MCP361: Assignment 1

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Question 1: Assignment Problem

Problem Formulation

The goal is to minimize the total cost of assigning 5 tasks to 5 people. Each person can be assigned to only one task, and each task needs exactly one person.

- **Decision Variables:**

$$x_{ij} = \begin{cases} 1, & \text{if person } i \text{ is assigned to task } j \\ 0, & \text{otherwise} \end{cases}$$

  Here, x_{ij} is a binary variable representing whether person i is assigned to task j.

- **Objective Function:**

$$\text{Minimize} \quad \sum_{i=1}^{5} \sum_{j=1}^{5} c_{ij} x_{ij}$$

  where c_{ij} is the cost of assigning person i to task j. The objective function aims to minimize the total cost across all assignments.

- **Constraints:**

$$\sum_{j=1}^{5} x_{ij} = 1, \quad \forall i \quad \text{(Each person is assigned exactly one task)}$$

$$\sum_{i=1}^{5} x_{ij} = 1, \quad \forall j \quad \text{(Each task is assigned to exactly one person)}$$

Results

The optimal allocation matrix is:

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The minimum cost achieved is 210.

Interpretation

The allocation matrix indicates that each person is assigned to a unique task, minimizing the total cost. The constraints ensure that every person is assigned to exactly one task and each task is covered by one person.

Question 2: Production Allocation Problem

Part (a) Formulation

The aim is to maximize profits from producing three grades of automotive batteries across three locations.

- **Decision Variables:**

$$x_{ij} = \text{number of batteries of grade } i \text{ produced at location } j$$

  Here, x_{ij} represents the quantity of batteries of grade i produced at facility j

- **Objective Function:**

$$\text{Maximize} \quad \sum_{i=1}^{3}\sum_{j=1}^{3} p_i x_{ij}$$

  where p_i is the profit per unit of battery of grade i. The objective function aims to maximize the total profit from the production.

- **Constraints:**

$$\sum_{j=1}^{3} x_{ij} \leq \text{demand}_i, \quad \forall i \quad \text{(Total production of each grade should not exceed its demand)}$$

$$\sum_{i=1}^{3} x_{ij} \leq \text{capacity}_j, \quad \forall j \quad \text{(Total production at each location should not exceed its capacity)}$$

$$\sum_{i=1}^{3} \text{lead}_i \times x_{ij} \leq \text{max\_lead}_j, \quad \forall j \quad \text{(Total lead time must not exceed the max allowed per location)}$$

Results

The optimal allocation matrix is:

$$\begin{bmatrix} 171 & 16 & 98 \\ 377 & 392 & 126 \\ 0 & 0 & 0 \end{bmatrix}$$

The maximum profit achieved is 12,370

Observation on Economy Type Batteries

The optimal allocation matrix shows no production of economy-type batteries (x_{3j}), as indicated by the third row of zeros:

$$\begin{bmatrix} 0.0 & 0.0 & 0.0 \end{bmatrix}$$

This indicates that the maximum profit can be generated without using the batteries of the Economy type.

Part (b) Economy Grade Constraint

An additional constraint ensures that at least 40% of total production is of the economy grade.

- **Additional Constraint:**

$$\sum_{j=1}^{3} x_{3j} \geq 0.4 \times \left(\sum_{i=1}^{3} \sum_{j=1}^{3} x_{ij} \right)$$

  where x_{3j} represents the number of economy batteries produced at facility j. This constraint ensures that the economy batteries make up at least 40% of the total production.

Results

The new allocation matrix is:

$$\begin{bmatrix} 476 & 0 & 199 \\ 0 & 0 & 0 \\ 0 & 450 & 0 \end{bmatrix}$$

The revised maximum profit is 11,250

Observation on Standard Type Batteries

The allocation matrix shows no production of standard type batteries (x_{2j}):

$$\begin{bmatrix} 0.0 & 0.0 & 0.0 \end{bmatrix}$$

This indicates that the constraint requiring at least 40 % economy type batteries led to the exclusion of standard type batteries, resulting in a reduced profit of INR 11,250.