

• By the division algorithm, $p=3a$, $p=3a+1$ or $p=3a+2$

If $p=3a \therefore 3|p$ and $3|p(p+1)(p+2)$

If $p=3a+1 \therefore p+2=(3a+1)+2=3(a+1) \therefore 3|(p+2)$ and $3|p(p+1)(p+2)$

If $p=3a+2 \therefore p+1=(3a+2)+1=3(a+1) \therefore 3|(p+1)$ and $3|p(p+1)(p+2)$

Either way, $3|p(p+1)(p+2)$

$\therefore 2, 3$ are co-prime to each other, $\therefore 2 \times 3 = 6 | p(p+1)(p+2)$

Prob 3: Prove the product of any four consecutive integers is divisible by 24.

• Suppose, we have four consecutive integers $p, p+1, p+2, p+3$.

• Let $n = p(p+1)(p+2)(p+3)$ • By division algorithm, we have $p=4a, 4a+1, 4a+2, 4a+3$.

If $p=4a$, then $4|p$ and $\therefore 4|n$

If $p=4a+1$, then $p+3=4a+1+3=4(a+1) \therefore 4|(p+3)$ and $\therefore 4|n$

If $p=4a+2$, then $p+2=4a+2+2=4(a+1) \therefore 4|(p+2)$ and $\therefore 4|n$

If $p=4a+3$, then $p+1=4a+3+1=4(a+1) \therefore 4|(p+1)$ and $\therefore 4|n$

$\therefore p(p+1)(p+2)(p+3)$ is divisible by 4.

Suppose $p=4a \therefore p+2=4a+2=2(2a+1) \therefore 2|(p+2)$ when $p=4a$

Suppose $p=4a+1 \therefore p+1=4a+2=2(2a+1) \therefore 2|(p+1)$ when $p=4a+1$

Suppose $p=4a+2=2(2a+1) \therefore 2|p$ when $p=4a+2$

Suppose $p=4a+3 \therefore p+3=4a+6=2(2a+3) \therefore 2|(p+3)$ when $p=4a+3$

In each case, we have 2 different no.s divisible by 2, out of which

1 is divisible by 4. \therefore In the prime factorization of $p(p+1)(p+2)(p+3)$