$

$$37. 2+(-1)=1 \text{ ?} \quad 38. 1 < 0 \text{ (Substitution of eq 37 in 36)}$$

$$39. 1 > 0 \text{ (Thm 2.1.8(b))} \quad 40. \text{By Trichotomy prop, } 1 < 0 \text{ is a contradiction}$$

$$41. \therefore 1 < \frac{1}{2} \text{ is F} \quad 42. \text{Let, } 1 = \frac{1}{2} \text{ (hold)} \quad 43. 1 \cdot 2 = \frac{1}{2} \cdot 2 \text{ (Substitution of eq 42)}$$

$$44. 1 \cdot 2 = 1 \text{ (Transitivity of eq on 43, 6)}$$

$$45. 2 = 1 \cdot 2 \text{ (Symmetry of eq 31)} \quad 46. 2 = 1 \text{ (Transitivity of eq 45, 44)}$$

$$47. 2+(-1)=1+(-1) \text{ (Substitution of eq 46)} \quad 48. 2+(-1)=0 \text{ (Transitivity of eq 47, 35)}$$

$$49. 1=2+(-1) \text{ (Symmetry of eq 37)} \quad 50. 1=0 \text{ (Transitivity of eq 49, 48)}$$

$$51. 1 \neq 0 \text{ (M3)} \quad 52. \therefore 1=0 \text{ is F, and } 1 = \frac{1}{2} \text{ is F}$$

$$53. \text{By the Trichotomy prop, } 1 > \frac{1}{2}$$

$$54. 1 \cdot a > \frac{1}{2} \cdot a \text{ (Thm 2.1.7(c))} \quad 55. 1 \cdot a = a \text{ (M3)} \quad 56. a > \frac{1}{2} \cdot a \text{ (Substitute eq 55 on 54)}$$

$$\boxed{\therefore a > \frac{1}{2} \cdot a > 0}$$

\therefore If it is claimed a is the smallest +ve real no, we can exhibit a smaller +ve no. $\frac{1}{2}a$.

This observation leads to the next result, which will be used frequently as a method of proof. For instance, to prove that a number $a > 0$ is actually equal to zero, we see that it suffices to show that a is smaller than any arbitrary positive number.