$$f(t) = \begin{cases} \frac{1}{3}, & 0 \leq t < 3 \\ 0, & 3 \leq t \end{cases}$$

$$\int_0^{\infty} e^{-st} f(t)\, dt =$$

$$\int_3^{\infty} e^{-st} (0)\, dt + \int_0^{3} e^{-st} \left(\frac{1}{3}\right) dt =$$

$$\lim_{b \to \infty} \int_3^{b} 0\, dt + \frac{1}{3} \int_0^{3} e^{-st}\, dt =$$

$$\lim_{b \to \infty} [C]\Big|_3^{b} + \frac{1}{3} \left[-\frac{e^{-st}}{s} \right]\Bigg|_0^{3} =$$

$$0 + \frac{1}{s} - \frac{e^{-3s}}{s} =$$

$$F(s) = \frac{1}{s} - \frac{e^{-3s}}{s} = \frac{1 - e^{-3s}}{s}$$

$$\lim_{s \to 0} (1 - e^{-3s}) = 0, \quad \lim_{s \to 0^+} \frac{1}{s} = 0 \Rightarrow \lim_{s \to 0} \frac{3e^{-3s}}{1} = 3$$

STUDENTZ

Sin puntos ordinarios, entonces

$$2\sum_{n=0}^{\infty}(n+1)(n+2)a_{n+2}x^n +$$

$$x\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n +$$

$$\sum_{n=0}^{\infty}a_n x^n = 0$$

$$\sum_{n=0}^{\infty}(n+1)(n+2)2a_{n+2}x^n + \sum_{n=0}^{\infty}(n+1)a_{n+1}x^{n+1} + \sum_{n=0}^{\infty}a_n x^n = 0$$

$$\sum_{n=0}^{\infty}(n+1)(n+2)2a_{n+2}x^n + \sum_{n=1}^{\infty}(n)a_n x^n + \sum_{n=0}^{\infty}a_n x^n = 0$$

$$4a_2 + a_0 + \sum_{n=1}^{\infty}\left[(n+1)(n+2)2a_{n+2} + a_n n + a_n\right]x^n = 0$$

$$4a_2 + a_0 = 0, \quad \boxed{a_2 = -\frac{a_0}{4} = -1}$$

$$2(n+1)(n+2)a_{n+2} + a_n n + a_n = 0, \quad n \geq 1, n \neq 2$$

$$a_{n+2} = \frac{-a_n - a_n n}{2(n+1)(n+2)}$$

$$\boxed{a_3 = \frac{-a_1 - a_1(1)}{12} = -1}$$

$r, K \in \mathbb{R}$

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right), \quad P(0) = 1$$

$$\frac{dP}{dt} = rP - \frac{rP^2}{K}$$

Se dice que EDO es separable si:

$$(1) \quad \frac{dP}{dt} = \frac{f(t)}{g(P)} = \frac{1}{\frac{KrP - rP^2}{K}}$$

Multiplicamos $g(P)$ e integramos:

$$\int g(P)\frac{dP}{dt}\,dt = \int f(t)\,dt$$

Si $G(P)$ es la antiderivada de $g(P)$:

$$\frac{dG}{dP} = g(P)$$

$$(2) \quad \Rightarrow \frac{dG}{dt} = \frac{dG}{dP}\frac{dP}{dt} = g(P)\frac{dP}{dt}$$

$$\Rightarrow \int \frac{dG}{dt}\,dt = \int f(t)\,dt$$

Por TFC: $G(P) = \int f(t)\,dt$

Por def. de $G(P)$:

$$\int g(P)\, dP = \int f(x)\, dx$$

$$\int \frac{1}{\left(1-\frac{P}{K}\right)P}\, dx = \int r\, dx$$

$$-K \int \frac{1}{P(P-K)}\, dP = rx + C_1$$

$$U = 1 - \frac{1}{P} \Rightarrow \frac{1}{P^2}\, dP = \frac{1}{K}\, dU$$

$$-K \int \frac{1}{\left(1-\frac{K}{P}\right)P^2}\, dP = rx + C_1$$

$$-K\left(\frac{1}{K}\right)\int \frac{1}{U}\, du = rx + C_1$$

$$-\ln\left(1-\frac{K}{P}\right) = rx + C_2$$

$$\ln\left(1-\frac{K}{P}\right) = -rx + C_3$$

$$\ln\left|\frac{K}{P} - 1\right| = -rx + C_3$$

$$\frac{K}{P} - 1 = e^{-rx + C_3}$$

$$P(t) = \frac{K}{Ge^{-rx} + 1}$$