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STAT2203 – Probability Models and Data Analysis for Engineering

STAT2203 Assignment 2

Question 1

Let *X* be the event that the ball falls in box *x*. Where *X ~ Bin(4, 0.5)*

SubQuestion A

$$\mathbb{P}(X = x) = \binom{4}{x} \left(\frac{1}{2}\right)^x \times \left(1 - \frac{1}{2}\right)^{4-x}$$

$$= \binom{4}{x} \left(\frac{1}{2}\right)^4$$

$$= \frac{1}{16} \binom{4}{x}$$

$$\mathbb{P}(X = 3) = \frac{1}{16} \binom{4}{3}$$

$$= \frac{4}{16} = \frac{1}{4}$$

SubQuestion B

$$\mathbb{P}(X = x) = \binom{4}{x} \left(\frac{1}{2}\right)^x \times \left(1 - \frac{1}{2}\right)^{4-x}$$

$$= \binom{4}{x} \frac{1}{2^x} \times \frac{1}{2^{4-x}}$$

$$= \binom{4}{x} \frac{2^x}{2^4 2^x}$$

$$= \frac{1}{16} \binom{4}{x}$$

Question 2

Part 1

We are given:

$$\mathbb{P}(X = 1) = r$$
$$\mathbb{P}(X = 0) = 1 - r$$
$$\mathbb{P}(Y = 1) = s$$
$$\mathbb{P}(Y = 0) = 1 - s$$

Therefore the following table can be generated:

Y\X	0	1
0	(1-r)(1-s)	r(1-s)
1	s(1-r)	rs

Table 1: Combinations of P(X=x)P(Y=y)

Using the above table, we can see that U and V will both take on values *{0, 1}*

U,V	X,Y	Solution
0,0	0,0	(1-r)(1-s)
0,1	1,0 + 0,1	r(1-s) + s(1-r)
1,0	*Not possible*	0
1,1	1,1	rs

Therefore the following joint pmf table can be calculated

U\V	0	1
0	*(1-r)(1-s)*	*r(1-s) + s(1-r)*
1	*0*	*rs*

Part 2

$$\mathbb{P}(V = 1, U = 1) = \mathbb{P}(V = 1)\mathbb{P}(U = 1)$$
$$rs = (rs)(r + s - rs)$$
$$1 = r + s - rs$$
$$1 - s = r - rs$$
$$= r(1 - s)$$
$$1 = r$$
$$\mathbb{P}(V = 0, U = 0) = \mathbb{P}(V = 0)\mathbb{P}(U = 0)$$
$$(1 - r)(1 - s) = (1 - rs)(1 - s - r + rs)$$
$$1 - r - s + rs = 1 - s - r + rs - rs + rs^2 + sr^2 + (rs)^2$$
$$1 + rs = 1 + rs^2 sr^2 - (rs)^2$$
$$1 + s = 1 + s^2 + s - s^2 \qquad \text{(When } r \text{ is 1)}$$
$$1 + s = 1 + s$$
$$\mathbb{P}(V = 1, U = 0) = \mathbb{P}(U = 0)\mathbb{P}(V = 1)$$
$$r + s - 2rs = (1 - rs)(r + s - rs)$$
$$r + s - 2rs = r + s - rs - r^2s - rs^2 + r^2s^2$$
$$rs = -r^2s - rs^2 + r^2s^2$$
$$s = -s - s^2 + s^2 \qquad \text{(When } r \text{ is 1)}$$
$$s = -s$$
$$s = 0$$
$$\mathbb{P}(U = 1, V = 0) = \mathbb{P}(U = 1)\mathbb{P}(V = 0)$$
$$0 = rs(1 - s - r + rs) \qquad \text{(When } r \text{ is 1, When } s \text{ is 0)}$$
$$0 = 0$$

Question 3

$$L(\theta; p) = \prod_{i=1}^{5} (1-p)^{x_i} p$$

$$l(\theta; p) = \sum_{i=1}^{5} \log\left((1-p)^{x_i-1}\right) \log(p)$$

$$= n \log(p) + \sum_{i=1}^{5} (x_i - 1) \log(1-p)$$

$$= n \log(p) + \log(1-p) \sum_{i=1}^{5} (x_i - 1)$$

$$\frac{d}{dp} \quad \frac{n}{p} - \frac{\sum_{i=1}^{5}(x_i - 1)}{1-p}$$

$$0 = \frac{n}{p} - \frac{\sum_{i=1}^{5}(x_i - 1)}{1-p}$$

$$\frac{n}{p} = \frac{\sum_{i=1}^{5}(x_i - 1)}{1-p}$$

$$\frac{n}{p} - n = -n \sum_{i=1}^{5} (x_i)$$

$$\frac{n}{p} = \sum_{i=1}^{5} (x_i)$$

$$p = \frac{n}{\sum_{i=1}^{5}(x_i)} = \frac{1}{\bar{x}}$$

Question 4

```
 1  function result = ass2q1
 2    N=1e4;
 3    results = 1:N;
 4    for i = 1:N;
 5      results(i) = drop_ball(0, 0);
 6    endfor
 7    hist(results, 0:4);
 8    xlabel("Ball Fall Position");
 9    ylabel("Number of balls in position");
10  endfunction
11
12  function result = drop_ball(level, position)
13    if (level >= 4)
14      result = position;
15    else
16      direction = rand > 0.5;
17      result = drop_ball(level + 1, position + direction);
18    endif
19  endfunction
```
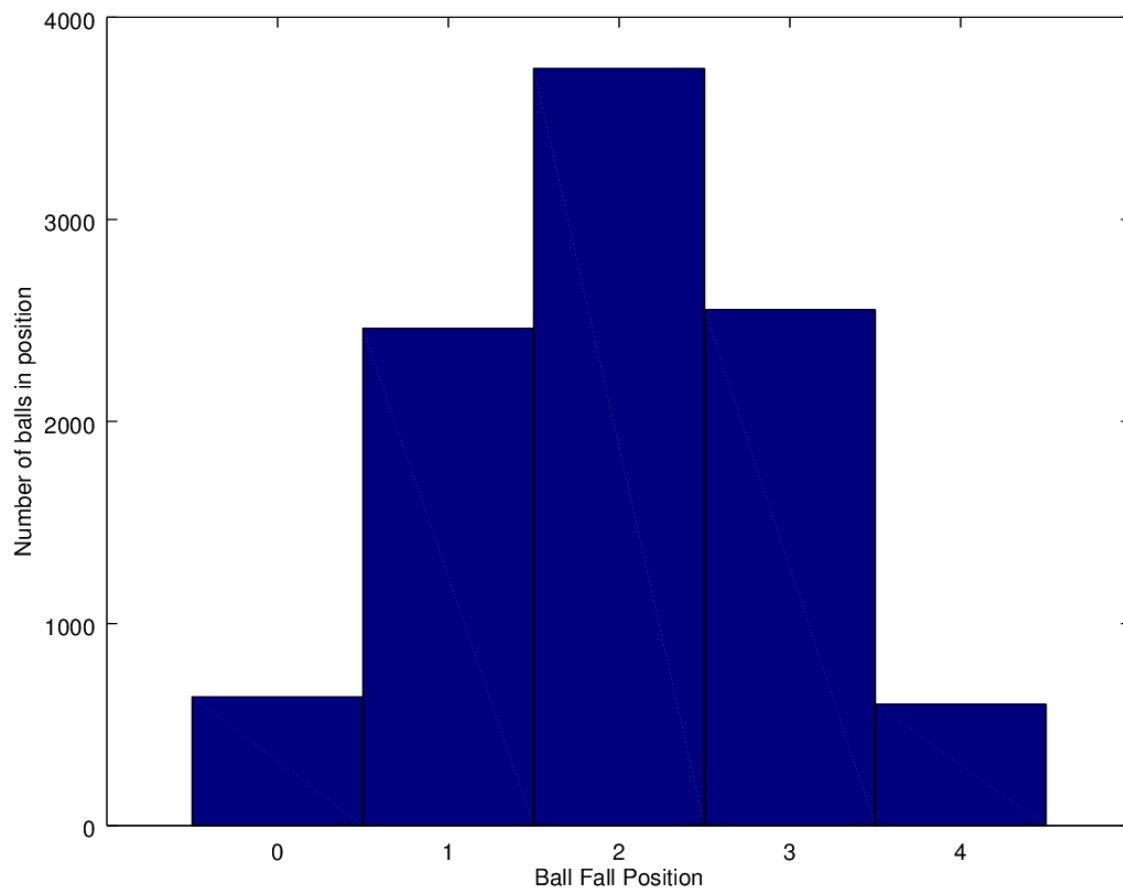
Figure 1: Result for Question 4

Question 5

```
1   function result = ass2q2
2     N = 1e2;
3     totals = zeros(N, N);
4     for i = 1:N
5       for j = 1:N
6         totals(i, j) = calcRS(i/N, j/N);
7       endfor
8     endfor
9     mesh(1:N, 1:N, totals);
10    title("Question 5");
11    xlabel("Value for r");
12    ylabel("Value for s");
13    zlabel("Resulting probability");
14  endfunction
15
16  function result = calcRS(r, s)
17    result = sumRS(0, 0, r, s) + sumRS(0, 1, r, s) + sumRS(1, 0, r, s) +
18      sumRS(1, 1, r, s);
19  endfunction
20
21  function result = sumRS(u, v, r, s)
22    result = abs(probUV(u, v, r, s) - (probU(u, r, s) * probV(v, r, s)));
23  endfunction
24
25  function result = probUV(u, v, r, s)
26    if (u == 1 && v == 1)
27      result = r * s;
28    elseif (u == 0 && v == 0)
29      result = (1 - r) * (1 - s);
30    elseif (u == 0 && v == 1)
31      result = r(1-s) + s(1-r);
32    elseif (u == 1 && v == 0)
33      result = 0;
34    else
35      # Should not reach this
36      result = -1;
37    endif;
38  endfunction
39
40  function result = probU(u, r, s)
41    result = probUV(u, 0, r, s) + probUV(u, 1, r, s);
42  endfunction
43
44  function result = probV(v, r, s)
45    result = probUV(0, v, r, s) + probUV(1, v, r, s);
46  endfunction
```
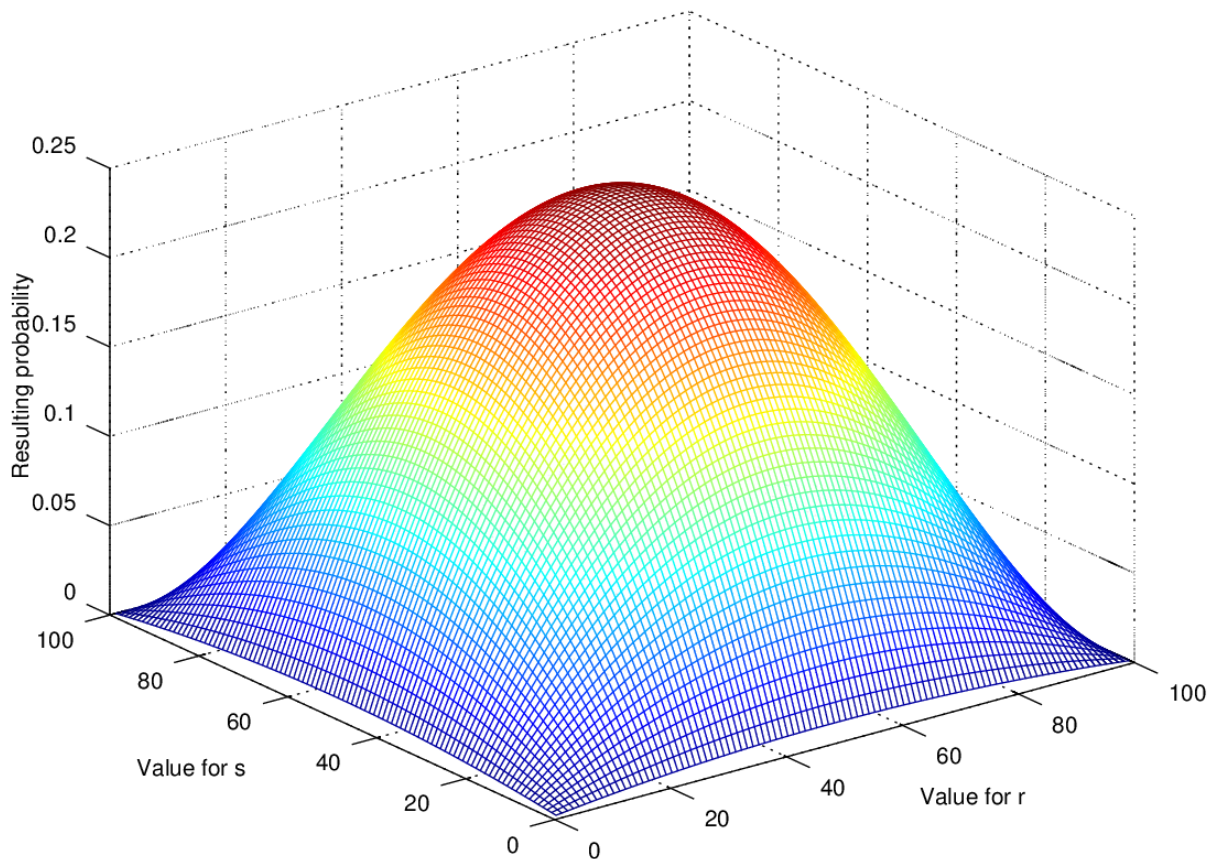
Figure 2: Result for Question 5

Question 6

```
 1  function result = ass2q6
 2    N=5e4;
 3    totals=1:N;
 4    for i = 1:N
 5      totals(i) = pmf(randi(5));
 6    endfor
 7    hist(totals, 0:0.1:0.4);
 8    xlabel("pmf result");
 9    ylabel("Number of results");
10  endfunction
11
12  function result = pmf(x)
13    result = (1/3) * (2/3)^(x - 1);
14  endfunction
```
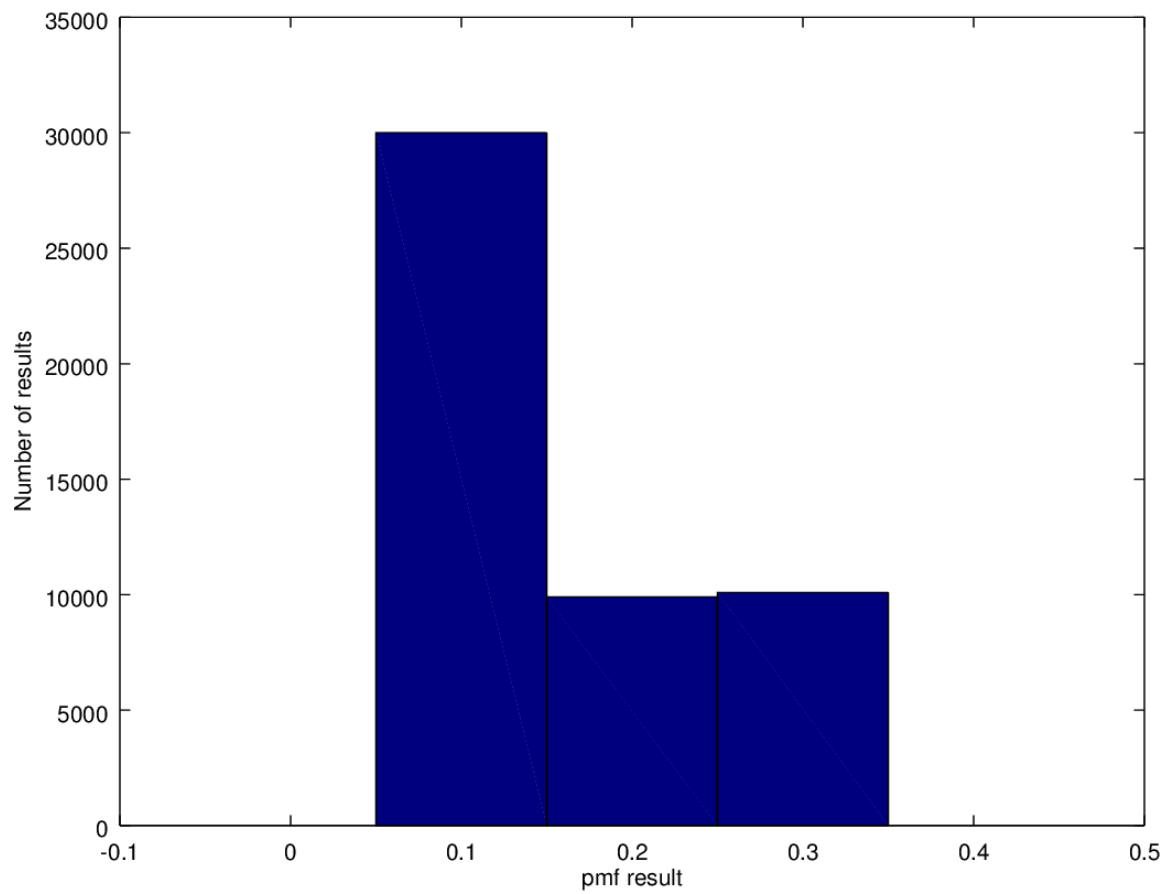
Figure 3: Result for Question 6