

we must have atleast three 2's present, \therefore It must be divisible by 8.

\therefore any four consecutive integers has two sets of three consecutive integers, by Prob 2, $p(p+1)(p+2)(p+3)$ is divisible by 3.

\therefore 8 and 3 are co-prime to each other, $\therefore 8 \times 3 = 24 \mid p(p+1)(p+2)(p+3)$

From here, it's very easy to show that the product of any 5 consecutive integers is divisible by 120.

Prob 5: Find the smallest natural number n such that $n!$ is divisible by 990.

Ans: $990 = 9 \times 10 \times 11 = 9 \times 10 \times 11$.

$\therefore n!$ must contain 9, 10 and 11 atleast once to be divisible by 990.

$\therefore n$ must be 11, since 11! is the smallest such factorial.

Prob 6: How many zeros are there at the end of the decimal representation of the number $100!$?

Ans: $100!$ contains atleast 11 0's. This is because the number 100! has 1 occurrence each of the ~~1st~~ 1st 10 multiples of 10 at the end.

We can also get zero at the end if we multiply a 5 by a even no., or more specifically, a 5 by a 2.

Let down the prime factorization of all the numbers from 1 to 100 apart from the multiples of 10.

Observation

5 15 = 3.5 25 = 5.5 35 = 5.7 45 = 3.3.5 55 = 5.11
65 = 5.13 75 = 3.5.5 85 = 5.17 95 = 5.19

Notice, we have 12 5's among all those no.'s. \therefore We can easily get 12 more 0's, \therefore we have 50 even no.s, there must be atleast 50 2's as a bare minimum in the prime factorization.