

statement $f(n) = \Omega(g(n))$ to be informative, $g(n)$ should be as large a function of n as possible for which the statement $f(n) = \Omega(g(n))$ is true. So while we shall say that $3n+5 = \Omega(n)$ and that $6 \cdot 2^n + n^2 = \Omega(2^n)$, we shall almost never say that $3n+5 = \Omega(1)$ or that $6 \cdot 2^n + n^2 = \Omega(1)$ even though both these statements are correct.

Theorem 1.3: If $f(n) = a_m n^m + \dots + a_1 n + a_0$ and $a_m > 0$, then $f(n) = \Omega(n^m)$

Proof: (Also Ex-1.5.3: Prob 3) Since a_m is positive, we can make the following claim: $f(n) = a_m n^m + \dots + a_1 n + a_0 \gg n^m$.

• The proof will be by mathematical induction on m .

Base Case: $m=0$. $f(n) = a_0$ R.H.S. $a_0 n^0 = a_0 \therefore \text{L.H.S.} \gg \text{R.H.S.}$
 a_0 must be positive $\therefore f(n)$ is non-negative
 \therefore Base Case holds

I.H: For $m=K$, let $f(n) = a_K n^K + a_{K-1} n^{K-1} + \dots + a_1 n + a_0 \gg n^K$
 We know, $a_K > 0$.

Induction Step: For $m=K+1$. $f(n) = a_{K+1} n^{K+1} + a_K n^K + a_{K-1} n^{K-1} + \dots + a_1 n + a_0$
 Let, $a_{K+1} > 0$. If $a_K > 0$, then by I.H. $f(n) \gg n^K$.
 $f(n) \gg a_{K+1} n^{K+1} + n^K = n^K (a_{K+1} n + 1) \gg n^K \cdot n$ (\therefore)

(Note: The proof is not correct)

Defn: [Theta] $f(n) = \Theta(g(n))$ (read as "f of n is theta of g of n") iff there exist positive constants c_1, c_2 and n_0 such that $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n, n \geq n_0$. \square .

• For function sum we had determined that $T_{\text{sum}}(n) = 2n+3$. So, $T_{\text{sum}}(n) = \Theta(n)$.
 $T_{\text{row}}(n) = 2n+2 = \Theta(n)$ and $T_{\text{col}}(\text{rows}, \text{cols}) = 2 \cdot \text{rows} \cdot \text{cols} + 2 \cdot \text{rows} + 1 = \Theta(\text{rows} \cdot \text{cols})$

• The asymptotic complexity (i.e. the complexity in terms of O, Ω and Θ) can be determined quite easily without determining the exact step count. This is usually done by first determining the asymptotic complexity of each statement (or group of statements) in the program and then adding up these complexities.