. Let $|F_x|=2$, and let $P_y, P_z \in F_x$ where $1 \leq y, z \leq 6$, wlog.

. Suppose $\{P_w, P_u, P_v\}$ are the set of people P_x doesn't know.

. If ($P_w \notin F_u$ and $P_u \notin F_w$), then (P_x, P_w, P_u); if ($P_w \notin F_v$ and $P_v \notin F_w$), then (P_w, P_v, P_x); if ($P_u \notin F_v$ and $P_v \notin F_u$), then (P_x, P_u, P_v) respectively form a set where none knows each other.

. The above condition represents if there is atleast one such pair in the set P_x doesn't know who don't know each other, we are done.

Else, $\{P_w, P_u, P_v\}$ represent a set of people who all know each other.

Prob 30: A warehouse contains 200 boots of size 41, 200 boots of size 42 and 200 boots of size 43. Of these 600 boots, there are 300 left boots and 300 right boots. Prove that one can find among these boots at least 100 usable pairs.

Proof: . L_{41} : no. of left boots of size 41 R_{41} : no. of right boots of size 41.

L_{42}, R_{42}, L_{43} and R_{43} are defined similarly.

$\boxed{L_{41}+R_{41}=200}$ $\boxed{L_{42}+R_{42}=200}$ $\boxed{L_{43}+R_{43}=200}$

$\boxed{L_{41}+L_{42}+L_{43}=300}$ $\boxed{R_{41}+R_{42}+R_{43}=300}$

. We need to prove that $\min(L_{41},R_{41})+\min(L_{42},R_{42})+\min(L_{43},R_{43}) \geq 100$

. Suppose, for the sake of contradiction, $\min(L_{41},R_{41})+\min(L_{42},R_{42})+\min(L_{43},R_{43}) < 100$.

. All the minimums can't be (L_{41},L_{42},L_{43}) or (R_{41},R_{42},R_{43}) due to given constraint.

~~Suppose for the sake of~~

. Wlog, let $\min(L_{41},R_{41})=L_{41}$ $\min(L_{42},R_{42})=L_{42}$ and $\min(L_{43},R_{43})=R_{43}$

$L_{41}+L_{42}+R_{43}<100$ or, $L_{41}+L_{42}+200-L_{43}<100$

or, $L_{43}-L_{41}-L_{42} > 100$