$\boxed{\text{Case 3:}}$ $p^2-q^2=(6k_1+1)^2-(6k_2+5)^2=\underbrace{(36k_1^2+12k_1)}_{\text{div. by 24 (shown prev)}}-\underbrace{(36k_2^2+60k_2+24)}_{\text{div. by 24}}$

$\boxed{\text{Case 4:}}$ $p^2-q^2=(6k_1+5)^2-(6k_2+5)^2=(36k_1^2+60k_1)-(36k_2^2+60k_2)$

↳ easy to show its div. by 24

$\boxed{\text{Prob 22:}}$ The natural numbers x, y and z satisfy the equation $x^2+y^2=z^2$. Prove that atleast one of them is divisible by 3.

- If, $3|x$ or $3|y$, we are done.
- Suppose $3\nmid x$ and $3\nmid y$. By division algorithm, we get the following cases: ① $x=3p+1$, $y=3q+1$ ② $x=3p+2$, $y=3q+1$ ③ $x=3p+1$, $y=3q+2$ ④ $x=3p+2$, $y=3q+2$

1. $x^2+y^2=(3p+1)^2+(3q+1)^2=9p^2+6p+1+9q^2+6q+1=3(3p^2+2p+3q^2+2q)+2$

   $z^2=3(3p^2+2p+3q^2+2q)+2$

   $\therefore$ z^2 gives remainder 2 when divided by 3.

   $\therefore$ $3\nmid z$ (Easy to show)

2. $x^2+y^2=(3p+2)^2+(3q+1)^2=9p^2+12p+4+9q^2+6q+1$
   $=3(3p^2+4p+3q^2+2q+1)+2$

   Again $3\nmid z$

3. Symmetrical to 2

4. $x^2+y^2=(3p+2)^2+(3q+2)^2$
   $=9p^2+12p+4+9q^2+12q+4$
   $=3(3p^2+4p+3q^2+4q+2)+2$

$\boxed{\therefore 3\nmid z}$

~~If $x^2+y^2=z^2$, if atleast one of them is div by 3, that can't be z~~ ~~V.G. Obs~~