Proof by Contradiction

- Suppose each of the 15 boys denoted by B_i, $1 \leq i \leq 15$ gather different no. of nuts.
- The sum of the nuts turn out to be lowest when the nuts are picked from the set $\{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$, and each of the values appear once, as the nuts picked.
- The sum turns out to be: $0+1+2+3+\ldots\ldots+14 = \frac{15\times 14}{2} = 105$.
- The no. of nuts gathered is 100 originally (Contradiction)

Prob 26: The digits $1, 2, \ldots\ldots, 9$ are divided into three groups. Prove that the product of the numbers in one of the groups must exceed 71, i.e. ≥ 72. **

- Prime factorization of 72: $(2\times 2\times 2)\times(3\times 3) = 8\times 9$
- It's clear from this observation that 9 and 8 must be in separate groups. Suppose 9 goes to Group 1 and 8 goes to Group 2 (W.log)
- My goal is to delay the inevitable as long as possible, i.e. having a group where product of no.s is ≥ 72. → Imp.
- My goal for now is to make the product of no.s in Groups 1 and 2 as large as possible but it should be less than 72.
- Group 1: $\because 9\times 8 = 72$. The largest no. under 8 is 7. Put 7 in Group 1. $\therefore$ For now, $9\times 7 = 63$
- Group 2: $\because 8\times 9 = 72$, The largest no. under 9 is 8, ~~put 8 in~~ so $\because 4\times 2 = 8$, Put 4 and 2 in Group 2. For now, $8\times(4\times 2) = 64$
- The no.s left over are: 1, 3, 5, 6.
  Apart from 1, putting any of the no.s in Group 1 and 2 ~~will exceed to~~