

a remainder of 4 when divided by 5, and a remainder of 5 when divided by 6.

Let n be that number which is smallest.

$n = 6x_1 + 5$, for some $x_1 \in \mathbb{N}$.

$= 5(x_1 + 1) + x_1$. By given condn, $x_1 = 5x_2 + 4$ for some $x_2 \in \mathbb{N}$.

$$= 5(x_1 + 1) + 5x_2 + 4 = 5(x_1 + 1 + x_2) + 4 = 5(x_1 + 1 + \frac{x_1 - 4}{5}) + 4$$

$$= 4(x_1 + 1 + \frac{x_1 - 4}{5} + 1) + (x_1 + 1 + \frac{x_1 - 4}{5})$$

$$= 4(x_1 + 2 + \frac{x_1 - 4}{5}) + (\frac{6x_1 + 1}{5}). \text{ By given condn, } \frac{6x_1 + 1}{5} = 4x_3 + 3 \text{ for some } x_3 \in \mathbb{N}$$

$$= 4(x_1 + 2 + \frac{x_1 - 4}{5}) + (4x_3 + 3) = 4(x_1 + 2 + \frac{x_1 - 4}{5} + x_3) + 3$$

$$= 4(x_1 + 2 + \frac{x_1 - 4}{5} + \frac{6x_1 - 14}{20}) + 3 = 4(x_1 + 2 + \frac{x_1 - 4}{5} + \frac{3x_1 - 7}{10}) + 3$$

$$= 4(\frac{10x_1 + 20 + 2x_1 - 8 + 3x_1 - 7}{10}) + 3 = 4(\frac{15x_1 + 5}{10}) + 3 = 4(\frac{3x_1 + 1}{2}) + 3$$

$$= 3(\frac{3x_1 + 1}{2} + 1) + \frac{3x_1 + 1}{2} = 3(\frac{3x_1 + 3}{2}) + \frac{3x_1 + 1}{2}$$

$$\text{By given condn, } \frac{3x_1 + 1}{2} = 3x_4 + 2 \quad x_4 = \sqrt{\frac{x_1 - 1}{2}}$$

$$= 3(\frac{3x_1 + 3}{2}) + 3x_4 + 2$$

$$= 3(\frac{3x_1 + 3}{2} + x_4) + 2 = 3(\frac{3x_1 + 3}{2} + \frac{x_1 - 1}{2}) + 2 = 3(\frac{4x_1 + 2}{2}) + 2$$

$$= 3(2x_1 + 1) + 2 = 2(2x_1 + 1 + 1) + (2x_1 + 1) = 2(2x_1 + 2) + (2x_1 + 1)$$

~~2.4~~

$$\bullet n = 5(x_1 + 1) + x_1 = 5(5x_2 + 5) + (5x_2 + 4) = 30x_2 + 29 = 4(7x_2 + 7) + (2x_2 + 1)$$

\bullet By given condn, $2x_2 + 1 = 4x_3 + 3$ for some $x_3 \in \mathbb{N}$.

$$\therefore x_2 = 2x_3 + 1 \quad n = 30x_2 + 29 = 30(2x_3 + 1) + 29 = 60x_3 + 59$$

$$n = 60x_3 + 59 = 3(20x_3 + 19) + 2$$

$$= 2(30x_3 + 29) + 1$$

$\therefore n = 60x_3 + 59$ is the ideal form

The smallest such n is 59, when $x_3 = 0$