# MCP361: Assignment 1

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## Question 1: Assignment Problem

### Problem Formulation

The goal is to minimize the total cost of assigning 5 tasks to 5 people. Each person can be assigned to only one task, and each task needs exactly one person.

- **Decision Variables:**

$$x_{ij} = \begin{cases} 1, & \text{if person } i \text{ is assigned to task } j \\ 0, & \text{otherwise} \end{cases}$$

  Here, $x_{ij}$ is a binary variable representing whether person $i$ is assigned to task $j$.

- **Objective Function:**

$$\text{Minimize} \quad \sum_{i=1}^{5} \sum_{j=1}^{5} c_{ij} x_{ij}$$

  where $c_{ij}$ is the cost of assigning person $i$ to task $j$. The objective function aims to minimize the total cost across all assignments.

- **Constraints:**

$$\sum_{j=1}^{5} x_{ij} = 1, \quad \forall i \quad \text{(Each person is assigned exactly one task)}$$

$$\sum_{i=1}^{5} x_{ij} = 1, \quad \forall j \quad \text{(Each task is assigned to exactly one person)}$$

### Results

The optimal allocation matrix is:

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The minimum cost achieved is 210.

### Interpretation

The allocation matrix indicates that each person is assigned to a unique task, minimizing the total cost. The constraints ensure that every person is assigned to exactly one task and each task is covered by one person.

## Question 2: Production Allocation Problem

### Part (a) Formulation

The aim is to maximize profits from producing three grades of automotive batteries across three locations.

- **Decision Variables:**

  $$x_{ij} = \text{number of batteries of grade } i \text{ produced at location } j$$

  Here, $x_{ij}$ represents the quantity of batteries of grade $i$ produced at facility $j$

- **Objective Function:**

  $$\text{Maximize} \quad \sum_{i=1}^{3}\sum_{j=1}^{3} p_i x_{ij}$$

  where $p_i$ is the profit per unit of battery of grade $i$. The objective function aims to maximize the total profit from the production.

- **Constraints:**

  $$\sum_{j=1}^{3} x_{ij} \le \text{demand}_i, \quad \forall i \quad \text{(Total production of each grade should not exceed its demand)}$$

  $$\sum_{i=1}^{3} x_{ij} \le \text{capacity}_j, \quad \forall j \quad \text{(Total production at each location should not exceed its capacity)}$$

  $$\sum_{i=1}^{3} \text{lead}_i \times x_{ij} \le \text{max\_lead}_j, \quad \forall j \quad \text{(Total lead time must not exceed the max allowed per location)}$$

### Results

The optimal allocation matrix is:

$$\begin{bmatrix} 171 & 16 & 98 \\ 377 & 392 & 126 \\ 0 & 0 & 0 \end{bmatrix}$$

The maximum profit achieved is 12,370

## Observation on Economy Type Batteries

The optimal allocation matrix shows no production of economy type batteries ($x_{3j}$), as indicated by the third row of zeros:

$$\begin{bmatrix} 0.0 & 0.0 & 0.0 \end{bmatrix}$$

This implies that the solution maximizes profit by focusing on other battery types, which are more profitable within the given constraints. The absence of economy type batteries suggests that their production is not optimal under the current setup.

## Part (b) Economy Grade Constraint

An additional constraint ensures that at least 40% of total production is of the economy grade.

- **Additional Constraint:**

  $$\sum_{j=1}^{3} x_{3j} \geq 0.4 \times \left( \sum_{i=1}^{3} \sum_{j=1}^{3} x_{ij} \right)$$

  where $x_{3j}$ represents the number of economy batteries produced at facility $j$. This constraint ensures that the economy batteries make up at least 40% of the total production.

## Results

The new allocation matrix is:

$$\begin{bmatrix} 476 & 0 & 199 \\ 0 & 0 & 0 \\ 0 & 450 & 0 \end{bmatrix}$$

The revised maximum profit is 11,250

## Observation on Standard Type Batteries

The allocation matrix shows no production of standard type batteries ($x_{2j}$):

$$\begin{bmatrix} 0.0 & 0.0 & 0.0 \end{bmatrix}$$

This indicates that the constraint requiring at least 40 % economy type batteries led to the exclusion of standard type batteries, resulting in a reduced profit of INR 11,250.