

Suggested Wording for Invariant (chatgpt)

- Just before the start of the j th iteration of the inner loop, for all K such that $\text{startB} \leq K < j$, the product terms $\text{terms}[i].\text{coef} * \text{terms}[K].\text{coef} * 2^{(\text{terms}[i].\text{expon} + \text{terms}[K].\text{expon})}$ have been correctly computed and stored in the global $\text{terms}[]$ array by the $\text{attach}()$ function in their original order.

Notes:

- This form clarifies which coefficients and which exponents are being multiplied.
- It avoids ambiguity around whether you are referring to the K th term in the resulting product or the K th index of polynomial B .
- It keeps invariant inductive: stating correctness upto $j-1$.

Initialization: Just before the 1st iteration of the loop, $j = \text{startB}$. The range $\text{startB} \leq K < j = \text{startB}$ doesn't make sense. \therefore Loop invariant trivially holds.

Maintenance: Let, just before the start of the j th iteration of the loop, where $j = x'$, $\text{startB} \leq x' < \text{finishB}$, the loop invariant holds. $\therefore \forall K, \text{startB} \leq K < x'$, the product terms $\text{terms}[i].\text{coef} * \text{terms}[K].\text{coef} * 2^{(\text{terms}[i].\text{expon} + \text{terms}[K].\text{expon})}$ have been correctly computed and stored in the global $\text{terms}[]$ array by the attach function in their original order.

Now, $j = x'$. By original assumption, we know, $\forall K, \text{startB} \leq K < x'$ and $\forall l, x' \leq l < \text{finishB}$,

$$\boxed{\text{terms}[K].\text{expon} + \text{terms}[x']. \text{expon} + \text{terms}[l].\text{expon}} \quad (1)$$

$$\therefore \boxed{\begin{array}{l} \text{terms}[i].\text{expon} + \text{terms}[K].\text{expon} + \text{terms}[i].\text{expon} + \text{terms}[x']. \text{expon} \\ \text{terms}[i].\text{expon} + \text{terms}[l].\text{expon} \end{array}} \quad (2)$$