

$\therefore 3 \nmid (a^2 + b^2) \therefore$ Not possible

Case 6: $a = 3K_1 + 1$ $b = 3K_2 + 2$ $a^2 + b^2 = 9K_1^2 + 6K_1 + 1 + 9K_2^2 + 12K_2 + 4$
 $= 9(3K_1^2 + 2K_1 + 3K_2^2 + 4K_2 + 1) + 2 \therefore 3 \nmid (a^2 + b^2) \therefore$ Not possible

Case 7: $a = 3K_1 + 2$ $b = 3K_2$ (Symm to Case 3)

Case 8: $a = 3K_1 + 2$ $b = 3K_2 + 1$ (Symm to Case 6)

Case 9: $a = 3K_1 + 2$ $b = 3K_2 + 2$ $a^2 + b^2 = 9K_1^2 + 12K_1 + 4 + 9K_2^2 + 12K_2 + 4$
 $= 9(3K_1^2 + 4K_1 + 3K_2^2 + 4K_2 + 2) + 2$

$\therefore 3 \nmid (a^2 + b^2) \therefore$ Not possible

The only possible case is 1, where a and b are both divisible by 3 and therefore $a^2 + b^2$ is divisible by 9.

By the division algorithm, a can be $7x$ or $7x+1$ or $7x+2$ or $7x+3$ or $7x+4$ or $7x+5$ or $7x+6$ and y can be $7y, \dots, 7y+6$, where $x, y \in \mathbb{N}$.

$a = 7x$ $b = 7y$, $a^2 + b^2 = (7x)^2 + (7y)^2 = 49(x^2 + y^2) = 7(7x^2 + 7y^2)$

$a = 7x$ $b = 7y+1$, $a^2 + b^2 = (7x)^2 + (7y+1)^2 = 49x^2 + 49y^2 + 14y + 1 = 7(7x^2 + 7y^2 + 2y) + 1$

$\therefore 7 \nmid (a^2 + b^2)$ (Not possible) The reverse or symmetric case is also not possible.

$a = 7x$ $b = 7y+2$, $a^2 + b^2 = (7x)^2 + (7y+2)^2 = 49x^2 + 49y^2 + 28y + 4 = 7(7x^2 + 7y^2 + 4y) + 4$

$\therefore 7 \nmid (a^2 + b^2) \therefore$ Not possible

The symmetric case is also not possible.

$a = 7x$ $b = 7y+3$ $a^2 + b^2 = (7x)^2 + (7y+3)^2 = 49x^2 + 49y^2 + 42y + 9 = 7(7x^2 + 7y^2 + 6y + 1) + 2$

$\therefore 7 \nmid (a^2 + b^2) \therefore$ Not possible

The symmetric case is also not possible

$a = 7x$ $b = 7y+4$ $a^2 + b^2 = (7x)^2 + (7y+4)^2 = 49x^2 + 49y^2 + 56y + 16 = 7(7x^2 + 7y^2 + 8y + 2) + 2 \therefore 7 \nmid (a^2 + b^2) \therefore$ Not possible
 (The symmetric case is not possible)