

$$1) K^{2K} + K^{2K} \sqrt{(K+1)^{2K}} \sqrt{(K+1)^{2K+1}} \quad \square.$$

$$\boxed{n^{2^n} + 6 \cdot 2^n \gg n^{2^n} \text{ for all } n \gg 0.}$$

$$\therefore \boxed{n^{2^n} \sqrt{n^{2^n} + 6 \cdot 2^n} \sqrt{2n^{2^n}} \text{ for all } n \gg 3}$$

$$c_1 = 1, c_2 = 2, n_0 = 3 \quad \therefore n^{2^n} + 6 \cdot 2^n = \theta(n^{2^n})$$

$$g) n^3 + 10^6 n^2 = \theta(n^3) \quad n^3 + 10^6 n^2 \gg n^3 \text{ for all } n \in \mathbb{N}, n \gg 0$$

$$n^3 + 10^6 n^2 = n^3 + (1000n)^2 \sqrt{n^3 + 10^6 n^3} = (10^6 + 1)n^3$$

$$n^3 \sqrt{n^3 + 10^6 n^2} \sqrt{(10^6 + 1)n^3}, n \gg 1, n \in \mathbb{N}$$

$$\therefore c_1 = 1, c_2 = 10^6 + 1, n_0 = 1 \quad \boxed{\therefore n^3 + 10^6 n^2 = \theta(n^3)}$$

$$h) 6n^3 / (\log n + 1) = O(n^3)$$

$$\text{Suppose, } \frac{6n^3}{\log n + 1} > 6n^3 \text{ for } n \in \mathbb{N}, n \gg 1$$

$$\log n + 1 > 1 \Rightarrow \log n > 0 \Rightarrow n > 1 \text{ (Contradiction)}$$

$$\therefore \frac{6n^3}{\log n + 1} \sqrt{6n^3} \text{ for } n \in \mathbb{N}, n \gg 1$$

$$n_0 = 1, c = 6 \quad \therefore 6n^3 / (\log n + 1) = O(n^3)$$

$$i) n^{1.001} + n \log n = \theta(n^{1.001}) \quad n^{1.001} + n \log n \gg n^{1.001} \text{ for } \forall n \in \mathbb{N}, n \gg 1$$

$$n^{1.001} + n \log n = (n^{\frac{1}{1000}})^{1001} + n \log n \sqrt{(n^{\frac{1}{1000}})^{1001} + (n^{\frac{1}{1000}})^{1001}} \quad \forall n \in \mathbb{N}, n \gg 2^{1000}$$

$$\boxed{n^{1.001} \sqrt{n^{1.001} + n \log n} \sqrt{2(n^{\frac{1}{1000}})^{1001}} \quad c_1 = 1, c_2 = 2, n_0 = 2^{1000}}$$

$$\therefore n^{1.001} + n \log n = \theta(n^{1.001})$$

$$j) n^K + n + n^K \log n = \theta(n^K \log n) \text{ for all } K \gg 1$$

$$n^K + n + n^K \log n \gg n^K \log n \text{ for } \forall n \in \mathbb{N}, n \gg 1. K \gg 1 \text{ is given.}$$

$$n^K + n + n^K \log n \sqrt{n^K \log n + n^K \log n + n^K \log n} = 3n^K \log n, \forall n \in \mathbb{N}, n \gg 1$$

$$n^K \log n \sqrt{n^K + n + n^K \log n} \sqrt{3n^K \log n}, n \gg 1 \quad \therefore c_1 = 1, c_2 = 3, n_0 = 1$$

$$\therefore n^K + n + n^K \log n = \theta(n^K \log n)$$