- Observe: $7^1=7$, $7^2=49$ $7^3=343$ 7^4 ends with 1, 7^5 ends with 7 and so on.
- We can easily show by induction on K, where $K \geq 1$, that the last digit of the no. 7^K will be either 7, 9, 3 or 1. Each of the numbers having last digit 7, 9, 3 or 1 will have K of the form $4p+1, 4p+2, 4p+3, 4p$, where $p \in \mathbb{N}$.

$\boxed{777 = 4\times 194+1}$ $\quad 7^{777} = 7^{776}\cdot 7 = (7^4)^{194}\cdot 7$

ends with 1

$7 \therefore$ ends with 7

Prob 31: Find the remainder when 2^{100} is divided by 3.

$$2^{100} = 4^{50} = (3+1)^{50} = (3+1)(3+1)\ldots\ldots(3+1)$$

- Ofc, we can apply the binomial thm, but its intuitive to see that only the term 1^{50} will not contain any 3 in its prime factorization, the rest of terms must contain atleast one 3 as it's factor.

$\therefore$ The remainder will be 1.

Prob 32: Find the remainder when the number 3^{1989} is divided by 7.

$$3^{1989} = 3^{1988}\cdot 3 = 9^{994}\cdot 3 = (7+2)^{994}\cdot 3$$

- Applying the intuition used in Prob 31, we will now try to find the remainder when $2^{994}\cdot 3$ is divided by 7.

$$2^{994}\cdot 3 = 2^{993}\cdot 6 = 8^{331}\cdot 6 = (7+1)^{331}\cdot 6$$

- Applying the ideas of Prb 31, the remainder will be 6.

Prob 33: Prove that $2222^{5555} + 5555^{2222}$ is divisible by 7

$$2222^{5555} = (7\times 317+3)^{5555} + (7\times 793+4)^{2222}$$

- Using the concept used in Prob 31, we will try to show that $3^{5555}+4^{2222}$ is divisible by 7. $\quad 3^{5555} = 3^{5554}\cdot 3 = 9^{2777}\cdot 3$

$$=(7+2)^{2777}\cdot 3$$