

**Loop invariant:** The element  $\text{searchnum}$  is not present in  $\text{list}[0 \dots \text{left}-1]$  and in  $\text{list}[\text{right}+1, n-1]$ .

**Initialization:** ~~Before~~ Before the first iteration of the loop,  $\text{left}=0$  and  $\text{right}=n-1$ .  $\text{searchnum} \notin \text{list}[0 \dots -1]$  and  $\notin \text{list}[n, n-1]$ , trivially, because the ranges do not make sense.

**Maintenance:** Suppose, after a particular iteration of the loop, the loop invariant holds.  $\therefore \text{searchnum} \notin \text{list}[0 \dots \text{left}-1]$  and  $\text{searchnum} \notin \text{list}[\text{right}+1, n-1]$ . Now, let  $\text{left} \leq \text{right}$  holds. In line 5, we compute  $\text{middle} = (\text{left} + \text{right})/2$ . If  $\text{list}[\text{middle}] < \text{searchnum}$ , then it must be the case that if  $\text{searchnum}$  is present in  $\text{list}[\ ]$ , it will be present in  $\text{list}[\text{middle}+1, \text{right}]$ .  $\therefore$  We initialize  $(\text{left} = \text{middle} + 1)$  in line 7 ( $\because \text{list}[\ ]$  is sorted in ascending order).

Before the start of the next iteration, ~~it's~~ it's easy to see that  $\text{searchnum} \notin \text{list}[0 \dots \text{left}-1]$  (Combine our maintenance assumption of prev iteration.  $\text{searchnum} \notin \text{list}[0 \dots \text{old-left}-1]$  and  $\text{searchnum} \notin \text{list}[\text{old-left}, \text{old-middle}]$  which we proved above) and  $\text{searchnum} \notin \text{list}[\text{right}+1, n-1]$  (prev assumption).  $\therefore$  Loop invariant holds in this case.

• We can similarly prove that loop invariant will hold if  $\text{list}[\text{middle}] > \text{searchnum}$ .

**Termination:** The function will terminate in two cases:

1) If  $\text{list}[\text{middle}] = \text{searchnum}$ . So, then  $\text{searchnum}$  has been found, and we return the posn.  $\text{middle}$ .

2) When the condn.  $\text{left} > \text{right}$  is reached. Just before this iteration,  $\text{left} \leq \text{right}$  would hold.  $\text{middle} = (\text{left} + \text{right})/2$ .

If,  $\text{left} > \frac{\text{left} + \text{right}}{2}$ ,  $\therefore$  then  $\text{left} > \text{right}$  (Contradiction)  
 $\therefore \text{left} \leq \frac{\text{left} + \text{right}}{2}$  Similarly,  $\frac{\text{left} + \text{right}}{2} \leq \text{right}$