

before the start of this iteration.

- If the if condn on line 12 would not hold, then it's trivial to see that before the start of the next iteration when  $j = x+1$ , the loop invariant still holds.

**Termination:** The inner loop terminates when  $j = n+1$ . By line 4, we know that  $n$  stores the no. of non-zero elements of matrix  $a$ , and hence also of matrix  $b$ . By loop invariant, we can say that  $\forall k, 1 \leq k \leq n$ , if  $a[k].col == i$ , then:

- 1) The corresponding transposed entry  $\langle a[k].col, a[k].row, a[k].value \rangle$  has been correctly stored in the next available index of  $b[]$  (current  $b$ ).

∴ All the non-zero elements having column  $i$  in  $a[]$  have been transposed.

- 2) For all the non-zero elements in  $b$  having row  $i$  (i.e. column  $i$  in  $a$ ), the ~~column indices of each~~ elements are arranged in  $b[]$  such that their respective column indices are in ascending order.  $\square$

**Outer loop invariant:** At the start of each iteration of the outer loop when  $i = c$ , where  $0 \leq c \leq a[0].col$ , for all columns  $k$ , where  $0 \leq k \leq c$ , all the <sup>non-zero</sup> <sup>of  $a[]$</sup>  elements in these columns have been transposed correctly to their corresponding <sup>indices</sup> ~~positions~~ <sup>of  $b[]$</sup>   $\langle a[k].col, a[k].row, a[k].value \rangle$  in  $b[]$ . We ensure that the elements are arranged in ascending order of their row indexes.

### Refined Outer Loop Invariant (ChatGPT)

1. At the start of each iteration of the outer loop where  $i = c$ ,  $0 \leq c \leq a[0].col$ , the transpose of all non-zero elements in columns  $0$  to  $c-1$  of matrix  $a[]$  have been correctly placed in  $b[]$  as triples  $\langle row, col, value \rangle$ ,