Problem 32: Is it possible to arrange the numbers from 1 through 9 in a sequence so that there are oddly many numbers between 1 and 2, between 2 and 3, ..., and between 8 and 9?

Ans:) Let us denote the parity P_i of the posn. of integer $i \in \{1, 2, \ldots, 9\}$ in a sequence.

∵ no. of integers are odd between 1 and 2,

∴ posn 1 ± (odd+1) = posn. 2

or, posn 1 ± even = posn. 2 ∴ P_1 and P_2 must have the same parity.

. Similarly, (P_2, P_3), (P_3, P_4), ..., (P_8, P_9) all have same parity in pairs.

∴ All the integers have same parity of posn. in the sequence.

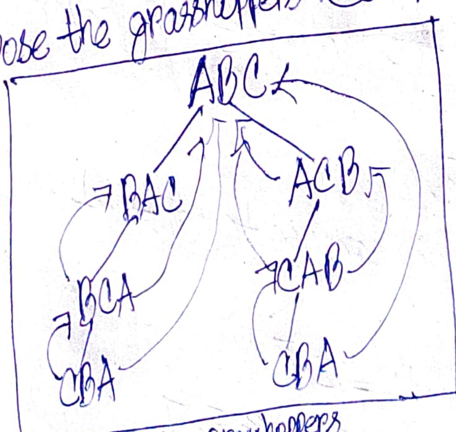
. But in any arbitrary arrangement of no.s from 1 to 9, 5 have odd parity of posn. and 4 have even parity of posn.

∴ The given arrangement is not possible.

Problem 30: Three grasshoppers play leapfrog along a line. At each turn, one grasshopper leaps over another, but not over two others. Can the grasshoppers return to their initial positions after 1991 leaps?

Ans: **Assumption:** I tried solving the problem using relative posn. of grasshopper, not how much distance they covered.

. Suppose the grasshoppers A, B, C have initial ordering ABC.



It's clear, we can only return to our original state by an even no. of moves.

. Each branch of a tree represents the new states we can reach from the original state

∵ 1991 is odd, grasshoppers can't return to original posn. by 1991 leaps.