Claim: the recursive_binomial_coefficient(n, K) function correctly computes ${}^{n}C_{K}$, $n \geq K$

Proof: We can prove this by using generalized strong mathematical induction on (n, K).

Base Case: • $K=0$. We know, ${}^{n}C_{0} = \frac{n!}{(n-0)!\,0!} = 1$. We have correctly returned 1 in line 4.

• now $n=K$. We know, ${}^{n}C_{K} = \frac{n!}{(n-K)!\,K!} = \frac{n!}{n!\,0!} = 1$. We have correctly returned 1 in line 6.

Induction hypothesis: Let, $\forall a,b \in N$, $\forall a \leq n$ and $\forall b \leq K$, the recursive_binomial_coefficient(a,b) function correctly computes ${}^{a}C_{b}$, $a \geq b$.

Induction Step: Let, $a=n+1$ and $b=K+1$. $\because a \neq b$, We compute line 7 as: recursive_binomial_coefficient(a,b) = recursive_binomial_coefficient(a-1,b) + recursive_binomial_coefficient(a-1,b-1)

or, recursive_binomial_coefficient(n+1, K+1) = recursive_binomial_coefficient(n,K+1) + recursive_binomial_coefficient(n,K)

• By induction hyp, recursive_binomial_coefficient(n, K) = ${}^{n}C_{K}$

• ~~By induction hyp~~ We need to compute recursive_binomial_coefficient(n,K+1). When, it is called, it is computed as: recursive_binomial_coefficient(n,K+1) = recursive_binomial_coefficient(n-1, K+1) + $\boxed{\text{recursive\_binomial\_coefficient}(n-1, K)}$

↳ By I.H., ${}^{n-1}C_{K}$.