Problem: $\frac{dM}{dt} + b\frac{M}{E(t)} = a$

$$M' + b\frac{M}{E(t)} = a$$

We need μ such that

$$(\mu M)' = \mu' M + \mu M'$$

and

$$\mu M' + b\frac{M}{E(t)}\mu = a\mu$$

We can see

$$\mu M' + \frac{b}{E(t)} M\mu = \mu' M + \mu M'$$

$$\frac{b}{E(t)} M\mu = \mu' M$$

$$\frac{b}{E(t)}\mu = \mu'$$

Separable differential equation:

$$\int \frac{b}{E(t)}\, dt = \int \frac{1}{\mu}\, dt$$

$$\int \frac{b}{E(t)}\, dt = \ln(\mu) + C_0$$

But $C_0 = 0$

$$\ln(\mu) = \int \frac{b}{E(t)} \, dt$$

$$\mu(t) = e^{\int \frac{b}{E(t)} dt}$$

We know $b = 100$

and

$$\frac{dE}{dt} = r_{in} - r_{out}$$

$$\frac{dE}{dt} = 50 - 100$$

$$\frac{dE}{dt} = -50$$

Direct differential equation method

$$E(t) = -50t + C$$

However

$$E(0) = 6000 = C$$

then substitute $E(t) = -50t + 6000$ and $b=100$

$$\mu(t) = e^{\int \frac{100}{-50t+6000} dt} \quad , \; U = -50t + 6000$$

$$\mu(t) = e^{\ln|(-50t+6000)^{-2}|}$$

$$\mu(t) = (-50t + 6000)^{-2}$$