will have product ~~than~~ greater than 72, of the respective groups.

. So, even if now we put 3,5 and 6 in one group, the product is $3.5.6=90$ which is inevitably greater than 72.

$\boxed{(9\times a)\times(8\times b)\times(x)=\text{const.}}$. If both $9\times a$ and $8\times b$ decrease, then x increases.

. Use this basic fact to write the proof a bit more rigorously

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Prob 21: Prove that there exists an integer whose decimal representation consists entirely of 1's, and which is divisible by 1987.

. Suppose, let us consider the series $S:$ $1, 11, 111, 1111, \ldots\ldots, \underbrace{1111\ldots\ldots1}_{(1987\ 1\text{'s})}$

. If a term a_i, where a_i consists of i 1's is div by 1987, we are already done.

. Suppose, for the sake of contradiction, the terms a_i gives remainders r_i on dividing by 1987 where $1 \leq r_i \leq 1986$, $r_i \in \mathbb{N}$. (By div algorithm)

. Since there are 1987 terms, by pigeonhole principle, $\exists\, a_i, a_j \in S$ s.t. $a_i > a_j$ (wlog)

$$a_i = 1987q_i + r \qquad a_j = 1987q_j + r$$

$$(a_i - a_j) = 1987(q_i - q_j)$$

or, $(\underbrace{11111\ldots\ldots1}_{(i)} - \underbrace{1111\ldots\ldots1}_{(j)}) = 1987(q_i - q_j)$

or, $1987 \mid (\underbrace{111\ldots\ldots1}_{(i-j)}\underbrace{00\ldots\ldots0}_{j})$ or, $1987 \mid (\underbrace{111\ldots\ldots1}_{(i-j)} \times 1\underbrace{00\ldots0}_{(j)})$

. 2 and 5 are not present in the prime factorization of 1987. (Observe)

. $\therefore$ There is no way a no. of the form 10^j will be div by 1987.

. $\therefore$ The no. having $(i-j)$ 1's must be div by 1987 (Euclid's div lemma)

(This is a contradiction to our assumption)

Prob 28: Prove that among any 6 people there are either 3 people, each of whom knows the other two, or 3 people, each of whom does not know the other ~~two~~ two.