Quiz 8: Solutions

Let $\mathbf{X} = [X_1, X_2, X_3]$. Consider a multivariate normal distribution $P(\mathbf{X}|\mu, \Sigma) = \mathcal{N}(\mathbf{x}; \mu, \Sigma)$ with the parameters $\mu = [1, 2, -1]$ and $\Sigma = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 2 & 0.5 \\ 0.5 & 0.5 & 4 \end{bmatrix}$. Use this distribution to answer the following questions.

Question 1: [33 points] What is the mean of X_2?

Solution: Since the distribution is multivariate normal, the marginal distribution of X_2 is $\mathcal{N}(\mathbf{x}_2; \mu_2, \Sigma_{22})$. The mean of X_2 is thus $\mu_2 = 2$.
Rubric:

- Full points for correct answer, 2.
- No points for incorrect answer.

Question 2: [33 points] What is the standard deviation of X3?

Solution Since the distribution is multivariate normal, the marginal distribution of X_3 is $\mathcal{N}(\mathbf{x}_3; \mu_3, \Sigma_{33})$. This means the variance of X_3 is $\Sigma_{33} = 4$. The standard deviation of X_3 is that $\sqrt{4} = 2$.
Rubric:

- Full points for correct answer, 2.
- Half points for giving the variance, 4.
- No points for other incorrect answers.

Question 3: [34 points] What is the mean of X_1 given $X_3 = -17$?

To find the conditional mean, we need to use the Gaussian conditioning formula. We let $A = \{1\}$ and $B = \{3\}$. We have:

$$\begin{aligned} \mu_{1|3} &= \mu_1 + \Sigma_{13}(\Sigma_{33})^{-1}(x_3 - \mu_3) && (1) \\ &= 1 + (0.5)(4)^{-1}((-17) - (-1)) && (2) \\ &= 1 + (0.5)(0.25)(-16) && (3) \\ &= 1 + (-2) = -1 && (4) \end{aligned}$$

Rubric:

- Full points for correct answer.
- 75% of points for answer $1 + (0.5)(4)^{-1}(-18) = -1.25$
- 50% of points for answer $(-1) + (0.5)(4)^{-1}(-18) = -3$

- 50% of points for answer $1 + (0.5)(4)(-16) = -31$
- 25% of points for answer $1 + (0.5)(4)(-18) = -35$
- 10% of pints for answer $1 + 4 * (-16) = -63$
- 10% of points for the answer $\mu_1 = 1$.