$$f(x) = \begin{cases} ax^2 + bx + c & x \le 0 \\ d|x| + c & 0 \le x \le g \\ hx^3 + i & x \ge g \end{cases}$$

To find the sub-differential $\partial f(x)$, we differentiate piece-wise. All the components are convex $(x^2, |x|, x^3)$

① $\frac{\partial}{\partial x}(ax^2 + bx + c) = \frac{\partial}{\partial x}(ax^2) + \frac{\partial}{\partial x}(bx) + \frac{\partial}{\partial x}(c)$

$= 2ax + b \quad \therefore \boxed{g_1 = 2ax + b}$

② $d|x| + c$

The function $y = |x|$ is not differentiable at 0.

$\therefore$ we differentiate piece-wise for $x > 0$ and $x < 0$

$\frac{\partial}{\partial x}$ For $x < 0$, $\frac{\partial}{\partial x}|x| = -1 \Rightarrow -d \quad \left[\because \frac{\partial}{\partial x} d|x| = d \cdot \frac{\partial}{\partial x}|x|\right]$

and $x > 0$, $\frac{\partial}{\partial x}|x| = 1 \Rightarrow d$

$\therefore \partial f(x) = [-d, d]$ at $x = 0$

$$\boxed{\therefore g_2 = \begin{cases} [-d, d] & \text{at } x = 0 \\ d & \text{at } x > 0 \end{cases}}$$

③ $\frac{\partial}{\partial x}(hx^3 + i) = \frac{\partial}{\partial x}(hx^3) + \frac{\partial}{\partial x}(i)$

$= 3hx^2 + 0 \Rightarrow \boxed{g_3 = 3hx^2}$

The sub-differential function of $f(x)$ is given by

$$\partial f(x) = \begin{cases} 2ax + b & x \le 0 \\ [-d, d] & x = 0 \\ d & 0 < x \le g \\ 3hx^2 & x \ge g \end{cases}$$