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**STAT2203** – Probability Models and Data Analysis for Engineering

STAT2203 Assignment 3

# Question 1

$$\mathbb{E}\left((X - \mathbb{E}(X))^2\right)$$
$$= \mathbb{E}(X^2 - 2X\mathbb{E}(X)\mathbb{E}(X)^2)$$
$$= \mathbb{E}(X^2) - 2\mathbb{E}(X\mathbb{E}(X)) + \mathbb{E}(X)^2 \quad (\text{Using } \mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y))$$
$$= \mathbb{E}(X^2) - 2\mathbb{E}(X)^2 + \mathbb{E}(X)^2$$
$$= \mathbb{E}(X^2) - \mathbb{E}(X)^2$$

# Question 2

Let $S$ be the system success
Let $X$ be component 1's lifetime
Let $Y$ be component 2's lifetime
Let $Z$ be component 3's lifetime

Since:

$$\lambda = \frac{1}{\bar{x}}$$

Therefore,

$$X \sim Exp\left(\frac{1}{5}\right)$$

$$Y \sim Exp\left(\frac{1}{3}\right)$$

$$Z \sim Exp\left(\frac{1}{3}\right)$$

$$S = X \cap (Y \cup Z)$$

$$\mathbb{P}(S \geq s) = \mathbb{P}(X \geq s) \cap (\mathbb{P}(Y \geq s) \cup \mathbb{P}(Z \geq s))$$
$$= \mathbb{P}(X \geq s) \cap (\mathbb{P}(Y \geq s) + \mathbb{P}(Z \geq s) - \mathbb{P}(Y \geq s)\mathbb{P}(Z \geq s))$$
$$= \mathbb{P}(X \geq s)\mathbb{P}(Y \geq s) + \mathbb{P}(X \geq s)\mathbb{P}(Z \geq s) - \mathbb{P}(X \geq s)\mathbb{P}(Y \geq s)\mathbb{P}(Z \geq s)$$
$$= \left(-e^{-\frac{s}{5}}\right)\left(-e^{-\frac{s}{3}}\right) + \left(-e^{-\frac{s}{5}}\right)\left(-e^{-\frac{s}{3}}\right) - \left(-e^{-\frac{s}{5}}\right)\left(-e^{-\frac{s}{3}}\right)\left(-e^{-\frac{s}{3}}\right)$$
$$= e^{\frac{8}{15}s} + e^{\frac{8}{15}s} - e^{\frac{13}{15}s}$$
$$= 2e^{\frac{8}{15}s} - e^{\frac{13}{15}s}$$