Project 5
Using Linear Programming Models to maximize profits

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Introduction

This report provides an analysis of inventory management practices for a Company focusing on the optimal order quantity for each product to maximize its profits. This section introduces the context and the need for linear programming in optimizing the product mix of a hardware company. It outlines the company's decision to analyze product lines and its constraints - budget, storage space, and marketing requirements. We aim to maximize profitability while considering budget, inventory space, and specific inventory requirements.

Table 1: Inventory Management and Storage Analysis

Items	Count	Cost Price ($)	Selling Price ($)	Profit per unit	Storage Space		Total Space Required
					Long in ft	Wide in ft	
Pressure washer - x1	1	330	499.99	169.99	5	5	25
Go-kart- x2	1	370	729.99	359.99	8	5	40
Generator - x3	1	410	700.99	290.99	5	5	25
Case of 1 Water Pump- x4	1	127	269.99	142.99	1.25	1.25	1.5625
Shelf Space	82				30	5	150
Monthly budget		170,000					

Table 1 provides an overview of the inventory items, their associated costs and profits, as well as the storage space required for effective management.

Analysis

1. Mathematical Formulation:

Mathematical functions present an analysis of a linear programming problem aimed at maximizing profit while adhering to budget constraints, inventory space limitations, and specific requirements regarding space utilization and item ratios. The linear programming model for maximizing profit is formulated as follows:

Objective Function:

Maximize Profit: 169.99 x1 + 359.99 x2 + 290.99 x3 + 142.99 x4 = Z

Subject to Constraints:

Budget: 330 x1 + 370 x2 + 410 x3 + 127 x4 <= 170000
Inventory Space: 25 x1 + 40 x2 + 25 x3 + 1.25 x4 <= 12300
Requirement 1: x1 + x2 <= .30(x1 + x2 + x3 + x4)
Requirement 2: 2 * x4 <= x3

2. Excel Solver Setup:

The linear programming problem was set up in an Excel workbook with the Solver tool used to find the optimal solution. The Solver was configured with the objective function, decision variables, and constraints as per the mathematical formulation. The optimal solution for each of the product is mentioned below in Table 2.

Table 2: Units to be sold for each product for maximum profit

Products	Pressure Washer	Go kart	Generator	Water Pump	Z (Total Profit)
Optimal Units	0.00	155.18	237.77	118.88	142050.70

The optimal monthly profit calculated by the model is $142,050.70, which is the sum of profits generated by each product as per their respective optimal selling quantities. The solution for Pressure Washers is 0 units, which indicates that under the current cost structure and constraints, including the profit margin of $169.99 per unit, it is not profitable to sell Pressure Washers. The model suggests allocating resources to other products to maximize overall profitability.

3. Sensitivity Report:

A sensitivity report was generated using the Solver, providing critical insights into how changes in the selling prices and resource availability could impact the optimal solution.

Variable Cell

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
Pressure Washer	0	-110.0715237	169.99	110.0715237	1E+30
Go kart	155.179067	0	359.99	205.8402439	76.73878564
Generator	237.7692613	0	290.99	98.20490541	131.8664063
Water Pump	118.8846306	0	142.99	196.4098108	89.11965734

Constraints

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Inventory Space	12300	3.841502841	12300	6078.378378	3099.894403
Requirement 1	1.629179331	0	0	0	1E+30
Requirement 2	0	-33.68339104	0	314.5454545	974.1201949
Budget	170000	0.557648341	170000	57280	56225

4. Optimal Solutions and Analysis:

The optimal solution obtained indicates that the company should not produce Pressure Washers, as indicated by a decision variable value of zero. The recommended quantities for following optimal inventory levels to maximize profits, given the current selling prices and other constraints:

- Pressure Washers (x1): 0 units
- Go-karts (x2): 155.18 units
- Generators (x3): 237.77 units
- Water Pumps (x4): 118.88 units

The optimal monthly profit, considering the suggested inventory levels, is calculated to be $142,050.70.

5. Selling Price Adjustment for Pressure Washer:

The sensitivity report revealed that Pressure Washers has an optimal value of zero. This indicates that its selling price would need to be adjusted to make its production profitable.

The sensitivity report indicates a reduced cost of -$110.07 for Pressure Washers, which are not produced in the optimal solution. To make producing Pressure Washers viable, the selling price must be increased by at least $110.07. Therefore, the new minimum selling price should be $499.99 (current price) + $110.07 = $610.06.

6. Budget Allocation Analysis:

Based on the shadow price for the budget constraint, the report assesses the impact of increasing the budget. It provides a recommendation on whether additional funds should be allocated and the expected increase in net monthly profit.

The shadow price of the budget constraint is $0.557648341. This indicates that for every additional dollar allocated to the budget, the profit could increase by approximately $0.56. Since the allowable increase in the budget is $57,280, the company could expect an increase in net monthly profit by $57,280 * $0.557648341 ≈ $31,944.48.

Therefore, if the company has the means to do so, allocating an additional $57,280 could be beneficial.

7. Warehouse Size Recommendation:

The sensitivity report evaluates the inventory space which denotes whether the company should rent a smaller or larger warehouse. It recommends an ideal warehouse size based on the solution's space requirements and discusses how changing the warehouse size would affect monthly profitability.

The shadow price for Inventory Space is $3.841502841, with an allowable increase of 6078 square feet. This suggests that for each additional square foot of space added to the warehouse, the profit could increase by approximately $3.84. If the company were to expand the warehouse by 6078 square feet, the potential profit increase could be 6078 * $3.841502841 ≈ $23,344.49. However, this expansion should only be considered if the company projects an increase in demand or plans to diversify the product range that would require additional space.

Given the current data, there's no indication that renting a smaller warehouse would benefit the company, as the available space is fully utilized (no slack). Renting a larger warehouse may only be recommended based on future business expansion plans and the need for additional inventory space.

Conclusion

The linear programming analysis conducted for the hardware company's product mix strategy has provided a clear directive towards optimizing the monthly profit. The model has successfully identified an optimal mix of Go-karts, Generators, and Water Pumps, while justifiably excluding Pressure Washers from immediate production due to its current unprofitability.

The generated sensitivity report was pivotal in determining the economic viability of each product, highlighting the potential for increased profitability with adjusted selling prices, particularly for the Pressure Washer. It also provided valuable insights into how additional budget allocation could lead to a significant profit increase, justifying a strategic investment decision.

Moreover, the analysis suggested that the current warehouse space is effectively utilized. Any considerations for expansion should be driven by concrete evidence of increased demand or product diversification to ensure that the additional space translates into a corresponding increase in profitability.

References

- *Linear Equation Word Problems - Examples & Practice - Expii*. (n.d.). Expii. https://www.expii.com/t/linear-equation-word-problems-examples-practice-4365