• $b=8K+4$ $\quad 8+3b^2 = 8+3\cdot16(2K+1)^2 = 8+48(4K^2+4K+1)$

$= 4(2+48K^2+48K+12)$

$= 4(48K^2+48K+14)$

$= 8(24K^2+24K+7) = 8(2(12K^2+12K+3)+1)$

↳ not div. by 2

~~by division algo~~

$\therefore 8+3b^2$ is not a perfect square

• $b=8K+5$ $\quad 8+3b^2 = 8+3(8K+5)^2 = 8+3(64K^2+25+80K) = 8+192K^2+75+240K$

$= 4(48K^2+60K+20)+3$

↳ can't be a^2

• $b=8K+6$ $\quad 8+3b^2 = 8+3(8K+6)^2 = 8+3(64K^2+36+96K) = 192K^2+288K+116$

$= 4(48K^2+72K+29)$

$= 4(2(24K^2+36K+14)+1)$

↳ not div. by 4

$\therefore 8+3b^2$ is not a perfect square

• $b=8K+7$ $\quad 8+3b^2 = 8+3(8K+7)^2 = 8+3(64K^2+49+112K) = 8+192K^2+147+336K$

$= 4(48K^2+84K+38)+3$

$\therefore 8+3b^2$ is not a perfect square

$\boxed{\therefore 8+3b^2 \text{ is not a perfect square in any case}}$

Prob 41: If $p, 4p^2+1, 6p^2+1$ are prime numbers, find p.

• By division algorithm and $\because p$ is a prime, p can be $10K+1, 10K+3, 10K+7, 10K+9$, $K \in N$

• $p=10K+1$ $\quad 4p^2+1 = 4(10K+1)^2+1 = 400K^2+80K+5 = 5(80K^2+16K+1)$

↳ not prime

• $p=10K+3$ $\quad 6p^2+1 = 6(10K+3)^2+1 = 600K^2+360K+55 = 5(120K^2+72K+11)$

↳ not prime

• $p=10K+7$ $\quad 6p^2+1 = 6(10K+7)^2+1 = 600K^2+840K+50 = 10(60K^2+84K+5)$

↳ not prime