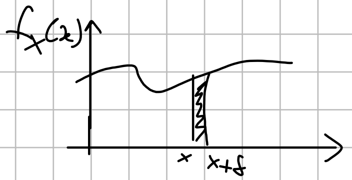
$$f_X(x)\cdot\delta \approx Pr(X\in[x,x+\delta])$$



$$f_{X|A}(x)\cdot\delta \approx Pr(X\in[x,x+\delta]/A)$$

$$\forall B:\quad Pr[X\in B/A] = \int_B f_{X|A}(x)\,dx$$

$$f_{X|A}(x)\geq 0$$

$$\int_{-\infty}^{\infty} f_{X|A}(x)\,dx = 1$$

$$Pr(X\in[x,x+\delta]/A) = \frac{Pr[\overbrace{X\in[x,x+\delta]\cap A}^{\subset A}]}{Pr[A]} = \frac{f_X(x)\,\delta}{Pr[A]}$$

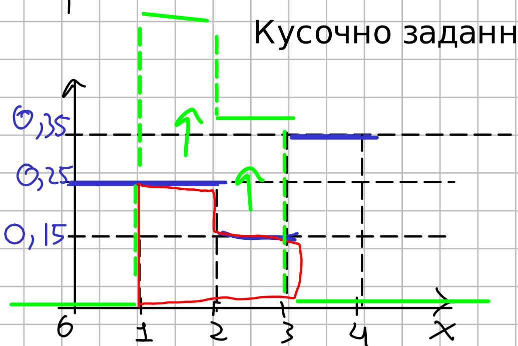
$$f_{X|A}(x) = \begin{cases} \dfrac{f_X(x)}{Pr[A]}, & x\in A \\ 0, & x\notin A \end{cases}$$

$$F_{X|A}(x) = Pr(X\leq x\,|A)$$

$$E[X|A] \overset{def}{=} \int_{-\infty}^{+\infty} x\cdot f_{X|A}(x)\,dx$$

$$E[\underbrace{g(X)}_{=Y}|A] = \int_{-\infty}^{+\infty} g(x)\,f_{X|A}(x)\,dx$$

Кусочно заданное равномерное распределение



$A=[1,3]$

$Pr(A)=0,25\cdot 1+0.15\cdot 1=0,4$

$$f_{X|A}(x) = \begin{cases} 0, & x\in[1,3] \\ \dfrac{0,25}{4}=0,625 & x\in[1,2] \\ \dfrac{0.15}{0,4}=0,375 & x\in(2,3] \end{cases}$$

Свойство беспамятства экспоненциального распределения

$$X\sim Exp(\lambda) \Longleftrightarrow f_X(x)=\lambda e^{-\lambda x},\quad x\geq 0$$

$A\subset R$

событие $A \Leftrightarrow X\in A$

$$\Omega=\{\dots\} \xrightarrow{X} R$$

$$A\subset\Omega \longrightarrow Event(A)\subset R$$

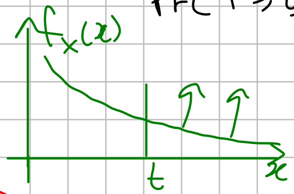
— Отступление про то, что мы отошли от Омеги, перешли к просто вещ. прямой?

Вернёмся к Exp → время жизни лампочки

$T \sim Exp(\lambda)$

$t \qquad 1 - F_{(T-t)|T\geq t}(x) = Pr(T-t \geq x \mid T \geq t) = \dfrac{Pr(T-t\geq x \cap T\geq t)}{Pr(T\geq t)} =$

$$= \frac{Pr(T-t\geq x)}{Pr(T\geq t)} = \frac{e^{-\lambda(x+t)}}{e^{-\lambda t}} = e^{-\lambda x} = 1 - F_x(x)$$



$\{A_i\}$ $\qquad Pr(B) = Pr(A_1)\cdot Pr(B|A_1) + \ldots + Pr(A_n)\cdot Pr(B|A_n) + \ldots$

$p_X(x) = Pr(A_1)\cdot p_{X|A_1}(x) + \ldots + Pr(A_n)\cdot p_{X|A_n}(x)$ – в дискретном случае

Что в непрерывном?

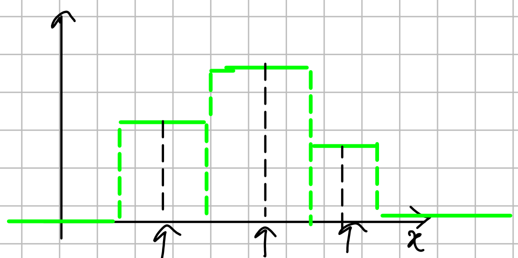
$$F_X(x) = Pr(A_1)\cdot F_{X|A_1}(x) + \ldots + Pr(A_n)\cdot F_{X|A_n}(x) + \ldots$$

$\quad || \qquad\qquad\qquad \downarrow \frac{dF}{dx}$

$Pr(X \leq x)$

$$f_x(x) = Pr(A_1)\cdot f_{X|A_1}(x) + \ldots + Pr(A_n)\cdot f_{X|A_n}(x) + \ldots$$

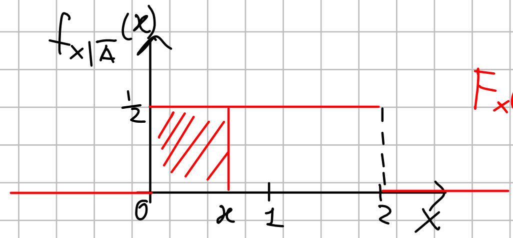
$$E[X] = Pr(A_1)\cdot E[X|A_1] + \ldots + Pr(A_n)E(X|A_n) + \ldots$$



мотожиди

1) орёл → $X = 1$

решка → 2) $X \sim U(0,2)$



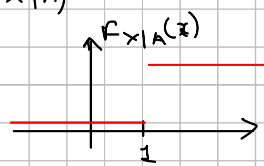
$F_x(x) = Pr(A)\cdot F_{X|A}(x) +$
$+ Pr(\bar{A})\cdot F_{X|\bar{A}}(x)$

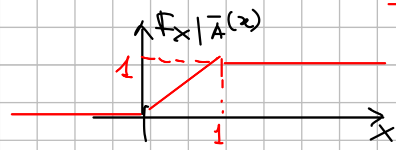
A = выпал орёл

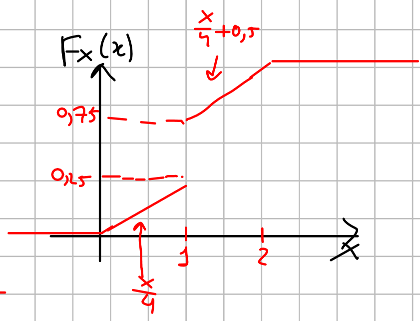
$$F_{X|A}(x) = \begin{cases} 0, & x < 1 \\ 1, & x \geq 1 \end{cases} \qquad F_{X|\bar{A}}(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{2}, & x \in [0,2] \\ 1, & x > 2 \end{cases}$$

$\quad ||$

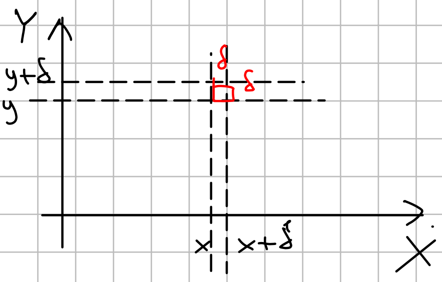
$Pr(X \leq x | A)$







$$f_{X,Y}(x,y)\cdot\delta^2 = Pr[X\in[x,x+\delta]\cap Y\in[y,y+\delta]]$$



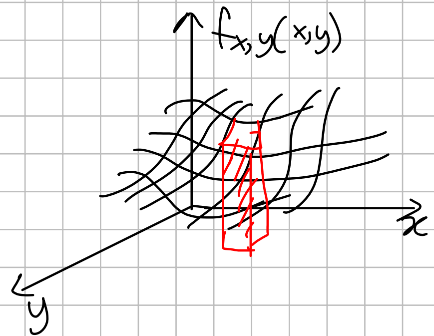
$$f_{X,Y}(x,y):\ \forall A\subset\mathbb{R}^2 \quad Pr((X,Y)\in A) = \iint_A f_{X,Y}(x,y)\,dx\,dy$$

$\downarrow$

Совместная плотность вероятности

1) $f_{X,Y}(x,y)\geq 0$

2) $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx\,dy = 1$



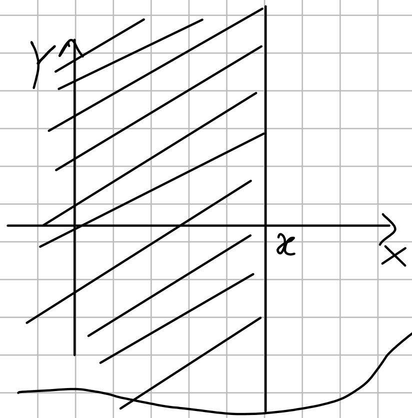
$$Pr(X=Y)=0$$

Маргинальные плотности вероятности

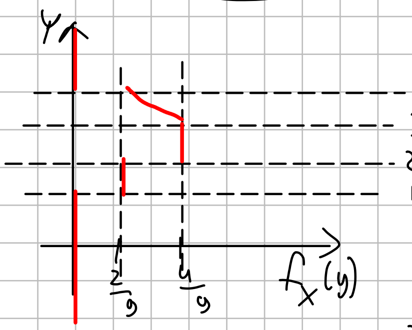
$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$

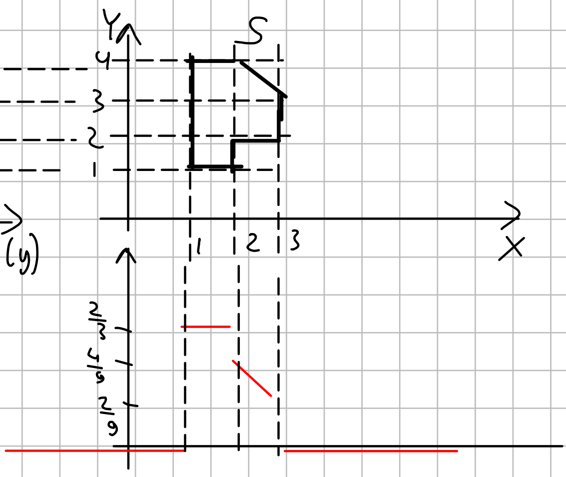
$$F_X(x) = Pr(X\leq x) = \int_{-\infty}^{x}\boxed{\int_{-\infty}^{\infty} f_{X,Y}(t,y)\,dy}\,dt$$

$$f_X(x) = F'_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$



$$f_{X,Y}(x,y) = \begin{cases} \frac{2}{9}, & (x,y)\in S \\ 0, & (x,y)\notin S \end{cases}$$





$$\iiint_{\mathbb{R}^3} f_{X,Y,Z}(x,y,z)\,dx\,dy\,dz = 1$$

$$F_{X,Y}(x,y) = Pr[X\leq x\cap Y\leq y]$$

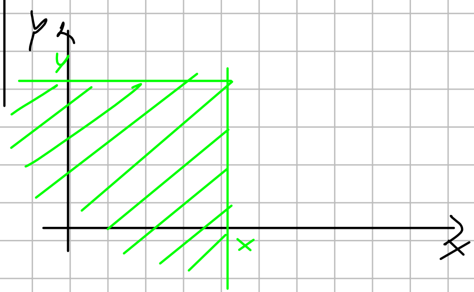
$$\frac{\partial^2 F_{X,Y}(x,y)}{\partial x\,\partial y} = f_{X,Y}(x,y)$$

$$Z = g(X,Y)$$

$$E[Z] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)\cdot f_{X,Y}(x,y)\,dx\,dy$$

$$E(X+Y) = E[X]+E[Y]$$

$$F_{X,Y}(x,y) = Pr[X\leq x\cap Y\leq y]$$



$$p_{X,Y}(x,y)$$

$$p_{X|A}(x) = \frac{p_X(x)\cdot[x\in A]}{\Pr(A)}$$

$[x \in A]$: $=1$, выполн.; $=0$, если не вып.

$$p_{X|Y}(x|y) = \frac{p_{X,Y}(x,y)}{p_X(y)}$$

$$f_{X,Y}(x,y)$$

$$f_{X|A}(x) = \frac{f_X(x)\cdot[x\in A]}{\Pr[A]}$$

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} \quad f_Y(y) > 0$$

$$\Pr\left(X\in[x, x+\delta] \mid Y\in[y, y+\varepsilon]\right) = \frac{\Pr(X_{\dots} \cap Y_{\dots})}{\Pr(Y_{\dots})} = \frac{f_{X,Y}(x,y)\cdot\delta\varepsilon}{f_Y(y)\cdot\varepsilon} = f_{X|Y}(x,y)\cdot\delta$$

$\forall A \subset \mathbb{R}$ $\quad \Pr[X\in A \mid Y=y] = \int_A f_{X|Y}(x|y)\,dx$

$\forall y : f_Y(y) \neq 0$

$$\int_{-\infty}^{\infty} f_{X|Y}(x|y)\,dx = 1$$

$$\int_{-\infty}^{\infty} \frac{f_{X,Y}(x,y)}{f_Y(y)}\,dx$$