SDS358: Applied Regression Analysis

Day 5: Simple Linear Regression

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Agenda for Today:

- Simple Regression
  - Intro & Similarity/Difference to r
  - The Linear equation and interpretation
  - Diagnostics

Today's Question:

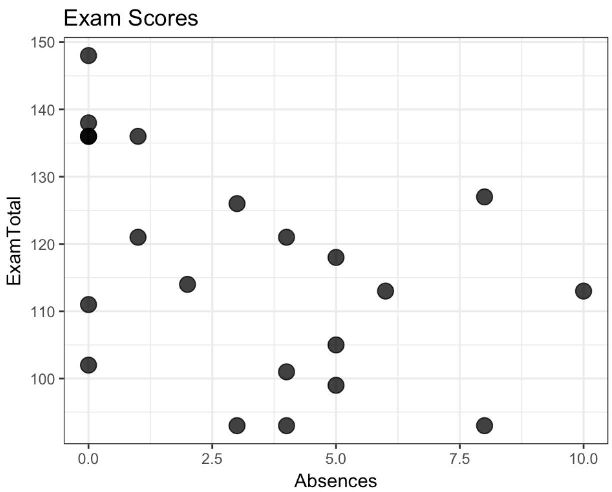
What is the Simple Linear Model predicting cost from carat weight in a sample of diamonds? *Are there any outliers in the data?*

Recap: Correlation

- We use correlation (Pearson Correlation) to:
- Determine the relationship between two *quantitiative* variables.
- Give that relationship definition.
  - Size of r
- However, only *linear* relationships are correctly captured by r
  - That's why we use the scatterplot

Since we know r…

What happens when we see this?



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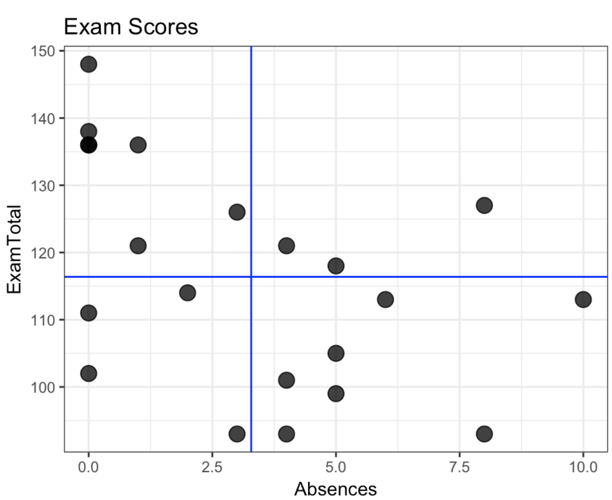
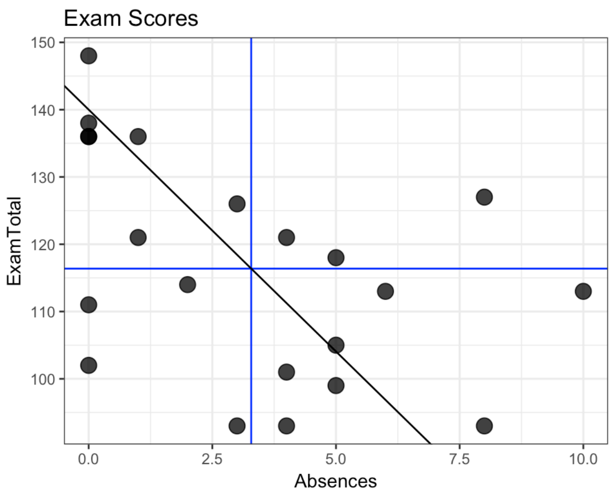


Since we know r…

What happens when we see this?



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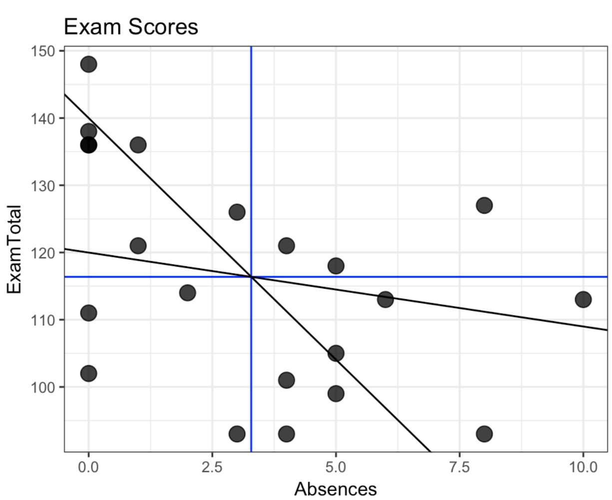
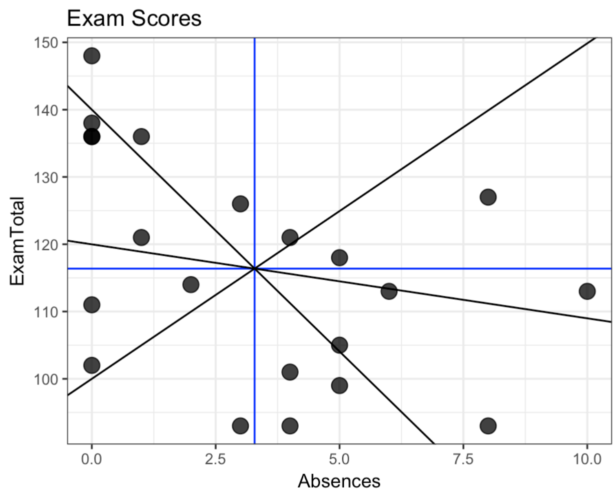


Since we know r…

What happens when we see this?

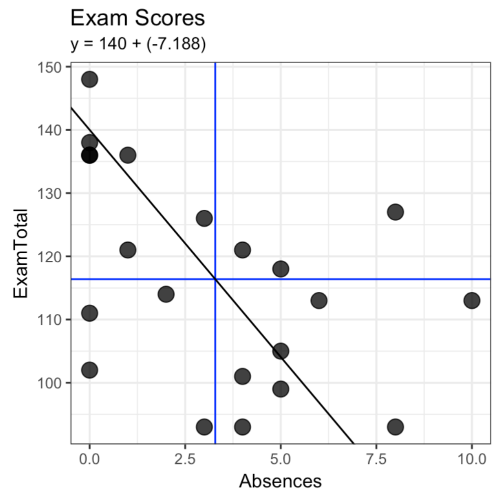


The Line of Best Fit

- "Find me the line that goes through the mean of x and the mean of y, that best fits the data."
- What's your definition of "fit?"
    - Residuals

A few things to notice:

- Each point does not fall directly on the line (any line).
- This "miss" is called the residual.
- Here's a *random line*



The Line

- We'll formally define a line as:

$$\hat{y} = b_0 + b_1 x_1$$

The Residual(s)

- We have two choices:
- Calculate **vertical** offset of each point to the line.
- Vertical offset provides a fit that *estimates y for a given x*

$$residual = e_i = (y - \hat{y})$$

- Calculate the **perpencicular** offset of each point to the line.

$$d_i = \frac{|y_i - (a + bx_i)|}{\sqrt{1 - b^2}}$$

The Line of Best fit

- The best fitting line is the one that is closest to the data points.
- As statisticians think:

Because the line can be used to predict a value of Y based on any X (with a corresponding Y), the best fitting line is a line that has the lowest amount of error from the predicted Y and the actual Y.

- In other words, the best line is the one that minimizes the residuals for all points (the *squared* residuals)

Lest see this in action!

R Script for residuals

This will make it a whole lot easier….

- Instead of doing this for every set of data (and we could)…
- We can simply turn to some basic algebra:

$$b_1 = r\left(\frac{S_y}{S_x}\right)$$

$$b_0 = \bar{y} - b_1\bar{x}$$

An Important Difference

- With r, we just get a relationship
  - It's symmetrical

$$COR_{(x,y)} = COR_{(y,x)}$$

- But symmetry fails with the linear equation of simple regression (and rightly so)
  - There are *two* possible lines:
    - x predicting y
    - y predicting x

Two Lines (and separate parameters)

- We can have R help us to find the "line of best fit" with the `lm()` function:
- So, we can *technically* have *two* lines:

```
lm(ExamTotal ~ Absences, data=school)
```

```
##
## Call:
## lm(formula = ExamTotal ~ Absences, data = school)
##
## Coefficients:
## (Intercept)     Absences
##     124.409       -2.443
```

Two Lines (and separate parameters)

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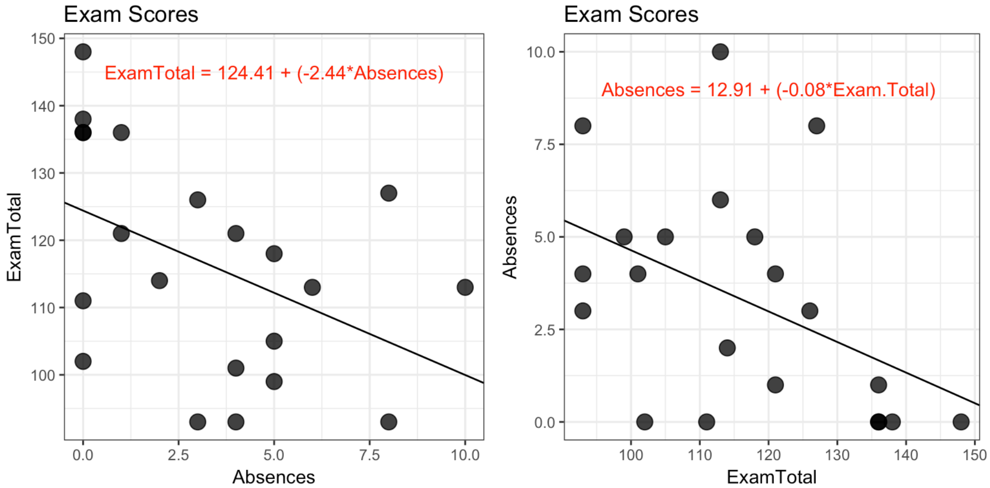
- So, we can *technically* have *two* lines:

```
lm(Absences ~ ExamTotal, data=school)
```

```
##
## Call:
## lm(formula = Absences ~ ExamTotal, data = school)
##
## Coefficients:
## (Intercept)    ExamTotal
##     12.9102      -0.0827
```

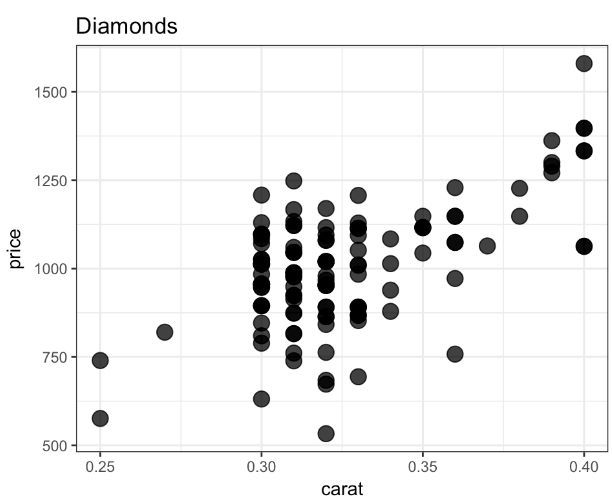

Two Lines (and separate parameters)



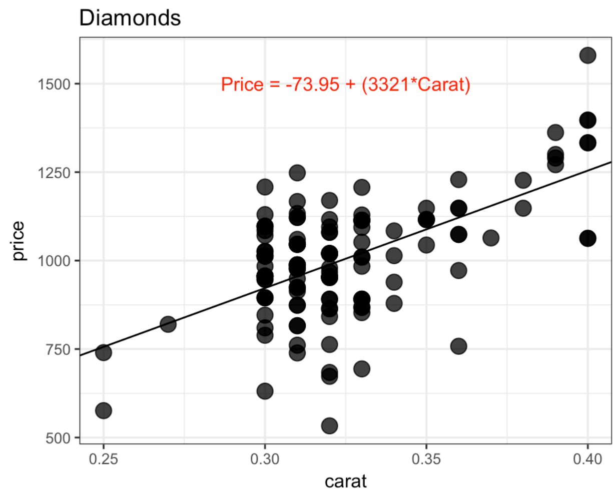
Interpretation

- The intercept and the slope have *very* specific interpretations.
  - Intercept: Value of y at an x of **zero**.
  - Slope: How much the predicted value of y changes given a **single** unit change in x.
- Let's try some new "enlightening" data

Diamonds



Diamonds



Interpretation:
- Slope
- Intercept

Assumptions of Simple Linear Regression

- Quantitative Data for both variables.
- The relationship is linear in nature.
- Residuals
  - Independence: Error associated with each data point is independent of every other value.
  - For a given value of x, e has a normal distribution, meaning:
    - The population mean of e is 0.
    - For a given value of x, the population variance of e is σ_e^2
  - Homoscedasticity
  - No Outliers

Summary:

- How Pearson r and Simple Linear Regression are the same and different.
- Definition of residuals
- The true meaning of the line of best fit.
- Interpretation for the Intercept and the Slope