

We know, by basic exponent law,  $x^m \times x^n = x^{m+n}$

∴ The exponent of the term created by multiplying ~~an~~ two other terms having exponents  $\text{terms}[i].\text{expon}$  and  $\text{terms}[j].\text{expon}$  is  $\text{terms}[i].\text{expon} + \text{terms}[j].\text{expon}$ . It's easy to see that the coefficient of the product term is

$\text{terms}[i].\text{coef} * \text{terms}[j].\text{coef}$ . By eqn(2), there can be no other coefficients for this exponent which can be added up.

• On line 10, we input  $\text{terms}[i].\text{coef} * \text{terms}[j].\text{coef}$  as coefficient and  $\text{terms}[j].\text{expon} + \text{terms}[i].\text{expon}$  as exponent. By correctness of the attach function, the new term is added to the global  $\text{terms}[]$  array. By eqn(2), we know the original order is maintained.

• ~~The~~ Before the start of the next iteration  $i$  updates to  $i+1$ . It's easy to see now that the loop invariant is still maintained.

**Termination:** The loop terminates when  $j = \text{finishB} + 1$ . By the loop invariant,  $\forall K, \text{startB} < K < \text{finishB} + 1$ , the product terms  $\text{terms}[i].\text{coef} * \text{terms}[K].\text{coef} * x^{(\text{terms}[i].\text{expon} + \text{terms}[K].\text{expon})}$  have been correctly computed and stored in the global  $\text{terms}[]$  array by the  $\text{attach}()$  function in their original order with polynomial B.  
∴ The partial product term corresponding to  $\text{terms}[i]$  has been correctly computed and stored in the terms array.  $\square$

**Outer loop invariant:** Just before the start of the  $i$ th iteration of the outer loop, for all  $K$  such that  $\text{startA} < K < i$ , the ~~correct~~ <sup>each of</sup> sum of the partial products corresponding to  $\text{terms}[K]$  with polynomial B has been correctly computed and stored in polynomial  $x$ . The polynomial  $x$  is represented in the terms array from index  $\text{startx}$  to  $\text{finishx}$ .