

Now we shall try to show  $2p^2 + 2p \neq 6p^2 + 9p + 3$   
 $\downarrow$   
 $2p(p+1)$   $\rightarrow 3(2p^2 + 3p + 1)$   
 $3(p+1)(2p+1)$

New way (Deserves a better idea)

$a^2 = 8 + 3b^2$ . By div algo,  $b$  can be  $8K, 8K+1, \dots, 8K+7$  for some  $K \in \mathbb{N}$

$b = 8K, 8 + 3b^2 = 8 + 192K^2 = 8(1 + 24K^2) = 8(2(12K^2) + 1)$

$\rightarrow$  not divisible by 2

~~Again by div algo,  $K$  can be  $2p$  or  $2p+1$ ,  $2p+2$~~   
 ~~$K = 2p, 24K^2 + 1 = 96p^2 + 1$~~   $\therefore 8 + 3b^2$  can't be a perfect square

$b = 8K+1, 8 + 3b^2 = 8 + 3(8K+1)^2 = 8 + 3(64K^2 + 16K + 1) = 192K^2 + 48K + 8 + 3$   
 $= 4(48K^2 + 12K + 2) + 3$

$a = 4p, 4p+1, 4p+2$  or  $4p+3$  for some  $p \in \mathbb{N}$ .

$a^2 = 16p^2, 16p^2 + 8p + 1 = 4(4p^2 + 2p) + 1, 16p^2 + 16p + 4 = 4(4p^2 + 4p + 1),$   
 $16p^2 + 24p + 9 = 4(4p^2 + 6p + 2) + 1 \quad \therefore a^2 \neq 8 + 3b^2$  for this case

$b = 8K+2 \quad 8 + 3b^2 = 8 + 3(64K^2 + 4 + 32K) = 8 + 192K^2 + 12 + 96K$   
 $= 192K^2 + 96K + 20$

$= 4(48K^2 + 2K + 5)$

We shall try to show  $48K^2 + 2K + 5$  is not divisible by 4

$K = 2K_1$  or  $2K_1+1$  for some  $K_1 \in \mathbb{N}$ . (Div algo)

$K = 2K_1, 48K^2 + 2K + 5 = 192K_1^2 + 4K_1 + 5 = 4(48K_1^2 + K_1 + 1) + 1$

$K = 2K_1+1, 48K^2 + 2K + 5 = 48(4K_1^2 + 4K_1 + 1) + 2(2K_1+1) + 5$   
 $= 4(48K_1^2 + 49K_1 + 13) + 3$

$\therefore 4 \nmid (48K^2 + 2K + 5)$

$b = 8K+3 \quad 8 + 3b^2 = 8 + 3(8K+3)^2 = 8 + 3(64K^2 + 48K + 9)$   
 $= 4(48K^2 + 36K + 8) + 3$

$\therefore a^2 \neq 8 + 3b^2$  here