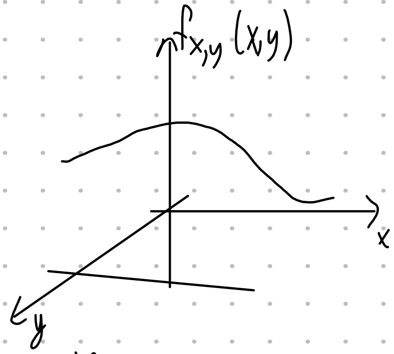
$$\Pr(X \in A \mid Y=y) = \int_A f_{X|Y}(x|y)\,dx$$

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$f_{X|Y}(x|y) \geq 0$

$$\int_{-\infty}^{\infty} f_{X|Y}(x|y)\,dx = 1$$



$$f_{X,Y}(x,y) = f_Y(y)\cdot f_{X|Y}(x|y) = f_X(x)\cdot f_{Y|X}(y|x)$$

$$f_X(x) = \int_{-\infty}^{\infty} f_Y(y)\, f_{X|Y}(x|y)\,dy$$

$$E[X \mid Y=y] = \int_{-\infty}^{\infty} x\cdot f_{X|Y}(x|y)\,dx \qquad E[g(X) \mid Y=y] = \int_{-\infty}^{\infty} g(x) f_{X|Y}(x|y)\,dx$$

$$E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\,dx = \int_{-\infty}^{\infty} dx\cdot x \int_{-\infty}^{\infty} dy\, f_Y(y) f_{X|Y}(x|y) = \int_{-\infty}^{\infty} dy\, f_Y(y) \underbrace{\int_{-\infty}^{\infty} x\, f_{X|Y}(x|y)\,dx}_{E(X|Y=y)}$$

$$= \int_{-\infty}^{\infty} f_Y(y)\, E[X \mid Y=y]\,dy$$

Независимость

$p_{X,Y}(x,y) = p_X(x)\cdot p_Y(y)$ – в дискретном случае?

X, Y - независ. $\overset{def}{\Longleftrightarrow} f_{X,Y}(x,y) = f_X(x)\cdot f_Y(y)$

Св-ва: 1) $E[X\cdot Y] = E(X)\cdot E(Y)$

2) $Var(X+Y) = Var(X) + Var(Y)$

3) $g, h \Rightarrow g(X), h(Y)$ – независ.

ТРАМВАИ

$X = U(0, 11)$

$Y = U(0, 17)$

$E(\min(X, Y))$

$$\Pr(Z \leq z) = \Pr(X \leq z \vee Y \leq z) = 1 - \Pr(X > z \wedge Y > z) =$$

* независ.

$$= 1 - \Pr(X>z)\cdot \Pr(Y>z) = 1 - (1 - F_X(z))(1 - F_Y(z))$$

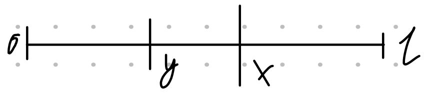
$$* = \int_z^{+\infty}\int_z^{+\infty} \underbrace{f_{X,Y}(x,y)}_{= f_X(x) f_Y(y)}\,dx\,dy = \underbrace{\int_z^{\infty} dx\, f_X(x)}_{\Pr(X>z)} \cdot \underbrace{\int_z^{+\infty} f_Y(y)\,dy}_{\Pr(Y>z)}$$

$$F_X(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{11}, & x \in (0, 11) \\ 1, & x > 11 \end{cases}$$

$$F_Y(y) = \begin{cases} 0, & y < 0 \\ \frac{y}{17}, & y \in (0, 17) \\ 1, & y > 17 \end{cases}$$

$$E[Z] = \int_0^{+\infty}(1-F_Z(z))dz = \int_0^{11}(1-F_X(z))(1-F_Y(z))dz = \int_0^{11}\left(1-\frac{z}{11}\right)\left(1-\frac{z}{17}\right)dz = 4.31$$

Ломаем палку (2 раза)

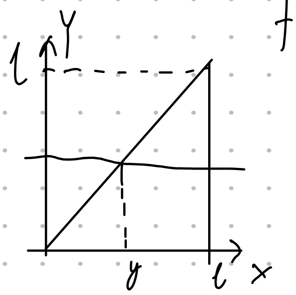


$$f_{X,Y}(x,y) = \underbrace{f_X(x)}_{=\frac{1}{l},\ x\in(0,l)} \cdot \underbrace{f_{Y|X}(y|x)}_{\frac{1}{x}} = \frac{1}{lx}$$

$X \sim U(0,l)$

$Y \sim U(0,x)$

$E(Y)$



$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dx = \int_y^l \frac{dx}{ly} = \frac{1}{l}\cdot\ln\left(\frac{l}{y}\right)$$

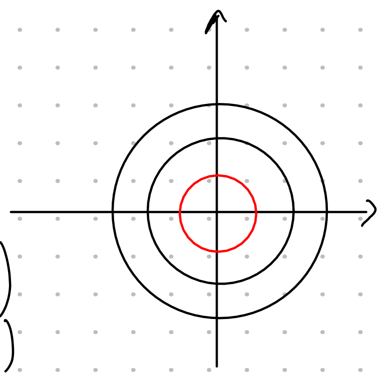
$$E[Y] = \int_{-\infty}^{\infty} y f_Y(y)dy = \int_0^l \frac{y}{l}\cdot\underbrace{\ln\left(\frac{l}{y}\right)}_{=\ln l - \ln y}dy =$$

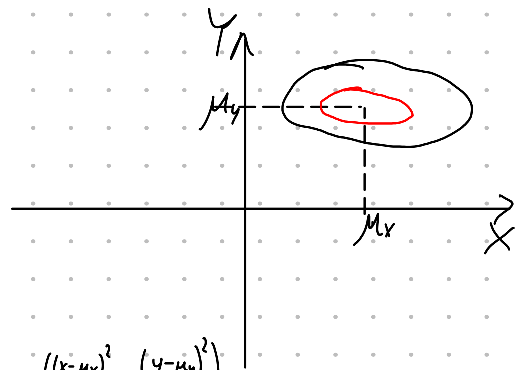
$$= \int_0^l \frac{1}{y}\ln\left(\frac{l}{y}\right)d\left(\frac{y^2}{2l}\right) = \frac{y^2}{2l}\cdot\ln\left(\frac{l}{y}\right)\Bigg|_0^l + \int_0^l \frac{\cancel{y^2}}{2l}\,\frac{y}{l}\left(\frac{l}{\cancel{y^2}}\right)dy = 0 + \int_0^l \frac{y\,dy}{2l} = \frac{l}{4}$$ – В тупую посчитали

$$E[Y] = \int_{-\infty}^{\infty} \underbrace{f_X(x)}_{=\frac{1}{l}}\cdot \underbrace{E[Y|X=x]}_{\frac{x}{2}}dx = \int_0^l \frac{1}{l}\cdot\frac{x}{2}dx = \frac{l}{4}$$

$X, Y \sim N(0,1)$

$$f_{X,Y}(x,y) = f_X(x)\cdot f_Y(y) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\cdot\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{y^2}{2}} = \frac{1}{2\pi}e^{-\frac{x^2+y^2}{2}}$$





$X \sim N(\mu_x, \sigma_x^2)$

$Y \sim N(\mu_y, \sigma_y^2)$

$$f_{X,Y}(x,y) = f_X(x)\cdot f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma_x}e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}\frac{1}{\sqrt{2\pi}\sigma_y}\cdot e^{-\frac{(y-\mu_y)^2}{2\sigma_y^2}} = \frac{1}{2\pi\sigma_x\sigma_y}\cdot e^{-\left(\frac{(x-\mu_x)^2}{2\sigma_x^2}+\frac{(y-\mu_y)^2}{2\sigma_y^2}\right)}$$

Формула Байеса

A_i – разбиение Ω

$$Pr(A_i|B) = \frac{Pr(A_i)\cdot Pr(B|A_i)}{\sum_j Pr(A_j)\cdot Pr(B|A_j)}$$

Про дискретные

$$P_{X,Y}(x|y) \quad \Big| \quad \cancel{P_Y(y)}\cdot P_{X|Y}(x|y) = \frac{p_X(x)\cdot p_{Y|X}(y|x)}{p_Y(y)} = \frac{p_X(x)\cdot p_{Y|X}(y|x)}{\sum_X p_X(x)\cdot p_{Y|X}(y|x)}$$

$$f_Y(y) = f_{X|Y}(x|y) = f_X(x) f_{Y|X}(y|x)$$

$$f_{X|Y}(x|y) = \frac{f_X(x) f_{Y|X}(y|x)}{\int_{-\infty}^{\infty} f_X(x)\cdot f_{Y|X}(y|x)dx}$$

X – дискр.
Y – непр.

$$Pr(X=x \cap y \le Y \le y+\delta) = Pr[X=x]\cdot Pr(Y\in(y,y+\delta)\,|\,\underbrace{X=x}) \approx p_X(x)\cdot f_{Y|X}(y|x)\cdot\delta$$

$$\| $$

$$Pr(Y=[y,y+\delta))\cdot \underbrace{Pr(X=x\,|\,Y\in[y,y+\delta])}_{p_{X|Y}(x|y)} \approx f_Y(y)\cdot\delta\cdot p_{X|Y}(x|y)$$

$$p_X(x) f_{Y|X}(y|x) = f_Y(y)\cdot p_{X|Y}(x|y) \Rightarrow p(x|y) = \frac{p_X(x)\cdot f_{Y|X}(y|x)}{\sum_{x'} p_X(x')\cdot f_{Y|X}(y|x')}$$ – потом пройдём

$$\Downarrow$$

$$f_{Y|X}(y|x) = \frac{f_Y(y)\cdot p_{X|Y}(x|y)}{\int_{-\infty}^{\infty} f_Y(y')\cdot p_{X|Y}(x|y')dy'}$$

$X \to +1$, $p=\frac12$; $X \to -1$, $p=\frac12$

$Pr[X=+1|Y=y]$

$$p_{X|Y}(x|y) = \frac{\frac12 \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-1)^2}{2}}}{\frac12 e^{-\frac{(y-1)^2}{2}}\cdot\frac{1}{\sqrt{2\pi}} + \frac12\cdot\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{(y+1)^2}{2}}} =$$

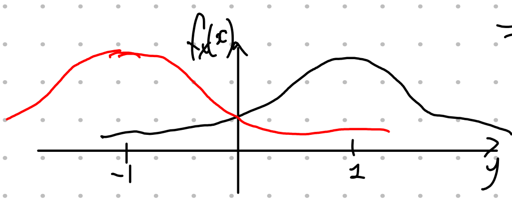
$$= \frac{1}{1+e^{\frac{(y-1)^2-(y+1)^2}{2}}} = \frac{1}{1+e^{-2y}}$$

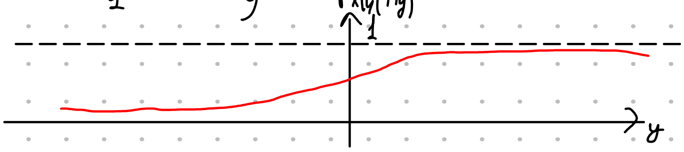
$Z \sim N(0,1)$

$p_X(+1) = \frac12$

$Y = X + Z$

$x = +1 \Rightarrow Y \sim N(1,1)$





Теперь: Наблюдали дискретную, хотим узнать непрерывную

$Y \sim U(0,1) = \Pr(X=1)$

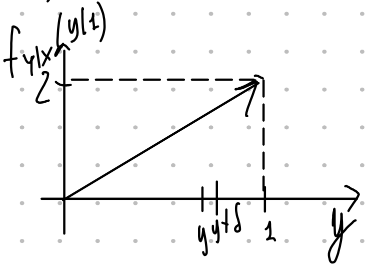
$X \begin{cases} 1 \text{ - орёл} \\ 0, \text{ решка} \end{cases}$

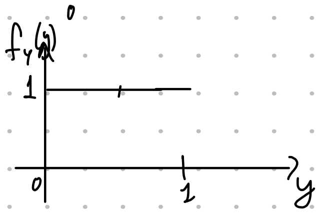
$$f_{Y|X}(y|x) = \frac{f_Y(y)\cdot p_{X|Y}(x|y)}{\int_{-\infty}^{\infty} f_Y(y')\cdot p_{X|Y}(x|y')\,dy'} = f_{Y|X}(y|1) = \frac{f_Y(y)\cdot \overbrace{p_{X|Y}(1|y)}^{y}}{\int_{-\infty}^{\infty} f_Y(y')\cdot p_{X|Y}(x|y')\,dy} =$$

$f_Y(y) = 1 \ (y\in[0,1])$

предполагаем

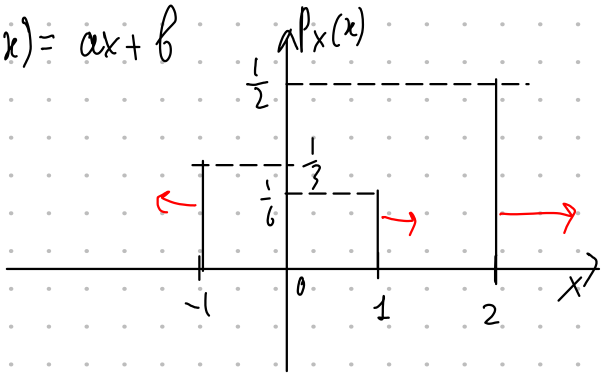
$$= \frac{1\cdot y}{\int_0^1 1\cdot y'\,dy'} = \frac{y}{y'^2\cdot\frac{1}{2}\Big|_0^1} = 2y$$





$X \quad g(x) \Longrightarrow Y = g(X)$ – с. в.

$g(x) = ax + b$



$y(x) = 2x - 3$

$Y = 2X - 3$