Prob 32: Prove that we can choose a subset of a set of ten given integers, such that their sum is divisible by 10.

Proof: Suppose, the set of integers are: $S = \{a_1, a_2, a_3, \dots\dots, a_{10}\}$.

- We pick 10 different sums: $a_1, a_1+a_2, \dots\dots, a_1+a_2+\dots\dots+a_i, \dots\dots, a_1+a_2+\dots\dots+a_{10}$.
- By division algorithm, any integer $n \in \mathbb{Z}$ can be written as $10q+r$, where $q \in \mathbb{Z}, r \in \mathbb{Z}, 0 \leq r \leq 9$.
- If any one of the above sums is already divisible by 10, we are done.
- Else, by pigeonhole principle, there exists $i, j \in \mathbb{N}$, $1 \leq i < j \leq 10$, s.t. $(a_1+a_2+\dots\dots+a_i) = 10q_1+r$ and $(a_1+a_2+\dots\dots+a_j) = 10q_2+r$, i.e. they leave the same remainder when divided by 10.
- $\boxed{\therefore (a_{i+1}+a_{i+2}+\dots\dots+a_j) = 10(q_2-q_1)}$, i.e. it is div by 10.

  $(a_{i+1}, a_{i+2}, \dots\dots, a_j)$ is a subset whose sum is divisible by 10.