

Maintenance: Let, before the start of the loop iteration when  $j=K$ , the loop invariant holds.  $\therefore \forall x, 0 \leq x < j, x \in N, c[i][x] = a[i][x] + b[i][x]$

Now, on line 6,  $c[i][K]$  stores the sum of  $a[i][K]$  and  $b[i][K]$ . In the next iteration,  $j=K+1$ . It's easy to see that now, the loop invariant holds  $\therefore \forall x, 0 \leq x < j, c[i][x] = a[i][x] + b[i][x]$

Termination: The inner loop terminates when  $j = \text{cols}$ .  $\therefore$  By loop invariant,  $\forall x, 0 \leq x < \text{cols} - 1, x \in N, c[i][x] = a[i][x] + b[i][x]$ .

$\therefore$  The sum of the corresponding matrix elements of  $a$  and  $b$  for row  $i$  has been correctly computed and stored in the  $i$ th row of  $c$ .  $\square$

### Outer loop invariant

Just before the start of the loop iteration when  $i=1$ , where  $0 \leq i < \text{rows}$ , all the corresponding elements of  $a$  and  $b$  for rows  $0$  to  $(i-1)$  have been added and stored in matrix  $c$ .

Initialization:  $i=0$ . The row indices of  $0$  to  $-1$  doesn't make sense.  $\therefore$  The claim trivially holds.

Maintenance:  ~~$i=1$~~   $i=1$ . Before the start of this iteration, let the loop invariant holds.  $\therefore \forall x \forall y, \text{where } 0 \leq x < i \text{ and } 0 \leq y < \text{cols}$ ,

$$c[x][y] = a[x][y] + b[x][y]. \text{ When } i=1, \text{ by the correctness proof}$$

of inner loop done above, we know that  $\forall y, 0 \leq y < \text{cols}$ ,

$$c[1][y] = a[1][y] + b[1][y]. \text{ Before the start of the next iteration, } i=i+1. \therefore \text{ Loop invariant}$$

still holds and it's easy to see.

Termination: The outer loop terminates when  $i = \text{rows}$ .  $\therefore$  By loop invariant,  $\forall x \forall y, 0 \leq x < \text{rows}, 0 \leq y < \text{cols}$ ,