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**STAT2203** – Probability Models and Data Analysis for Engineering

# STAT2203 Assignment 1

# Question 1

Let $S$ be the event the system succeeds
Let $A_i$ be the event that a joint is successful
Let $L_i$ be the event that an O-ring is successful

$$\begin{aligned}
\mathbb{P}(L_i) &= 1 - \mathbb{P}(L_i^c) \\
&= 1 - 0.1 && \text{(From question)} \\
&= 0.9 \\
\mathbb{P}(A_i^c) &= \mathbb{P}(L_i \cap L_i) \\
&= \mathbb{P}(L_i^c)\mathbb{P}(L_i^c) \\
&= 2 \times 0.1 \\
&= 0.01 \\
\mathbb{P}(A_i) &= 0.99 \\
\mathbb{P}(S) &= \mathbb{P}(A_i \cap A_i \cap A_i \cap A_i \cap A_i \cap A_i) \\
&= \mathbb{P}(A_i)^6 \\
&= 0.99^6 \\
&= 0.9415 \\
\mathbb{P}(S^c) &= 1 - \mathbb{P}(S) \\
&\approx 0.0585
\end{aligned}$$

Therefore the probability that the system will fail is 5.85%

# Question 2

Let $R_i$ be the event that $i$ is received
Let $S_i$ be the event that $i$ is sent
From the question:

$$\begin{aligned}
\mathbb{P}(S_0) &= 0.5 \\
\mathbb{P}(S_1) &= 0.5 \\
\mathbb{P}(R_1 \mid S_1) &= 0.9 \\
\mathbb{P}(R_0 \mid S_0) &= 0.95
\end{aligned}$$

Using Baynes rule:

$$\begin{aligned}
\mathbb{P}(S_0 \mid R_1) &= \frac{\mathbb{P}(R_1 \mid S_0)\mathbb{P}(S_0)}{\mathbb{P}(R_1 \mid S_0)\mathbb{P}(S_0) + \mathbb{P}(R_1 \mid S_1)\mathbb{P}(S_1)} \\
&= \frac{(1 - 0.95) \times 0.5}{(1 - 0.95) \times 0.5 + 0.9 \times 0.5} \\
&= \frac{1}{19} \approx 0.0526
\end{aligned}$$

Therefore, given we receive a 1 it is 5.26% likely that a 0 was sent

# Question 3

Let $A$ be the event that the sum of numbers is 1

$$\Omega = \{-1, 0, 1\}$$

> Assume that the each number is draw at equal probability

## Without Replacement

$$\begin{matrix}
(1,0), & (1,-1), & (0,1), & (0,-1), & (-1,0), & (1,1) \\
1 & 0 & 1 & -1 & -1 & 0
\end{matrix}$$

Therefore:

$$\mathbb{P}(A) = \frac{\mid A \mid}{\mid \Omega \mid} = \frac{2}{6} = \frac{1}{3}$$

## With Replacement

$$\begin{array}{ccccccccc} (1,1), & (1,0), & (1,-1), & (0,1), & (0,0), & (0,-1), & (-1,1), & (-1,0), & (-1,-1) \\ 2 & 1 & 0 & 1 & 0 & -1 & 0 & -1 & -2 \end{array}$$

Therefore:

$$\mathbb{P}(A) = \frac{\mid A \mid}{\mid \Omega \mid} = \frac{2}{9}$$