• Since the no. of checkers are odd, there must be atleast one checker which can't be paired off with another checker for symmetry's sake.

∴ That checker must lie on the diagonal.

Prob 14: Let us now assume that the positions of the checkers in Prob 13 are symmetric with respect to both diagonals of the checkerboard. Prove that one of the checkers is placed in the center square.

Ans:) Two checkers are symmetric with respect to both diagonals iff they lie on one particular diagonal equidistant from the center square of a 25×25 checkerboard.

$(i-(j-K), K) = (j, i)$ $\qquad$ $(i+(j-K), K) = (j, i)$

$i-j+K = j$ $\qquad (K = i)$ $\qquad$ $K = i \quad i+j-K = j$

$i-j+i = j \qquad 2i = 2j, \underline{i=j}$ $\qquad$ $\underline{K=i}$

$\boxed{\begin{array}{l} i+(j-K) = K \\ i+j-i = i \quad i=j \end{array}}$

it

$\boxed{(j,i) = (j,i)}$

1st diag $\quad$ 2nd diag

∴ ∵ there are 25 checkers, which is odd, 1 checker has no other similar checker which can be put as symmetric with respect to both diagonals.

∴ It must lie in the middle square.

Prob 15: In each box of a 15×15 square table one of the numbers 1, 2, 3, ..., 15 is written. Boxes which are symmetric to one of the main diagonals contain equal numbers, and no row or column contains two copies of the same number. Show that no two of the numbers along the main diagonal are the same.

Ans:) Suppose any two of the no.s on the main diagonal are equal.

∴ $\exists x_i, x_i \in \{1, 2, ..., 15\}$ which does not lie on the diagonal.