Theorem 1.2: If $f(n) = a_m n^m + \dots\dots + a_1 n + a_0$, then $f(n) = O(n^m)$.
(f is non-negative)

Proof: **Claim:** $f(n) \leq (|a_m| + |a_{m-1}| + \dots\dots + |a_1| + |a_0|)\, n^m$; for all $m \geq 0$ and $n \geq 0$.

Proof: The proof will be by mathematical induction on m.

Base Case: $m = 0$. $f(n) = a_0$ R.H.S. $|a_0| \cdot n^0 = |a_0|$. It's a standard result by defn. that $a_0 \leq |a_0|$. ∴ Base Case holds.

I.H: Let, for some $m = K$, $(a_K n^K + a_{K-1} n^{K-1} + \dots\dots + a_1 n + a_0) \leq (|a_K| + |a_{K-1}| + \dots\dots + |a_1| + |a_0|)\, n^K$ holds.

Induction Step: $m = K+1$. $f(n) = (a_{K+1} n^{K+1} + a_K n^K + a_{K-1} n^{K-1} + \dots + a_1 n + a_0)$

$\leq a_{K+1} n^{K+1} + (|a_K| + |a_{K-1}| + \dots\dots + |a_1| + |a_0|)\, n^K$ (By I.H.)

$\leq a_{K+1} n^{K+1} + (|a_K| + |a_{K-1}| + \dots\dots + |a_1| + |a_0|)\, n^{K+1}$ ($\because n \geq 0$)

$\leq (|a_{K+1}| + |a_K| + \dots\dots + |a_1| + |a_0|)\, n^{K+1}$ (By defn.)

- We can give a ~~the claim or the~~ similar proof for $(n+1)$. From there, we can conclude that for all non-negative m, n, our inequality holds. We already can see that when $n = 0$, $f(n) = 0$ and R.H.S. $= 0$. ∴ Base Case holds □

- The case when $n \geq 0$ and $m \leq 0$ can be proved similarly. The proof will be by mathematical induction on $\boxed{m_1 = -m}$.

So, $f(n) = O(n^m)$. □

Defn: [Omega] $f(n) = \Omega(g(n))$ (read as "f of n is omega of g of n") iff there exist positive constants c and n_0 such that $f(n) \geq c\, g(n)$ for all n, $n \geq n_0$. □

- As in the case of the "big oh" notation, there are several functions $g(n)$ for which $f(n) = \Omega(g(n))$. $g(n)$ is only a lower bound on $f(n)$. For the