

7) Compare the two functions  $n^2$  and  $20n+4$  for various values of  $n$ . Determine when the second function becomes smaller than the first

$n^2 \geq 20n+4$   ~~$n^2 \geq 20n+4$~~ . When?

**Claim:**  $n^2 \geq 20n+4$  for  $n \in \mathbb{N}$ ,  $n \geq 21$

**Proof:** **Base Case:**  $n=21$ .  $n^2=21^2=441$   $20n+4=424$ .  $\therefore$  Base Case holds

**I.H:**  $n=k$ , and  $k^2 \geq 20k+4$  holds for  $k \geq 21$ .

**Induction Step:**  $n=k+1$ .  $(k+1)^2 = k^2 + 2k + 1 \geq (20k+4) + (2k+1) = 22k+5$   
 $= 20k + (2k+5) \geq 20k + (2 \times 21 + 3) = 20k + 47 \geq 20k + 24$   
 $(\because k \geq 21)$   $= 20(k+1) + 4$

**$\therefore n^2 \geq 20n+4$  when  $n \geq 21$ .**

$n=20$ ,  $20^2=400$   $20n+4=404$ .