

$\therefore \forall x, x \in \mathbb{N}, 0 \leq x < l, c[i][x] = a[i][x] * b[x][j]$ holds

• Now, when $k=l$, by line 8, $c[i][j] = c[i][j] + a[i][l] * b[l][j]$.

• Now, the loop variable k gets updated to $(l+1)$. It's easy to see that

$\forall x, x \in \mathbb{N}, 0 \leq x < l+1, c[i][j] = a[i][x] * b[x][j]$ holds.

\therefore Loop invariant still holds

Termination: The loop terminates when $k = \text{MAX_SIZE}$.

\therefore By loop invariant, $c[i][j] = \sum_{k=0}^{\text{MAX_SIZE}-1} a[i][k] * b[k][j]$. \therefore The element

of matrix c at row i , column j stores the sum of the products of row i of matrix a and column j of matrix b 's corresponding elements. \square
This is in accordance to definition.

Loop (j) invariant

• Just before the start of the iteration when $j=m$, the elements of matrix c in row i , from indices 0 to $m-1$, have been correctly computed.

Initialization: ~~when~~ $j=0$. $\therefore m=0$, the indices 0 to -1 doesn't make sense.

\therefore The loop invariant trivially holds.

Maintenance: Let, the loop invariant hold just before when $j=m$. $\therefore \forall x, x \in \mathbb{N}, 0 \leq x < m$, the matrix element $c[i][x]$ is correctly computed. That means, according to correctness of Loop(K), $c[i][x] = \sum_{k=0}^{\text{MAX_SIZE}-1} a[i][k] * b[k][x]$

• Now, $j=m$. On line 6, we initialize $c[i][m]$ to 0 . By correctness of

Loop(K), $c[i][m] = \sum_{k=0}^{\text{MAX_SIZE}-1} a[i][k] * b[k][m]$. j now gets updated to $(m+1)$.

It's easy to see that $\forall x, x \in \mathbb{N}, 0 \leq x < m+1$, the matrix element $c[i][x]$ is correctly computed.