

$\therefore x^2 \neq 3y^2 + 2$  for this case

$$x = 2k_1, y = 2k_2 + 1 \quad x^2 = 4k_1^2 \quad 3y^2 + 2 = 3(2k_2 + 1)^2 + 2 = 12k_2^2 + 12k_2 + 5 = 4(3k_2^2 + 3k_2 + 1) + 1$$

$\therefore x^2 \neq 3y^2 + 2$  for this case

$$x = 2k_1 + 1, y = 2k_2 \quad x^2 = 4k_1^2 + 4k_1 + 1 = \boxed{4(k_1^2 + k_1) + 1}$$

$$3y^2 + 2 = 12k_2^2 + 2 = 2(6k_2^2 + 1) = 2(\boxed{3k_2^2 + 1}) = \boxed{4(3k_2^2 + 2)}$$

$\therefore x^2 \neq 3y^2 + 2$  for this case

$$x = 2k_1 + 1, y = 2k_2 + 1 \quad x^2 = 4k_1^2 + 4k_1 + 1 = \boxed{4(k_1^2 + k_1) + 1}$$

$$3y^2 + 2 = 3(2k_2 + 1)^2 + 2 = 12k_2^2 + 12k_2 + 5 = 4(3k_2^2 + 3k_2 + 1) + 1$$

• We will try showing that  $\boxed{k_1^2 + k_1 \neq 3(k_2^2 + k_2) + 1}$

By division algorithm,  $k_1$  and  $k_2$  can be  $2p, 2p+1$  and  $2q, 2q+1$  for some  $p, q \in \mathbb{N}$ .

$$k_1 = 2p, k_2 = 2q \quad k_1^2 + k_1 = 4p^2 + 2p = 2(2p^2 + p) \quad 3(k_2^2 + k_2) + 1 = 3(4q^2 + 2q) + 1 = 2(6q^2 + 3q) + 1$$

$\therefore k_1^2 + k_1 \neq 3(k_2^2 + k_2) + 1$  for this case

$$k_1 = 2p, k_2 = 2q + 1 \quad k_1^2 + k_1 = 4p^2 + 2p = 2(2p^2 + p)$$

$$3(k_2^2 + k_2) + 1 = 3(4q^2 + 4q + 1 + 2q + 1) + 1 = 12q^2 + 18q + 7 = 2(6q^2 + 9q + 3) + 1$$

$\therefore k_1^2 + k_1 \neq 3(k_2^2 + k_2) + 1$  for this case

$$k_1 = 2p + 1, k_2 = 2q \quad k_1^2 + k_1 = 4p^2 + 4p + 1 + 2p + 1 = 2(2p^2 + 3p + 1)$$

$$3(k_2^2 + k_2) + 1 = 3(4q^2 + 2q) + 1 = 2(6q^2 + 3q) + 1$$

$\therefore k_1^2 + k_1 \neq 3(k_2^2 + k_2) + 1$  for this case

$$k_1 = 2p + 1, k_2 = 2q + 1 \quad k_1^2 + k_1 = 4p^2 + 4p + 1 = 4(p^2 + p) + 1$$

$$3(k_2^2 + k_2) + 1 = 3(4q^2 + 4q + 1 + 2q + 1) + 1 = 12q^2 + 18q + 7 = 2(6q^2 + 9q + 3) + 1$$