Daniel **Fitzmaurice**
(43961229)

University of Queensland

STAT2203 – Probability Models and Data Analysis for Engineering

STAT2203 Assignment 4

Question 1

Part A

Let C_1 be circle with radius 2
Let C_2 be circle with radius 1.5
The area of C_1 and C_2 are 4pi and 2.25pi respectively
Therefore:

$$\begin{aligned}\mathbb{P}(R > 1.5) &= \frac{\mathbb{P}(C_1) - \mathbb{P}(C_2)}{\mathbb{P}(C_1)} \\ &= \frac{4\pi - 2.25\pi}{4\pi} \\ &= 0.4375\end{aligned}$$

Part B

Using the information above, replacing 1.5 with *r*

$$\begin{aligned}\mathbb{P}(R > r) &= \frac{4\pi - \pi r^2}{4\pi} \\ &= \frac{4 - r^2}{4} \\ &= 1 - \frac{r^2}{4} \\ F_R(r) = \mathbb{P}(R \leq r) &= 1 - \mathbb{P}(R > r) \\ &= 1 - \left(1 - \frac{r^2}{4}\right) \\ &= \frac{r^2}{4} \\ f_R(r) &= \frac{d}{dr}\frac{r^2}{4} \\ &= \begin{cases} \frac{r}{2} & 0 \leq r \leq 2 \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

Question 2

Part A

Because *f{X,Y}(x,y)* = $f_X(x)f_Y(y)$ where $f_X = 2x$ when $0 \leq x \leq 1$ and $f_Y(y) = 2y$ when $0 \leq y \leq 1$

Part B

$$
\begin{aligned}
\mathbb{P}(2X > Y) &= \mathbb{P}\left(X > \frac{Y}{2}\right) \\
\mathbb{P}\left(X > \frac{Y}{2}\right) &= 1 - \mathbb{P}\left(X \leq \frac{Y}{2}\right) \\
&= 1 - \int_0^1 \int_0^{\frac{y}{2}} 4xy \quad dxdy \\
&= 1 - \int_0^1 \left[4y\frac{x^2}{2}\right]_0^{\frac{y}{2}} \quad dy \\
&= 1 - \int_0^1 \left[2yx^2\right]_0^{\frac{y}{2}} \quad dy \\
&= 1 - \int_0^1 \frac{2y^3}{4} \quad dy \\
&= 1 - \int_0^1 \frac{y^3}{2} \quad dy \\
&= 1 - \left[\frac{y^4}{8}\right]_0^1 \\
&= 1 - \frac{1}{8} \\
&= \frac{7}{8}
\end{aligned}
$$

Question 3

$$
\begin{aligned}
\mathbb{P}(U \leq u) &= \mathbb{P}(\frac{X}{X+Y} \leq u) \\
&= \mathbb{P}(X \leq u(X+Y)) \\
&= \mathbb{P}(X \leq uXuY) \\
&= \mathbb{P}(X - uX \leq uY) \\
&= \mathbb{P}(X(1-u) \leq uY) \\
&= \mathbb{P}\left(X\frac{1-u}{u} \leq Y\right) \\
&= \mathbb{P}\left(Y \geq X\frac{1-u}{u}\right)
\end{aligned}
$$

Since *f{X,Y}(x,y)* = $f_X(x)f_Y(y)$:

$$
\begin{aligned}
\mathbb{P}\left(Y \geq X\frac{1-u}{u}\right) &= \int_0^\infty \int_{x\frac{(1-u)}{u}}^\infty f_{X,Y}(x,y) \quad dydx \\
&= \int_0^\infty \int_{x\frac{1-u}{u}}^\infty f_X(x)f_Y(y) \quad dydx \\
&= \int_0^\infty \int_{x\frac{1-u}{u}}^\infty e^{-x}e^{-y} \quad dydx \\
&= \int_0^\infty e^{-x} \int_{x\frac{1-u}{u}}^\infty e^{-y} \quad dydx \\
&= \int_0^\infty e^{-x} \left[-e^{-y}\right]_{x\frac{1-u}{u}}^\infty \quad dx \\
&= \int_0^\infty e^{-x} \left(0 + e^{-x\frac{1-u}{u}}\right) \quad dx \\
&= \int_0^\infty e^{-x} e^{\frac{-x}{u}} e^{x} \quad dx \\
&= \int_0^\infty e^{\frac{-x}{u}} \quad dx \\
&= \left[-ue^{\frac{-x}{u}}\right]_0^\infty \quad dx \\
&= ue^0 \\
&= u
\end{aligned}
$$

Question 4

```
function result = ass4q4
    N = 1e7;
    R = 2.*sqrt(rand(N,1));
    I = (R > 1);
    mean(I);
endfunction
```

This yields a response of approximately 0.74992

Question 5

```
function result = ass4q5
    N = 1e7;
    X = sqrt(rand(N,1));
    Y = sqrt(rand(N,1));
    I = (2.*X>Y);
    mean(I);
endfunction
```

This yields a response of approximately 0.8751

Question 6

```
function result = ass4q6
    N = 1e7;
    X = -log(rand(N,1));
    Y = -log(rand(N,1));
    U = X./(X+Y);
    hist(U, (0:0.05:0.0975));
    ylabel("U Frequencies");
    xlabel("u");
endfunction
```

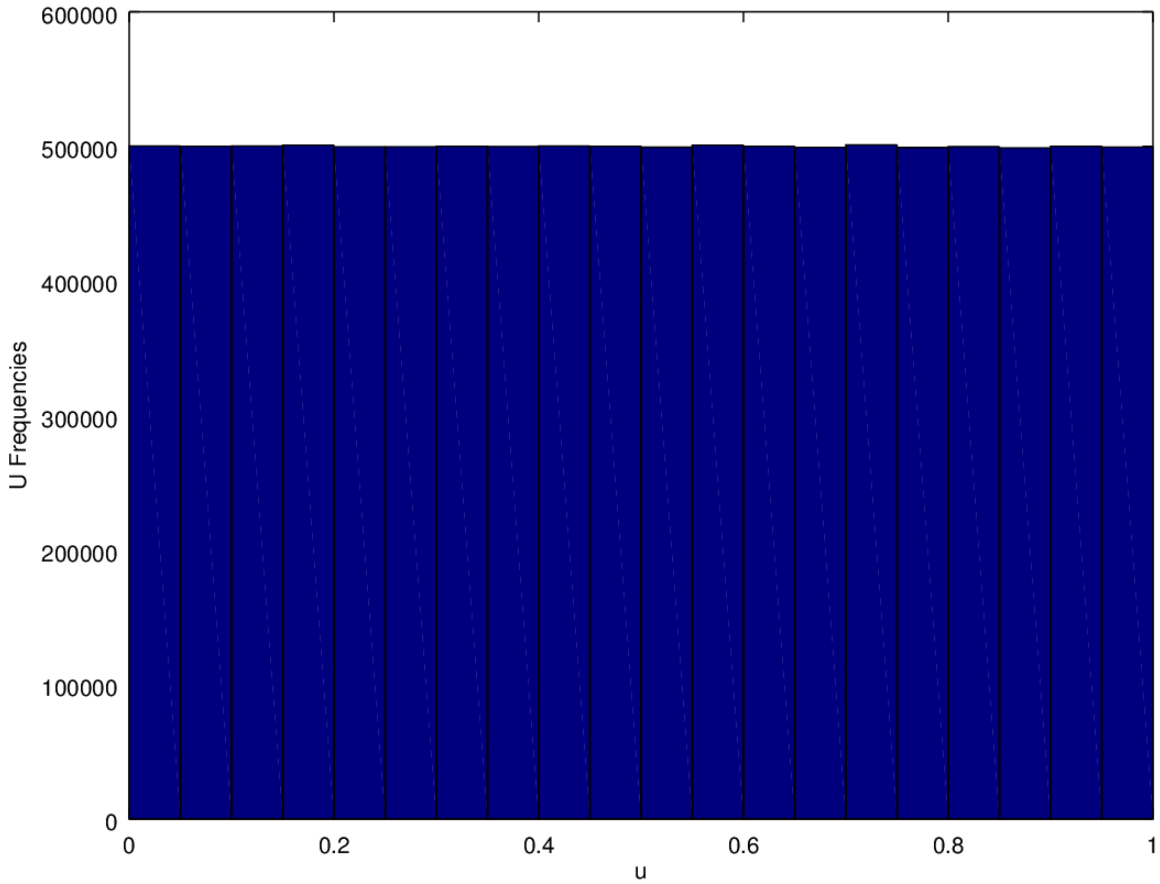

Figure 1: Question 6 answer