Interesting obs

$50 = 5 \cdot 10$ $\therefore$ 50 multiplied by a 2 will give 2 0's, not 1.

$\therefore$ The set of ten multiples of 10 will give 12 0's, not 11 at the end

$\boxed{\therefore \text{Total no. of 0's at the end: } 12+12=24}$

Prob 11: The numbers a and b satisfy the equation $56a=65b$. Prove that $a+b$ is compositive.

$56=2\cdot2\cdot2\cdot7$ $65=5\cdot13$. $\because \gcd(a,b)=1$ $\therefore$ a and b must be a multiple of $5\cdot13$ and $2\cdot2\cdot2\cdot7$ respectively. $\because$ a and b must satisfy the eqn. $56a=65b$, $\therefore$ $\because$ a and b must satisfy the eqn. must be the same.

The extra multiple used on both sides of the eqn. must be the same.

i.e. $a=n\cdot5\cdot13$ $b=n\cdot2\cdot2\cdot2\cdot7$ for some $n\in\mathbb{N}$

$\therefore a+b=n(56+65)=n\cdot121=\boxed{n\cdot11\cdot11}$

$\therefore$ a+b is composite

Prob 12: Find all solutions in natural numbers of the equations a) $x^2-y^2=31$ b) $x^2-y^2=303$

$(x+y)(x-y)=31\cdot1$

$x+y=31$
$x-y=1$
$x=16, y=15$ ✓

$x+y=1$
$x-y=31$
$x=16$ $y=-15$ X

$(x+y)(x-y)=303=101\cdot3$
$=303\cdot1$

$x+y=101$
$x-y=3$
$x=52$ $y=49$ ✓

$x-y=101$
$x+y=3$
$x=52$ $y=-49$ X

$x+y=303$
$x-y=1$
$x=152$
$y=151$

Prob 13: Find the integer roots of the equation $x^3+x^2+x-3=0$

$x^3+x^2+x=3 \Rightarrow x(x^2+x+1)=3$

$x=1$
$x^2+x+1=3$

$x=\cancel{3}$ 1 ✓

$x^2+x+1=3$
$x^2+x-2=0$
$x^2+2x-x-2=0$
$(x-1)(x+2)=0$