

For the matrix b , the following criterion must hold:

- 1) The triples $\langle \text{row}, \text{col}, \text{value} \rangle$ are organized such that the row indices are in ascending order.
- 2) For ^{all} the ~~column~~ triples in any row, the columns must also be in ascending order.

Proof of Correctness

Inner loop invariant: For the given column i of matrix a , $\forall K$ where $1 \leq K \leq j$, if i is the corresponding column of the matrix element $a[K]$, then the corresponding correct ~~position~~ row, column no. in matrix b has been ~~found~~ computed. The criterion in matrix b that the row indices are in ascending order and for triples _{$\langle \text{row}, \text{col}, \text{value} \rangle$} of a given row, the columns are in ascending order is maintained.

Precise Inner Loop Invariant (ChatGPT)

- At the start of each iteration of the inner loop (indexed by j), for all K such that $1 \leq K \leq j$, if $a[K].\text{col} == i$, then:
 - 1) The corresponding transposed entry $\langle a[K].\text{col}, a[K].\text{row}, a[K].\text{value} \rangle$ has been correctly stored in the next available index of $b[\]$.
 - 2) The sequence of elements inserted into $b[\]$ so far maintains ascending order of row indices (i.e. $a[K].\text{col}$), and within a given row, ascending order of column indices (i.e. $a[K].\text{row}$), as induced by the order and inner loop traversal order.

Initialization: Before the start of the 1st iteration of the loop, $j=1$ (line 11). The range $1 \leq K \leq j=1$ is empty and doesn't make sense. The invariant automatically holds since the premise of the implication is False. (A universal quantifier over an empty set is vacuously true).