• **Assumption:** Each person has someone sitting diametrically opposite
** to that person.
• Suppose, we have x girl pairs, y boy pairs and z mixed pairs.
• Given, $\boxed{2x+2y+(z+z)=100} \Rightarrow \underbrace{(2x+z)}_{\text{no. of girls}}+\underbrace{(2y+z)}_{\text{no. of boys.}}=100$

• Given, $2x+z<2y+z \Rightarrow \boxed{x<y}$
$\therefore$ no. of girl pairs must be strictly less than no. of boy pairs.
• In the worst case, there exist no girl pairs, so still there must exist atleast 1 boy pair.

Prob 25: Fifteen boys gathered 100 nuts. Prove that some pair of boys gathered an identical number of nuts.
• Suppose we have 15 boys $B_1, B_2, \ldots\ldots, B_{14}, B_{15}$. Suppose, we consider the boys $B_1, B_2, \ldots\ldots, B_{14}$.
• Two possible cases arises: i) $\exists\, i,j,\ 1\leq i,j\leq 14,\ i\neq j$ such that B_i and B_j gather the same no. of nuts. Then we are already done, and it doesn't matter how many nuts B_{15} has (provided constraint is satisfied)
ii) Each of B_1 to B_{14} has different no. of nuts.
$\boxed{S = Nuts(B_1)+Nuts(B_2)+\ldots\ldots+Nuts(B_{14})\leq 100}$
• The lowest value of S arises when each of the B_i's, $1\leq i\leq 14$ take the values from the set $\{0,1,2,3,4,5,6,7,8,9,10,11,12,13\}$ exactly once.
Then $S=\frac{13\times 14}{2}=91$.
• Then B_{15} has $100-91=9$ nuts, and in this case, B_{15} has the same no. of nuts as one of the B_i's, $1\leq i\leq 14$.
(This approach leads to nowhere)