$= 2(80x^3+120x^2+60x+4y^3+11) = \boxed{2(4(20x^3+30x^2+15x+y^3+2)+3)}$

↳ not div. by 8 ∴ 16 also

- $a=2x+1, b=2y+1$ $\quad 20a^3+b^3+2 = 20(2x+1)^3+(2y+1)^3+2$

$= 160x^3+240x^2+120x+8y^3+12y^2+6y+23$

$= \boxed{2(80x^3+120x^2+60x+4y^3+6y^2+3y+11)+1}$

↳ not div. by 2 ∴ 16 also

$16 \nmid 20a^3+b^3+2$

∴ The given no. is not a perfect cube

$\boxed{\text{Prob 44:}}$ Prove that the number $6n^3+3$ cannot be a perfect sixth power ~~for~~ of an integer for any natural number n.

- $6n^3+3 = 3(2n^3+1)$
- We will try to show, $\forall n$, $2n^3+1$ is not divisible by 243.
- By division algorithm, n can be $9K, 9K+1, 9K+2, \ldots\ldots, 9K+8$. for some $K \in \mathbb{N}$.
- $n=9K$, $2n^3+1 = 2(9K)^3+1 = 2\cdot 729K^3+1 = 243(6K^3)+1$

~~$n=9K+1$, $2n^3+1 = 2(9K+1)^3+1 = 2\cdot(729K^3+18K+1)+1 = 3(486K^3+12K+1)$~~

~~$= 3(3(81K^3+2K)+1) = \boxed{3(3(162K^3+4K)+1)}$ ↳ not div. by 3^2, 3^5 is out of sight~~

~~$n=9K+2$, $2n^3+1 = 2(9K+2)^3+1 = 2(729K^3+36K$~~

- $n=9K+1$, $2n^3+1 = 2(9K+1)^3+1 = 2\cdot 729K^3+6\cdot 81K^2+2\cdot 27K+3$

$= 3(486K^3+162K^2+18K+1) = \boxed{3(9(54K^3+18K^2+2K)+1)}$

↳ not div by 3^3

- $n=9K+2$, $2n^3+1 = 2(9K+2)^3+1 = 2\cdot 729K^3+243K^2\cdot 4+162K+17$

$= \boxed{9(162K^3+27\cdot 4K^2+18K+1)+8}$

↳ not div by 3^2

- $n=9K+3$, $2n^3+1 = 2(9K+3)^3+1 = 2\cdot 729K^3+2\cdot 81K^2\cdot 9+2\cdot 9K\cdot 27+55$

$= \boxed{27(54K^3+54K^2+18K+2)+1}$ ↗ not div by 3^3