

**Claim:** For a constant  $b'$ , and  $\forall a' \leq n$ , the recursive binomial coefficient  $(a', b')$  function correctly computes  $a' C_{b'}$ .

**Proof:** The proof is by mathematical induction on  $a'$ .

**Base Case:**  $a' = b'$ . We return 1, justification given before.

**I.H.:** Let, the recursive binomial coefficient  $(a', b')$  correctly compute  $a' C_{b'}$ .

**Induction Step:** By line 7,  
recursive binomial coefficient  $(a'+1, b') =$  recursive binomial coefficient  $(a', b')$  + recursive binomial coefficient  $(a', b'-1)$   
computed as  $a' C_{b'}$  by I.H.  $\rightarrow$  computed as  $a' C_{b'-1}$  by Strong I.H.

$\therefore$  The function correctly computes  $(a'+1) C_{b'}$   $\square$

$$\therefore \text{recursive binomial coefficient}(n, k+1) = n C_{k+1}$$

$$\text{So, recursive binomial coefficient}(n+1, k+1) = n C_{k+1} + n C_k = (n+1) C_{k+1}$$

$\therefore$  The function is correct  $\square$

11) [Towers of Hanoi] There are three towers and 64 disks of different diameters placed on the first tower. The disks are in order of decreasing diameter as one scans up the tower. Monks were reputedly supposed to move the disk from tower 1 to tower 3 obeying the rules:

- Only one disk can be moved at any time
- No disk can be placed on top of a disk with a smaller diameter

Write a recursive function that prints out the sequence of moves needed to accomplish this task.