UNIVERSITY OF WARSAW

Determination of Thermal Conductivity of Snow by Linear Probe Technology

Summer 2022

Geophysical Laboratory I

Joydeep Sarkar
Registration Number - 437956

Supervised by
dr hab. Konrad Kossacki

Abstract

In this Geophysics Laboratory, we aim to calculate the thermal conductivity of snow, without any sampling from the test material. A linear probe setup was used for the measurements and based on the available data we obtain the value of thermal conductivity. There are situations when it becomes technically impossible to undertake samples from the required materials or objects. For example, when investigating any astrophysical object such as asteroids or planets and then using this technique, the properties of the object are analyzed at the very site, where the observations are carried out.

Contents

1 Introduction 4

2 Experiment 4
2.1 Interpretation of the Experiment 4
2.2 Data 5

3 Data Analysis and Results 5
3.1 Graphs 6
3.2 Calculations 7
3.3 Discussion 8

4 Conclusion 8

1 Introduction

Snow covers quite a small part of the earth's surface. From mountains to glaciers and sometimes in the form of snowfall. Interestingly, the presence of snow around an object (lake, river or anything) reduces the flow of heat by a considerable amount. It's property is thus significant towards the heat flow balance of earth. As a result, it's necessary to understand the thermal conductivity of snow[1].

Thermal conductivity describes the extent of conductivity of a material or it's capability to transfer heat.

$$q = -k\frac{dT}{dZ} \tag{1}$$

In the above equation, k is the thermal conductivity, q is the heat flow of heat and $\frac{dT}{dZ}$ is the temperature gradient. In our experiment, we focus towards understanding the thermal conductivity of snow without taking samples from the provided material. A linear probe setup is used to take measurements. This method is utilised with a further focus towards understanding the properties of astrophysical bodies like comets, planet with the cosmic probes.

2 Experiment

The experiment involves heater of the length $l = 458$ mm, which was inserted into the snow stored in the standard freezer. At the beginning it was not powered and the temperature decreased significantly. When the heating was switched on at a voltage of 21.9 V with current of 0.49 A, the temperature started increasing. The wire used for the experiment has a radius r, density ρ_w, and specific heat c_w with thermal capacity per unit length $S = 2\pi r^2 c_w \rho_w$. The wire is assumed as a perfect heat conductor while neglecting the heat resistance between the wire and the medium. We know that power dissipated per unit length is q, therefore the temperature T of the wire will vary with time t in form of equation[2] below

$$T - T_0 = \frac{2q\alpha^2}{\pi^3\lambda}\int_0^\infty \frac{1-\exp(-atu^2/r^2)}{u^3\Delta(u,\alpha)}du \tag{2}$$

T_0 denotes the initial temperature at the time when the power is just switched on, $a = \lambda/\rho c$ is the thermal diffusivity, and $\alpha = 2\rho c/\rho_w c_w$

2.1 Interpretation of the Experiment

Let us imagine a linear heater of the radius negligible when compared to the length. We know the dissipated power per unit of the length is q, the thermal conductivity of the surrounding medium is λ or k and the initial temperature is $T(0)$ or T_0. The temperature of the heater increases with time and the function $T(t)$ is initially very complex, but asymptotically becomes simple and therefore equation 2 takes the general form[2] in the following way

$$T(t) - T(0) = \frac{q}{4\pi\lambda}\left[-0.5772 + \log\frac{4at}{r^2}\right] \tag{3}$$

$$T(t) - T(0) = \frac{q}{4\pi\lambda}\left[-0.5772 + \log\frac{4a}{r^2} + \log t\right] \tag{4}$$

$$T(t) - T(0) = A + \frac{q}{4\pi\lambda}\log t \tag{5}$$

$$T(t) - T(0) = A + B\log t \tag{6}$$

Thus, this allows us to directly determine λ from the slope of $T(\log t)$.

2.2 Data

The output of the experiment provides us with a range of values necessary for interpretation and further towards understanding the nature of thermal conductivity.

	Seconds	Temp	Temperature(K)
0	0	-16.102	256.898
1	15	-16.020	256.980
2	30	-15.828	257.172
3	45	-15.617	257.383
4	60	-15.406	257.594
...	...	...	...
66	990	-5.364	267.636
67	1005	-5.258	267.742
68	1020	-5.152	267.848
69	1035	-5.093	267.907
70	1040	-5.204	267.796

Figure 1: Values Obtained from the Experiment

3 Data Analysis and Results

To analyse the data, one can use any programming language or software. For my convenience, I have used python and visualized the code in pandas dataframe. Before analysing on the data, we just simplify it a little by converting it into the log format and obtaining the difference between initial and final temperature obtained in the experiment.

	Seconds	Temp	Temperature(K)	T-T(0)	log(Seconds)
1	15	-16.020	256.980	0.082	2.708050
2	30	-15.828	257.172	0.274	3.401197
3	45	-15.617	257.383	0.485	3.806662
4	60	-15.406	257.594	0.696	4.094345
5	75	-15.193	257.807	0.909	4.317488
...	...	...	...	...	...
66	990	-5.364	267.636	10.738	6.897705
67	1005	-5.258	267.742	10.844	6.912743
68	1020	-5.152	267.848	10.950	6.927558
69	1035	-5.093	267.907	11.009	6.942157
70	1040	-5.204	267.796	10.898	6.946976

Figure 2: Modified Values

3.1 Graphs

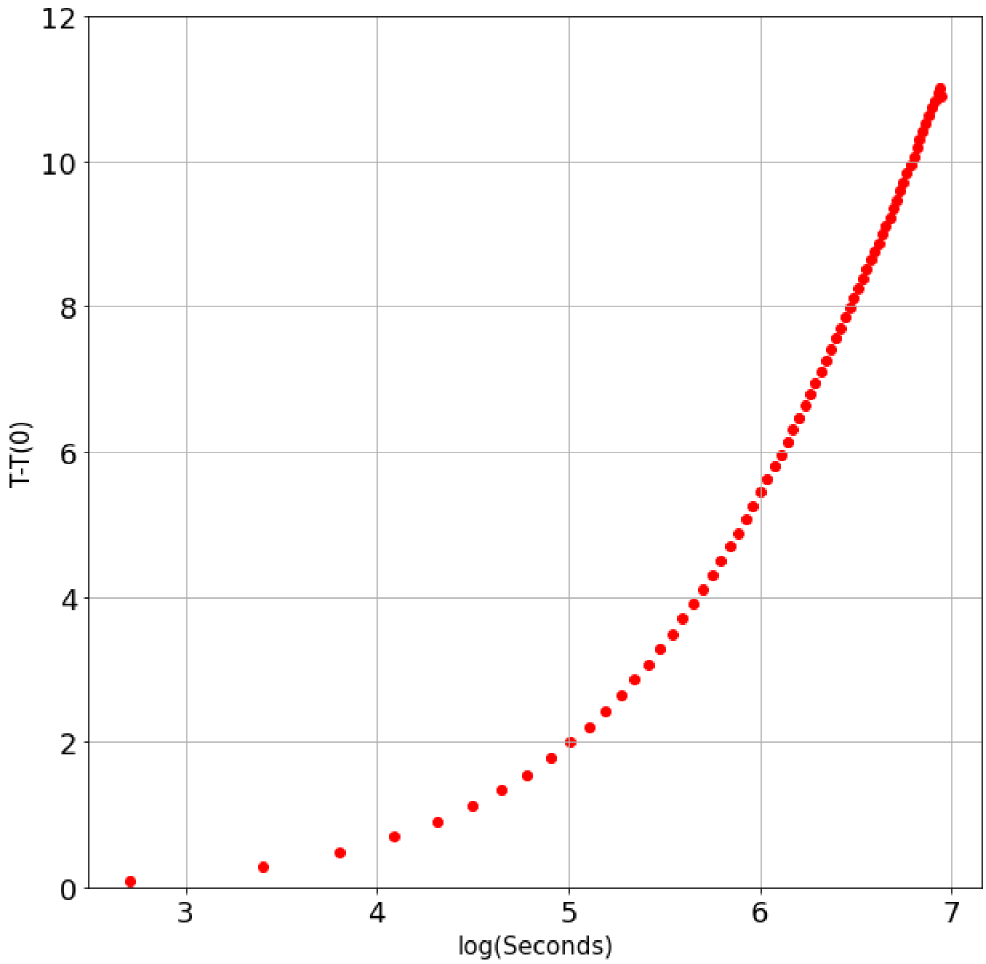

Figure 3: Variation of temperature with time

- Equation 6 is the asymptotic form of general relation i.e. it becomes applicable after some period. Thus, if we take a look at the plot in figure 3, we can decide how many points should be skipped until the rest look linear. This remaining set of data points need to be used to fit the linear function. According to the plot the possible candidate can be the data set corresponding to $\log(t)$ within the range on x axes, such as 6.5 - 7.0 or to the end.

- Thus, using the linear relation between temperature and time, we can now fit in a linear equation to the obtained curve to obtain the values of coefficients and further comparing it to equation 6 to obtain the values of thermal conductivity.

- According to the discussion in point 1, we fit only certain number of data points to the function. Depending on the number of data points it influences the values of coefficients obtained and furthermore towards the uncertainity. Therefore, we can try to explore the change in values of the coefficients. This is illustrated in the Table 1.

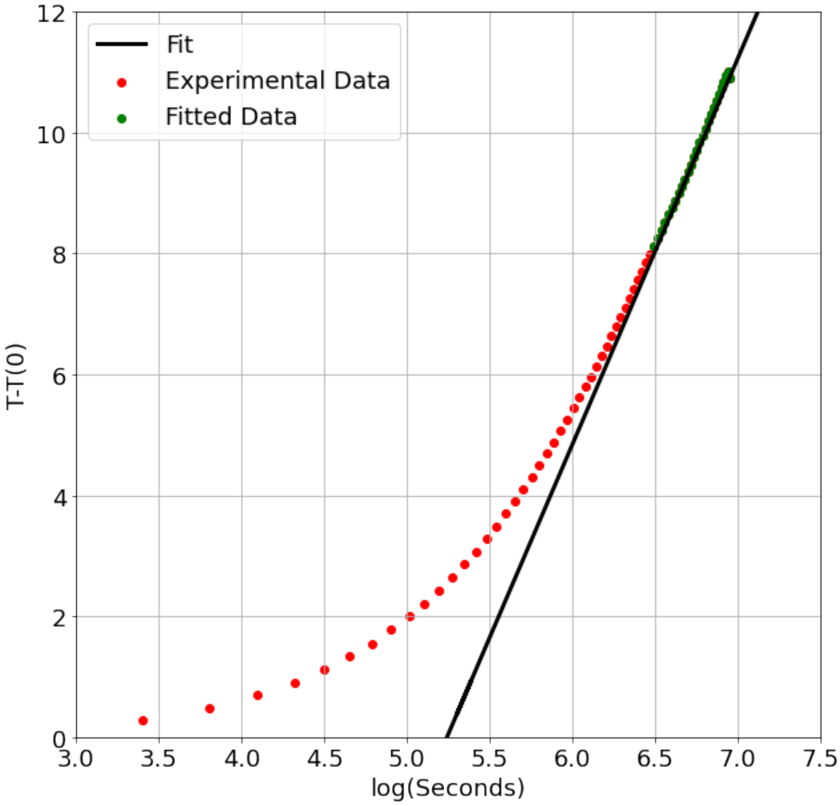

Figure 4: Curve fit to the experimental data

3.2 Calculations

In order to obtain the value of λ or thermal conductivity of snow, we use the values of the coefficients(in this case, we use the coefficient B) obtained by linear curve fitting of the plot in figure 5 and then equate both sides using equation 7. Where,

$$T(t) - T(0) = A + B \log t \tag{7}$$

is of the form $y = ax + b$. Therefore, using the values obtained from curve fit (Figure 5), we can estimate,

$$\begin{aligned} y &= T(t) - T(0) \\ a &= B \\ x &= \log t \end{aligned} \tag{8}$$

The value of coefficient B obtained from the curve fit is,

$$\begin{aligned} B &= 6.388828189813921 \\ B &= \frac{q}{4\pi\lambda} \\ B &= \frac{VI/L}{4\pi\lambda} \\ \lambda &= \frac{0.49 \times 21.9}{4\pi \times 0.498 \times 6.388828189813921} \\ \lambda &= 0.2918\, Wm^{-1}K^{-1} \end{aligned} \tag{9}$$

But, there was an uncertainity/error obtained during curve fit such that $\Delta B = 0.12711913$. Therefore, the uncertainity in λ is given by the differentiation of the 3rd term in equation 9.

$$\Delta\lambda = \frac{1}{4\pi BL}\left[DL \times V \times \frac{I}{L} + DI \times V + DV \times I + DB \times V \times \frac{I}{B}\right]$$
$$\Delta\lambda = 0.0338968493. \tag{10}$$

Where, DL denotes the uncertainity/error of the length l, DI is the error for the current I, DV is for uncertainity in voltage, V and DB is the uncertainity for the coefficient B. The values include, $DL = 0.001$, $V = 21.9$, $I = 0.49$, $L = 0.498$, $DV = 0.05$, $DB = 0.12711913$. Therefore, the value of thermal conductivity obtained from the dataset is 0.2918 $Wm^{-1}K^{-1}$, with an error/uncertainity of $\pm$ 0.0338968493.

We, have also calculated curve fit values of the coefficients based on a varied number of data points. Figure 2 shows for the case, when we fitted only 31 data points. Table 1 below, shows the values for the coefficients and lambda for different data points in our dataset. We can therefore see the difference in the obtained values of lambda, from 31 data points to 22 data points.

Data Points	Coefficient Value B	Error - ΔB	λ	$\Delta\lambda$
31	6.388	0.127	0.2918	0.033896
27	6.43	0.116	0.266814	0.0330
22	6.56	0.0722	0.261	0.0308657

Table 1: Values of Curve Fit for different data points

3.3 Discussion

The following table represents the thermal conductivity of a few materials from everyday lives.

Material	Thermal Conductivity(W/mK)
Air (gas)	0.024
Glass	0.935
Snow	0.05 - 0.85

Table 2: Values of Thermal Conductivity of different materials[3][4][5]

4 Conclusion

The values obtained in Table 1 are well within the limits of current calculated values[4][5] of thermal conductivity. [4][5] provides us with values of thermal conductivity for the cases of seasonal snow, compacted snow and undisturbed snow. We can also see from Table 1 and curve fitting method that with variability in the number of data points taken, provides us with more prominent values of λ and with lower values of uncertainity.

References

[1] S. H. K. Morris, "The thermal conductivity of seasonal snow," *Journal of Glaciology, Vol. 43, No. 143, 1997*, vol. 43, no. 143, pp. 26–41, 1997. [Online]. Available: https://doi.org/10.3189/S0022143000002781

[2] T. S. G. K. M. Banaszkiewicz, K. Seiferlin and N. Kömle, "A new method for the determination of thermal conductivity and thermal diffusivity from linear heat source measurements," *Review of Scientific Instruments*, 1997. [Online]. Available: http://dx.doi.org/10.1063/1.1148365

[3] B. J. Michael Smith1, "The sybil attack," in *A NEW SET OF THERMAL CONDUCTIVITY MEASUREMENTS*, ser. IPTPS '01. Banff, Canada: Springer, Berlin, 7-8 March 2014, pp. 507–510.

[4] "Measuring the thermal conductivity of snow with the tls-100." [Online]. Available: https://thermtest.com/application/snow-thermal-conductivity-tls

[5] F. Riche and M. Schneebeli, "Thermal conductivity of snow measured by three independent methods and anisotropy considerations," *The Cryosphere*, no. 7, p. 217–227, 2013. [Online]. Available: https://tc.copernicus.org/articles/7/217/2013/tc-7-217-2013.pdf