

Case 8: $p = 6k_1 + 5$ $q = 6k_2 + 5$ $r = 6k_3 + 5$

$$q = p + d \Rightarrow d = 6(k_2 - k_1)$$

$$r = p + 2d \Rightarrow 6k_3 + 5 = 6k_1 + 5 + 12(k_2 - k_1) = 6(2k_2 - k_1) + 5$$

$\therefore 6 \mid d$

Prob 28: Find the last digit of the number 1989^{1989}

- It's intuitive to see that the last digit of 1989^{1989} is the same as that of 9^{1989} .
- Observe: $9^1 = 9$, $9^2 = 81$, $9^3 = 729$ 9^4 will end with 1 and so on.
- We can easily show by induction on K , where $K \geq 1$, that the last digit of the number 9^K will either end with 9 or 1.
- After every 2 powers, the same pattern of last digit repeats (Observation)
- $\therefore 1989$ is an odd no., $\therefore 1989^{1989}$ will end with 9.

Prob 29: Find the last digit of the number 2^{50} .

$2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, and so on.

- We can easily show by induction on K , where $K \geq 1$, that the last digit of the number 2^K will be either 2, 4, 8, or 6. Each of the numbers having last digit 2 will have K of the form $4p+1$, having last digit 4 will have K of the form $4p+2$, having last digit 8 will have K of the form $4p+3$, having last digit 6 will have K of the form $4p$, where $p \in \mathbb{N}$

$50 = (4 \times 12) + 2$

$2^{50} = 2^{48} \cdot 2^2$
 \downarrow ends with 6 \downarrow ends with 4

Prob 30: What is the last digit of 777^{777} ?

- It's intuitive to see that the last digit of 777^{777} is the same as that of 777 .