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0.1 Assessment

- Online Quizzes (10% = Best 10 × 1%)
Due Mondays at 8am
- Mid-semester exam (10% or 20%)
Saturday (centrally scheduled - sometime week 5 to 7)
Multiple-choice, open-book
- Prac Exam (Pass/Fail)
Held during Monday/Wednesday Learning Lab sessions in week 6
You must pass in order to pass the course

- Project (20%)
Develop a microcontroller program
- Final Exam (50% or 60%)
Short answer, problem solving, open-book

Chapter 1

Lecture Notes

1.1 Bits, Bytes and Binary

1.1.1 Structured Computer Organization

Level 5: Problem-oriented language level

Level 4: Assembly language level

Level 3: Operating system machine level

Level 2: Instruction set architecture level

Level 1: Microarchitecture level

Level 0: Digital Logic level

1.1.2 Unsigned Number in Binary

Each bit position has a value $\rightarrow 2^n$ (starting at zero). Add all values of the positions together and that's unsigned value.

1.1.3 Converting Decimal to Binary

- Method 1
rewrite n as sum of powers of 2 (by repeatedly subtracting largest power of 2 not greater than n)
Assemble binary number from 1's in bit positions corresponding to those powers of 2, 0's elsewhere
- Method 2
Divide n by 2
Remainder of division (0 or 1) is next bit
Repeat with $n = \text{quotient}$

Note 1: Example

Convert 53 to binary

$$\begin{array}{l} \frac{53}{2} = 26 \text{ rem } 1 \Rightarrow 1 \\ \frac{26}{2} = 13 \text{ rem } 0 \Rightarrow 0 \\ \frac{13}{2} = 6 \text{ rem } 1 \Rightarrow 1 \\ \frac{6}{2} = 3 \text{ rem } 0 \Rightarrow 0 \\ \frac{3}{2} = 1 \text{ rem } 1 \Rightarrow 1 \\ \frac{1}{2} = 1 \text{ rem } 1 \Rightarrow 1 \end{array}$$

$\therefore 53 \equiv 0b110101$

1.1.4 Least and Most Significant Bits

Most Significant Bit (MSB): Bit that's worth the most, the left-most bit

Least Significant Bit (LSB): Bit that's worth the least, the right-most bit

Note 2: Radices

- **Radix:** number system base
- A radix- k number system
 k different symbols to represent digits 0 to $k - 1$
Value of each digit is (from the right) $k^0, k^1, k^2, k^3, \dots$
- Often convenient to deal with
 - Octal** (radix-8) - Symbols: 0, 1, 2, 3, 4, 5, 6, 7
One octal digit corresponds to 3 bits
 - Hexadecimal** (radix-16) - Symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
One hexadecimal digit corresponds to 4 bits (useful)

Note 3: Radix Identification

- Hexadecimal
 - Leading 0x (C, Atmel AVR)
 - Trailing h (Some assembly languages)
 - Leading \$ (Atmel AVR Assembly)
- Octal
 - Leading 0 (C, Atmel AVR)
 - Trailing q (Some assembly languages)
 - Leading @ (Some assembly languages)
- Binary
 - Leading 0b (Atmel AVR Assembly, Some C)
 - Trailing b (Some assembly languages)
 - Leading % (some assembly languages)

1.1.5 Conversions

Easiest to convert from most formats to binary then to the desired format.

Octal

From Binary: Group bits into series of 3 and then convert to decimal (0b010 = 02)

To Binary: Convert each octal number to binary and append

Hex

From Binary: Group bits into series of 4 and then convert to hex with overflow being apart of the alphabet (0b1100 = 0xC)

To Binary: Convert each hex number to binary and append

Decimal

From Binary: Add together the powers of two at each position n (0b1010 = $2^3 + 2^1 = 10$)

To Binary: Starting with LSB, divide by 2 with the remainder being bit value at position. ($9 = 9/2 = 4\text{rem}1, 4/2 = 2\text{rem}0, 2/2 = 1\text{rem}0, 1/2 = 0\text{rem}1. \therefore 9 = 0b1001$)

1.1.6 Negative Numbers

Signed Magnitude

Leftmost bit is the sign bit, true is negative and false is positive

One's Complement

Leftmost bit = sign-bit (as per signed magnitude), true is negative and false is positive. If negative all bits are inverted

Two's Complement

MSB signifies if negative, true is negative and false is positive. To negate invert all bits and add decimal 1.

█ Allows addition without requiring conversion

Excess 2^{m-1}

e.g. for 8 bits, excess-128. Add 128 to the original bit and convert to binary

1.2 Logic Gates

NOT Gate: Inverts the signal (i.e. input is true, output is false)

AND Gate: Output is true only if **all** inputs are true

NAND Gate: Opposite of AND, always true unless all inputs are true

OR Gate: Output is true when at **least one** input is true

XOR Gate: Output is true if only one input is true

Note 4: XOR Multiple Inputs

For more than 2 inputs, XOR is true if there is an odd number of inputs true. Also referred to as the "odd function"

1.2.1 Logic Functions

- Logic functions can be expressed as expressions involving:

variables (literals), e.g. A B X

functions, e.g. +, \oplus , \bar{A}

- Rules about how this works called **Boolean algebra**

- Variables and functions can only take on values **0** or **1**

Conventions

Inversion: \overline{A} (overline of A)

AND: dot(.) or implied by adjacency. $AB = A.B$

OR: plus sign. $OR(A, B, C) = A + B + C$

XOR: $OR(A, B) = A \oplus B = \overline{A}B + A\overline{B}$

NAND: \overline{ABC}

NOR: $\overline{A + B}$

Representations of Logic Functions

There are four representations of logic functions (assume function of n inputs)

- **Truth Table**

Lists output for all 2^n combinations of in-

puts

- **Boolean Function** (or equation)

Describes the conditions under which the function output is true

- **Logic Diagram**

Combination of logic symbols joined by wires

- **Timing Diagram**

1.2.2 Logic Function Implementation

Any logic function can be implemented as the OR of AND combinations of the inputs. Called **sum of products**.

Table 1.1: Boolean Identities

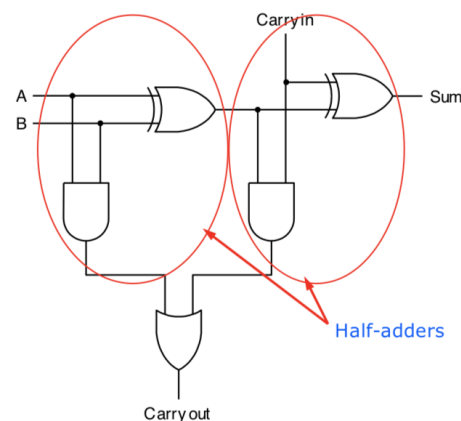
Name	AND Form	OR Form
Identity Law	$1A = A$	$0 + A = A$
Null Law	$0A = 0$	$1 + A = 1$
Idempotent Law	$AA = A$	$A + A = A$
Commutative Law	$AB = BA$	$A + B = B + A$
Associative Law	$(AB)C = A(BC)$	$(A + B) + C = A + (B + C)$
Distributive Law	$A + BC = (A + B)(A + C)$	$A(B + C) = AB + AC$
Absorption Law	$A(A + B) = A$	$A + AB = AB$
De Morgan's Law	$\overline{AB} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A}\overline{B}$

1.3 Binary Arithmetic

1.3.1 Equivalent Circuits

All circuits can be constructed from NAND and NOR gates

1.3.3 Full Adder



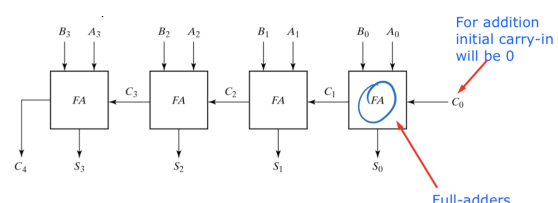
1.3.2 Overflow

Overflow with two's complement addition:

- Carry into sign-bit is different to the carry out of the sign-bit
- Equivalently, overflow occurs if
 - Two negatives added together give a positive, or
 - Two positives added together give a negative

1.3.4 Binary Adder

Can cascade full adders to make binary adder. This is a **ripple-carry adder**.



1.4 Combination Logic

1.4.1 Combinational Circuits

Each output can be expressed as a function of n input variables. Can write truth table also:

- n input columns
- m output columns
- 2^n rows (i.e. possible input combination)

Note 5: Multiplexer (or Mux)

- 2^n data inputs
- 1 output
- n control (or **select**) inputs - that **select** one of the inputs to be “sent” or “steered” to the output

Note 6: Decoder

Converts n -bit input to a logic-1 on exactly one of 2^n outputs

1.4.2 Timing Diagram

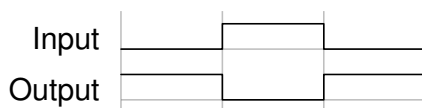


Figure 1.1: Timing Diagram of an inverter

There is a slight delay in logic timings in reality

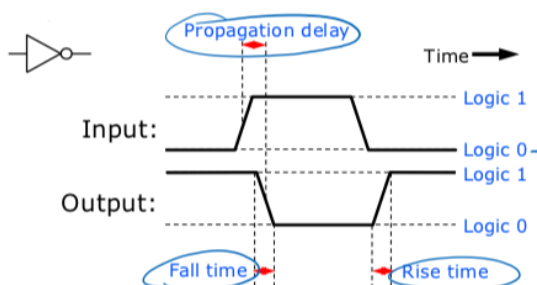


Figure 1.2: Reality of Timing

Propagation delay: time for change in input to affect output

Fall time: time taken for output to fall from 1 to 0

Rise time: time for output to rise from 0 to 1

1.5 Flip-flops

1.5.1 D Flip Flop

- **D** is input
- **Q** is output
- **CLK** (clock) is control input

Q copies the value of D (and remembers it) whenever CLK goes from 0 to 1 (**rising edge**).

Characteristic Table

Characteristic table defines operation of flip-flop in tabular form

Table 1.2: D Flip-Flop Characteristic Table

D	Q(t+1)
0	0
1	1

1.5.2 Flip-Flops Vs Latches

- The last few slides show **latches**
These are **level-triggered** devices
- Remember we want to capture the input value at rising **edge** (a short instant)!
- Any devices based on edges are referred to as **flip-flops**

These are **edge-triggered** devices

1.6 Shift Registers

1.6.1 Combinational vs Sequential Circuits

- **Combinational** Circuits
 - Logic gates only (no flip-flops)
 - Output is uniquely determined by the inputs
i.e. you'll always get the same output for a given set of inputs
- **Sequential** Circuits
 - Include flip-flops
 - Output determined by current inputs and current **state** (values in the flip-flops)
 - Output can change when clock “ticks” (rising edge)

Sequential Circuits

- **State** is value stored in flip-flops

- Output depends on input and state or sometimes just the state
- Next state depends on inputs and state

Synchronous Sequential Circuit

- Storage elements (flip-flops) can only change at discrete instants of time

1.6.2 Registers

- A **register** is a group of flip-flops
 n -bit register consists of n flip-flops capable of storing n bits
- A register is a sequential circuit *without* any combinational logic

Shift Register

A shift register is a register which is capable of shifting its binary information in one or both directions