Two facts which can be easily shown

(a) If some natural number is divisible by two relatively prime numbers p and q, then it is divisible by their product pq.

(b) If the number pA is divisible by q, where p and q are relatively prime, then A is divisible by q.

- Greatest Common Divisor (G.C.D.) gcd(x,y): the greatest natural no. which divides both of them.
- Least Common Multiple (L.C.M.) lcm(x,y): the least natural no. which is divisible by both of them.

12) $A = 2^3 \cdot 3^{10} \cdot 5 \cdot 7^2$ $\quad B = 2^5 \cdot 3 \cdot 11$ $\quad gcd(A,B) = 2^3 \cdot 3 = 24$ (Common part / Intersection)

13) $A = 2^8 \cdot 5^3 \cdot 7$ $\quad B = 2^5 \cdot 3 \cdot 5^7$ $\quad lcm(A,B) = 2^8 \cdot 3 \cdot 5^7 \cdot 7$ (Union)

Prob 1: Given two different prime numbers p and q, find the number of different divisors of the number $p^n q^m$.

Ans: $p^n q^m$ has p n times and q m times. For each choice of p^i, we have (m+1) choices of q as a divisor of the form $p^i q^j$.

$\therefore$ Total no. of divisors: $(n+1)(m+1)$

Prob 2: Prove that the product of any three consecutive natural numbers is divisible by 6.

Ans: Suppose we have three consecutive natural numbers $p, p+1$ and $p+2$.

Let, $n = p(p+1)(p+2)$. By the division algorithm, $p = 2a$ or $p = 2a+1$

- If $p = 2a$, $2|p$ $\therefore$ $2|p(p+1)(p+2)$ when $p = 2a$
- If $p = 2a+1$, $p+1 = 2a+1+1 = 2(a+1)$ $\quad \therefore 2|(p+1)$

$\therefore 2|p(p+1)(p+2)$ when $p = 2a+1$ $\qquad \therefore$ Either way, $2|p(p+1)(p+2)$