$n=5K+3$, $n^4+4=(5K+3)^4+4=(5K)^4+4(5K)^3\cdot 3+6(5K)^2\cdot 3^2+4(5K)\cdot 3^3+85$

$=5(125K^4+300K^3+270K^2+81K+17)$

$n=5K+4$, $n^4+4=(5K+4)^4+4=(5K)^4+4(5K)^3\cdot 4+6(5K)^2\cdot 4^2+4(5K)\cdot 4^3+260=5(125K^4+400K^3+480K^2+256K+52)$

Prob 18: Prove that n^2+1 is not divisible by 3 for any integer n.

By division algorithm, $n=3K$ or $3K+1$ or $3K+2$, $K\in N$

$n=3K$, $n^2+1=(3K)^2+1=3(3K^2)+1$

$n=3K+1$, $n^2+1=(3K+1)^2+1=9K^2+6K+2=3(3K^2+2K)+2$

$n=3K+2$, $n^2+1=(3K+2)^2+1=9K^2+12K+5=3(3K^2+4K+1)+2$

$\therefore$ None of the cases yield a n^2+1 which is divisible by 3.

Prob 20: Prove that n^3-n is divisible by 24 for any odd n.

$n^3-n=n(n-1)(n+1)$ $\because n$ is odd, $\therefore$ By div algo, $n=2K_1+1$, $K_1\in Z$

$=(2K_1+1)(2K_1)(2K_1+2)=\boxed{4(K_1)(K_1+1)(2K_1+1)}$

- We will try to show $K_1(K_1+1)(2K_1+1)$ is divisible by 6.
- By division algorithm, $K_1=2K_2$ or $2K_2+1$ $K_2\in Z$
- Either way, its easy to see $K_1(K_1+1)(2K_1+1)$ is divisible by 2
- By division algorithm, $K_1=3K_2$ or $3K_2+1$ or $3K_2+2$. $K_2\in Z$
- If $K_1=3K_2$, $3|K_1$ $\therefore 3|K_1(K_1+1)(2K_1+1)$
- If $K_1=3K_2+1$, $2K_1+1=2(3K_2+1)+1=3(2K_2+1)$ $\therefore 3|(2K_1+1)$ and $3|K_1(K_1+1)(2K_1+1)$
- If $K_1=3K_2+2$, $K_1+1=3(K_2+1)$ $3|(K_1+1)$ $\therefore 3|K_1(K_1+1)(2K_1+1)$

$\because$ 2 and 3 are co-prime, $\therefore 6|K_1(K_1+1)(2K_1+1)$

$\boxed{4(K_1)(K_1+1)(2K_1+1)=4\cdot 6\cdot K=24K}$ for some $K\in Z$.