$\boxed{\text{Observation:}}$ Either one of or both x and y are div by 2.

- Suppose, both of x and y are div by 2, and atleast 1 of x and y is div by 3. It's easy to see that xy is div by 12 in this situation.
- Suppose exactly one of x or y is div by 2 and atleast 1 of both is div. by 3.

We will try to show the no div by 2 is also div by 4.

Take $\boxed{\text{Case 2:}}$ $x^2+y^2 = 4(m^2+n^2+n)+1$

- By div algorithm, z can 2o or 2o+1, $o \in N$

$z = 2o$, $z^2 = 4o^2$ ✗

$z = 2o+1$, $z^2 = \boxed{4(o^2+o)+1}$

$m^2+n^2+n = o^2+o$. By div algo, m can be $2a_1$ or $2a_1+1$, n can be $2b_1$ or $2b_1+1$

$m=2a_1$, $n=2b_1$, $m^2+n^2+n = 4a_1^2+4b_1^2+2b_1 = 2(2a_1^2+2b_1^2+b_1)$

$m=2a_1$ $n=2b_1+1$ $m^2+n^2+n = 4a_1^2+4b_1^2+6b_1+2 = 2(2a_1^2+2b_1^2+3b_1+1)$

$m=2a_1$ $n=2b_1+1$ $m^2+n^2+n = 4a_1^2+4a_1+4b_1^2+2b_1+1 = 2(2a_1^2+2a_1+2b_1^2+b_1)+1$

$m=2a_1+1$ $n=2b_1$ $m^2+n^2+n = 4a_1^2+4a_1+4b_1^2+6b_1+3 = 2(2a_1^2+2a_1+2b_1^2+3b_1+1)+1$

$m=2a_1+1$ $n=2b_1+1$

By div algorithm, o can be $2c_1$ or $2c_1+1$

$o=2c_1$ $o^2+o = 4c_1^2+2c_1 = 2(2c_1^2+c_1)$

$o=2c_1+1$ $o^2+o = 4c_1^2+6c_1+2 = 2(2c_1^2+3c_1+1)$

Infeasible cases.

$\therefore$ In the feasible cases, $2|m$.

$\therefore x = 2m = 2(2a_1) = 4a_1$

$\therefore 4|x$.

Case 3 can be solved similarly

$\boxed{\therefore xy \text{ is div by } 12}$

$\boxed{\text{Prob 48:}}$ Seven natural numbers are such that the sum of any six of them is divisible by 5. Prove that each of these numbers is divisible by 5.