• $x=2p_1,\ y=2p_2+1$ $\quad 4(3x^2+3y^2)=4(3(2p_1)^2+3(2p_2+1)^2)$
$=4(12p_1^2+12p_2^2+12p_2+3)=4(2(6p_1^2+6p_2^2+6p_2+1)+1)$
↳ not div. by 8

• $x=2p_1+1\ y=2p_2+1$
(Symmetric to prev case)

• $x=2p_1+1\ y=2p_2+1$ $\quad 4(3x^2+3y^2)=4(3(2p_1+1)^2+3(2p_2+1)^2)$
$=4(12p_1^2+12p_1+12p_2^2+12p_2+6)=8(6p_1^2+6p_1+6p_2^2+6p_2+3)$
↳ div. by 8

• $6(x+y+1)=6(2p_1+1+2p_2+1+1)=6(2(p_1+p_2+1)+1)$
↳ not div. by 8

$\therefore 6(2x^2+x+2y^2+y+1)$ is not div. by 8.

$\therefore (a^3+b^3+4)$ is not a perfect cube for $\forall a,b \in N$

Prob 42: Prove that the number 100........00500.......001 (100 zeros in each group) is not a perfect cube.

$100........00500......001 = 1\times10^{202}+5\times10^{101}+1$
$=10(10^{67})^3+\frac{(10^{34})^3}{2}+1$

$10a^3+\frac{b^3}{2}+1=\boxed{\frac{20a^3+b^3+2}{2}}$ I will try to prove $20a^3+b^3+2$ is not div. by 16.

$a=2x$ or $2x+1$, $b=2y$ or $2y+1$, for some $x,y\in N$.

• $a=2x, b=2y$ $\quad 20a^3+b^3+2=160x^3+8y^3+2$
$=2(80x^3+4y^3+1)$
$=2(4(20x^3+y^3)+1)$
↳ not div. by 4 $\therefore$ 16 also

• $a=2x, b=2y+1$
$20a^3+b^3+2=20(2x)^3+(2y+1)^3+2=160x^3+8y^3+12y^2+6y+3$
$=\boxed{2(80x^3+4y^3+6y^2+3y+1)+1}$
↳ not div. by 2 $\therefore$ 16 also

• $a=2x+1, b=2y$ $\quad 20a^3+b^3+2=20(2x+1)^3+(2y)^3+2$
$=160x^3+240x^2+120x+22+8y^3$