7.11 Ф-я от м-цы, привод. к ж.ф.

$$f(x) = \sum_{m=0}^{\infty} C_m x^m \qquad |x| < R$$

$$f(A) = \sum_{m=0}^{\infty} C_m A^m$$

$$A = T J T^{-1}$$

$$J = \begin{pmatrix} \boxed{} & & \\ & \boxed{} & \\ & & \boxed{} \end{pmatrix}$$

$$\downarrow$$

$$f(J) = \begin{pmatrix} \boxed{f(J_r(\lambda))} & \\ & \ddots \end{pmatrix}$$

$$f(J_r(\lambda)) = ?$$

$$J_r(\lambda) = \lambda E_r + I_r$$

$$E_r = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{r\times r} \qquad I_r = \begin{pmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ 0 & & & 0 \end{pmatrix}_{r\times r}$$

$$f(J_r(\lambda)) = \sum_{m=0}^{\infty} c_m \, J_r^m(\lambda)$$

$$(\lambda E + I)^m = \sum_{k=0}^{m} C_m^k I^k \lambda^{m-k} =$$

$$I = \begin{pmatrix} 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix} \quad I^2 = \begin{pmatrix} 0&0&1&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 0&0&0&0 \end{pmatrix} \quad I^3 = \begin{pmatrix} 0&0&0&1 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{pmatrix}$$

$$= \begin{pmatrix} \lambda^m & m\lambda^{m-1} & \frac{m(m-1)}{2!}\lambda^{m-2} & \frac{m(m-1)(m-2)}{3!}\lambda^{m-3} & \cdots \\ & \lambda^m & m\lambda^{m-1} & \ddots & \ddots \\ & & \ddots & \ddots & \ddots \\ & & & m\lambda^{m-1} & \\ 0 & & & \lambda^m & m\lambda^{m-1} \\ & & & & \lambda^m \end{pmatrix}$$

$$I^4 = 0 \qquad \boxed{I_r^r = 0}$$

$$C_m^2 = \frac{m(m-1)}{2!}$$

7.11 ф-я от м-цы, привод. к ж. ф.

$$f(x) = \sum_{m=0}^{\infty} C_m x^m \qquad |x| < R$$

$$f'(x) = \sum_{m=1}^{\infty} C_m m x^{m-1}$$

$$f''(x) = \sum_{m=2}^{\infty} C_m m(m-1) x^{m-2}$$

$$f(tJ_n(\lambda)) = \begin{pmatrix} \sum\limits_{m=0}^{\infty} C_m(\lambda t)^m & \frac{t}{1!}\sum\limits_{m=1}^{\infty} C_m m(\lambda t)^{m-1} & \frac{t^2}{2!}\sum\limits_{m=2}^{\infty} C_m m(m-1)(\lambda t)^{m-2} & \frac{t^3}{3!}\sum\limits_{m=3}^{\infty} C_m m(m-1)(m-2)(\lambda t)^{m-3} & \ldots \\ & \ddots & \ddots & \ddots & \ddots \\ 0 & & & & \end{pmatrix}$$

$$f(At) = T\, f(Jt)\, T^{-1}$$

$$f(tJ_r(\lambda)) = \sum_{m=0}^{\infty} c_m\, J_r^m(\lambda)\, t^m$$

$t \in \mathbb{R}$

$$(\lambda E + I)^m = \sum_{k=0}^{m} C_m^k\, I^k\, \lambda^{m-k} =$$

$$= \begin{pmatrix} f(\lambda t) & t\,\frac{f'(\lambda t)}{1!} & t^2\frac{f''(\lambda t)}{2!} & t^3\frac{f'''(\lambda t)}{3!} & \cdots \\ & \ddots & \ddots & \ddots & \ddots \\ & & \ddots & \ddots & \ddots \\ & 0 & & \ddots & \vdots \\ & & & & f(\lambda t) \end{pmatrix}$$

$$f^{(k)}(\lambda t) = \left(f^{(k)}(x)\right)\Big|_{x=\lambda t}$$

Пример:

$$\tau = \begin{pmatrix} \lambda & 1 & & 0 \\ & \lambda & 1 & \\ & & \lambda & 1 \\ 0 & & & \lambda \end{pmatrix}$$

$$\boxed{\begin{aligned} A &= T \tau T^{-1} \\ f(A) &= T f(\tau) T^{-1} \end{aligned}}$$

$$\sin(\tau t) = \begin{pmatrix} \sin(\lambda t) & t\cos\lambda t & -\frac{t^2}{2!}\sin\lambda t & -\frac{t^3}{3!}\cos\lambda t \\ 0 & \sin\lambda t & t\cos\lambda t & -\frac{t^2}{2!}\sin\lambda t \\ 0 & 0 & \sin\lambda t & t\cos\lambda t \\ 0 & 0 & 0 & \sin\lambda t \end{pmatrix}$$

$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$