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STAT2203 – Probability Models and Data Analysis for Engineering

STAT2203 Assignment 1

Question 1

Let S be the event the system succeeds
Let A_i be the event that a joint is successful
Let L_i be the event that an O-ring is successful

$$\begin{aligned}
\mathbb{P}(L_i) &= 1 - \mathbb{P}(L_i^c) \\
&= 1 - 0.1 && \text{(From question)} \\
&= 0.9 \\
\mathbb{P}(A_i^c) &= \mathbb{P}(L_i \cap L_i) \\
&= \mathbb{P}(L_i^c)\mathbb{P}(L_i^c) \\
&= 2 \times 0.1 \\
&= 0.01 \\
\mathbb{P}(A_i) &= 0.99 \\
\mathbb{P}(S) &= \mathbb{P}(A_i \cap A_i \cap A_i \cap A_i \cap A_i \cap A_i) \\
&= \mathbb{P}(A_i)^6 \\
&= 0.99^6 \\
&= 0.9415 \\
\mathbb{P}(S^c) &= 1 - \mathbb{P}(S) \\
&\approx 0.0585
\end{aligned}$$

Therefore the probability that the system will fail is 5.85%

Question 2

Let R_i be the event that i is received
Let S_i be the event that i is sent
From the question:

$$\begin{aligned}
\mathbb{P}(S_0) &= 0.5 \\
\mathbb{P}(S_1) &= 0.5 \\
\mathbb{P}(R_1 \mid S_1) &= 0.9 \\
\mathbb{P}(R_0 \mid S_0) &= 0.95
\end{aligned}$$

Using Baynes rule:

$$\begin{aligned}
\mathbb{P}(S_0 \mid R_1) &= \frac{\mathbb{P}(R_1 \mid S_0)\mathbb{P}(S_0)}{\mathbb{P}(R_1 \mid S_0)\mathbb{P}(S_0) + \mathbb{P}(R_1 \mid S_1)\mathbb{P}(S_1)} \\
&= \frac{(1-0.95) \times 0.5}{(1-0.95) \times 0.5 + 0.9 \times 0.5} \\
&= \frac{1}{19} \approx 0.0526
\end{aligned}$$

Therefore, given we receive a 1 it is 5.26% likely that a 0 was sent

Question 3

Let A be the event that the sum of numbers is 1

$$\Omega = \{-1, 0, 1\}$$

> Assume that the each number is draw at equal probability

Without Replacement

$$\begin{matrix} (1,0), & (1,-1), & (0,1), & (0,-1), & (-1,0), & (1,1) \\ 1 & 0 & 1 & -1 & -1 & 0 \end{matrix}$$

Therefore:

$$\begin{aligned}\mathbb{P}(A) &= \frac{\mid A \mid}{\mid \Omega \mid} \\ &= \frac{2}{6} = \frac{1}{3}\end{aligned}$$

With Replacement

$$\begin{array}{ccccccccc} (1,1), & (1,0), & (1,-1), & (0,1), & (0,0), & (0,-1), & (-1,1), & (-1,0), & (-1,-1) \\ 2 & 1 & 0 & 1 & 0 & -1 & 0 & -1 & -2 \end{array}$$

Therefore:

$$\begin{aligned}\mathbb{P}(A) &= \frac{\mid A \mid}{\mid \Omega \mid} \\ &= \frac{2}{9}\end{aligned}$$

Question 4

```
function result = ass1
  N = 1e3;
  x = linspace(0, 1, N);
  y = [1:N];
  for i = 1:N
    y(i) = systemFailure(x(i));
  end
  plot(x, y, "linewidth", 2, x, x, "linewidth", 2);
  xlabel("Probability of ring failure");
  ylabel("Probability of system failure");
endfunction

function result = linkSuccess(p)
  result = 1 - p;
endfunction

function result = sealSuccess(p)
  result = 1 - (1 - linkSuccess(p))^2;
endfunction

function result = systemSuccess(p)
  result = sealSuccess(p)^6;
endfunction

function result = systemFailure(p)
  result = 1 - systemSuccess(p);
endfunction
```

By running the above code, we get the output provided below. The point at which `systemFailure(p) = p` occurs at points `[0, 0.18, 1]` as visible from the diagram below.

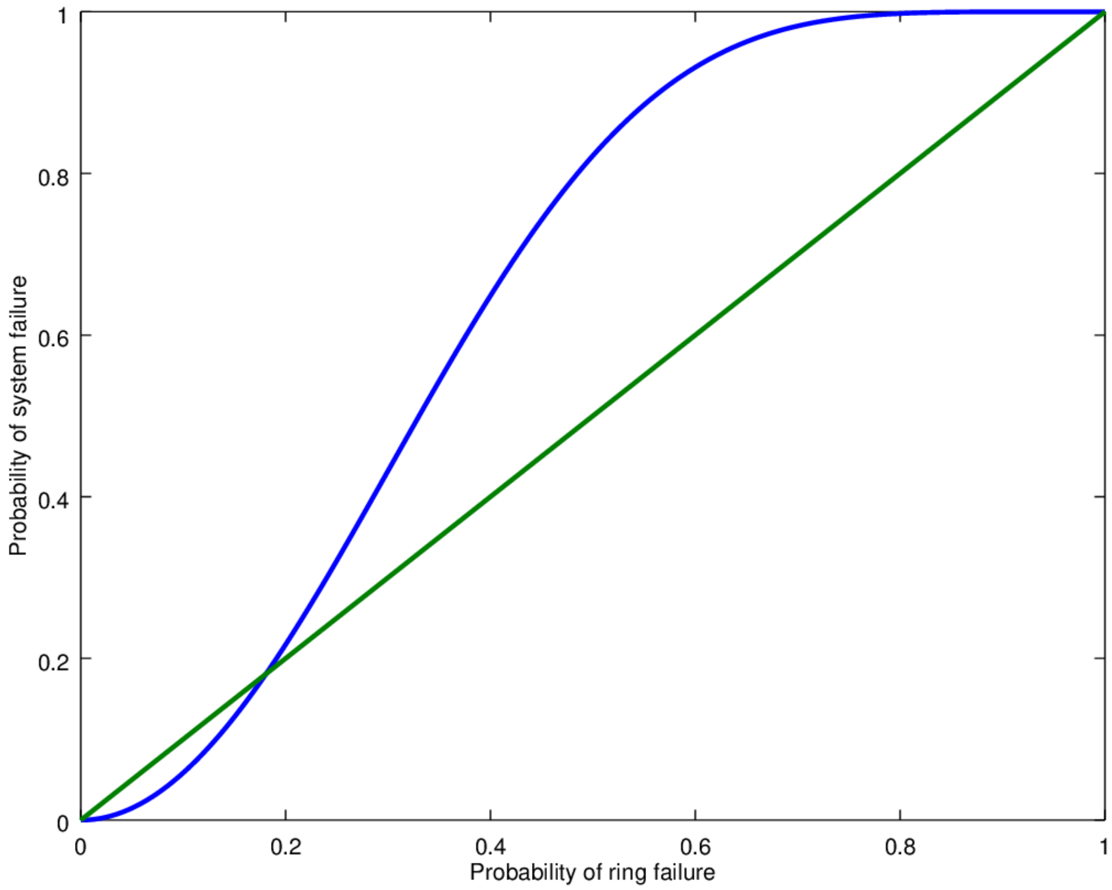

Figure 1: Question 4 Answer

Question 5

```
function result = ass1q5
  N = 1e4;
  received = 0;
  sent0 = 0;
  for i = 1:N
    [s, r] = get_receive_bit();
    if r == 1
      received += 1;
      if s == 0
        sent0 += 1;
      endif
    endif
  end
  printf("%d:%d\n", received, sent0);
endfunction

function [sent, received] = get_receive_bit
  bit = get_sent_bit();
  if bit == 1
    if rand >= 0.9
      received = bit;
    else
      received = 0;
    endif
  else
    if rand >= 0.95
      received = bit;
    else
      received = 1;
    endif
  endif
  sent = bit;
endfunction

function result = get_sent_bit
  if rand >= 0.5
    result = 1;
  else
    result = 0;
  endif
endfunction
```

Based on the code above, the output by running `ass1q5`, we can expect an answer close to `5241:4728`

Question 6

```
function ass1q6
  N=1e4;
  result = [-3:3];
  for i = 1:N
    result(i) = sumWithReplace();
  endfor
  result;
  hist(result, -3:3);
  xlabel("Sum of draw with replace");
  ylabel("Number of sums");
endfunction

function result = sumWithReplace
  result = sum(int8(rand(3, 1) * 2) - 1);
endfunction
```

The above code generates the following histogram:

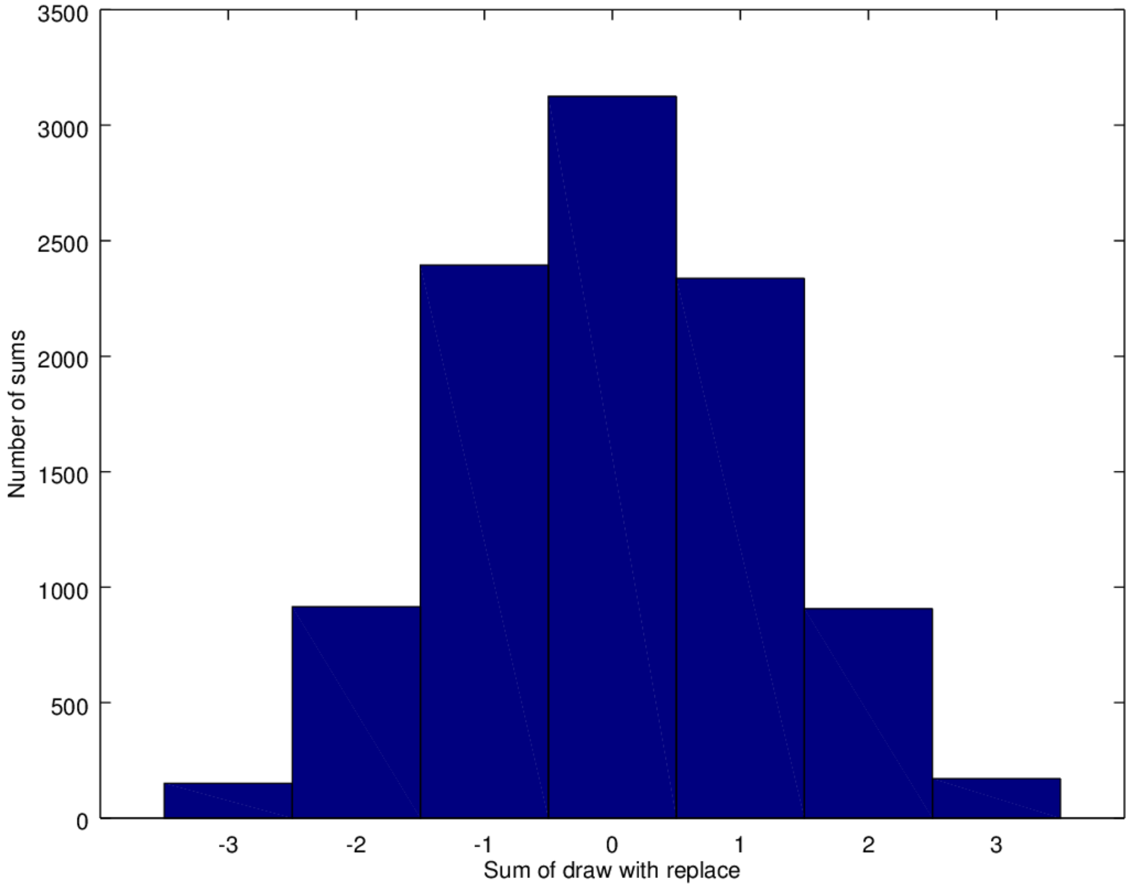

Figure 2: Question 6 Answer