

We know, $\text{bin_coeff}(l+1, l+1)$ returns 1 by line 4.

By I.H., $\text{bin_coeff}(l+1, l)$ has maximum call stack depth 1.

The function $\text{bin_coeff}(l+1, l)$ is also on stack, \therefore the max. call stack depth of $\text{bin_coeff}(l+2, l+1)$ is $l+1 = (K+1)$. \square

\therefore Maximum call stack depth of recursive binomial coefficient $(K+1, K)$ is $K = n-1$. \therefore Base case holds.

I.H.: For $\forall l \in \mathbb{N}$, $l < n$, the recursive binomial coefficient (l, K) function has a maximum call stack depth of $(l-1)$, for all $K < l$.

Induction Step: $l = n$. By line 7, we call: $\text{bin_coeff}(n, K) = \text{bin_coeff}(n-1, K) + \text{bin_coeff}(n-1, K-1)$. By I.H., both $\text{bin_coeff}(n-1, K)$ and $\text{bin_coeff}(n-1, K-1)$ has a max. call stack depth of $(n-1)-1 = (n-2)$. $\therefore \text{bin_coeff}(n, K)$ has a max. call stack depth of $(n-2)+1 = (n-1)$. \square

• Space complexity of recursive binomial coefficient is: $c(n-1) = O(n)$.
• It's easy to see that the iterative version takes up a constant space.
The only unknowns are n, K both take 4 bytes each (constant).

Iterative binomial coefficient $(n, K) = 0$

4) Determine the space complexity of the function created in Exercise 5, Sec-1.3 (pigeonhole principle)

Soln: The function $\text{int } f(\text{int } x)$ has no structure variables, depending on input. x takes up constant space, the return address takes up const. space.

Sf(I) = 0