Prob 5: Twenty-five crates of apples are delivered to a store. The apples are of three different sorts, and all the apples in each crate are of the same sort. Show that among these crates there are atleast nine containing the same sort of apple.

- Suppose, for now, take 1 crate aside. We have 24 crates of apples.

Case 1: Among 24 crates, there are atleast 9 containing the same sort of apple. So, we have already got the soln, and adding 1 more crate doesn't change it.

Case 2: Among 24 crates, each sort of apple has atmost 8 crates.
This is only possible when $\boxed{24 = 8_{sort1} + 8_{sort2} + 8_{sort3}}$

∵ There are only 3 sorts, when we add back 1 more crate, we get one sort of apple in exactly 9 crates.

Generalized pigeon-hole principle

- If we must put $Nk+1$ or more pigeons into N pigeonholes, then some pigeonhole must contain atleast $k+1$ pigeons. (Easy to show)

Prob 6: In the country of Courland there are M football teams, each of which has 11 players. All the players are gathered at an airport for a trip to another country for an important game, but they are travelling on "standby". There are 10 flights to their destination, and it turns out that each flight has room for exactly M players. One football player will take his own helicopter to the game, rather than travelling standby on a plane. Show that atleast one whole team will be sure to get to the important game.

Pf by contradiction: Suppose, no whole team reaches the game

- Let n_i denote the no. of players from each team who reach the final destination.