$\gcd(a,b) = a_1^{\min(p_1,r_1)} a_2^{\min(p_2,r_2)} \cdots\cdots a_x^{\min(p_x,r_x)}$

$\text{lcm}(a,b) = a_1^{\max(p_1,r_1)} a_2^{\max(p_2,r_2)} \cdots\cdots a_x^{\max(p_x,r_x)} b_1^{q_1} b_2^{q_2} \cdots\cdots b_y^{q_y} \cdots$

$c_1^{s_1} c_2^{s_2} \cdots\cdots c_z^{s_z}$

$\boxed{\text{Notice, } \gcd(a,b)\cdot \text{lcm}(a,b) = a\cdot b}$

- To divide a natural number N by the natural number m with a remainder means to represent N as $N = Km + r$, where $0 \leq r < m$. The number r is the remainder when N is divided by m.

(very powerful)

Lemma on Remainders: The sum/product of any two natural numbers has the same remainder, when divided by 3, as the sum/product of their remainders.

$N_1 = 3K_1 + r_1 \qquad N_2 = 3K_2 + r_2 \qquad 0 \leq r_1 < 3 \quad 0 \leq r_2 < 3$

$N_1 + N_2 = 3(K_1+K_2) + (r_1+r_2)$

$\boxed{\text{Now, } (r_1+r_2) = 3K + r} \quad 0 \leq r < 3$

$\boxed{N_1 + N_2 = 3(K_1+K_2+K) + r \quad 0 \leq r < 3}$

$N_1 \cdot N_2 = 9K_1K_2 + 3K_1r_2 + 3K_2r_1 + r_1r_2$

$= 3(3K_1K_2 + K_1r_2 + K_2r_1) + r_1r_2$

$\boxed{r_1 \cdot r_2 = 3K + r \quad 0 \leq r < 3}$

$\boxed{N_1 \cdot N_2 = 3(3K_1K_2 + K_1r_2 + K_2r_1 + K) + r \quad 0 \leq r < 3}$

Here, the no. 3 can be changed to any other no, the same proof follows through.

(Imp)

Prob 15: Find the remainder which a) the number $1989 \cdot 1990 \cdot 1991 + 1992^3$ gives when divided by 7