

• by division algorithm, x can be $3a, 3a+1, 3a+2$ and y can be $3b, 3b+1, 3b+2$ for some $a, b \in \mathbb{N}$.

Case 1: $x=3a, y=3b \quad x^2+y^2 = 9a^2+9b^2 = 3(3a^2+3b^2)$ (Feasible)

Case 2: $x=3a, y=3b+1 \quad x^2+y^2 = 9a^2+9b^2+6b+1 = 3(3a^2+3b^2+2b)+1$ (Feasible)

Case 3: $x=3a, y=3b+2 \quad x^2+y^2 = 9a^2+9b^2+12b+4 = 3(3a^2+3b^2+4b+1)+1$ (Feasible)

Case 4: $x=3a+1, y=3b$ (Symmetric to case 2)

Case 5: $x=3a+1, y=3b+1 \quad x^2+y^2 = 9a^2+6a+9b^2+6b+2 = 3(3a^2+2a+3b^2+2b)+2$ (Not Feasible)

Case 6: $x=3a+1, y=3b+2 \quad x^2+y^2 = 9a^2+6a+9b^2+12b+5 = 3(3a^2+2a+3b^2+4b+1)+2$ (Not Feasible)

Case 7: $x=3a+2, y=3b$ (Symmetric to case 3)

Case 8: $x=3a+2, y=3b+1$ (Symmetric to case 6)

Case 9: $x=3a+2, y=3b+2 \quad x^2+y^2 = 9a^2+12a+9b^2+12b+8 = 3(3a^2+4a+3b^2+4b+2)+2$ (Not Feasible)

Observation: Each of the feasible cases has either x or y (or both) divisible by 3

• by division algorithm, $\exists m, n \in \mathbb{N}$ s.t. x is $2m$ or $2m+1$, y is $2n$ or $2n+1$

1) $x=2m, y=2n \quad x^2+y^2 = 4m^2+4n^2 = 4(m^2+n^2)$

2) $x=2m, y=2n+1 \quad x^2+y^2 = 4m^2+4n^2+4n+1 = 4(m^2+n^2+n)+1$

3) $x=2m+1, y=2n$ (Symmetric to Case 2)

4) $x=2m+1, y=2n+1 \quad x^2+y^2 = 4m^2+4m+4n^2+4n+2 = 2(2m^2+2m^2+2n^2+2n+1)$

Not Feasible $= 2(2(m^2+n^2+m+n)+1) \rightarrow$ not div by 4