

• Suppose rev_8 and n_8 have same parity. $\therefore C_7 = 1$ to make \oplus sum at 8th posn. odd. Also, rev_{10} and n_{10} have same parity $\therefore C_9 = 1$ to make sum at 10th posn. odd.

• Suppose rev_7 and n_7 have opposite parity. $\therefore rev_{11}$ and n_{11} have opposite parity. $\therefore C_{10} = 0$ to make sum at 11th posn. odd.

$$\begin{aligned} (C_7 + rev_8 + n_8)/10 &= C_8 & C_7 = C_9 = 1 & rev_8 = n_{10}, n_8 = rev_{10} \\ (C_9 + rev_{10} + n_{10})/10 &= C_{10} & \therefore C_8 &= C_{10} \end{aligned}$$

But, $C_{10} = 0, C_8 = 1$

$\therefore rev_7, n_7, rev_{11}, n_{11}$ have same parity

$\therefore C_{10} = 1$ and $C_8 = 1$ to make sum at 11th and 8th posn. odd respectively.

(The cases for the lower bits gets repeated)

Problem 25: There are 100 soldiers in a detachment, and every evening three of them are on duty. Can it happen that after a certain period of time each soldier has shared duty with every other soldier exactly once?

Ans:). Suppose, we take aside one soldier. We have 99 soldiers left.

• For 49 days, we form 49 disjoint pairs, and put them to duty along with the original soldier.

• But on the 50th day, we take the 99th soldier, original soldier. We must take atleast 1 of the remaining 98 soldiers.

\therefore A single soldier couldn't have shared duty with every other soldier exactly once.

\therefore It's impossible.