MCP361 Assignment 3

Sanchit | 2021ME21063

1 Wagner-Whitin Procedure

The Wagner-Whitin algorithm is used for solving dynamic lot-sizing problems where the goal is to minimize the total cost of production, including setup and inventory holding costs. This algorithm is particularly useful in cases where demand varies over time, and constant setup and holding costs are present.

1.1 Procedure Overview

The Wagner-Whitin procedure operates as follows:

1. **Initialization:** Define the number of periods N, and initialize the cost and decision arrays. Let Z_t be the minimum total cost of satisfying demand up to period t, and j_t be the period from which the production should have started to reach this minimum cost.

2. **Cost Calculation:** For each period t, calculate the minimum cost by evaluating all possible production starting points k from earlier periods up to t. The cost includes the setup cost for producing in period k and the holding cost for inventory carried over from k to t.

3. **Update Costs:** Update the cost and decision arrays based on the calculated minimum costs. The cost for period t is the minimum of the previously computed costs plus the cost of setting up and holding inventory.

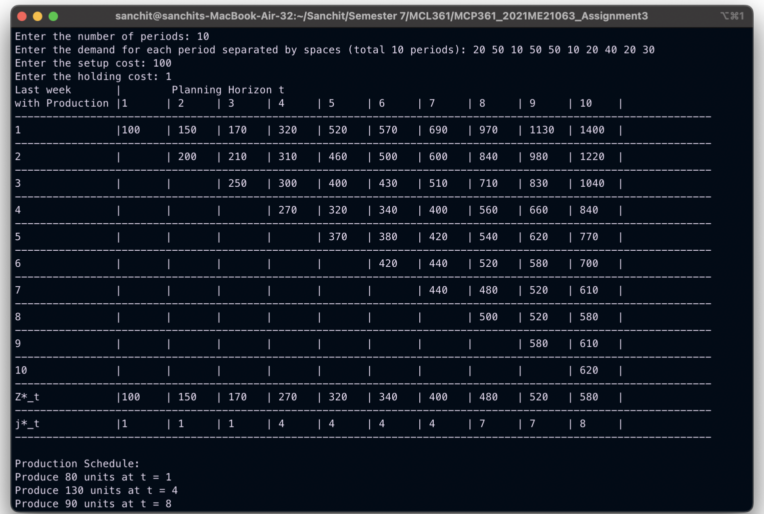
4. **Backtracking:** To determine the optimal production schedule, backtrack using the decision array j_t to find out the periods where production should be scheduled to minimize costs.

The Wagner-Whitin procedure ensures that the total cost of meeting demand across all periods is minimized by optimally scheduling production runs.

2 Question 1: Lot-Sizing Problem

2.1 Problem Details

The problem involves forecasting demand over a specified number of periods and determining the optimal production schedule to minimize the total cost. We used the Wagner-Whitin algorithm to calculate setup and holding costs, generate the cost table, and derive the production schedule.



```
sanchit@sanchits-MacBook-Air-32:~/Sanchit/Semester 7/MCL361/MCP361_2021ME21063_Assignment3
Enter the number of periods: 10
Enter the demand for each period separated by spaces (total 10 periods): 20 50 10 50 50 10 20 40 20 30
Enter the setup cost: 100
Enter the holding cost: 1
Last week        |       Planning Horizon t
with Production |1      | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
------------------------------------------------------------------------------------------------
1               |100    | 150   | 170   | 320   | 520   | 570   | 690   | 970   | 1130  | 1400  |
------------------------------------------------------------------------------------------------
2               |       | 200   | 210   | 310   | 460   | 500   | 600   | 840   | 980   | 1220  |
------------------------------------------------------------------------------------------------
3               |       |       | 250   | 300   | 400   | 430   | 510   | 710   | 830   | 1040  |
------------------------------------------------------------------------------------------------
4               |       |       |       | 270   | 320   | 340   | 400   | 560   | 660   | 840   |
------------------------------------------------------------------------------------------------
5               |       |       |       |       | 370   | 380   | 420   | 540   | 620   | 770   |
------------------------------------------------------------------------------------------------
6               |       |       |       |       |       | 420   | 440   | 520   | 580   | 700   |
------------------------------------------------------------------------------------------------
7               |       |       |       |       |       |       | 440   | 480   | 520   | 610   |
------------------------------------------------------------------------------------------------
8               |       |       |       |       |       |       |       | 500   | 520   | 580   |
------------------------------------------------------------------------------------------------
9               |       |       |       |       |       |       |       |       | 580   | 610   |
------------------------------------------------------------------------------------------------
10              |       |       |       |       |       |       |       |       |       | 620   |
------------------------------------------------------------------------------------------------
Z*_t            |100    | 150   | 170   | 270   | 320   | 340   | 400   | 480   | 520   | 580   |
------------------------------------------------------------------------------------------------
j*_t            |1      | 1     | 1     | 4     | 4     | 4     | 4     | 7     | 7     | 8     |
------------------------------------------------------------------------------------------------

Production Schedule:
Produce 80 units at t = 1
Produce 130 units at t = 4
Produce 90 units at t = 8
```

Figure 1: Results of Wagner-Whitin Procedure for Question 1

3 Question 2: Cost Calculation and Application

In this question, we solved a lot-sizing problem with specific setup and holding cost parameters provided. We calculated the setup and holding costs based on the given parameters and used these costs to solve the Wagner-Whitin problem.

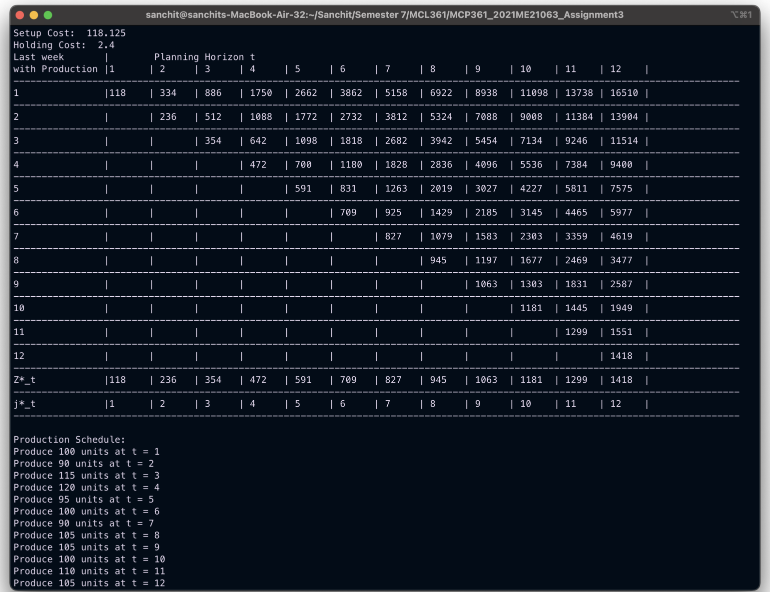
3.1 Cost Calculation

The setup cost calculated for the problem is \$118.125, and the holding cost is \$2.4. These costs reflect the total expenses associated with setting up production runs and holding inventory over the specified periods.

3.2 Cost Table Analysis

The cost table provides detailed insights into the minimum total cost for each period, considering different scenarios of production planning.

> The Wagner–Whitin algorithm tells us that the minimum total setup plus inventory carrying cost in this example is given by $Z^*_{12} = \$1,418$. We note that this is indeed lower than the cost achieved by either the lot-for-lot or fixed order quantity solutions we considered earlier. The optimal lot sizes are determined from the j^*_t values.



```
sanchit@sanchits-MacBook-Air-32:~/Sanchit/Semester 7/MCL361/MCP361_2021ME21063_Assignment3
Setup Cost:  118.125
Holding Cost:  2.4
Last week        |        Planning Horizon t
with Production |1      | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    |
----------------------------------------------------------------------------------------------------------------
1               |118    | 334   | 886   | 1750  | 2662  | 3862  | 5158  | 6922  | 8938  | 11098 | 13738 | 16510 |
----------------------------------------------------------------------------------------------------------------
2               |       | 236   | 512   | 1088  | 1772  | 2732  | 3812  | 5324  | 7088  | 9008  | 11384 | 13904 |
----------------------------------------------------------------------------------------------------------------
3               |       |       | 354   | 642   | 1098  | 1818  | 2682  | 3942  | 5454  | 7134  | 9246  | 11514 |
----------------------------------------------------------------------------------------------------------------
4               |       |       |       | 472   | 700   | 1180  | 1828  | 2836  | 4096  | 5536  | 7384  | 9400  |
----------------------------------------------------------------------------------------------------------------
5               |       |       |       |       | 591   | 831   | 1263  | 2019  | 3027  | 4227  | 5811  | 7575  |
----------------------------------------------------------------------------------------------------------------
6               |       |       |       |       |       | 709   | 925   | 1429  | 2185  | 3145  | 4465  | 5977  |
----------------------------------------------------------------------------------------------------------------
7               |       |       |       |       |       |       | 827   | 1079  | 1583  | 2303  | 3359  | 4619  |
----------------------------------------------------------------------------------------------------------------
8               |       |       |       |       |       |       |       | 945   | 1197  | 1677  | 2469  | 3477  |
----------------------------------------------------------------------------------------------------------------
9               |       |       |       |       |       |       |       |       | 1063  | 1303  | 1831  | 2587  |
----------------------------------------------------------------------------------------------------------------
10              |       |       |       |       |       |       |       |       |       | 1181  | 1445  | 1949  |
----------------------------------------------------------------------------------------------------------------
11              |       |       |       |       |       |       |       |       |       |       | 1299  | 1551  |
----------------------------------------------------------------------------------------------------------------
12              |       |       |       |       |       |       |       |       |       |       |       | 1418  |
----------------------------------------------------------------------------------------------------------------
Z*_t            |118    | 236   | 354   | 472   | 591   | 709   | 827   | 945   | 1063  | 1181  | 1299  | 1418  |
----------------------------------------------------------------------------------------------------------------
j*_t            |1      | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    |
----------------------------------------------------------------------------------------------------------------

Production Schedule:
Produce 100 units at t = 1
Produce 90 units at t = 2
Produce 115 units at t = 3
Produce 120 units at t = 4
Produce 95 units at t = 5
Produce 100 units at t = 6
Produce 90 units at t = 7
Produce 105 units at t = 8
Produce 105 units at t = 9
Produce 100 units at t = 10
Produce 110 units at t = 11
Produce 105 units at t = 12
```

Figure 2: Results of Wagner-Whitin Procedure for Question 2

3.3 Optimal Production Schedule

The optimal production schedule derived from the Wagner-Whitin procedure is as follows:

- Produce 100 units at $t = 1$

- Produce 90 units at $t = 2$
- Produce 115 units at $t = 3$
- Produce 120 units at $t = 4$
- Produce 95 units at $t = 5$
- Produce 100 units at $t = 6$
- Produce 90 units at $t = 7$
- Produce 105 units at $t = 8$
- Produce 105 units at $t = 9$
- Produce 100 units at $t = 10$
- Produce 110 units at $t = 11$
- Produce 105 units at $t = 12$

This production schedule is derived to minimize the total cost of setup and holding over the 12-month period. It is evident from the results that the optimal production quantities vary over time, reflecting the changing demand and cost considerations.

3.4 Conclusion

The Wagner-Whitin algorithm has provided us with a cost-effective production schedule for the given parameters. The calculated setup cost of \$118.125 and holding cost of \$2.4 are integral to determining the total cost over the 12-month period.

The optimal production schedule reveals that we should produce varying quantities at different periods to minimize overall costs. For instance, we produce 100 units in the first period, 90 units in the second period, and so forth, with production quantities tailored to meet the demand while balancing setup and holding costs.

The final total cost, represented by $Z_{12}^* = \$1,418$, demonstrates that our approach using the Wagner-Whitin algorithm achieves a more efficient cost management strategy compared to simpler methods like lot-for-lot or fixed order quantity solutions. By strategically planning production to cover future demand efficiently, we achieve a balance between minimizing setup and holding costs, resulting in an optimized inventory management strategy over the entire planning horizon.