K) $10n^3+15n^4+100n^2 2^n = O(n^2 2^n)$

Let, $10n^3 \leq n^2 \cdot 2^n \Rightarrow n^2 \cdot (10n - 2^n) \leq 0$ $\because n \geq 0 \therefore (10n-2^n) \leq 0$.

$\boxed{10n \leq 2^n}$ $\boxed{\text{Claim:}}$ $10n \leq 2^n$ for $n \geq 6$

$\boxed{\text{Proof:}}$ $\boxed{\text{Base Case:}}$ $n=6$ $10 \cdot 6 = 60$ $2^6 = 64$ $60 \leq 64$ $\therefore$ Base Case holds

$\boxed{\text{I.H:}}$ For $n=K$, $10K \leq 2^K$ $\boxed{\text{Induction Step:}}$ $n=K+1$ $10(K+1) = 10K+10$
$\leq 10 + 2^K$ (By I.H.)
$\because K \geq 6 \therefore 10 + 2^K \leq 2^K + 2^K = 2^{K+1}$

$\boxed{\therefore 10(K+1) \leq 2^{K+1}}$ $\boxed{\therefore 10n \leq 2^n}$ $\square$ $\boxed{\therefore 10n^3 \leq n^2 \cdot 2^n}$ for $n \geq 6$

$\boxed{\text{Claim:}}$ $15n^2 \leq 2^n$ for $n \geq 11$

$\boxed{\text{Base Case:}}$ $2^{11} = 2048$ $15 \times 11^2 = 1815$ $\therefore$ Base Case holds
$1815 \leq 2048$

$\boxed{\text{I.H:}}$ For $n=K, K \geq 11$, $15K^2 \leq 2^K$
$\boxed{\text{Induction Step:}}$ $n=K+1$ $15(K+1)^2 = 15K^2 + (30K+15) \leq 2^K + (30K+15)$ (By I.H.)
$= 2^K + 15(2K+1) \leq 2^K + 2^K = 2^{K+1}$

$\therefore 15n^2 \leq 2^n$ or, $\boxed{15n^4 \leq n^2 \cdot 2^n}$ for $n \geq 11$.

$\boxed{100n^2 2^n \leq 100n^2 2^n}$ $\forall n \in N$.

$\boxed{\therefore 10n^3+15n^4+100n^2 2^n \leq 102n^2 2^n \quad \forall n \in N, n \geq 11}$ $c=102, n_0=11$

$\therefore 10n^3+15n^4+100n^2 2^n = O(n^2 \cdot 2^n)$

2) Show that the following statements are incorrect:

a) $10n^2+9 = O(n)$. Suppose, it is true. $\therefore \exists c, n_0$ which are positive constants
s.t. $10n^2+9 \leq cn$ for $\forall n \in N, n \geq n_0$
or, $\boxed{10n + \frac{9}{n} \leq c}$ $\rightarrow$ Not Possible, $\because$ if $n \to \infty$, $10n + \frac{9}{n} \to \infty$