• $a=7x+4$ $b=7y+5$ $a^2+b^2=49x^2+56x+16+49y^2+70y+25$
$=7(7x^2+8x+7y^2+10y+5)+6$

$7 \nmid (a^2+b^2)$ $\therefore$ (Not possible). The symmetric case is also not possible

• $a=7x+4$ $b=7y+6$ $a^2+b^2=49x^2+56x+16+49y^2+84y+36$
$=7(7x^2+8x+7y^2+12y+7)+3$

$7 \nmid (a^2+b^2)$ Not possible. The symmetric case is also not possible

• $a=7x+5$ $b=7y+5$ $a^2+b^2=49x^2+70x+25+49y^2+70y+25$
$=7(7x^2+10x+7y^2+10y+7)+1$

$7 \nmid (a^2+b^2)$ Not possible

$a=7x+5$ $b=7y+6$ $a^2+b^2=49x^2+70x+25+49y^2+84y+36$
$=7(7x^2+10x+7y^2+12y+8)+5$

$7 \nmid (a^2+b^2)$ $\therefore$ Not possible. The symmetric case is also not possible.

• $a=7x+6$ $b=7y+6$ $a^2+b^2=49x^2+84x+36+49y^2+84y+36$
$=7(7x^2+12x+7y^2+12y+10)+2$

$7 \nmid (a^2+b^2)$ $\therefore$ Not possible

The only possible case is when both a and b are divisible by 7.
$\boxed{\therefore a^2+b^2 \text{ is divisible by } 49}$

So, we conclude a^2+b^2 is div by $49 \cdot 9 = 441$, $\boxed{\because \gcd(9,49)=1}$

Prob 24: Given natural numbers a, b and c such $a+b+c$ is divisible by 6, prove that $a^3+b^3+c^3$ is also divisible by 6.

$\because a+b+c$ is div by 6, $\therefore \exists K_1, K_2, K_3 \in \mathbb{N}$, s.t. 1) $a=6K_1$ $b=6K_2$ $c=6K_3$

2) $a=6K_1$ $b=6K_2+1$ $c=6K_3+5$ 3) $a=6K_1$ $b=6K_2+2$ $c=6K_3+4$

4) $a=6K_1$ $b=6K_2+3$ $c=6K_3+3$ 5) $a=6K_1+1$ $b=6K_2+1$ $c=6K_3+4$

6) $a=6K_1+1$ $b=6K_2+2$ $c=6K_3+3$ 7) $a=6K_1+2$ $b=6K_2+2$ $c=6K_3+2$

(The rest of the cases are symmetric to these cases.)