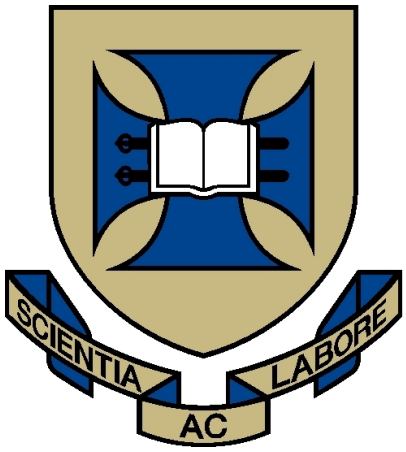
Daniel **Fitzmaurice**
(43961229)



University Of Queensland

STAT2203 – Probability Models and Data Analysis for Engineering

STAT2203 Assignment 3

Question 1

$$
\begin{aligned}
&\mathbb{E}\left((X-\mathbb{E}(X))^2\right) \\
&= \mathbb{E}(X^2 - 2X\mathbb{E}(X)\mathbb{E}(X)^2) \\
&= \mathbb{E}(X^2) - 2\mathbb{E}(X\mathbb{E}(X)) + \mathbb{E}(X)^2 \quad (\text{Using } \mathbb{E}(aX+bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)) \\
&= \mathbb{E}(X^2) - 2\mathbb{E}(X)^2 + \mathbb{E}(X)^2 \\
&= \mathbb{E}(X^2) - \mathbb{E}(X)^2
\end{aligned}
$$

Question 2

Let *S* be the system success
Let *X* be component 1's lifetime
Let *Y* be component 2's lifetime
Let *Z* be component 3's lifetime

Since:

$$\lambda = \frac{1}{\bar{x}}$$

Therefore,

$$
\begin{aligned}
X &\sim Exp\left(\frac{1}{5}\right) \\
Y &\sim Exp\left(\frac{1}{3}\right) \\
Z &\sim Exp\left(\frac{1}{3}\right) \\
S &= X \cap (Y \cup Z) \\
\mathbb{P}(S \geq s) &= \mathbb{P}(X \geq s) \cap (\mathbb{P}(Y \geq s) \cup \mathbb{P}(Z \geq s)) \\
&= \mathbb{P}(X \geq s) \cap (\mathbb{P}(Y \geq s) + \mathbb{P}(Z \geq s) - \mathbb{P}(Y \geq s)\mathbb{P}(Z \geq s)) \\
&= \mathbb{P}(X \geq s)\mathbb{P}(Y \geq s) + \mathbb{P}(X \geq s)\mathbb{P}(Z \geq s) - \mathbb{P}(X \geq s)\mathbb{P}(Y \geq s)\mathbb{P}(Z \geq s) \\
&= \left(-e^{-\frac{s}{5}}\right)\left(-e^{-\frac{s}{3}}\right) + \left(-e^{-\frac{s}{5}}\right)\left(-e^{-\frac{s}{3}}\right) - \left(-e^{-\frac{s}{5}}\right)\left(-e^{-\frac{s}{3}}\right)\left(-e^{-\frac{s}{3}}\right) \\
&= e^{\frac{8}{15}s} + e^{\frac{8}{15}s} - e^{\frac{13}{15}s} \\
&= 2e^{\frac{8}{15}s} - e^{\frac{13}{15}s}
\end{aligned}
$$

Question 3

Part A

Set F(x) to be one and measure the area under the curve

$$
\begin{aligned}
F(x) &= \int_0^{\frac{1}{2}} \alpha(1-x)dx \\
&= \int_0^{\frac{1}{2}} \alpha \quad dx - \int_0^{\frac{1}{2}} \alpha x \quad dx \\
&= \left[\alpha x - \alpha\frac{x^2}{2}\right]_0^{\frac{1}{2}} \\
1 &= \frac{\alpha}{2} - \frac{\alpha}{8} \\
&= \alpha\frac{3}{8} \\
\alpha &= \frac{8}{3}
\end{aligned}
$$

Part B

$$\begin{aligned}
\mathbb{P}\left(\frac{1}{3} < X < \frac{1}{2}\right) &= \int_{\frac{1}{3}}^{\frac{1}{2}} \frac{8}{3}(1-x) \quad dx \\
&= \frac{8}{3}\int_{\frac{1}{3}}^{\frac{1}{2}} (1-x) \quad dx \\
&= \frac{8}{3}\left[x - \frac{x^2}{2}\right]_{\frac{1}{3}}^{\frac{1}{2}} \\
&= \frac{8}{3}\left(\left(\frac{3}{8}\right) - \left(\frac{1}{3} - \frac{1}{18}\right)\right) \\
&= \frac{8}{3}\left(\frac{3}{8} - \frac{5}{18}\right) \\
&= \frac{8}{3}\frac{7}{72} \\
&= \frac{7}{27}
\end{aligned}$$

Question 4

Part A

The variance of an Exp is

$$\frac{1}{\lambda^2}$$

Therefore since lambda is 1, the variance is 1

Part B

$$\begin{aligned}
Var(Z_n) &= Var\left(\frac{X_1 + \ldots + X_n - \mathbb{E}(X_1 + \ldots + X_n)}{\sqrt{Var(X_1 + \ldots + X_n)}}\right) \\
&= Var\left(\frac{1}{\sqrt{n}}(X_1 + \ldots + X_n) - \frac{n}{\sqrt{n}}\right) \\
&= \frac{Var(X_1 + \ldots + X_n)}{n} \qquad (a^2 Var(X) = Var(aXb)) \\
&= \frac{n}{n} \\
&= 1
\end{aligned}$$

Question 5

```
function result = ass3q5
  N=1e3;
  numOfInsideRange = 0;
  sumOfInsideRange = 0;
  for i = 1:N
    tempVal = outcome(rand);
    if (tempVal > 1/3 && tempVal < 1/2)
      numOfInsideRange++;
      sumOfInsideRange += tempVal;
    endif
  endfor
  avg = sumOfInsideRange/numOfInsideRange
endfunction

function result = outcome(y)
  lambda = 8/3;
  a = -lambda/2;
  b = lambda;
  c = -y;
  result = (sqrt(b^2 - 4ac) - b)/(2*a);
endfunction
```

The above code, when run, gives an approximate answer of 0.40

Question 6

```
function result = ass3q6
  N=1e3;
  results1 = 1:N;
  results10 = 1:N;
  for i = 1:N
    results1(i) = simVariable(1);
    results10(i) = simVariable(10);
    results100(i) = simVariable(100);
  endfor
  subplot(3, 1, 1);
  hist(results1, -4:.5:4);
  title("Z = 1");
  ylabel("Num of hits");
  xlabel("x of Xn");
  subplot(3, 1, 2);
  hist(results10, -4:.5:4);
  title("Z = 10");
  ylabel("Num of hits");
  xlabel("x of Xn");
  subplot(3, 1, 3);
  hist(results100, -4:.5:4);
  title("Z = 100");
  ylabel("Num of hits");
  xlabel("x of Xn");
endfunction

function result = simVariable(n)
  X = -log(rand(1, n));
  Z = (sum(X) - n)/sqrt(n);
  result = Z;
endfunction
```

The above code outputs the following image:

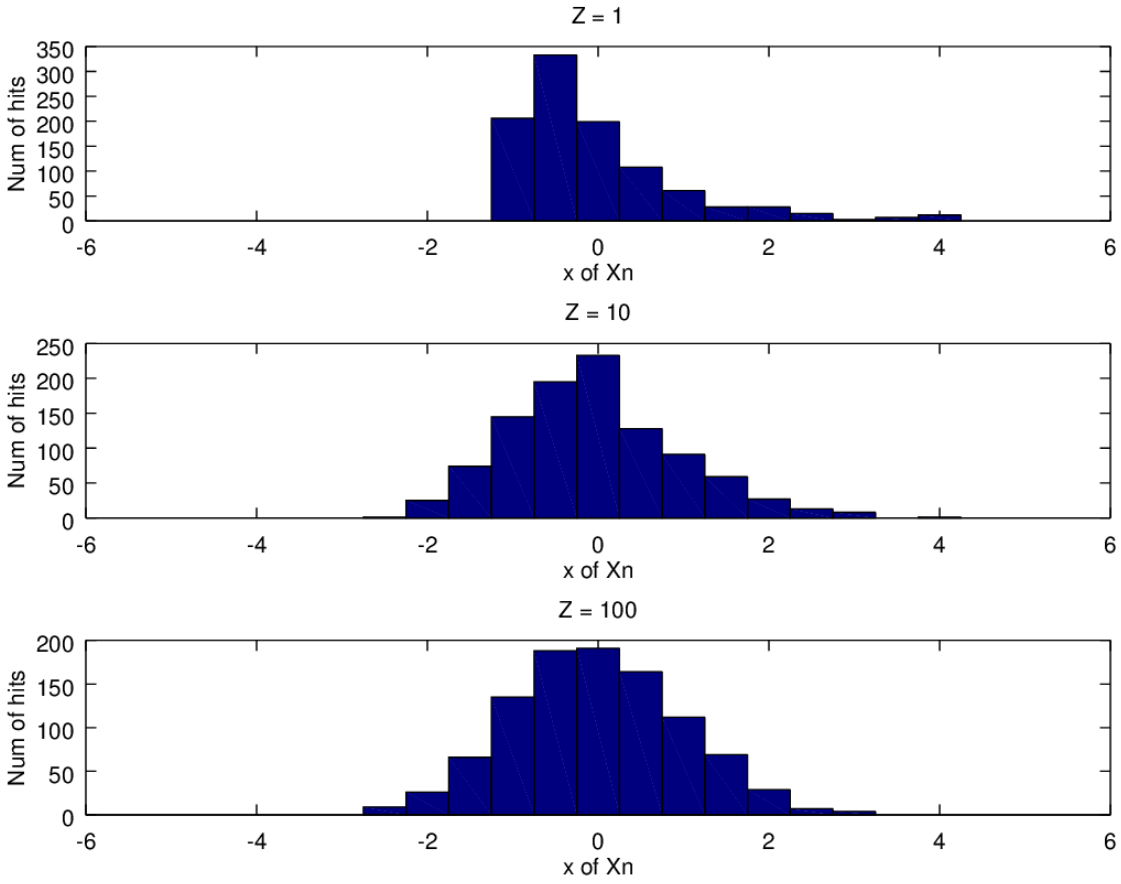


Figure 1: Question 6 answer