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STAT2203 – Probability Models and Data Analysis for Engineering

STAT2203 Assignment 3

Question 1

$$
\begin{aligned}
&\mathbb{E}\left((X-\mathbb{E}(X))^2\right) \\
&= \mathbb{E}(X^2 - 2X\mathbb{E}(X)\mathbb{E}(X)^2) \\
&= \mathbb{E}(X^2) - 2\mathbb{E}(X\mathbb{E}(X)) + \mathbb{E}(X)^2 \quad (\text{Using } \mathbb{E}(aX+bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)) \\
&= \mathbb{E}(X^2) - 2\mathbb{E}(X)^2 + \mathbb{E}(X)^2 \\
&= \mathbb{E}(X^2) - \mathbb{E}(X)^2
\end{aligned}
$$

Question 2

Let S be the system success
Let X be component 1's lifetime
Let Y be component 2's lifetime
Let Z be component 3's lifetime

Since:

$$\lambda = \frac{1}{\bar{x}}$$

Therefore,

$$
\begin{aligned}
X &\sim Exp\left(\frac{1}{5}\right) \\
Y &\sim Exp\left(\frac{1}{3}\right) \\
Z &\sim Exp\left(\frac{1}{3}\right) \\
S &= X \cap (Y \cup Z) \\
\mathbb{P}(S \geq s) &= \mathbb{P}(X \geq s) \cap (\mathbb{P}(Y \geq s) \cup \mathbb{P}(Z \geq s)) \\
&= \mathbb{P}(X \geq s) \cap (\mathbb{P}(Y \geq s) + \mathbb{P}(Z \geq s) - \mathbb{P}(Y \geq s)\mathbb{P}(Z \geq s)) \\
&= \mathbb{P}(X \geq s)\mathbb{P}(Y \geq s) + \mathbb{P}(X \geq s)\mathbb{P}(Z \geq s) - \mathbb{P}(X \geq s)\mathbb{P}(Y \geq s)\mathbb{P}(Z \geq s) \\
&= \left(-e^{-\frac{s}{5}}\right)\left(-e^{-\frac{s}{3}}\right) + \left(-e^{-\frac{s}{5}}\right)\left(-e^{-\frac{s}{3}}\right) - \left(-e^{-\frac{s}{5}}\right)\left(-e^{-\frac{s}{3}}\right)\left(-e^{-\frac{s}{3}}\right) \\
&= e^{\frac{8}{15}s} + e^{\frac{8}{15}s} - e^{\frac{13}{15}s} \\
&= 2e^{\frac{8}{15}s} - e^{\frac{13}{15}s}
\end{aligned}
$$

Question 3

Part A

Set F(x) to be one and measure the area under the curve

$$
\begin{aligned}
F(x) &= \int_0^{\frac{1}{2}} \alpha(1-x)dx \\
&= \int_0^{\frac{1}{2}} \alpha \quad dx - \int_0^{\frac{1}{2}} \alpha x \quad dx \\
&= \left[\alpha x - \alpha\frac{x^2}{2}\right]_0^{\frac{1}{2}} \\
1 &= \frac{\alpha}{2} - \frac{\alpha}{8} \\
&= \alpha\frac{3}{8} \\
\alpha &= \frac{8}{3}
\end{aligned}
$$

Part B

$$
\begin{aligned}
\mathbb{P}\left(\frac{1}{3} < X < \frac{1}{2}\right) &= \int_{\frac{1}{3}}^{\frac{1}{2}} \frac{8}{3}(1-x) \quad dx \\
&= \frac{8}{3}\int_{\frac{1}{3}}^{\frac{1}{2}} (1-x) \quad dx \\
&= \frac{8}{3}\left[x - \frac{x^2}{2}\right]_{\frac{1}{3}}^{\frac{1}{2}} \\
&= \frac{8}{3}\left(\left(\frac{3}{8}\right) - \left(\frac{1}{3} - \frac{1}{12}\right)\right) \\
&= \frac{8}{3}\left(\frac{3}{8} - \frac{1}{4}\right) \\
&= \frac{8}{3}\frac{1}{8} \\
&= \frac{1}{3}
\end{aligned}
$$

Question 4

Part A

The variance of an Exp is

$$\frac{1}{\lambda^2}$$

Therefore since lambda is 1, the variance is 1

Part B

$$
\begin{aligned}
Var(Z_n) &= Var\left(\frac{X_1 + \ldots + X_n - \mathbb{E}(X_1 + \ldots + X_n)}{\sqrt{Var(X_1 + \ldots + X_n)}}\right) \\
&= Var\left(\frac{1}{\sqrt{n}}(X_1 + \ldots + X_n) - \frac{n}{\sqrt{n}}\right) \\
&= \frac{Var(X_1 + \ldots + X_n)}{n} \qquad (a^2 Var(X) = Var(aXb)) \\
&= \frac{n}{n} \\
&= 1
\end{aligned}
$$