

Proof: The proof is by the method of strong mathematical induction on n .

Base Case: $n=0$. ~~Base Case~~ The function returns 0 in line 4, which is correct since the 0th Fibonacci no. is 0.

$n=1$. The function returns 1 in line 6, which is correct since the 1st Fibonacci number is 1.

Induction hypothesis: For all $n \leq K$, $n \in \mathbb{N}$, recursive_nthFibonacci(n) correctly computes the n th Fibonacci number.

Induction Step: Let, $n=K+1$. The function evaluates the apt else block in line 7. In line 8, recursive_nthFibonacci($K+1$) = recursive_nthFibonacci($(K+1)-1$) + recursive_nthFibonacci($(K+1)-2 = K-1$).

By induction hyp, both recursive_nthFibonacci(K) and recursive_nthFibonacci($K-1$) is computed correctly.

\therefore recursive_nthFibonacci($K+1$) is computed correctly, since we have implemented the correct recursive function call for the n th Fibonacci number \square

int iterative_nthFibonacci(int n)

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2. int f_i = 0;
3. if (n == 0)
4. return f_i;
5. int f_j, f_k;
6. f_j = 1;
7. if (n == 1)
8. return f_j;
9. for (int i = 2; i <= n; i++)
10. f_k = f_i + f_j;
11. f_i = f_j;
12. f_j = f_k;
13. return f_k;
```

Loop Invariant: At the start of each iteration with index i , $f_i = F(i-2)$ and $f_j = F(i-1)$

Initialization: Before the 1st iteration of the loop, we initialize f_i to 0 in line 3 and f_j to 1 in line 7.

Initially, index i is initialized to 2.
 $\therefore F(i-1) = F(1) = 1$ and $F(i-2) = F(0) = 0$.
We know these are True and we have respectively initialized f_j to 1 and f_i to 0. \therefore Loop invariant holds.