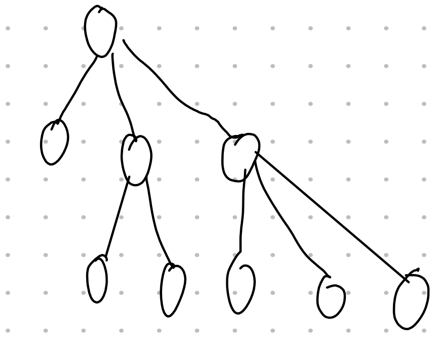


$$\boxed{a_n = \frac{1}{n}[s^{n-1}](\varphi(s))^n \qquad A(t) = \sum_{n=0}^{\infty} a_n t^n}$$

Формула обращения Лагранжа

$$T = t \times \mathrm{Seq}\, T$$

$$A(t) = t \cdot \varphi(A(t))$$

$$\varphi(s) = \frac{1}{1-s}$$

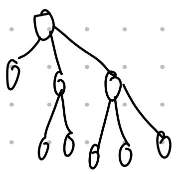
коэф. при s^(n-1)

$$a_n = \frac{1}{n}[s^{n-1}]\left(\frac{1}{1-s}\right)^n = \frac{1}{n}\binom{2n-2}{n-1}$$

$$(1-s)^{-n} = 1 - \binom{-n}{1}s + \binom{-n}{2}s^2 - \binom{-n}{3}s^3 + \ldots$$

$$(-1)^{n-1}\binom{-n}{n-1} = (-1)^{n-1}\frac{-n(-n-1)(-n-2)\ldots(-n-(n-1)+1)}{1\cdot 2\cdot 3\cdot\ldots\cdot(n-1)} = \frac{n(n+1)\ldots(n+n-2)}{1\cdot 2\cdot\ldots\cdot(n-1)}$$

$$(-1)^{n-1}\binom{-n}{n-1} = \frac{(2n-2)!}{(n-1)!(n-1)!} = \binom{2n-2}{n-1}$$



EGF

Формула Кэли: помеченн. подвешенн. дер. n^{n-2}

$$T = t \times \mathrm{Set}\, T$$

$$A(t) = t \cdot \varphi(A(t))$$

$$\varphi(s) = e^s$$

$$\frac{a_n}{n!} = \frac{1}{n}[s^{n-1}]e^{ns} \qquad a_n = n^{n-1}$$

выбираем за что подвесить

$$e^{ns} = 1 + \frac{1}{1!}(ns) + \frac{1}{2!}(ns)^2 + \frac{1}{3!}(ns)^3 + \ldots$$

$$[s^{n-1}]e^{ns} = \frac{n^{n-1}}{(n-1)!}$$

Производящие функции от нескольких переменных

k

$\underbrace{0\,1\,1\,0\ldots\ldots\,1}$

$z \quad uz$

$\underbrace{u^k z^n}$

$\binom{n}{k}$

OGF
EGF
MGF

n\k					
	1				
	1	1			
	1	2	1		
	1	3	3	1	
	1	4	6	4	1

$$A_k(t) = \sum_{n=0}^{\infty}\binom{n}{k}t^n$$

$$B_n(t) = \sum_{k=0}^{\infty}\binom{n}{k}t^k$$

u, z

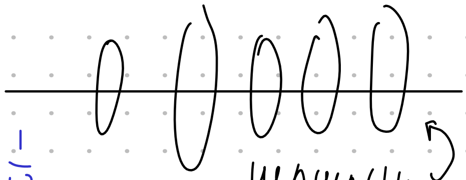
$$C(u,z) = \sum_{n,k}\binom{n}{k}z^n u^k = \frac{1}{1-z-uz}$$

$$\mathrm{Seq}\underbrace{\{z, uz\}}_{A} = [\,], [z], [uz], [zz], [z, uz], [uz, z], [uz, uz], \ldots$$

$$A(u,z) = z + uz$$

Числа Стирлинга I рода.

Перест. - помеч. $\quad Set\, Cyc\, Z = e^{\ln\frac{1}{1-z}} = \frac{1}{1-z}$



шашлык (allignment)

$$Set\, Cyc\, Z = \bigcup_{k=0}^{\infty} Set_{=k}\, Cyc\, Z$$

$$\bigcup_{k=0}^{\infty} Set_{=k}(u\times Cyc\, Z) \longmapsto \sum_{n,k} \begin{bmatrix} n\\ k\end{bmatrix} \frac{1}{n!} z^n u^k$$

$$Set_{=k}(A) = \frac{Seq_{=k} A}{k!} = \frac{A(z)^k}{k!} \Rightarrow \bigcup_{k=0}^{\infty} Set_{=k}(u\times Cyc\, Z) = \sum_{k=0}^{\infty} \frac{\left(u\ln\left(\frac{1}{1-z}\right)\right)^k}{k!} = e^{u\ln\frac{1}{1-z}} = (1-z)^{-u}$$

$$(1-z)^{-u} = \sum_{n,k} \frac{\begin{bmatrix} n\\ k\end{bmatrix}}{n!} z^n u^k$$

Числа Стирлинга 2 рода

$\begin{Bmatrix} n\\ k\end{Bmatrix} \quad Set\, Set_{>0}\, Z$

e^{e^z-1}

$$\bigcup_{k=0}^{\infty} Set_{=k}(u\times Set_{>0} Z) = \sum_{k=0}^{\infty} \frac{(u(e^z-1))^k}{k!} = e^{ue^z-u} = \sum_{n,k} \frac{\begin{Bmatrix} n\\ k\end{Bmatrix}}{n!} z^n u^k$$

$A \quad a_{n,k} = [z^n u^k] A(u,z)$

$$W_n = \frac{\sum_{k=0}^{\infty} k a_{n,k}}{\sum_{k=0}^{\infty} a_{n,k}} = \frac{[z^n]\left(\frac{\partial A(u,z)}{\partial u}\right)\Big|_{u=1}}{[z^n]\, A(1,z)}$$

Разбиение на слагаемые. Порядок важен

$Seq\, Seq_{>0}\, Z$

$Seq(u\times Seq_{>0}\, Z)$

$$\frac{\partial A(u,z)}{\partial u}\Big|_{u=1} = \frac{z(1-z)}{(1-z-uz)^2}\Big|_{u=1} = \frac{z(1-z)}{(1-2z)^2}$$

$$\frac{1}{1-z} - 1 = \frac{z}{1-z}$$

числ. $[z^n]\frac{z(1-z)}{(1-2z)^2} = \frac{2^n(n+1)}{4}$

$$A(u,z) \quad \frac{1}{1-\frac{uz}{1-z}} = \frac{1-z}{1-z-uz}$$

знам. $[z^n]\frac{1-z}{1-2z} = 2^n - 2^{n-1} = 2^{n-1}$

Среднее число слагаемых $\frac{2^n(n+1)}{2^{n-1}\cdot 4} = \frac{n+1}{2}$

Среднее # циклов в перест.

$$A(u,z) = (1-z)^{-u}$$

$$\frac{\partial A(u,z)}{\partial u} = \frac{\partial}{\partial u} e^{u \ln \frac{1}{1-z}} = \ln \frac{1}{1-z} e^{u \ln \frac{1}{1-z}}$$

$n=1$

числ.: $[z^n] \dfrac{\ln\left(\frac{1}{1-z}\right)}{1-z} = B(z)$

знам. $(1-z)^{-n}\Big|_{n=1} = \dfrac{1}{1-z}$

$[z^n] \dfrac{1}{1-z} = 1$

$$\left(z + \frac{1}{2}z^2 + \frac{1}{3}z^3 + \ldots + \frac{1}{k}z^k + \ldots\right)\left(1 + z + z^2 + \ldots\right) -$$

$$[z^n] B(z) = \sum_{k=1}^{n} \frac{1}{k} = H_n \sim \log n$$

$$A(z) = \sum_{n=0}^{\infty} a_n z^n$$

$$z \cdot A'(z) = \sum_{n=0}^{\infty} a_n \cdot n z^n$$

$A(1)$ $A'(1)$

$$A(s) = \sum_{n=1}^{\infty} a_n \frac{1}{n^s}$$