- No. of ways we can choose 6 no.s = ${}^7C_6 = 7$
- No. of different combinations a part. no. can be a part of = ${}^6C_5 = 6$
- Sum of all such combinations of 6 no.s = $6(a_1+a_2+\dots\dots+a_7)$, where $a_1, a_2, \dots\dots, a_7 \in \mathbb{N}$.
- $\because$ sum of any six natural numbers is div by 5,

$(a_1+a_2+a_3+a_4+a_5+a_6) = 5K_1$, $(a_1+a_2+a_3+a_4+a_5+a_7) = 5K_2$, $(a_1+a_2+a_3+a_4+a_6+a_7) = 5K_3$,

$(a_1+a_2+a_3+a_5+a_6+a_7) = 5K_4$, $(a_1+a_2+a_4+a_5+a_6+a_7) = 5K_5$, $(a_1+a_3+a_4+a_5+a_6+a_7) = 5K_6$

$(a_2+a_3+a_4+a_5+a_6+a_7) = 5K_7$, where $K_1, K_2, \dots\dots, K_7 \in \mathbb{N}$.

$\therefore 6(a_1+a_2+\dots\dots+a_7) = 5(K_1+K_2+\dots\dots+K_7)$

$\because \gcd(5,6)=1 \;\therefore 5 \mid (a_1+a_2+\dots\dots+a_7)$

$(a_1+a_2+\dots\dots+a_7) = 5 \cdot \dfrac{(K_1+K_2+\dots\dots+K_7)}{6} = 5P$, $P \in \mathbb{N}$.

$a_7 = 5(P-K_1)$ $a_6 = 5(P-K_2)$ $a_5 = 5(P-K_3)$ $a_4 = 5(P-K_4)$

$a_3 = 5(P-K_5)$ $a_2 = 5(P-K_6)$ $a_1 = 5(P-K_7)$

$\therefore$ Each of the no.s is div by 5

$\boxed{\text{Prob 49:}}$ For any $n>1$ prove that the sum of any n consecutive odd natural numbers is a composite number.

- Suppose, the n consecutive odd natural numbers are: $2K+1, 2K+3, 2K+5, \dots\dots, 2K+(2n-1)$.
- Adding them up, we get: $2nK + \boxed{(1+3+5+\dots\dots+(2n-1))} = 2nK + n^2 = n(2K+n)$

↳ High-school prereq.

$\boxed{\text{Prob 50:}}$ Find the smallest natural number which has a remainder of 1 when divided by 2, a remainder of 2 when divided by 3, a remainder of 3 when divided by 4,