

the first non-zero entry of row 0 in the transposed matrix should be placed. \therefore Loop invariant holds just before $i=1$.

- Maintenance:** Let the loop invariant hold just before the start of the iteration when $i=x$, where $1 \leq x \leq \text{numCols}$. Therefore, for all indices K where $0 \leq K < \text{startingPos}[x]$, $\text{startingPos}[K]$ correctly stores the index in array $b[]$ where the first non-zero entry of row K in the transposed matrix should be placed.
- By the definition of the transpose matrix, row x of matrix $b[]$ is actually the column x of matrix $a[]$. By the correctness of the loop in lines 10-11, $\text{rowTerms}[x-1]$ stores the no. of non-zero elements of column $(x-1)$ of $a[]$, hence it is also the no. of non-zero elements of row $(x-1)$ of $b[]$ (We intend to create matrix $b[]$ as the transpose of matrix $a[]$).
 - From the above justification, it's easy to see why assigning the value of $\text{startingPos}[x-1] + \text{rowTerms}[x-1]$ to the storage location $\text{startingPos}[x]$ is correct. (Justification of line 14)
 - The loop variable i updates to $(x+1)$ and it's easy to see that the loop invariant still holds.

Termination: The loop terminates when $i = \text{numCols}$. By the loop invariant, for all indices K where $0 \leq K < \text{numCols}$, $\text{startingPos}[K]$ correctly stores the index in array $b[]$ where the first non-zero entry of row K in the transposed matrix should be placed.

By the defn. of transpose, we know that column K of $a[]$ is row K of $b[]$. \therefore By criterion of $a[]$, $\text{startingPos}[K]$ correctly stores the index in array $b[]$ ^{of the element} having column K in array $a[]$ and minimum indexed row of all other non-zero elements of $a[]$ having column K . Since the index K is in the range 0 to $\text{numCols}-1$, $\text{startingPos}[]$ has taken into account all the columns of $a[]$ \square