The Pigeon Hole Principle (Chap-4)

. If we must put N+1 or more pigeons into N pigeon-holes, then some pigeon hole must contain two or more pigeons.

Proof by Contradiction: Suppose no more than one pigeon were in each hole. Then there would be no more than N pigeons altogether, which contradicts the assumption that we have N+1 pigeons.

Prob 1: A bag contains beads of two colors: black and white. What is the smallest no. of beads which must be drawn from the bag, without looking, so that among these beads there are two of the same color?

. Suppose, we arbitrarily pick two beads. There is a possibility that these two beads are of different colors. When we pick a third bead, that bead must have a color same as that of either of the previous two beads, since we have only 2 possible colors.

$\therefore$ The answer is 3

Prob 3: Given twelve integers, show that two of them can be chosen who difference is divisible by 11.

. Let, the 12 integers be $a_1, a_2, a_3, \ldots\ldots, a_{12}$.

. By the division algorithm, $\forall i \in \{1, 2, \ldots, 12\}$, $a_i = 11q_i + r_i$, where $\boxed{0 \leq r_i < 11}$

. Observe, there are 11 different remainders $(0, 1, 2, \ldots, 10)$ which a_i can give when divided by 11.

. In the worst case, 11 out of the 12 integers give these different remainders each.

. $\therefore \exists a_j, 1 \leq j \leq 12$ s.t. $r_i = r_j$ for some a_i, where $1 \leq i \leq 12, i \neq j$.

$a_j - a_i = 11(q_j - q_i) + \cancel{(r_j - r_i)}\,0$

. $a_i = 11q_i + r_i \quad a_j = 11q_j + r_j$

. $\boxed{\therefore 11 \mid a_j - a_i}$