

• On line 12, we initialize  $\text{startingPos}[0]$  to 1. By the description of  $\text{startingPos}[]$ , ~~and~~ and since we know that the non-zero elements of  $b[]$  are stored starting from index 1, this initialization is correct.

Loop invariant of Lines 13-14: Just before the start of the  $i$ th iteration of the loop, for all indices  $K$ , where  $1 \leq K < i < \text{numCols}$ , the ~~entry~~ <sup>entry</sup>  $\text{startingPos}[K]$  stores the index of the triple  $\langle b[x].\text{row}, b[x].\text{col}, b[x].\text{value} \rangle$ , where  $\text{startingPos}[K] = x$ ,  $b[x].\text{row} = K$ ,  $b[x].\text{col} = K_1$ , and  $\forall x'$ , where  $x < x'$ , if  $b[x'].\text{row} = K$ , then  $b[x'].\text{col} = K_2 \gg K_1 = b[x].\text{col}$ . Or in other words,  $\text{startingPos}[K]$  stores that ~~entry~~ non-zero element of  $b[]$  having row  $K$  and minimum indexed column. Hence,  $\text{startingPos}[K]$  has been computed correctly.

Rewritten (cleaner and Equivalent) Version by ChatGPT

Just before the start of the  $i$ th iteration of the loop, for all indices  $K$  where  $0 \leq K < i$ ,  $\text{startingPos}[K]$  correctly stores the index in array  $b[]$  where the first non-zero entry of row  $K$  in the transposed matrix should be placed. This ensures that all future elements belonging to row  $K$  in  $b[]$  will be inserted starting from this position in increasing column order.

Initialization: At the start of the 1st iteration of the loop,  $i = 1$ .

~~Loop invariant~~ Just before the start of this iteration, for all indices  $0 \leq K < i = 1$ , i.e. only for index 0, we have already justified before why  $\text{startingPos}[0]$  <sup>being</sup> initialized to 1 is correct.  
 $\therefore \text{startingPos}[0]$  correctly stores the index in array  $b[]$  where