mathematicians had for centuries.

- They knew they were choosing laws first, then retrofitting "meaning."
- That's why negative numbers were resisted for so long. They seemed to exist only as artifacts of rules, not as natural objects.
- It wasn't until the 19th century (Dedekind, Hamilton, Peano) that people said: "Arithmetic isn't about 'truth in nature'; it's about building a consistent system from axioms." This is the birth of structural mathematics.

4. The deeper philosophical question you are asking

(a) Should arithmetic reflect reality, or should it be a consistent symbolic system?

- The Ancients leaned toward reality: numbers.

IMPORTANT NOTES