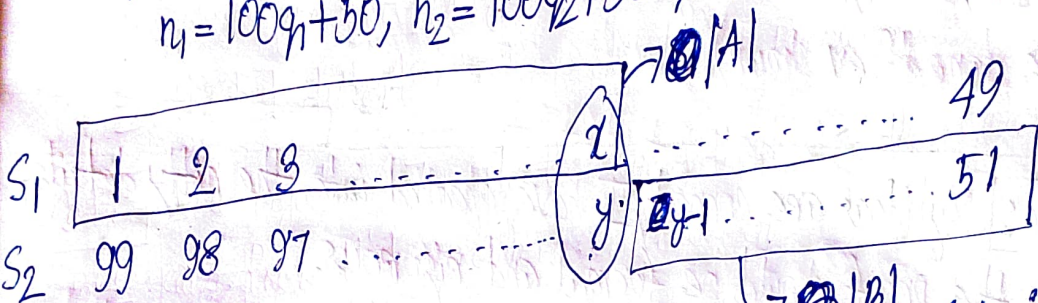


or, $n_1 + n_2 = 100(q_1 + q_2 + 1) + 0$ ~~if given remainder~~

$\therefore 100 | (n_1 + n_2)$. We can also see 0 and 50 can only be paired with 0 and 50 respectively.

\therefore Let, $n_1 = 100q_1 + 0$, $n_2 = 100q_2 + 0 \Rightarrow n_1 + n_2 = 100(q_1 + q_2) + 0$
 $n_1 = 100q_1 + 50$, $n_2 = 100q_2 + 50 \Rightarrow n_1 + n_2 = 100(q_1 + q_2 + 1) + 0$



We take two non-overlapping ~~subset~~ (in the sense of having no downward adjacent pairs) subsets $A \subseteq S_1$ and $B \subseteq S_2$.

~~Notice~~ $|S_1| = |S_2| = 49$

~~Notice~~ Here, $|A| + |B| = 49$

Two more integers can be there who give remainders 0 and 50 respectively.

Note: Since we have taken no integers who give same remainder till now, ~~no~~ no pair has their difference div. by 100.

$n_1 = 100q_1 + r_1$, $n_2 = 100q_2 + r_2$, $n_1 - n_2 = 100(q_1 - q_2) + (r_1 - r_2)$
 $100 | (n_1 - n_2)$ iff \checkmark

Diff of
 \therefore No pair of integers has their squares div by 100 till now.

If the 52nd integer gives remainder 0 or 50, we are done (Proved before)

~~If~~ If $a_{52} = 100q + r$, ~~then~~ $\forall a \in A, r > a$ and $r < 49$, then r can be downward paired with an element of set B. (\therefore then $r \in S_1 - A$ and we have taken all elements from set B.)