• If $\exists a \in A$, $r = a$, then we get repeated remainders for sure, and we are done (Prev proved)

• If $\forall a \in B, r > a, r \leq 99$, we form an upward pair with an element of set A. ($\because r \in S_2 - B$)

• If $\exists a \in B$, $r = a$, then we get repeated remainders for sure, and we are done (Prev shown)

Conclusion: My ideas are correct, I have checked into ~~the nitty details~~ but the proof should be formal and rigorous.

Prob 23: Each box in a 3x3 arrangement of boxes is filled with one of the numbers −1, 0, 1. Prove that of the eight possible sums along the rows, the columns, and the diagonals, two sums must be equal.

• The list of sums are

$$-1+-1+-1=-3$$
$$-1+-1+0=-2$$
$$-1+-1+1=-1$$
$$-1+0+0=-1$$
$$-1+0+1=0$$
$$0+0+0=0$$
$$0+0+1=1$$
$$0+1+1=2$$
$$1+1+1=3$$
$$-1+1+1=1$$

We see, the sums are in the set $S = \{-3, -2, -1, 0, 1, 2, 3\}$

$|S| = 7$

$\because$ We have 8 possible sums,

$\therefore$ By PgP, two sums must exist having same value.

Prob 24: Of 100 people seated at a round table, more than half are men. Prove that there are two men who are seated diametrically opposite each other.