- Since p is a prime no., using division algorithm, p can take the form of $6K+1$ or $6K+5$, where $K \in \mathbb{N}$.
- $p = 6K+1$ $p^2+2 = 36K^2+12K+3 = 3(12K^2+4K+1)$ (Not possible)
- $p = 6K+5$ $p^2+2 = 36K^2+60K+27 = 3(12K^2+20K+9)$ (Not possible)

$p = 2$ $p^2+2 = 6$ ✗

$\boxed{p = 3 \quad p^2+2 = 11 \quad p^3+2 = 29}$ → Only possible case

Prob 39: a) Can the sum of two perfect squares be another perfect square?

- Yes, take any Pythagorean triplet.

b) Can the sum of three squares of odd natural numbers be a perfect square?

- Let, x, y and z be odd natural numbers

$\therefore$ By division algorithm, x is $2K_1+1$, y is $2K_2+1$ and z is $2K_3+1$, for some $K_1, K_2, K_3 \in \mathbb{N}$.

- $x^2+y^2+z^2 = 4K_1^2+4K_1+4K_2^2+4K_2+4K_3^2+4K_3+3 = \boxed{4(K_1^2+K_1+K_2^2+K_2+K_3^2+K_3)+3}$
- Suppose, $\exists p \in \mathbb{N}$ s.t. $p^2 = x^2+y^2+z^2$
- By div algo, p can be $2K_4$ or $2K_4+1$.
- If $p = 2K_4$, $p^2 = 4K_4^2$. If $p = 2K_4+1$, $p^2 = 4K_4^2+4K_4+1 = \boxed{4(K_4^2+K_4)+1}$

In either case, p^2 can't be mapped to $x^2+y^2+z^2$

$\therefore$ Such a p doesn't exist

Prob 40: Prove that the sum of the squares of five consecutive natural numbers cannot be a perfect square.

- Let $n, n+1, n+2, n+3$ and $n+4$ be five consecutive natural numbers.

$n^2+(n+1)^2+(n+2)^2+(n+3)^2+(n+4)^2 = 5n^2+20n+30 = 5(n^2+4n+6)$

- If $5(n^2+4n+6)$ has to be a perfect square, n^2+4n+6 has to be divisible by 5.
- By division algorithm, $n = 5K$, or $5K+1$ or $5K+2$ or $5K+3$ or $5K+4$ for some $K \in \mathbb{N}$.