

where $row = a[j].col$, $col = a[j].row$, and $value = a[j].value$. Furthermore, these entries occupy the first $currentb$ positions of $b[]$ and are arranged such that:

- 1) The row fields in $b[i]$ to $b[currentb-1]$ are in ascending order
- 2) For entries with the same row, the col fields are in ascending order.

5. **Initialization:** ^{Just before} the start of the 1st iteration of the outer loop, $i=0$. The set of columns 0 to $c-1 = -1$ doesn't make sense and is empty. A universal quantifier over an empty set is vacuously true. \therefore Loop invariant trivially holds.

Maintenance: Let, the loop invariant hold just before the start of the iteration when $i=x$, $0 \leq x \leq a[0].col$. By the correctness of the inner loop, all non-zero matrix elements of $a[]$ having column x have been correctly transposed in $b[]$ as $\langle \text{row}, \text{col}, \text{value} \rangle$ where $row = a[j].col$, $col = a[j].row$, $value = a[j].value$ in the index $currentb$ (Lines 11-17). We note that the value of $currentb$ increases by 1 (line 16) ^{loop} with each entry being inserted in $b[]$. From the correctness of the inner loop, we have also seen that for all non-zero elements in b having row x (i.e. column x in a), the elements are arranged in $b[]$ such that their respective column indices are in ascending order.

• It's easy to see that x is the largest indexed column from 0 to x of $a[]$. Since, we now know that elements of column x has been correctly transposed in $b[]$, by using the loop invariant and correctness of inner loop, we can say: the transpose of all non-zero elements in columns 0 to x of matrix $a[]$ have been correctly placed in $b[]$ as triples $\langle row, col, value \rangle$, where $row = a[j].col$, $col = a[j].row$ and $value = a[j].value$. These entries occupy the first $currentb$ positions of $b[]$ and are arranged such that:

- 1) The row ^{fields} from $b[i]$ to $b[currentb-1]$ are in ascending order.
- 2) For entries with the same row, the col fields are in