$Y = aX + b$

дискр. $\quad a \neq 0 \quad$ дискр.

$p_Y(y) = Pr[Y=y] = Pr[aX+b=y] = Pr\left[X = \frac{y-b}{a}\right] = p_X\left(\frac{y-b}{a}\right)$ $\qquad$ Итого: $f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$

непр. 1) $a > 0$

$F_Y(y) = Pr(Y \le y) = Pr(aX + b \le y) = Pr\left(X \le \frac{y-b}{a}\right) = F_X\left(\frac{y-b}{a}\right)$

$f_Y(y) = f_X\left(\frac{y-b}{a}\right) \cdot \frac{1}{a}$

2) $a < 0$

$F_Y(y) = Pr(Y \le y) = Pr(aX + b \le y) = Pr\left(X \ge \frac{y-b}{a}\right) = 1 - F_X\left(\frac{y-b}{a}\right)$

$f_Y(y) = -f_X\left(\frac{y-b}{a}\right) \cdot \frac{1}{a}$

$X \sim N(\mu, \sigma^2)$

$Y = aX + b$

$$\left.f_Y(y) = \frac{1}{|a|} \cdot \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{\left(\frac{y-b}{a} - \mu\right)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}(|a|\sigma)} \cdot e^{-\frac{(y-(b+a\mu))^2}{2(a\sigma)^2}} \right\} Y \sim N\left(b + a\mu, (a\sigma)^2\right)$$

$Y = g(X)$

1) $F_Y(y) = Pr[Y \le y] = Pr[g(X) \le y]$

а) g – монотонно $\nearrow$, дифф-на

1) $F_Y(y) = Pr(Y \le y) = Pr(g(x) \le y) = Pr[X \le g^{-1}(y)] = F_X(g^{-1}(y))$

2) $f_Y(y) = f_X(g^{-1}(y)) \cdot \frac{1}{g'(g^{-1}(y))}$

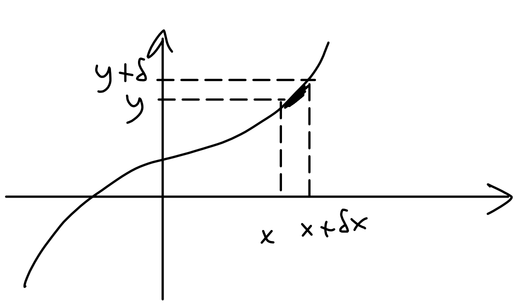
б) g – $\searrow$

— // — $\quad Pr[X \ge g^{-1}(y)] = 1 - F_X(g^{-1}(y))$

$f_Y(y) = -f_X(g^{-1}(y)) \cdot \frac{1}{g'(g^{-1}(y))}$

$f_Y(y) = f_X(g^{-1}(y)) \cdot \frac{1}{|g'(g^{-1}(y))|}$

$Y = g(X)$ $\qquad$ g – строго монотонна, диф-на



$f_Y(y) \cdot \delta_y \approx Pr[Y \in [y, y+\delta]] = Pr\left[X \in \left[x, x + \frac{\delta_y}{g'(x)}\right]\right] = f_X(g^{-1}(y)) \cdot \frac{\delta_y}{g'(g^{-1}(y))}$

$\delta_y \approx \delta_x \cdot g'(x)$

$\delta_x = \frac{\delta_y}{g'(x)}$

1) $F_Y(y) = Pr(g(x) \le y) = h(F_X(?))$

$Y = X^2$ $\qquad Pr(Y \le y) \xrightarrow{y<0} 0$

$\qquad \xrightarrow{y \ge 0} Pr(X^2 \le y) = Pr(X \in [-\sqrt{y}; \sqrt{y}]) = F_X(\sqrt{y}) - F_X(-\sqrt{y}) + \underbrace{p_X(-\sqrt{y})}_{=0 \text{ в непр. случае}}$

$f_Y(y) = \frac{f_X(\sqrt{y})}{2\sqrt{y}} - \frac{f_X(-\sqrt{y})}{(-2\sqrt{y})} = \frac{f_X(\sqrt{y}) + f_X(-\sqrt{y})}{2\sqrt{y}}$

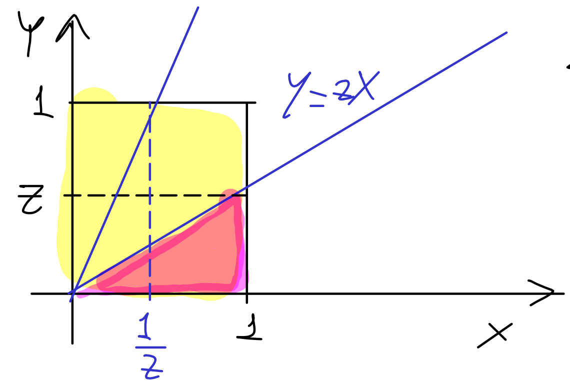
$Z = g(X, Y)$

1) $F_Z(z) = \Pr[Z \le z] = \Pr(g(X,Y) \le z)$

X, Y - независ, $\sim U(0,1)$

$Z = \dfrac{Y}{X}$

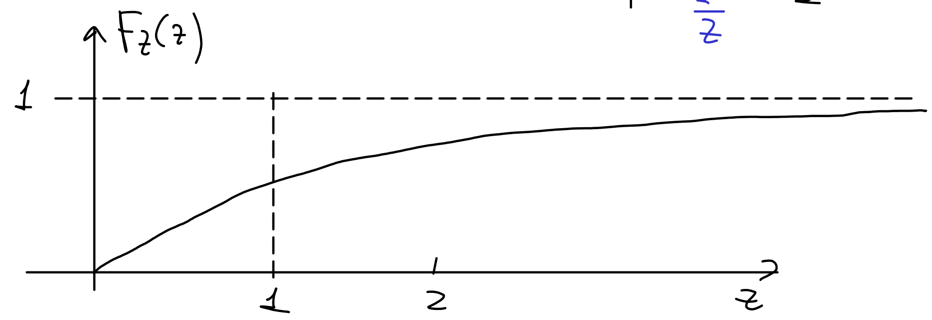
$\Pr\left[\dfrac{Y}{X} \le z\right]$



$z \le 0 \Rightarrow \Pr = 0$

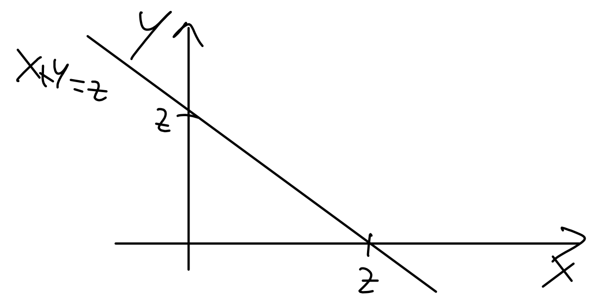
$0 < z \le 1 \Rightarrow \Pr\left(\frac{Y}{X} \le z\right) = \frac{z}{2}$

$z > 1 \Rightarrow 1 - \frac{1}{2z}$



$Z = X + Y$ $\quad X$ и Y - независ

1) $P_Z(z) = \sum_x \underbrace{P_X(x) P_Y(z-x)}_{\Pr[X=x \cap Y=z-x]}$



2) $f_{Z|X}(z|x) = f_{\underline{Y+X|X}}(z|x) = f_{Y+X}(z) = f_Y(z-x)$
$\quad \overset{\parallel}{Y+X}$

$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) \cdot f_{Z|X}(z|x)\,dx = \int_{-\infty}^{\infty} f_X(x) \cdot f_Y(z-x)\,dx$

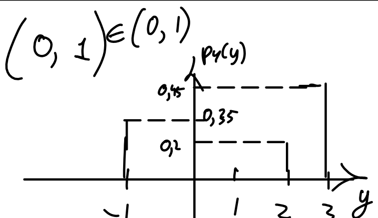
$X \sim N(\mu_X, \sigma_X^2)$

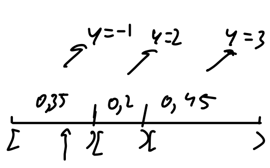
$Y \sim N(\mu_Y, \sigma_Y^2)$

$f_{X+Y}(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\,dx = \frac{1}{2\pi\sigma_X\sigma_Y} \int_{-\infty}^{+\infty} e^{-\left(\frac{(x-\mu_X)^2}{2\sigma_X^2} + \frac{(z-x-\mu_Y)^2}{2\sigma_Y^2}\right)} dx \overset{\dots}{=} \frac{1}{\sqrt{2\pi}\left(\sqrt{\sigma_X^2+\sigma_Y^2}\right)} \cdot e^{-\frac{(z-\mu_X-\mu_Y)^2}{2(\sigma_X^2+\sigma_Y^2)}}$

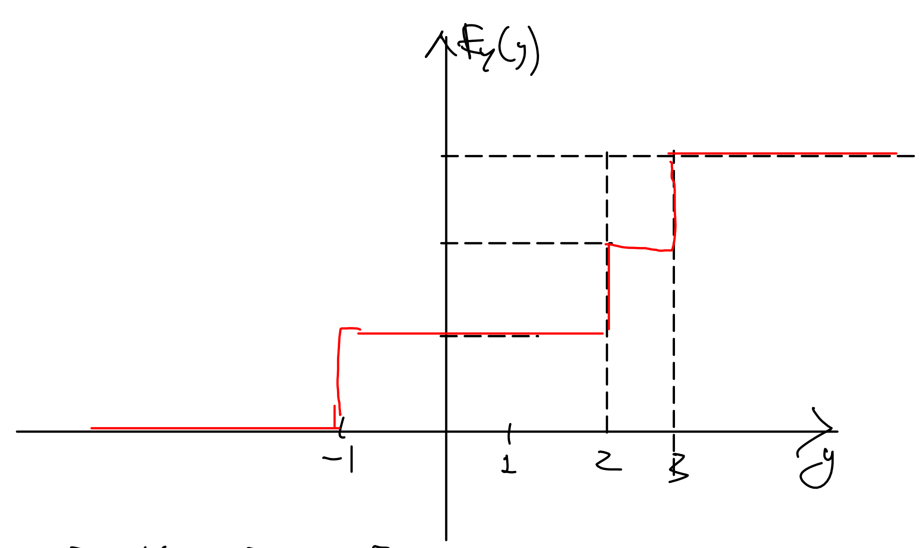
$X \sim U(0,1) \in (0,1)$

Y:

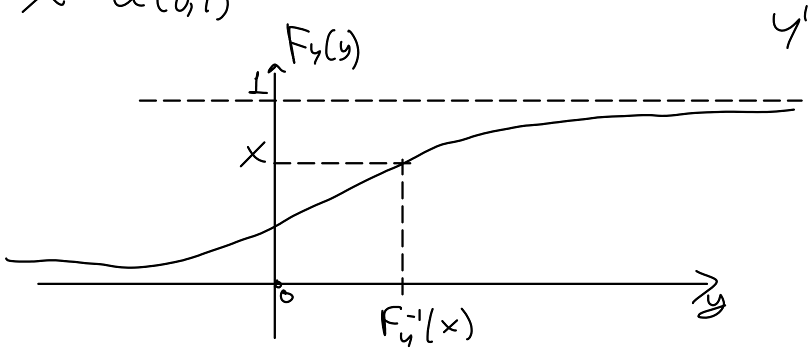




$P_Y(-1) = \Pr(X \in (0, 0.35)) = 0.35$



$X \sim U(0,1)$



$Y' = F_Y^{-1}(X)$

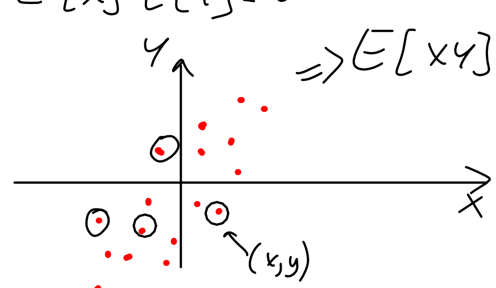
$F_{Y'}(y) = \Pr(Y' \le y) = \Pr[F_Y^{-1}(X) \le y] = \Pr[X \le F_Y(y)] = F_Y(y)$

сгенерировали X

$Y = F_X(X) \sim U(0,1)$

X, Y - независ. $\Rightarrow E[XY] = E[X] \cdot E[Y] = 0$

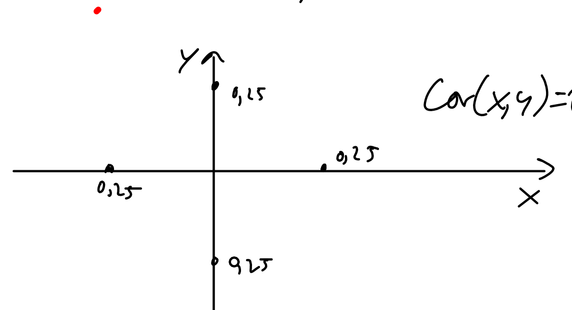
$// E[X] = E[Y] = 0$



$\Rightarrow E[XY] > 0$

$Cov(X,Y) = E\left[(X - E(X))(Y - E(Y))\right] =$

$Cov(X,X) = Var(X)$



$Cov(X,Y) = 0$, но X и Y зависимы

$$Cov(X,Y) = E[XY - XE[Y] - Y\cdot E(X) + E[X]E[Y]] = E[XY] - E[X]E[Y]$$

$$Cov(aX+b, Y) = E(aXY + bY) - E(aX+b)E(Y) = aE(XY) + \cancel{bE(Y)} - aE(X)E(Y) - \cancel{bE(Y)} = aE[XY] - aE(X)E(Y) = aCov(X,Y)$$

$$Cov(X+Y, Z) = E[(X+Y-E(X+Y))(Z-E(Z))] = E[((X-E[X]) + (Y-E[Y]))\cdot(Z-E(Z))] = Cov(X,Z) + Cov(Y,Z)$$

$$Var(X+Y) = E[(\underbrace{X+Y-E(X+Y)}_{=(X-E(X))+(Y-E(Y))})^2] = E[(X-E[X])^2] + E[(Y-E(Y))^2] + 2E[(X-E(X))(Y-E(Y))] = Var(X)+Var(Y)+2Cov(X,Y)$$

$$Cov(X_i,X_j) = E[X_iX_j] - \overbrace{E[X_i]E[X_j]}^{=0}$$

$$Var(X_1+\dots+X_n) = E[(X_1+X_2+\dots+X_n)^2] = E\left[\sum_{i=1}^{n} X_i^2 + \sum_{i\ne j} X_iX_j\right] = \sum_{i=1}^{n} Var(X_i) + \sum_{i\ne j} Cov(X_i,X_j)$$

// $\forall i \ E[X_i] = 0$

Если ненулевые матожидания, то нужно посчитать для сл.в. X'_i = X_i - E[X_i].
Там все будет так же