

Assignment-6

Parameter estimation

Q-1) $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$X_1, X_2, X_3 \dots X_n$ 2) sample of size n

$$L(X_1, X_2, X_3 \dots X_n) = f(x_1) \cdot f(x_2) \cdot \dots \cdot f(x_n)$$

$$\Rightarrow \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}} \right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}} \right) \dots$$

taking \ln on both sides

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^n \left(-\frac{(x_i-\mu)^2}{2\sigma^2} \right) \quad \text{--- (1)}$$

taking partial derivative w.r.t μ of the above eqn

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^n - \left(\frac{2(x_i-\mu)}{2\sigma^2} \right) = 0$$

$$\Rightarrow \sum_{i=1}^n (x_i - \mu) = 0$$

$$n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu$$

hence $\mu_1 = \bar{x}$ is therefore sample mean

taking derivative w.r.t σ^2 (eqn 1)

$$\frac{\partial \ln(L)}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \sum_{i=1}^n - \frac{(x_i-\mu)^2}{2\sigma^2} = 0$$

$$\Rightarrow -n + \sum_{i=1}^n - \frac{(x_i-\mu)^2}{\sigma^2} = 0$$

$$n = \sum_{i=1}^n \frac{(x_i-\mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n} \left(\sum_{i=1}^n (x_i - \mu)^2 \right)$$

$$\text{hence } \sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$$

Q-2) Binomial distribution $\rightarrow {}^n C_{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^n {}^n C_{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$$

log on both sides

$$\log L = \sum_{i=1}^n \left(\log({}^n C_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i} \right)$$

$$\log L = \sum_{i=1}^n \log({}^n C_{x_i}) + \log \theta \sum_{i=1}^n x_i +$$

$$\log (1-\theta) \sum_{i=1}^n (n-x_i)$$

diff. wrt θ

$$\frac{d \log L}{d \theta} = 0$$

$$\frac{1}{\theta} \sum_{i=1}^n x_i - \frac{1}{1-\theta} \sum_{i=1}^n (n-x_i) = 0$$

$$\frac{1}{\theta} \sum_{i=1}^n x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \sum_{i=1}^n x_i = 0$$

$$\frac{1}{\theta(1-\theta)} \sum_{i=1}^n x_i = \frac{n^2}{1-\theta} \Rightarrow \boxed{\theta^2 \sum_{i=1}^n x_i = n^2}$$