Prob 7: For some number n, can the number $n!$ have exactly five zero's at the end of its decimal representation?

Ans:) We will have a zero at the end of the decimal representation of a number if there is atleast one 5 and atleast one 2 in the prime decomposition of the no.

. Perform prime decomposition of no.s starting from 1, and multiply the no.s. We shall notice that 24! has four 5's in its prime decomposition (5, 10, 15, 20 contribute one 5 each). $\therefore$ 24! has four 0's at the end.

$25 = 5\times5$. $\therefore$ 25! must have 6 0's at the end.

$\therefore$ There does not exist a natural no. n with exactly five zero's at the end of its decimal representation.

Prob 8: Prove that if a number has an odd number of divisors, then it is a perfect square.

Ans:) Suppose, the prime decomposition of a number n is: $\boxed{p_1^{m_1} p_2^{m_2} \cdots\cdots p_x^{m_x}}$

. The no. of divisors of this number is: $(m_1+1)(m_2+1)\cdots\cdots(m_x+1)$
(Very easy to understand).

. Since, n has an odd number of divisors, the parity of each m_i, $1\leq i\leq x$ is even (Basic ideas of parity reqd. to understand this)

$\therefore n = p_1^{m_1} p_2^{m_2} \cdots\cdots p_x^{m_x} = (p_1^{m_1/2} p_2^{m_2/2} \cdots\cdots p_x^{m_x/2})^2 = x^2$, where $x\in\mathbb{Z}$
$(\because m_i/2 \in \mathbb{Z},\ 1\leq i\leq x)$

$\therefore n$ is a perfect square

Prob 14: Prove that any two natural numbers a and b satisfy the equation $\boxed{\gcd(a,b)\ \mathrm{lcm}(a,b) = ab}$

Ans:) Suppose, the only common prime factors of a and b are $a_1, a_2, \ldots, a_x$

Let the prime decomposition of a be: $a_1^{p_1} a_2^{p_2} \cdots\cdots a_x^{p_x} \cdot b_1^{q_1} b_2^{q_2} \cdots\cdots b_y^{q_y}$
and of b be: $a_1^{r_1} a_2^{r_2} \cdots\cdots a_x^{r_x} \cdot c_1^{s_1} c_2^{s_2} \cdots\cdots c_z^{s_z}$