

Useful Formulae and Other Information

$$\mathcal{L}x(s) = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt \quad \mathcal{L}^{-1}X(t) = x(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st} ds \quad \mathcal{L}_u x(s) = X(s) = \int_{0^-}^{\infty} x(t)e^{-st} dt$$

$$A_k = (v - p_k)F(v)|_{v=p_k} \quad A_{kl} = \frac{1}{(q_k - l)!} \left[\left[\frac{d}{dv} \right]^{q_k-l} [(v - p_k)^{q_k} F(v)] \right] \Big|_{v=p_k}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Bilateral Laplace Transform Properties

Property	Time Domain	Laplace Domain	ROC
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$	At least $R_1 \cap R_2$
Time-Domain Shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
Laplace-Domain Shifting	$e^{s_0t}x(t)$	$X(s - s_0)$	$R + \text{Re}\{s_0\}$
Time/Laplace-Domain Scaling	$x(at)$	$\frac{1}{ a }X\left(\frac{s}{a}\right)$	aR
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Time-Domain Convolution	$x_1 * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Laplace-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Time-Domain Integration	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{\text{Re}\{s\} > 0\}$

Property

Initial Value Theorem	$x(0^+) = \lim_{s \rightarrow \infty} sX(s)$
Final Value Theorem	$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$

Bilateral Laplace Transform Pairs

Pair	$x(t)$	$X(s)$	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\text{Re}\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\text{Re}\{s\} < 0$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$	$\text{Re}\{s\} > 0$
5	$-t^n u(-t)$	$\frac{n!}{s^{n+1}}$	$\text{Re}\{s\} < 0$
6	$e^{-at}u(t)$	$\frac{1}{s+a}$	$\text{Re}\{s\} > -a$
7	$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\text{Re}\{s\} < -a$
8	$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$	$\text{Re}\{s\} > -a$
9	$-t^n e^{-at}u(-t)$	$\frac{n!}{(s+a)^{n+1}}$	$\text{Re}\{s\} < -a$
10	$\cos(\omega_0 t)u(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\text{Re}\{s\} > 0$
11	$\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\text{Re}\{s\} > 0$
12	$e^{-at} \cos(\omega_0 t)u(t)$	$\frac{s+a}{(s+a)^2 + \omega_0^2}$	$\text{Re}\{s\} > -a$
13	$e^{-at} \sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(s+a)^2 + \omega_0^2}$	$\text{Re}\{s\} > -a$

Unilateral Laplace Transform Properties

Property	Time Domain	Laplace Domain
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$
Laplace-Domain Shifting	$e^{s_0t}x(t)$	$X(s - s_0)$
Time/Laplace-Domain Scaling	$x(at), a > 0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
Conjugation	$x^*(t)$	$X^*(s^*)$
Time-Domain Convolution	$x_1 * x_2(t)$, x_1 and x_2 are causal	$X_1(s)X_2(s)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$sX(s) - x(0^-)$
Laplace-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}X(s)$
Time-Domain Integration	$\int_{0^-}^t x(\tau)d\tau$	$\frac{1}{s}X(s)$

Property		
Initial Value Theorem	$x(0^+) = \lim_{s \rightarrow \infty} sX(s)$	
Final Value Theorem	$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$	

Unilateral Laplace Transform Pairs

Pair	$x(t), t \geq 0$	$X(s)$
1	$\delta(t)$	1
2	1	$\frac{1}{s}$
3	t^n	$\frac{n!}{s^{n+1}}$
4	e^{-at}	$\frac{1}{s+a}$
5	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
6	$\cos(\omega_0 t)$	$\frac{s}{s^2 + \omega_0^2}$
7	$\sin(\omega_0 t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$
8	$e^{-at} \cos(\omega_0 t)$	$\frac{s+a}{(s+a)^2 + \omega_0^2}$
9	$e^{-at} \sin(\omega_0 t)$	$\frac{\omega_0}{(s+a)^2 + \omega_0^2}$