

## Refined version of outer loop invariant (ChatGPT)

Just before the start of the  $i$ th iteration of the outer loop (for  $i$  such that  $\text{startA} \leq i \leq \text{finishA}$ ), the polynomial  $\left[ \sum_{k=\text{startA}}^{i-1} \text{terms}[k] \cdot B(x) \right]$  — that is, the sum of the partial products formed by multiplying each term  $\text{terms}[k]$  of polynomial A (for  $k < i$ ) with the polynomial B — has been correctly computed and stored in the terms array between indices  $\text{start}_x$  and  $\text{finish}_x$  as polynomial  $x$ .

**Initialization:**  $i = \text{startA}$ . The range  $k = \text{startA}$  to  $i-1$  ~~is not possible~~ does not make sense.  $\therefore$  The outer loop invariant trivially holds.

**Maintenance:** Let, the outer loop invariant hold just before the start of the iteration when  $i = x$ .  $\therefore \sum_{k=\text{startA}}^{x-1} \text{terms}[k] \cdot B(x)$  is correctly computed and has been stored in the terms array between indices  $\text{start}_x$  and  $\text{finish}_x$  as polynomial  $x$ .

- By correctness of inner loop (lines 9-10), the partial product  $\text{terms}[x'] * B(x)$  has been correctly computed and stored in the terms array by the ~~attach~~ <sup>leftmost</sup> function. We know, the current posn. of avail is ~~at~~ the ~~current~~ <sup>leftmost</sup> memory block just to the right of the last term of the polynomial  $\text{terms}[x'] * B(x)$ . (Refer to pf of correctness of inner loop)
- If  $i = \text{startA}$ , then only the 1st partial product term has been computed and we rightly stop further iteration of the loop and move on to the next iteration (lines 11-12).
- Otherwise,  $\text{finish}_y$  and  $\text{start}_y$  is initialized to the finishing and starting posns. of the polynomial  $\text{terms}[x'] * B(x)$ .