

• By original assumption, $\text{terms}[\text{startB}] \cdot \text{expon} > \text{terms}[i] \cdot \text{expon} > \text{terms}[\text{finishB}] \cdot \text{expon}$
for $\forall i, \text{startB} < i < \text{finishB}$ (Eqn. 1)

∴ $\text{terms}[x] \cdot \text{expon} + \text{terms}[\text{startB}] \cdot \text{expon} > \text{terms}[x] \cdot \text{expon} + \text{terms}[\text{finishB}] \cdot \text{expon}$
for $\forall i, \text{startB} < i < \text{finishB}$

We know, by law of exponents, $x^m \times x^n = x^{m+n}$, $m, n \in \mathbb{N}$. (Eqn. 2)

So, comparing eqn(1) and (2), we see that the no. of terms in $B(x)$ is the same as no. of terms in $\text{terms}[x'] * B(x)$

∴ $\text{finishy} - \text{starty} = \text{finishB} - \text{startB}$. We know, $\text{finishy} = \text{avail} - 1$
(justified before)

∴ $\text{starty} = (\text{avail} - 1) - \text{finishB} + \text{startB}$
 $= \text{avail} - (\text{finishB} - \text{startB} + 1)$. We have done just that in lines 13-14.

We know that $\sum_{k=\text{startA}}^{x'-1} \text{terms}[k] \cdot B(x) + \text{terms}[x] \cdot B(x) = \sum_{k=\text{startA}}^{x'} \text{terms}[k] \cdot B(x)$

→ Distributive law.

• By correctness of padd function, on line 15, we have passed the polynomials x and y ($\text{startx}, \text{finishx}$ and $\text{starty}, \text{finishy}$) to padd and it correctly computes $D = x + y$. It returns the pointers to the starting ($x.\text{startD}$) and ending ($x.\text{finishD}$) to the polynomial D in the global $\text{terms}[]$ array.

• Now, we initialize this sum as polynomial x on lines 16-17.

• The loop now gets updated to $x'+1$. It's easy to see that the loop invariant holds.

Termination: The loop terminates when $i = \text{finishA} + 1$. By the loop invariant, the polynomial $\sum_{k=\text{startA}}^{\text{finishA}+1-1} \text{terms}[k] \cdot B(x)$ has been correctly computed and

stored in the terms array between indices startx and finishx as polynomial x .

We know, by distributive law and mathematical induction that