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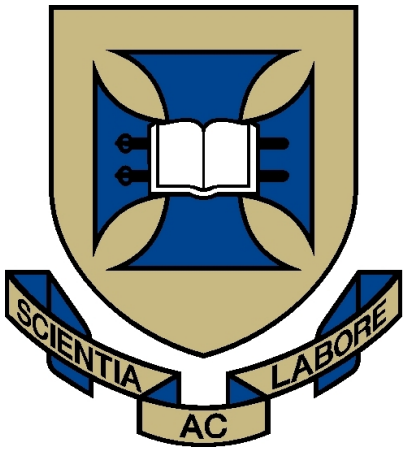

University Of Queensland

STAT2203 – Probability Models and Data Analysis for Engineering

STAT2203 Assignment 2

Question 1

Let *X* be the event that the ball falls in box *x*. Where *X ~ Bin(4, 0.5)*

SubQuestion A

$$\begin{aligned}\mathbb{P}(X=x) &= \binom{4}{x}\left(\frac{1}{2}\right)^{x} \times \left(1-\frac{1}{2}\right)^{4-x} \\ &= \binom{4}{x}\left(\frac{1}{2}\right)^{4} \\ &= \frac{1}{16}\binom{4}{x} \\ \mathbb{P}(X=3) &= \frac{1}{16}\binom{4}{3} \\ &= \frac{4}{16} = \frac{1}{4}\end{aligned}$$

SubQuestion B

$$\begin{aligned}\mathbb{P}(X=x) &= \binom{4}{x}\left(\frac{1}{2}\right)^{x} \times \left(1-\frac{1}{2}\right)^{4-x} \\ &= \binom{4}{x}\frac{1}{2^{x}} \times \frac{1}{2^{4-x}} \\ &= \binom{4}{x}\frac{2^{x}}{2^{4}2^{x}} \\ &= \frac{1}{16}\binom{4}{x}\end{aligned}$$

Question 2

Part 1

We are given:

$$\begin{aligned}\mathbb{P}(X=1) &= r \\ \mathbb{P}(X=0) &= 1-r \\ \mathbb{P}(Y=1) &= s \\ \mathbb{P}(Y=0) &= 1-s\end{aligned}$$

Therefore the following table can be generated:

Y\X	0	1
0	*(1-r)(1-s)*	*r(1-s)*
1	*s(1-r)*	*rs*

Table 1: Combinations of P(X=x)P(Y=y)

Using the above table, we can see that U and V will both take on values {*0, 1*}

U,V	X,Y	Solution
0,0	0,0	*(1-r)(1-s)*
0,1	1,0 + 0,1	*r(1-s) + s(1-r)*
1,0	*Not possible*	*0*
1,1	1,1	*rs*

Therefore the following joint pmf table can be calculated

U\V	0	1

0	*(1-r)(1-s)*	*r(1-s) + s(1-r)*
1	*0*	*rs*

Part 2

$$
\begin{aligned}
\mathbb{P}(V=1,U=1) &= \mathbb{P}(V=1)\mathbb{P}(U=1) \\
rs &= (rs)(r+s-rs) \\
1 &= r+s-rs \\
1-s &= r-rs \\
&= r(1-s) \\
1 &= r \\
\mathbb{P}(V=0,U=0) &= \mathbb{P}(V=0)\mathbb{P}(U=0) \\
(1-r)(1-s) &= (1-rs)(1-s-r+rs) \\
1-r-s+rs &= 1-s-r+rs-rs+rs^2+sr^2+(rs)^2 \\
1+rs &= 1+rs^2sr^2-(rs)^2 \\
1+s &= 1+s^2+s-s^2 && \text{(When } r \text{ is 1)} \\
1+s &= 1+s \\
\mathbb{P}(V=1,U=0) &= \mathbb{P}(U=0)\mathbb{P}(V=1) \\
r+s-2rs &= (1-rs)(r+s-rs) \\
r+s-2rs &= r+s-rs-r^2s-rs^2+r^2s^2 \\
rs &= -r^2s-rs^2+r^2s^2 \\
s &= -s-s^2+s^2 && \text{(When } r \text{ is 1)} \\
s &= -s \\
s &= 0 \\
\mathbb{P}(U=1,V=0) &= \mathbb{P}(U=1)\mathbb{P}(V=0) \\
0 &= rs(1-s-r+rs) && \text{(When } r \text{ is 1, When } s \text{ is 0)} \\
0 &= 0
\end{aligned}
$$

Question 3

$$
\begin{aligned}
L(\theta;p) &= \prod_{i=1}^{5}(1-p)^{x_i}p \\
l(\theta;p) &= \sum_{i=1}^{5}\log\left((1-p)^{x_i-1}\right)\log(p) \\
&= n\log(p) + \sum_{i=1}^{5}(x_i-1)\log(1-p) \\
&= n\log(p) + \log(1-p)\sum_{i=1}^{5}(x_i-1) \\
\frac{d}{dp} \quad \frac{n}{p} &- \frac{\sum_{i=1}^{5}(x_i-1)}{1-p} \\
0 &= \frac{n}{p} - \frac{\sum_{i=1}^{5}(x_i-1)}{1-p} \\
\frac{n}{p} &= \frac{\sum_{i=1}^{5}(x_i-1)}{1-p} \\
\frac{n}{p} - n &= -n\sum_{i=1}^{5}(x_i) \\
\frac{n}{p} &= \sum_{i=1}^{5}(x_i) \\
p &= \frac{n}{\sum_{i=1}^{5}(x_i)} = \frac{1}{\bar{x}}
\end{aligned}
$$

Question 4

```
function result = ass2q1
  N=1e4;
  results = 1:N;
  for i = 1:N;
    results(i) = drop_ball(0, 0);
  endfor
  hist(results, 0:4);
  xlabel("Ball Fall Position");
  ylabel("Number of balls in position");
endfunction

function result = drop_ball(level, position)
  if (level >= 4)
    result = position;
  else
    direction = rand > 0.5;
    result = drop_ball(level + 1, position + direction);
  endif
endfunction
```

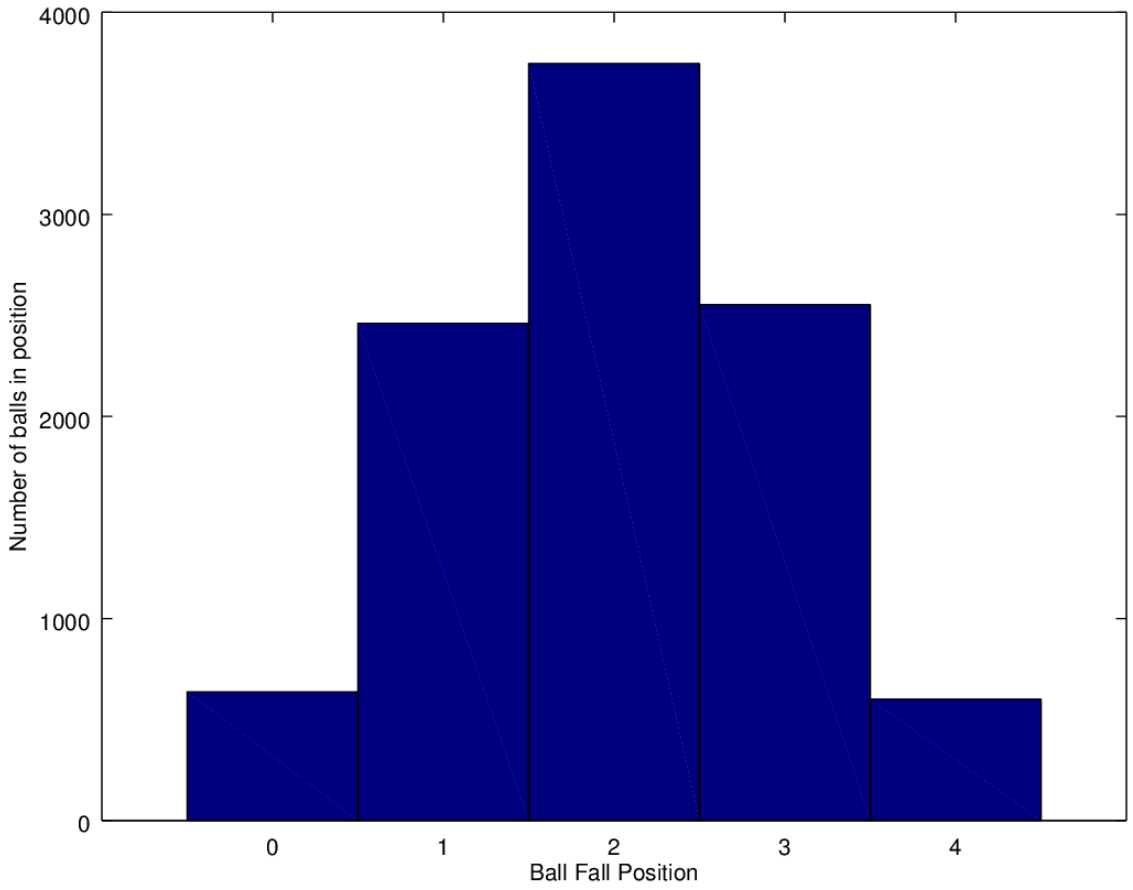

Figure 1: Result for Question 4

Question 5

```
function result = ass2q2
  N = 1e2;
  totals = zeros(N, N);
  for i = 1:N
    for j = 1:N
      totals(i, j) = calcRS(i/N, j/N);
    endfor
  endfor
  mesh(1:N, 1:N, totals);
  title("Question 5");
  xlabel("Value for r");
  ylabel("Value for s");
  zlabel("Resulting probability");
endfunction

function result = calcRS(r, s)
  result = sumRS(0, 0, r, s) + sumRS(0, 1, r, s) + sumRS(1, 0, r, s) +
    sumRS(1, 1, r, s);
endfunction

function result = sumRS(u, v, r, s)
  result = abs(probUV(u, v, r, s) - (probU(u, r, s) * probV(v, r, s)));
endfunction

function result = probUV(u, v, r, s)
  if (u == 1 && v == 1)
    result = r * s;
  elseif (u == 0 && v == 0)
    result = (1 - r) * (1 - s);
  elseif (u == 0 && v == 1)
    result = r(1-s) + s(1-r);
  elseif (u == 1 && v == 0)
    result = 0;
  else
    # Should not reach this
    result = -1;
  endif;
endfunction

function result = probU(u, r, s)
  result = probUV(u, 0, r, s) + probUV(u, 1, r, s);
endfunction

function result = probV(v, r, s)
  result = probUV(0, v, r, s) + probUV(1, v, r, s);
endfunction
```

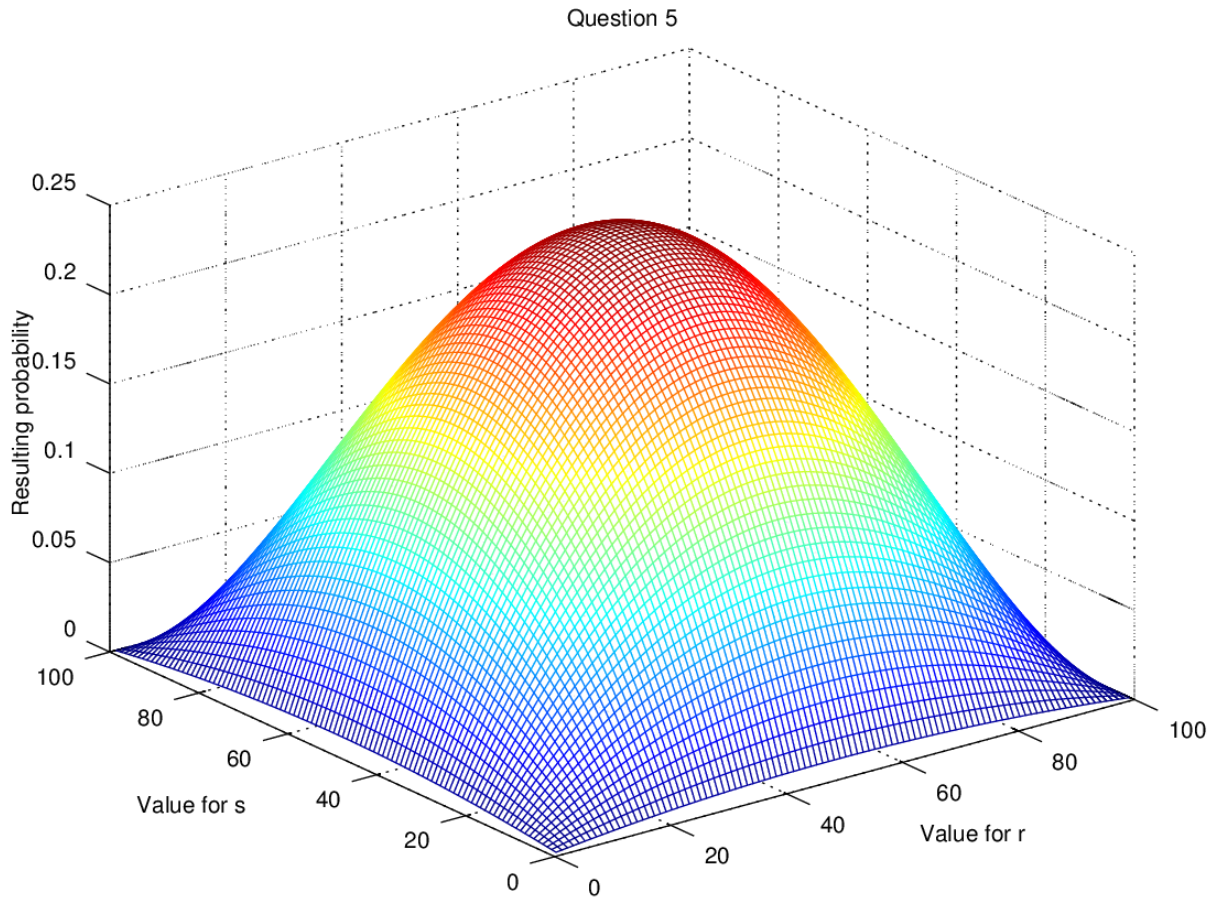

Figure 2: Result for Question 5

Question 6

```
1  function result = ass2q6
2    N=5e4;
3    totals=1:N;
4    for i = 1:N
5      totals(i) = pmf(randi(5));
6    endfor
7    hist(totals, 0:0.1:0.4);
8    xlabel("pmf result");
9    ylabel("Number of results");
10 endfunction
11
12 function result = pmf(x)
13   result = (1/3) * (2/3)^(x - 1);
14 endfunction
```

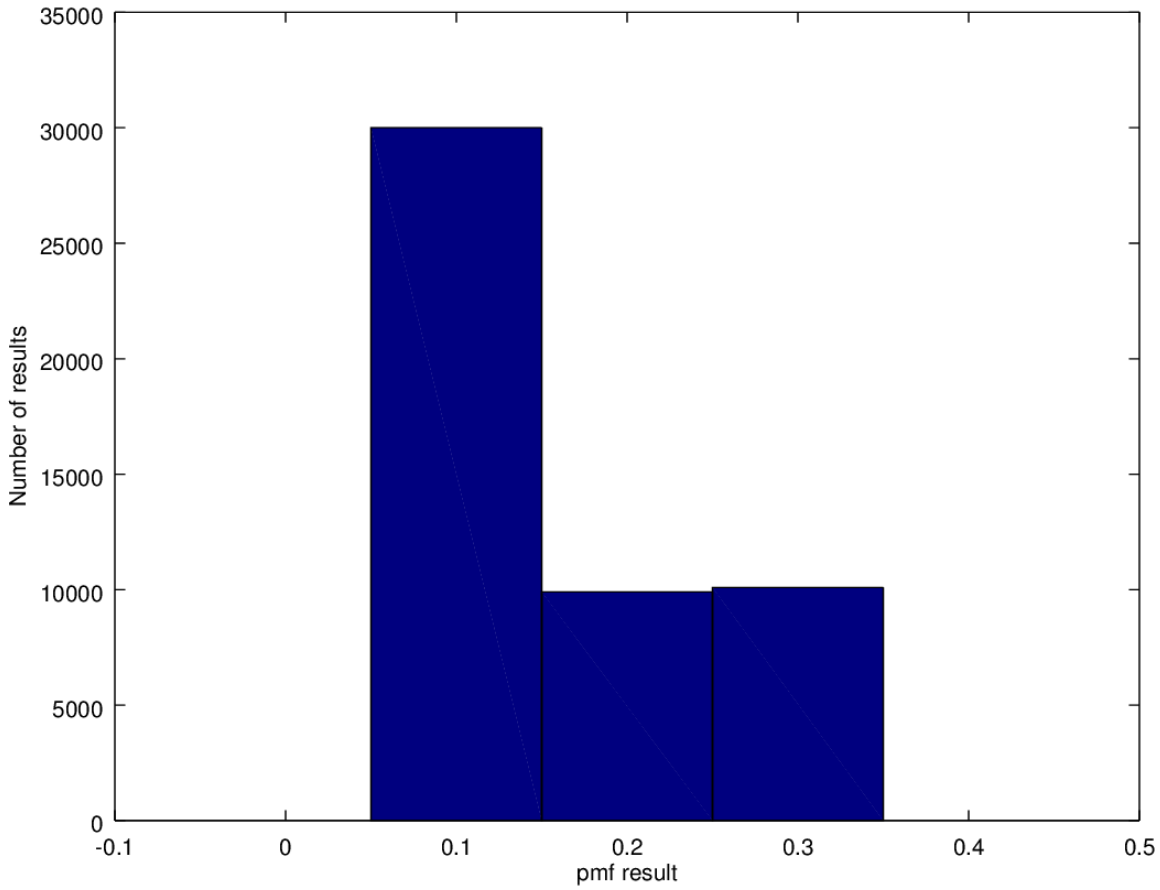

Figure 3: Result for Question 6