$\boxed{\text{or, } n_1+n_2 = 100(q_1+q_2+1)+0}$; given remainder

$\hookrightarrow \therefore 100 \mid (n_1+n_2)$ • We can also see 0 and 50 can only be paired with 0 and 50 respectively.

$\because$ Let, $n_1 = 100q_1+0$, $n_2 = 100q_2+0 \Rightarrow n_1+n_2 = 100(q_1+q_2)+0$

$n_1 = 100q_1+50$, $n_2 = 100q_2+50 \Rightarrow n_1+n_2 = 100(q_1+q_2+1)+0$

$\rightarrow |A|$

S_1	1	2	3	....	x	 49
					y	$y+1$ 51
S_2	99	98	97	....		

$\rightarrow |B|$

• We take two non-overlapping (in the sense of having no downward adjacents pairs) subsets $A \subseteq S_1$ and $B \subseteq S_2$.

$|S_1| = |S_2| = 49$.

$\boxed{\text{Here, } |A|+|B| = 49}$

• Two more integers can be there who give remainders 0 and 50 respectively.

• $\boxed{\text{Note:}}$ Since we have taken no integers who give same remainder till now, no pair has their difference div. by 100.

$n_1 = 100q_1+r_1$ $n_2 = 100q_2+r_2$ $n_1-n_2 = 100(q_1-q_2)+(r_1-r_2)$

$100 \mid (n_1-n_2)$ iff 0

$\boxed{\therefore \text{No pair of integers has their diff of squares div by 100 till now.}}$

• If the 52nd integer gives remainder 0 or 50, we are done (Proved before)

• If $a_{52} = 100q+r$, $\forall a \in A, r > a$ and $r < 49$, then r can be downward paired with an element of set B. ($\because$ then $r \in S_1 - A$) ~~and we have taken all elements from~~