Ex-1.5.3

1) Show that the following statements are correct.:

a) $5n^2-6n=\Theta(n^2)$. $5n^2-6n\leq 5n^2 \Rightarrow n\geq 0$.

$5n^2-6n\geq 4n^2 \Rightarrow n(n-6)\geq 0$. Either $n\geq 0$ and $n\geq 6$ or, $\boxed{n\leq 0}$ and $n\leq 6$ ↳ can't happen.

$\boxed{\therefore n\geq 6.}$

$\boxed{4n^2\leq 5n^2-6n\leq 5n^2}$ when $n\geq 6$

$c_1=4$ $c_2=5$ $n_0=6$ $\boxed{\therefore 5n^2-6n=\Theta(n^2)}$

b) $n!=O(n^n)$. Let, $n\geq 1$. We know $n\leq n, (n-1)<n, (n-2)<n \ldots\ldots (n-k)<n, \ldots\ldots 1<n$. $\therefore n(n-1)(n-2)\ldots\ldots 2\cdot 1\leq n^n$ or, $n!\leq n^n$, for $n\geq 1$.

$\therefore c=1, n_0=1$ $\boxed{\therefore n!=O(n^n)}$

c) $2n^2+n\log n=\Theta(n^2)$ Let, $2n^2+n\log n\leq 3n^2$ hold. $\therefore (n^2-n\log n)\geq 0$ or, $n(n-\log n)\geq 0$. $\therefore \boxed{n\geq 0}$ and $\boxed{n\geq \log n}$, or $\boxed{2^n\geq n}$

$\boxed{\text{Claim:}}$ For all $n\in N$, $2^n\geq n$.

$\boxed{\text{Proof:}}$ The proof is by mathematical induction on n

$\boxed{\text{Base Case:}}$ $n=0$. $2^n=2^0=1\geq 0=n$ $\therefore$ Base case holds

$\boxed{\text{I.H.:}}$ Let, for some $n=k$, $2^k\geq k$.

$\boxed{\text{Induction Step:}}$ $n=k+1$. $2^{k+1}=2\cdot 2^k\geq 2k$ (By I.H.) $=k+k\geq k+1$

$\therefore \forall n\in N, 2^n\geq n$ □

Let, $2n^2+n\log n\geq 2n^2 \Rightarrow n\log n\geq 0 \Rightarrow n\geq 0$ and $\log n\geq 0$, or $\boxed{n\geq 1}$

$\boxed{\therefore n\geq 1}$

$\therefore 2n^2\leq 2n^2+n\log n\leq 3n^2$ for $n\geq 1$

$c_1=2, c_2=3, n_0=1$ $\boxed{\therefore 2n^2+n\log n=\Theta(n^2)}$