- We need to show there exists atleast 1 student who has solved atleast 5 problems.
- Suppose, none of the 7 students solve more than 4 problems each.
- ∴ The students can solve atmost $7 \times 4 = 28$ problems.
- But, we have 29 problems given.
- ∴ Our assumption that everyone solves atmost 4 problems each is wrong
- ∴ $\exists$ atleast 1 student who has solved ≥ 5 problems.

Prob 17: In a brigade of 7 people, the sum of the ages of the members is 332 years. Prove that 3 members can be chosen so that the sum of their ages is no less than 142 years.

- Suppose, the ages of the 7 members are $a_1, a_2, a_3, a_4, a_5, a_6, a_7$.

$$\boxed{a_1 + a_2 + \ldots\ldots + a_6 + a_7 = 332}$$

- Suppose, for any 3 members, the sum of their ages is less than or equal to 141.
- We can have $^7C_3 = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35$ choices of 3 members.
- Pick any one member. It forms $^6C_2 = 15$ groups with others.
- ∴ Sum of all possible 3 member groups is : $15 \times 332 = 4980$
- But this condn. shows the sum can be atmost $141 \times 35 = 4935$

$4980 > 4935$

- ∴ There must exist atleast 1 3 member group whose sum of age is ≥ 142 years.

Prob 19: Prove that there exist two powers of 2 which differ by a multiple of 1987.

- The powers of 2 are represented as 2^K, $K \geq 0$ and $K \in N$.
- ~~Any natural no. n can be~~ By the division algorithm, $\forall n \in N$, $\exists q, r$. $0 \leq r < 1987$ s.t. $\boxed{n = 1987 \cdot q + r}$