$n=5K$, $n^2+4n+6=(5K)^2+4\cdot(5K)+6=25K^2+20K+6=5(5K^2+4K+1)+1$

$n=5K+1$, $n^2+4n+6=(5K+1)^2+4\cdot(5K+1)+6=25K^2+30K+11=5(5K^2+6K+2)+1$

$n=5K+2$, $n^2+4n+6=(5K+2)^2+4\cdot(5K+2)+6=25K^2+40K+18=5(5K^2+8K+3)+3$

$n=5K+3$, $n^2+4n+6=(5K+3)^2+4\cdot(5K+3)+6=25K^2+50K+27=5(5K^2+10K+5)+2$

$n=5K+4$, $n^2+4n+6=(5K+4)^2+4\cdot(5K+4)+6=25K^2+60K+38=5(5K^2+12K+7)+3$

∴ In neither case, $5 \mid n^2+4n+6$

∴ $5 \nmid n^2+4n+6$ ∴ $5(n^2+4n+6)$ is not a perfect square.

Prob 38: Prove that there are no natural numbers a and b such that $a^2-3b^2=8$

~~• By division algorithm, a can be $2x$ or $2x+1$. b can be $2y$ or $2y+1$, for some $x,y\in N$.~~

~~• $a=2x, b=2y$ $a^2-3b^2=4x^2-3\cdot 4y^2=4(x^2-3y^2)$~~

~~Prove, $x^2-3y^2\neq 2$. Suppose $x^2-3y^2=2$~~

~~$x^2=3y^2+2$~~

• We need to show that $\nexists a$ such that $\boxed{a^2=8+3b^2}$

• By division algorithm, a can be $2x$ or $2x+1$. b can be $2y$ or $2y+1$, for some $x,y\in N$.

• $a=2x, b=2y$ $a^2=4x^2$ $8+3b^2=8+12y^2=4(3y^2+2)$

We will now show that $x^2\neq 3y^2+2$

• By division algorithm, x can be $2K_1$ or $2K_1+1$. y can be $2K_2$ or $2K_2+1$, for some $K_1,K_2\in N$

$x=2K_1, y=2K_2$ $x^2=4K_1^2$ $3y^2+2=3\cdot 4K_2^2+2=2(1+6K_2^2)$

~~By division algorithm, K_2 can be $2p$ or $2p+1$ for some $p\in N$~~

$2(6K_2^2+1)=2\underbrace{(2\cdot 3K_2^2+1)}_{\text{not div by 2}}$