$\boxed{\text{Maintenance:}}$ Let, just before the start of the iteration when $j=x$, where $1 \leq x \leq n$, the loop invariant holds. $\therefore \forall K$ such that $1 \leq K < x$, if $a[K].col == i$, then:

1) The corresponding transposed entry $\langle a[K].col, a[K].row, a[K].value \rangle$ has been correctly stored in the next available index of $b[\,]$ (currentb)

2) The sequence of elements inserted into $b[\,]$ so far maintains ascending order of row indices (i.e. $a[K].col$), and within a given row, ascending order of column indices (i.e. $a[K].row$), as induced by the outer and inner loop traversal order.

• Now, let $j=x$. Let the if condition on line 12 hold. $\therefore a[x].col = i$.

Our goal now is to put the corresponding element $a[x].value$ in its transposed posn in matrix b. By defn. of transpose, $b[currentb].row = a[x].col = i$ and $b[currentb].col = a[x].row$. currentb is the available index in b. We have done just that in lines 13-15, along with the obvious assignment $b[currentb].value = a[x].value$. For further insertion into b, the available index currentb increases by 1 in line 16.

By the criterion of matrix a, $\forall K$ s.t. $1 \leq K < x$, $a[K].row \leq a[x].row$

Taking the column $a[x].col = i$, in the transposed matrix, $\forall K'$ s.t. $1 \leq K' < currentb$, for the row $b[currentb].row (= a[x].row)$, $b[currentb].col = a[x].row \geq b[K'].col = a[K].row$ for all K.

Since, we know that for all such K' the matrix b already maintains an ascending order of column indices for a given row, $\therefore \forall K', 1 \leq K' \leq currentb$, the matrix b still maintains an ascending order of column indices for a given row, as proved above.

$\therefore$ Now, the value of j updates to $x+1$. By loop invariant of previous iteration's start and our current justification, the loop invariant still holds