

Daniel **Fitz**  
(43961229)



University Of Queensland

**STAT2203** – Probability Models and Data Analysis for Engineering

STAT2203 Assignment 1

## Question 1

Let  $S$  be the event the system succeeds

Let  $A_i$  be the event that a joint is successful

Let  $L_i$  be the event that an O-ring is successful

$$\begin{aligned}\mathbb{P}(L_i) &= 1 - \mathbb{P}(L_i^c) \\ &= 1 - 0.1 \quad (\text{From question}) \\ &= 0.9\end{aligned}$$

$$\begin{aligned}\mathbb{P}(A_i^c) &= \mathbb{P}(L_i \cap L_i^c) \\ &= \mathbb{P}(L_i^c)\mathbb{P}(L_i^c) \\ &= 2 \times 0.1 \\ &= 0.01\end{aligned}$$

$$\mathbb{P}(A_i) = 0.99$$

$$\begin{aligned}\mathbb{P}(S) &= \mathbb{P}(A_i \cap A_i \cap A_i \cap A_i \cap A_i \cap A_i) \\ &= \mathbb{P}(A_i)^6 \\ &= 0.99^6 \\ &= 0.9415\end{aligned}$$

$$\begin{aligned}\mathbb{P}(S^c) &= 1 - \mathbb{P}(S) \\ &\approx 0.0585\end{aligned}$$

Therefore the probability that the system will fail is 5.85%

## Question 2

Let  $R_i$  be the event that  $i$  is received

Let  $S_i$  be the event that  $i$  is sent

From the question:

$$\begin{aligned}\mathbb{P}(S_0) &= 0.5 \\ \mathbb{P}(S_1) &= 0.5 \\ \mathbb{P}(R_1 | S_1) &= 0.9 \\ \mathbb{P}(R_0 | S_0) &= 0.95\end{aligned}$$

Using Bayes rule:

$$\begin{aligned}\mathbb{P}(S_0 | R_1) &= \frac{\mathbb{P}(R_1 | S_0)\mathbb{P}(S_0)}{\mathbb{P}(R_1 | S_0)\mathbb{P}(S_0) + \mathbb{P}(R_1 | S_1)\mathbb{P}(S_1)} \\ &= \frac{(1 - 0.95) \times 0.5}{(1 - 0.95) \times 0.5 + 0.9 \times 0.5} \\ &= \frac{1}{19} \approx 0.0526\end{aligned}$$

Therefore, given we receive a 1 it is 5.26% likely that a 0 was sent

## Question 3

Let  $A$  be the event that the sum of numbers is 1

$$\Omega = \{-1, 0, 1\}$$

| Assume that the each number is draw at equal probability

### Without Replacement

(1, 0),	(1, -1),	(0, 1),	(0, -1),	(-1, 0),	(1, 1)
1	0	1	-1	-1	0

Therefore:

$$\begin{aligned}\mathbb{P}(A) &= \frac{|A|}{|\Omega|} \\ &= \frac{2}{6} = \frac{1}{3}\end{aligned}$$

## With Replacement

(1, 1), (1, 0), (1, -1), (0, 1), (0, 0), (0, -1), (-1, 1), (-1, 0), (-1, -1)  
           2      1      0      1      0      -1      0      -1      -2

Therefore:

$$\begin{aligned}\mathbb{P}(A) &= \frac{|A|}{|\Omega|} \\ &= \frac{2}{9}\end{aligned}$$

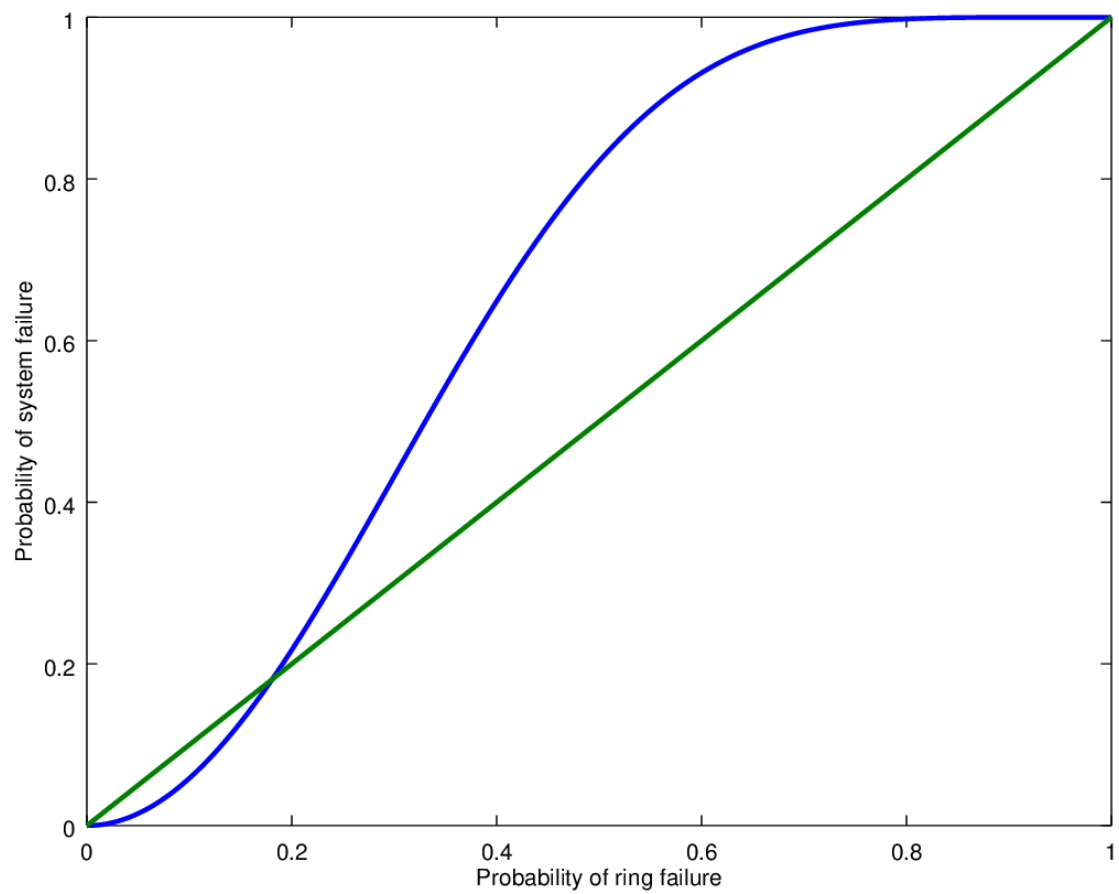
## Question 4

```

1 function result = ass1
2     N = 1e3;
3     x = linspace(0, 1, N);
4     y = [1:N];
5     for i = 1:N
6         y(i) = systemFailure(x(i));
7     end
8     plot(x, y, "linewidth", 2, x, x, "linewidth", 2);
9     xlabel("Probability of ring failure");
10    ylabel("Probability of system failure");
11 endfunction
12
13 function result = linkSuccess(p)
14     result = 1 - p;
15 endfunction
16
17 function result = sealSuccess(p)
18     result = 1 - (1 - linkSuccess(p))^2;
19 endfunction
20
21 function result = systemSuccess(p)
22     result = sealSuccess(p)^6;
23 endfunction
24
25 function result = systemFailure(p)
26     result = 1 - systemSuccess(p);
27 endfunction

```

By running the above code, we get the output provided below. The point at which `systemFailure(p) = p` occurs at points `[0, 0.18, 1]` as visible from the diagram below.



*Figure 1: Question 4 Answer*

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## Question 5

```
1 function result = ass1q5
2     N = 1e4;
3     received = 0;
4     sent0 = 0;
5     for i = 1:N
6         [s, r] = get_receive_bit();
7         if r == 1
8             received += 1;
9             if s == 0
10                sent0 += 1;
11            endif
12        endif
13    end
14    printf("%d:%d\n", received, sent0);
15 endfunction
16
17 function [sent, received] = get_receive_bit
18     bit = get_sent_bit();
19     if bit == 1
20         if rand >= 0.9
21             received = 0;
22         else
23             received = bit;
24         endif
25     else
26         if rand >= 0.95
27             received = 1;
28         else
29             received = bit;
30         endif
31     endif
32     sent = bit;
33 endfunction
34
35 function result = get_sent_bit
36     if rand >= 0.5
37         result = 1;
38     else
39         result = 0;
40     endif
41 endfunction
```

Based on the code above, the output by running `ass1q5`, we can expect an answer close to `247:4772`

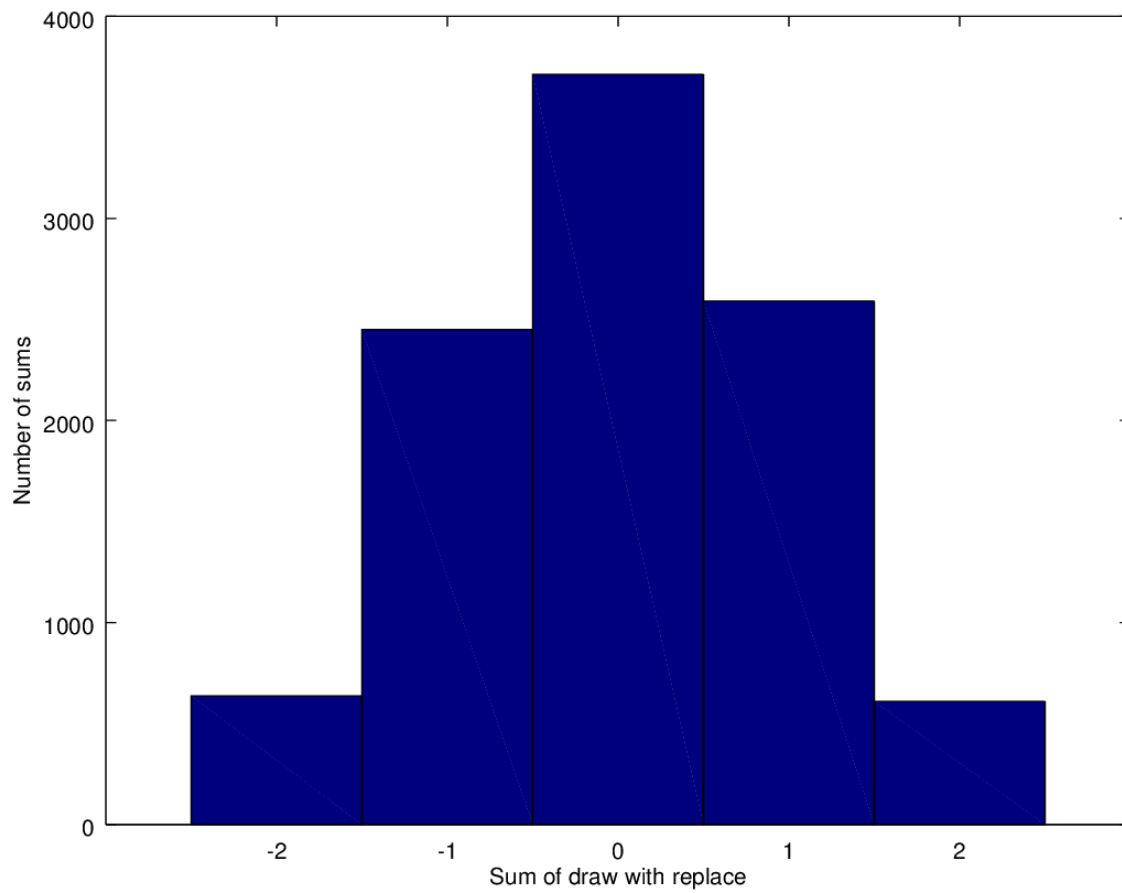
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## Question 6

```
1 function ass1q6
2     N=1e4;
3     result = [1:N];
4     for i = 1:N
5         result(i) = sumWithReplace();
6     endfor
7     result;
8     hist(result, -2:2);
9     xlabel("Sum of draw with replace");
10    ylabel("Number of sums");
11 endfunction
12
13 function result = sumWithReplace
14     result = sum(int8(rand(2, 1) * 2) - 1);
15 endfunction
```

---

The above code generates the following histogram:



*Figure 2: Question 6 Answer*