Quiz 5: Solutions

1 Question 1 (100 points)

Consider the optimization problem shown below when answering the following questions.

$$\min_{\mathbf{w},\epsilon} \frac{1}{2}\|\mathbf{w}\|_2^2 + \frac{C}{2}\|\epsilon\|_2^2$$
$$\text{s.t. } \epsilon_n = y_n - \mathbf{w}^T\mathbf{x}_n \text{ for } 1 \leq n \leq N$$

Note that C is a constant as are the y_n's and $\mathbf{x}_n$'s. The optimization variables are $\mathbf{w}$ and $\epsilon = [\epsilon_1, ..., \epsilon_N]$. Both $\mathbf{w}$ and the $\mathbf{x}_n$'s are column vectors.

Explain what model this optimization problem is learning and give an alternate unconstrained optimization problem that results in the same optimal parameters. Show your work and/or explain your answer.

Solution: Since this is an equality constrained problem, we can move the constraint directly into the objective function letting $\mathbf{y} = [y_1, ..., y_n]$ and $\mathbf{X} = [\mathbf{x}_1, ..., \mathbf{x}_N]$ we have:

$$\min_{\mathbf{w},\epsilon} \frac{1}{2}\|\mathbf{w}\|_2^2 + \frac{C}{2}\|\mathbf{y} - \mathbf{w}^T\mathbf{X}\|_2^2 \tag{1}$$

Expanding the second norm, we have:

$$\min_{\mathbf{w},\epsilon} \frac{1}{2}\|\mathbf{w}\|_2^2 + \frac{C}{2}\sum_{n=1}^{N}(y_n - \mathbf{w}^T\mathbf{x}_n)^2 \tag{2}$$

So, this optimization problem is learning a linear regression model $f(\mathbf{x}) = \mathbf{w}^T\mathbf{x}$.

Rubric:

- 50 points: An explanation indicating that a linear regression model is being learned.
- 25 points: A valid unconstrained form of the problem as in (1) or (2) or similar.
- 25 points: Some valid work or a brief explanation of how the equivalent form was obtained.