Prob 57: Find $\gcd(\underbrace{111\ldots\ldots111}_{100}, \underbrace{11\ldots\ldots11}_{60})$, where there are 100 1's in the decimal representation of the first number and 60 1's in the decimal representation of the second number.

$111\ldots111\ (100\ 1\text{'s}) = (10^{99}+10^{98}+\ldots+10^0)$ $\quad 11\ldots11\ (60\ 1\text{'s}) = (10^{59}+10^{58}+\ldots+10^0)$

- We will use Euclidean gcd algorithm.
- $10^{40}(10^{59}+10^{58}+\ldots+10^0)$ subtracted from $(10^{99}+10^{98}+\ldots+10^0)$ we will get $(10^{39}+10^{38}+\ldots+10^0)$.
- $(10^{59}+10^{58}+\ldots+10^0) - 10^{20}(10^{39}+10^{38}+\ldots+10^0) = (10^{19}+10^{18}+\ldots+10^0)$
- $(10^{39}+10^{38}+\ldots+10^0) - 10^{20}(10^{19}+10^{18}+\ldots+10^0) = (10^{19}+10^{18}+\ldots+10^0)$

$\therefore$ $111\ldots\ldots111\ (20\ 1\text{'s})$ is the gcd

Prob 51: Prove that if $(n-1)!+1$ is divisible by n, then n is a prime number.

- $\exists K \in N$ s.t. $\boxed{nK = (n-1)!+1}$ $\quad \therefore \boxed{\gcd(nK, (n-1)!) = 1}$ → Basic Prop of two consecutive integers

$\therefore$ nK and $(n-1)!$ are co-prime, i.e. they have no common factor.

- $(n-1)! = \boxed{(n-1)(n-2)\ldots\ldots 3\cdot 2\cdot 1}$ → Observe, all the primes less than n must be a factor of $(n-1)!$

- If n would had been composite, its prime factorization would be, $\boxed{2^{n_1}3^{n_2}5^{n_3}\ldots\ldots}$ → upto primes less than n, where atleast one of the powers must be greater than 0

- But then, p_i would had been the common factor of $nK, (n-1)!$, which is a contradiction. $\boxed{\therefore n_1 = n_2 = n_3 = \ldots\ldots = n_i = \ldots\ldots = 0}$

So, n must be prime