Prob 9: Given a convex 101-gon which has an axis of symmetry, prove that the axis of symmetry passes through one of its vertices. What can you say about a 10-gon with the same properties?

Ans :. An axis of symmetry divides the polygon into two similar halves. ∴ ~~In the case~~ We must have 50 vertices on one side and 50 vertices on the other side. The left out vertex will have the axis of symmetry passing through it.

- In case of a 10-gon, we will have 5 vertices on one side and 5 on the other. ∴ The axis of symmetry will not pass through any of the vertices.

(I will prove it later in a more rigorous manner after enhancing my geometry skills)

Prob 13: Twenty-five checkers are placed on a 25×25 checkerboard in such a way that their positions are symmetric with respect to one of its diagonals. Prove that atleast one of the checkers is positioned on that diagonal.

Ans : Let, each square be denoted by (i,j). (ith row, jth column)
$i \in \{1,2,\dots,25\}$ $j \in \{1,2,\dots,25\}$.

- Since, the checkers are posn symmetrically with respect to the main diagonal (assume), if a checker has posn. (i,j), there must exist one more checker in the posn. (j,i).
- Since, the no. of checkers are odd, there must be atleast one checker which can't be paired off with another checker for the sake of symmetry.
- ∴ That checker must lie on the main diagonal at (i,i) probably.

- Suppose the checkers are posn symmetrically with respect to the other diagonal. If a checker has posn. (i,j) and the diagonal passes through (i,K), if $(j>K)$, then the symmetric posn. is $(i-(j-K),K)$, if $(j<K)$, then the symmetric posn is $(i+(j-K),K)$, else it's ~~remains same~~ posn. becomes (i,i) in symmetry