Daniel **Fitz**
(43961229)

University Of Queensland

STAT2203 – Probability Models and Data Analysis for Engineering

STAT2203 Assignment 1

Question 1

Let S be the event the system succeeds
Let A_i be the event that a joint is successful
Let L_i be the event that an O-ring is successful

$$\mathbb{P}(L_i) = 1 - \mathbb{P}(L_i^c)$$
$$= 1 - 0.1 \qquad \text{(From question)}$$
$$= 0.9$$
$$\mathbb{P}(A_i^c) = \mathbb{P}(L_i \cap L_i)$$
$$= \mathbb{P}(L_i^c)\mathbb{P}(L_i^c)$$
$$= 2 \times 0.1$$
$$= 0.01$$
$$\mathbb{P}(A_i) = 0.99$$
$$\mathbb{P}(S) = \mathbb{P}(A_i \cap A_i \cap A_i \cap A_i \cap A_i \cap A_i)$$
$$= \mathbb{P}(A_i)^6$$
$$= 0.99^6$$
$$= 0.9415$$
$$\mathbb{P}(S^c) = 1 - \mathbb{P}(S)$$
$$\approx 0.0585$$

Therefore the probability that the system will fail is 5.85%

Question 2

Let R_i be the event that i is received
Let S_i be the event that i is sent
From the question:

$$\mathbb{P}(S_0) = 0.5$$
$$\mathbb{P}(S_1) = 0.5$$
$$\mathbb{P}(R_1 \mid S_1) = 0.9$$
$$\mathbb{P}(R_0 \mid S_0) = 0.95$$

Using Baynes rule:

$$\mathbb{P}(S_0 \mid R_1) = \frac{\mathbb{P}(R_1 \mid S_0)\mathbb{P}(S_0)}{\mathbb{P}(R_1 \mid S_0)\mathbb{P}(S_0) + \mathbb{P}(R_1 \mid S_1)\mathbb{P}(S_1)}$$
$$= \frac{(1 - 0.95) \times 0.5}{(1 - 0.95) \times 0.5 + 0.9 \times 0.5}$$
$$= \frac{1}{19} \approx 0.0526$$

Therefore, given we receive a 1 it is 5.26% likely that a 0 was sent

Question 3

Let A be the event that the sum of numbers is 1

$$\Omega = \{-1, 0, 1\}$$

Assume that the each number is draw at equal probability

Without Replacement

$(1, 0),$	$(1, -1),$	$(0, 1),$	$(0, -1),$	$(-1, 0),$	$(1, 1)$
1	0	1	-1	-1	0

Therefore:

$$\mathbb{P}(A) = \frac{|A|}{|\Omega|}$$
$$= \frac{2}{6} = \frac{1}{3}$$

With Replacement

$$(1,1), \quad (1,0), \quad (1,-1), \quad (0,1), \quad (0,0), \quad (0,-1), \quad (-1,1), \quad (-1,0), \quad (-1,-1)$$
$$2 \qquad 1 \qquad 0 \qquad 1 \qquad 0 \qquad -1 \qquad 0 \qquad -1 \qquad -2$$

Therefore:

$$\mathbb{P}(A) = \frac{|A|}{|\Omega|}$$
$$= \frac{2}{9}$$

Question 4

```
1  function result = ass1
2    N = 1e3;
3    x = linspace(0, 1, N);
4    y = [1:N];
5    for i = 1:N
6      y(i) = systemFailure(x(i));
7    end
8    plot(x, y, "linewidth", 2, x, x, "linewidth", 2);
9    xlabel("Probability of ring failure");
10   ylabel("Probability of system failure");
11 endfunction
12
13 function result = linkSuccess(p)
14   result = 1 - p;
15 endfunction
16
17 function result = sealSuccess(p)
18   result = 1 - (1 - linkSuccess(p))^2;
19 endfunction
20
21 function result = systemSuccess(p)
22   result = sealSuccess(p)^6;
23 endfunction
24
25 function result = systemFailure(p)
26   result = 1 - systemSuccess(p);
27 endfunction
```

By running the above code, we get the output provided below. The point at which `systemFailure(p) = p` occurs at points `0.18` as visible from the diagram below.
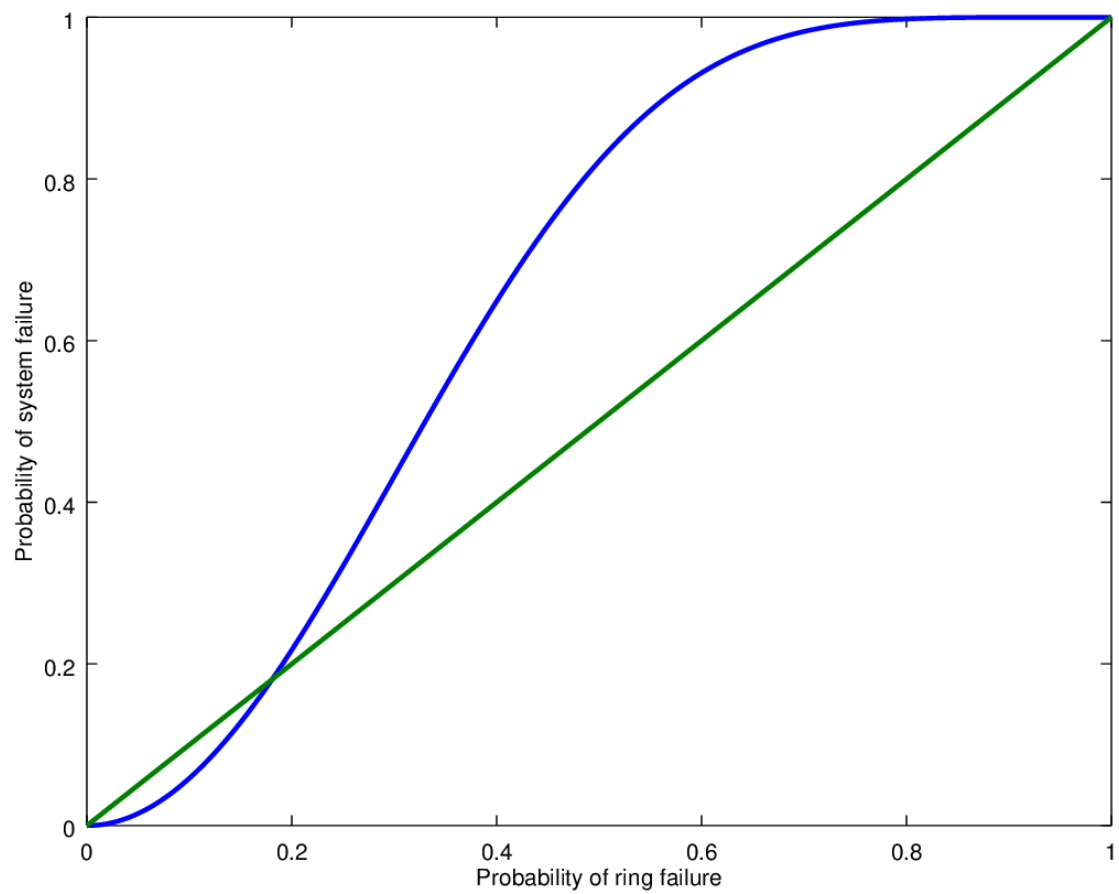
Figure 1: Question 4 Answer

Question 5

```octave
1   function result = ass1q5
2     N = 1e4;
3     received = 0;
4     sent0 = 0;
5     for i = 1:N
6       [s, r] = get_receive_bit();
7       if r == 1
8         received += 1;
9         if s == 0
10          sent0 += 1;
11        endif
12      endif
13    end
14    printf("%d:%d\n", received, sent0);
15  endfunction
16
17  function [sent, received] = get_receive_bit
18    bit = get_sent_bit();
19    if bit == 1
20      if rand >= 0.9
21        received = 0;
22      else
23        received = bit;
24      endif
25    else
26      if rand >= 0.95
27        received = 1;
28      else
29        received = bit;
30      endif
31    endif
32    sent = bit;
33  endfunction
34
35  function result = get_sent_bit
36    if rand >= 0.5
37      result = 1;
38    else
39      result = 0;
40    endif
41  endfunction
```

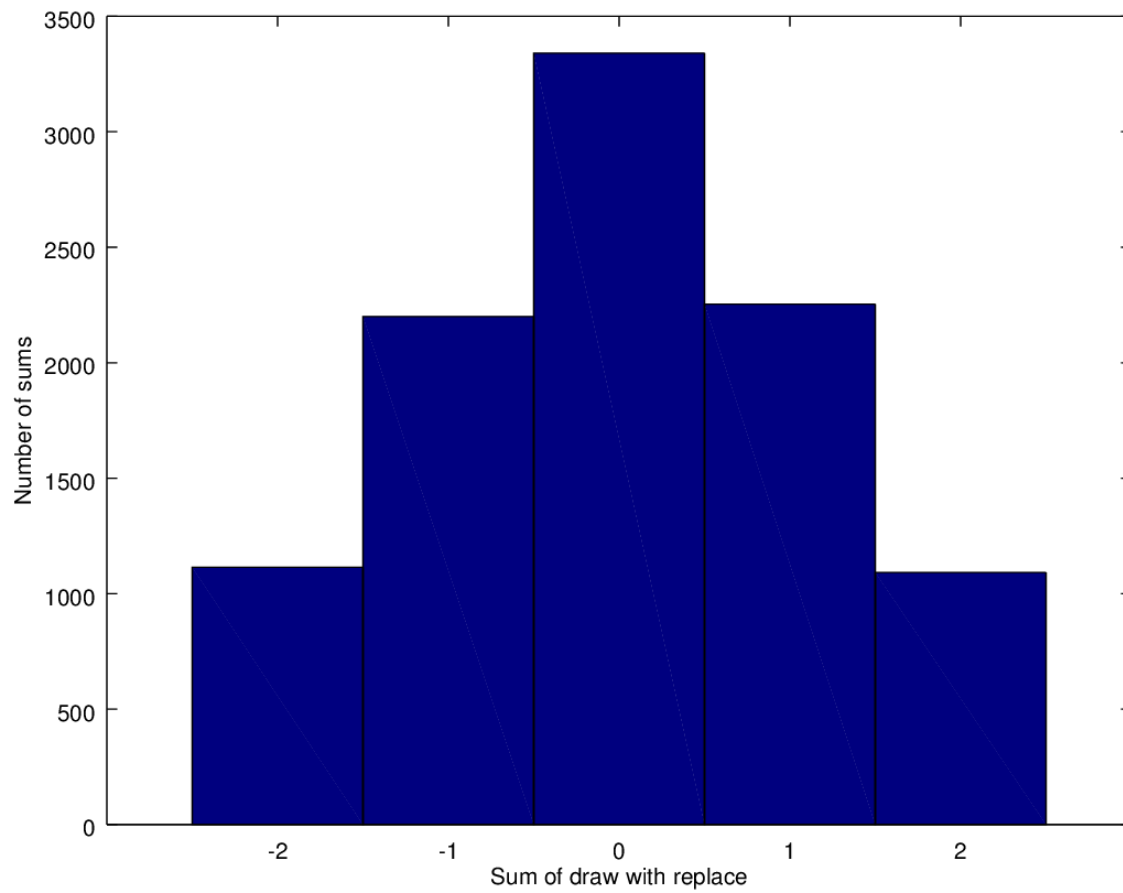Based on the code above, the output by running `ass1q5`, we can expect an answer close to `247:4772`

Question 6

```octave
1   function ass1q6
2     N=1e4;
3     result = [1:N];
4     for i = 1:N
5       result(i) = sumWithReplace();
6     endfor
7     result;
8     hist(result, -2:2);
9     xlabel("Sum of draw with replace");
10    ylabel("Number of sums");
11  endfunction
12
13  function result = sumWithReplace
14    result = sum(randi([-1, 1], 2, 1));
15  endfunction
```

The above code generates the following histogram:

Figure 2: Question 6 Answer