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STAT2203 – Probability Models and Data Analysis for Engineering

STAT2203 Assignment 2

Question 1

Let *X* be the event that the ball falls in box *x*. Where *X ~ Bin(4, 0.5)*

SubQuestion A

$$\begin{aligned}
\mathbb{P}(X = x) &= \binom{4}{x}\left(\frac{1}{2}\right)^x \times \left(1 - \frac{1}{2}\right)^{4-x} \\
&= \binom{4}{x}\left(\frac{1}{2}\right)^4 \\
&= \frac{1}{16}\binom{4}{x} \\
\mathbb{P}(X = 3) &= \frac{1}{16}\binom{4}{3} \\
&= \frac{4}{16} = \frac{1}{4}
\end{aligned}$$

SubQuestion B

$$\begin{aligned}
\mathbb{P}(X = x) &= \binom{4}{x}\left(\frac{1}{2}\right)^x \times \left(1 - \frac{1}{2}\right)^{4-x} \\
&= \binom{4}{x}\frac{1}{2^x} \times \frac{1}{2^{4-x}} \\
&= \binom{4}{x}\frac{2^x}{2^4 2^x} \\
&= \frac{1}{16}\binom{4}{x}
\end{aligned}$$

Question 2

Part 1

We are given:

$$\begin{aligned}
\mathbb{P}(X = 1) &= r \\
\mathbb{P}(X = 0) &= 1 - r \\
\mathbb{P}(Y = 1) &= s \\
\mathbb{P}(Y = 0) &= 1 - s
\end{aligned}$$

Therefore the following table can be generated:

Y\X	0	1
0	*(1-r)(1-s)*	*r(1-s)*
1	*s(1-r)*	*rs*

Table 1: Combinations of P(X=x)P(Y=y)

Using the above table, we can see that U and V will both take on values {*0, 1*}

U,V	X,Y	Solution
0,0	0,0	*(1-r)(1-s)*
0,1	1,0 + 0,1	*r(1-s) + s(1-r)*
1,0	*Not possible*	*0*
1,1	1,1	*rs*

Therefore the following joint pmf table can be calculated

U\V	0	1

0	*(1-r)(1-s)*	*r(1-s) + s(1-r)*
1	*0*	*rs*

Part 2

$$\begin{aligned}
\mathbb{P}(V=1,U=1) &= \mathbb{P}(V=1)\mathbb{P}(U=1) \\
rs &= (rs)(r+s-rs) \\
1 &= r+s-rs \\
1-s &= r-rs \\
&= r(1-s) \\
1 &= r \\
\mathbb{P}(V=0,U=0) &= \mathbb{P}(V=0)\mathbb{P}(U=0) \\
(1-r)(1-s) &= (1-rs)(1-s-r+rs) \\
1-r-s+rs &= 1-s-r+rs-rs+rs^2+sr^2+(rs)^2 \\
1+rs &= 1+rs^2sr^2-(rs)^2 \\
1+s &= 1+s^2+s-s^2 && \text{(When } r \text{ is 1)} \\
1+s &= 1+s \\
\mathbb{P}(V=1,U=0) &= \mathbb{P}(U=0)\mathbb{P}(V=1) \\
r+s-2rs &= (1-rs)(r+s-rs) \\
r+s-2rs &= r+s-rs-r^2s-rs^2+r^2s^2 \\
rs &= -r^2s-rs^2+r^2s^2 \\
s &= -s-s^2+s^2 && \text{(When } r \text{ is 1)} \\
s &= -s \\
s &= 0 \\
\mathbb{P}(U=1,V=0) &= \mathbb{P}(U=1)\mathbb{P}(V=0) \\
0 &= rs(1-s-r+rs) && \text{(When } r \text{ is 1, When } s \text{ is 0)} \\
0 &= 0
\end{aligned}$$

Question 3

$$
\begin{aligned}
L(\theta; p) &= \prod_{i=1}^{5} (1-p)^{x_i} p \\
l(\theta; p) &= \sum_{i=1}^{5} \log\left((1-p)^{x_i - 1}\right) \log(p) \\
&= n \log(p) + \sum_{i=1}^{5} (x_i - 1) \log(1-p) \\
&= n \log(p) + \log(1-p) \sum_{i=1}^{5} (x_i - 1) \\
\frac{d}{dp} & \quad \frac{n}{p} - \frac{\sum_{i=1}^{5} (x_i - 1)}{1-p} \\
0 &= \frac{n}{p} - \frac{\sum_{i=1}^{5} (x_i - 1)}{1-p} \\
\frac{n}{p} &= \frac{\sum_{i=1}^{5} (x_i - 1)}{1-p} \\
\frac{n}{p} - n &= -n \sum_{i=1}^{5} (x_i) \\
\frac{n}{p} &= \sum_{i=1}^{5} (x_i) \\
p &= \frac{n}{\sum_{i=1}^{5} (x_i)} = \frac{1}{\bar{x}}
\end{aligned}
$$