• Again, try finding the ~~remainder~~ fact whether $2^{2777}\cdot 3 + 4^{2222}$ is div by 7

$$2^{2777}\cdot 3+4^{2222} = 2^2\cdot 2^{2775}\cdot 3 + 2^{4444} = 12\cdot 8^{925} + 2\cdot 8^{1481}$$

$$= 12\cdot(7+1)^{925} + 2\cdot(7+1)^{1481}$$

• Again, using previous concept, $12\cdot 1^{925}+2\cdot 1^{1481} = 14$ which is really div by 7

F $\boxed{\text{Prob 34:}}$ Find the last digit of the number 7^{7^7}.

• Using the ideas used in Prob 30,

$$7^{7^7} = 7^{7\times7\times7\times7\times7\times7\times7} = 7^{(4+3)(4+3)\cdots\cdots(4+3)} = (7^4)^x\cdot 7^{3^7}$$

↳ has last digit 1

∴ We will try finding the last digit of 7^{3^7}

$$7^{3^7} = 7^{3^6\times 3} = 7^{9^3\times 3} = 7^{(8+1)(8+1)(8+1)\cdot 3} = (7^4)^x\cdot 7^3$$

↳ has last digit 1

F ∴ The last digit of 7^3 is 3

$\boxed{\text{Prob 36:}}$ Given the pair of prime numbers p and $8p^2+1$, find p.

• Since p is a prime no., using division algorithm, p can be of the form $6K+1$ or $6K+5$, for some $K\in\mathbb{N}$. (Key idea) (Very imp technique)

• $p=6K+1$ $\quad 8p^2+1 = 8(6K+1)^2+1 = 8\cdot 36K^2+8\cdot 12K+9 = 3(8\cdot 12K^2+8\cdot 4K+3)$

↳ not prime

• $p=6K+5$ $\quad 8p^2+1 = 8(6K+5)^2+1 = 8\cdot 36K^2+8\cdot 60K+201 = 3(8\cdot 12K^2+8\cdot 20K+67)$

↳ not prime

$p=2$, $8p^2+1 = 33$ → not prime

$\boxed{p=3,\ 8p^2+1 = 73}$ → only possible pair

$\boxed{\text{Prob 37:}}$ Given the pair of prime numbers p and p^2+2, prove that p^3+2 is also a prime number.