$$\begin{array}{r} 284 \\ 7\,\overline{)\,1989} \\ 14 \\ \hline 58 \\ -56 \\ \hline 29 \\ -28 \\ \hline 1 \end{array}$$

$1989=(7\cdot 284)+1 \quad 1990=(7\cdot 284)+2 \quad 1991=(7\cdot 284)+3$

$1992=(7\cdot 284)+4$

$1989\cdot 1990\cdot 1991 = (7\cdot 284+1)(7\cdot 284+2)(7\cdot 284+3)$

$= \boxed{7\cdot K_1+6}$

$1992^3=(7\cdot 284+4)^3 = 7K_2+4^3 = 7(K_2+9)+1$

Adding up, $7\cdot K_1+6+7(K_2+9)+1 = 7(K_1+K_2+10)+\boxed{0} \rightarrow$ rem.

b) The number $9^{100}$ gives when divided by 8.

$9^{100} = (8+1)(8+1)(8+1)\ldots\ldots$ (100 times)

$= \boxed{8K+1} \rightarrow$ remainder

**Prob 16:** Prove that the number $n^3+2n$ is divisible by 3 for any natural number $n$.

- By division algorithm, $n=3K$ or $3K+1$ or $3K+2$, $(K\in N)$

$n=3K$, $n^3+2n=(3K)^3+2(3K)=\boxed{3(9K^3+2K)}$

$n=3K+1$, $n^3+2n=(3K+1)^3+2(3K+1) = 27K^3+27K^2+9K+1+6K+2$

$=\boxed{3(9K^3+9K^2+5K+1)}$

$n=3K+2$, $n^3+2n=(3K+2)^3+2(3K+2) = 27K^3+54K^2+36K+8+6K+4$

$=3(9K^3+18K^2+14K+4)$

Key Idea: Case-by-Case analysis

**Prob 17:** Prove that $n^5+4n$ is divisible by 5 for any integer $n$.

- By division algorithm, $n$ can be $5K, 5K+1, 5K+2, 5K+3, 5K+4$, ~~$K\in N$~~. $(K\in N)$

$n^5+4n = n(n^4+4)$. If $n=5K$, we are done.

$n=5K+1$, $n^4+4=(5K+1)^4+4 = (5K)^4+4(5K)^3+6(5K)^2+4(5K)+5$

$=5(125K^4+100K^3+30K^2+4K+1)$

$n=5K+2$, $n^4+4=(5K+2)^4+4=(5K)^4+4(5K)^3\cdot 2+6(5K)^2\cdot 2^2+4(5K)\cdot 2^3+20$

$=5(125K^4+200K^3+120K^2+32K+4)$