• $2L_{43} > 400 \Rightarrow \boxed{L_{43} > 200} \rightarrow$ violates $\boxed{L_{43} + R_{43} = 200}$

(Contradiction)

Prob 31: The alphabet of a certain language contains 22 consonants and 11 vowels. Any string of these letters is a word in this language, so long as no two consonants are together and no letter is used twice. The alphabet is divided into 6 (non-empty) subsets. Prove that the letters in atleast one of these groups form a word in the language.

C: set of consonants
V: set of vowels

Proof: • We will represent the subsets as S_i, where $1 \leq i \leq 6$, $i \in \mathbb{N}$.

• For all S_i, if $|S_i| = n$, then $\langle a_1 a_2 \ldots\ldots a_n \rangle$ represents a permutation of the characters where $\forall j \in \mathbb{N}$, $1 \leq j \leq n$, $a_j \in S_i$.

• A permutation of characters $\langle P_1 P_2 \ldots\ldots P_m \rangle$ is a word iff $\forall j$, $1 \leq j \leq m-1$, (if $P_j \in C$, then $P_{j+1} \notin C$) and (if $P_{j+1} \in C$, then $P_j \notin C$) (By defn. of word)

• **Claim:** If for any S_i, where $1 \leq i \leq 6$, $i \in \mathbb{N}$, we get two subsets of S_i which are S_{ic} and S_{iv}, which are defined as follows: if character $c \in S_{ic}$, then $c \in C$ and if $c \in S_{iv}$, then $c \in V$, and $|S_{ic}| - |S_{iv}| > 1$, then there exists no permutation of characters $\langle a_1 a_2 \ldots\ldots a_m \rangle$ (if $|S_i| = m$) of S_i which is a word. (Not needed to show)

Note: By defn. of consonant and vowel, if $c \in V$ then $c \notin C$ and if $c \in C$ then $c \notin V$

Proof: • Suppose, $|S_{ic}| = a$ and $|S_{iv}| = b$, $|S_i| = a + b = m$.

• Let $\langle P_1, P_2, \ldots\ldots, P_a \rangle$ be an arbitrary permutation of S_{ic}.

• To satisfy the condition that an arbitrary permutation of S_i will be a word, there must exist atleast one vowel between any two characters in $\langle P_1, P_2, \ldots\ldots, P_a \rangle$