

$\text{all_comb}(\text{tval}, \text{begin}+1, n)$ will be called, $\text{begin} = -1$, and we know that our first call to all_comb has $\text{begin} = 0$. This is a contradiction.

\therefore In both cases, the function $\text{all_comb}(\text{tval}, \text{begin}, n)$, where $\text{begin} = 0$ is called 1 time, where $1 = 2^0 = 2^{\text{begin}}$. This is correct. \square (base case holds)

I.H.: For a particular $\text{begin} = k$, where $\text{begin} < n$, the function all_comb on the parameters $(\text{tval}, \text{begin}, n)$ is called 2^{begin} times.

Induction Step: Now, for the value $\text{begin} = k+1$. The if block from lines 4-9 doesn't call any instance of the function all_comb . Only the else block does that on lines 15 and 17. If on the current call, $\text{begin} = k+1$, then it's easy to see that in the previous call, $\text{begin} = k$. For each case, when $\text{begin} = k$, the function all_comb on the parameters $(\text{tval}, k+1, n)$ is called twice, i.e. on lines 15, 17. By I.H., the function $\text{all_comb}()$ on the parameters (tval, k, n) is called 2^k times.

\therefore The total no. of times the function all_comb on the parameters $(\text{tval}, k+1, n)$ is called $= 2 \cdot 2^k = 2^{k+1} = 2^{\text{begin}}$. \square

\therefore We have shown for a fixed n , and $\forall \text{begin} < n, \text{begin} \geq 0$, the function $\text{all_comb}(\text{tval}, \text{begin}, n)$ is called 2^{begin} times.

Now, we can do a similar proof when the no. of variables is $n+1$, and from here, our original claim follows. \square

\therefore Total no. of function calls: $2^0 + 2^1 + 2^2 + \dots + 2^n = (2^{n+1} - 1)$

We leave aside the original function call. \therefore Now it's $(2^{n+1} - 2) = 2(2^n - 1)$