

Suppose each row and column have ~~no~~ equal copies of the same no.
 $\therefore x_i$ must occur 15 times in the square table.
 By problem 13, one occurrence of x_i must be on the main diagonal,
 since 15 is odd. But this is a contradiction.
 \therefore No two no.s on the main diagonal are equal.

Prob 16: Can one make change of a 25-ruble bill, using in all ten bills each having a value of 1, 3 or 5 rubles?

Ans: No, since the sum of evenly many odd numbers is always even.

$$(2k_1+1) + (2k_2+1) + \dots + (2k_{10}+1) = 2(k_1 + \dots + k_{10}) + 10$$

$$= 2(k_1 + \dots + k_{10} + 5)$$

Prob 17: Pete bought a notebook containing 96 pages, and numbered them from 1 through 192. Victor tore out 25 pages of the notebook, and added up the 50 numbers he found on the pages. Could Victor have gotten 1990 as the sum?

Ans: **Observation:** Any particular page has an odd no. in the front and an even no. in the back.
 • On 25 pages, we will get 25 odd and 25 even no.s respectively.
 • Since the no. of odds are odd, the total sum must be odd.
 \therefore It's not possible.

Prob 18: The product of 22 integers is equal to 1. Show that their sum cannot be zero.

Ans: Since the no.s are integers, they can only take up the value +1 or -1.
 Since the product is +1, the no. of negative 1's must be even.
 • The summation of the integers can be: $\{-22, -18, -14, -10, -6, -2, 2, 6, 10, 14, 18, 22\}$
 when the no. of ~~no~~ -1's are: $\{22, 20, 18, 16, 14, 12, 10, 8, 6, 4, 2, 0\}$ respectively.