- $n=9k+4$, $2n^3+1 = 2(9k+4)^3+1 = 2\cdot 729k^3+2\cdot 3\cdot 81k^2\cdot 4+2\cdot 3\cdot 9k\cdot 16 + 129$

$= \boxed{27(54k^3+72k^2+32k+4)+17}$ → not div by 3^3

- $n=9k+5$, $2n^3+1 = 2(9k+5)^3+1 = 2\cdot 729k^3+2\cdot 3\cdot 81k^2\cdot 5+2\cdot 3\cdot 9k\cdot 25 + 251$

$= \boxed{27(54k^3+90k^2+50k+9)+8}$ → not div by 3^3

- $n=9k+6$, $2n^3+1 = 2(9k+6)^3+1 = 2\cdot 729k^3+2\cdot 3\cdot 81k^2\cdot 6+2\cdot 3\cdot 9k\cdot 36 + 433$

$= \boxed{81(18k^3+36k^2+24k+5)+28}$ → not div by 3^4

- $n=9k+7$, $2n^3+1 = 2(9k+7)^3+1 = 2\cdot 729k^3+2\cdot 3\cdot 81k^2\cdot 7+2\cdot 3\cdot 9k\cdot 49 + 687$

$= \boxed{27(54k^3+126k^2+98k+25)+12}$ ↳ not div by 3^3

- $n=9k+8$, $2n^3+1 = 2(9k+8)^3+1 = 2\cdot 729k^3+2\cdot 3\cdot 81k^2\cdot 8+2\cdot 3\cdot 9k\cdot 64 + 1025$

$= \boxed{27(54k^3+144k^2+128k+37)+26}$ ↳ not div by 3^3

$\therefore \forall n$, $2n^3+1$ is not div by 243.

$\therefore 6n^3+3$ is not a perfect sixth power of an integer for $\forall n$.

So, $6n^3+3$ cannot be a perfect sixth power of an integer for $\forall n$.

Prob 45: Given natural numbers $x, y,$ and z such that $x^2+y^2=z^2$, prove that xy is divisible by 12.

- First we will try to show, whether either of x or y must be div. by 3.

$z = 3c$ or $3c+1$ or $3c+2$ for some $c \in \mathbb{N}$ (By div. algorithm)

$z^2 = 9c^2 = 3(3c^2)$ or $9c^2+6c+1 = 3(3c^2+2c)+1$

or $9c^2+12c+4 = 3(3c^2+4c+1)+1$

$\therefore z^2$ always leaves $\{0,1\}$ as remainder when divided by 3.