Optimization on a manifold:

L(x,y,λ) = (R|t)' Q (R|t) + (R|t) L + cst + λ [Unit(Rx), Unit(Ry), Unit(Rz), Ortho(Rx,Ry), Ortho(Rx,Rz), Ortho(Ry,Rz)]

Let **Ax = b.** We solve for **x = [R|t,λ]** (we do not care about lambdas). We form the rows of **A,b** so that **Grad(L)_R|t = 0 *(Optimality)*** and **Partial(L)/Partial(λ) = 0 *(Stationarity), ****

See the notes on how we form the matrix next page.*

		A																		x	b	
		Rxr	*Rxs*	*Rxt*	*Ryr*	*Rys*	*Ryt*	*Rzr*	*Rzs*	*Rzt*	*Tx*	*Ty*	*Tz*	*λ0*	*λ1*	*λ2*	*λ3*	*λ4*	*λ5*			
Optimality	d/dRxr													2*Rxr0-1	0	0	Ryr0	Rzr0	0	Rxr	_QRxr	
	d/dRxs													2*Rxs0-1	0	0	Rys0	Rzs0	0	Rxs	_QRxs	
	d/dRxt													2*Rxt0-1	0	0	Ryt0	Rzt0	0	Rxt	_QRxt	
	d/dRyr													0	2*Ryr0-1	0	Rxr0	0	Rzr0	Ryr	_QRyr	
	d/dRys													0	2*Rys0-1	0	Rxs0	0	Rzs0	Rys	_QRys	
	d/dRyt	Q (symmetric, positive definite) with 2x on the diag												0	2*Ryt0-1	0	Rxt0	0	Rzt0	Ryt	_QRyt	L
	d/dRzr													0	0	2*Rzr0-1	0	Rxr0	Ryr0	Rzr	_QRzr	
	d/dRzs													0	0	2*Rzs0-1	0	Rxs0	Rys0	Rzs	_QRzs	
	d/dRzt													0	0	2*Rzt0-1	0	Rxt0	Ryt0	Rzt	_QRzt	
	d/dTx																			Tx	_QTx	
	d/dTy															0				Ty	_QTy	
	d/dTz																			Tz	_QTz	
Stationarity unit.	d/dl0	2*Rxr0	2*Rxs0	2*Rxt0	0	0	0	0	0	0										λ0	_(Rxr0 + Rxs0 + Rxt0) + 2*(Rxr0^2 + Rxs0^2 + Rxt0^2) + 1	
	d/dl1	0	0	0	2*Ryr0	2*Rys0	2*Ryt0	0	0	0										λ1	_(Ryr0 + Rys0 + Ryt0) + 2*(Ryr0^2 + Rys0^2 + Ryt0^2) + 1	
	d/dl2	0	0	0	0	0	0	2*Rzr0	2*Rzs0	2*Rzt0		0				0				λ2	_(Rzr0 + Rzs0 + Rzt0) + 2*(Rzr0^2 + Rzs0^2 + Rzt0^2) + 1	
ortho.	d/dl3	Ryr0	Rys0	Ryt0	Rxr0	Rxs0	Rxt0	0	0	0										λ3	Rxr0*Ryr0 + Rxs0*Rys0 + Rxt0*Ryt0	
	d/dl4	Rzr0	Rzs0	Rzt0	0	0	0	Rxr0	Rxs0	Rxt0										λ4	Rxr0*Rzr0 + Rxs0*Rzs0 + Rxt0*Rzt0	
	d/dl5	0	0	0	Rzr0	Rzs0	Rzt0	Ryr0	Rys0	Ryt0										λ5	Ryr0*Rzr0 + Rys0*Rzs0 + Ryt0*Rzt0	

	: gradient of Err = (R|t)*Q*(R|t) + L*(R|T) + csts	Note:	Taking the gradient Q becomes linear in R|t, L becomes a cst, and cst are zeroed
	: gradient of the linearized rotation constraints	Note:	These do not introduce constant terms since they are functions of lambdas
	: linearized rotation constraints (linear terms)	Note:	Just the linearization, no magic here...
	: there is no constraint in Tx,Ty, Tz	Note:	-
	: these are the lagrangian terms!	Note:	-

*How do we form the matrix:

We want to optimize the quadratic cost:
R*,t* = arg min_{R,t} Err(R,t)
subject to R is a rotation

R is (Rxr, Rxs, Rxt,
Ryr, Rys, Ryt,
Rzr, Rzs, Rzt)

We are given the quadratic cost
Err = (R|t)' Q (R|t) + L (R|t) + cst

R is a rotation if
|Rx| = | Ry | = |Rz| = 1
and Rx.Ry = 0, Rx.Rz = 0, Ry.Rz = 0

Let the lagrangian:
Lag(R,t,λ) = Err + λ0 (|Rx|-1)+ λ1(|Ry|-1) + λ2(|Rz|-1)
+ λ3(Rx*Ry) + λ4(Rx*Rz) + λ5(Ry*Rz)

We want to solve for R*,t*:
dLag / dR |R*,t* = 0 (Optimality = we are at a minimum)
and dLag / dλi |R*,t* = 0 (Stationarity = constraints are satisfied)

Note that dLag/dR is linear in R,t since Err is quadratic in and the constraints are quadratic and bilinear in R,t
To get a closed form solution, we then need dLag/dλi to be linear in R,t, this means the constraints need to be linear.
Then, we linearize, the constraints around R0 = (Rxr0 ... Rzt0), keeping only the first order terms:
Unitary constraints:
|Rx| ~ Rxr0 + Rxs0 + Rxt0
+ (Rxr-Rxr0)*2*Rxr0
+ (Rxs-Rxs0)*2*Rxs0
+ (Rxt-Rxt0)*2*Rxt0 + O(Rxr,Rxs,Rxt)
|Ry| ~ ...
|Rz | ~ ...

Orthogonality constraints:
Rx.Ry ~ Rxr0*Ryr0 + Rxs0*Rys0 + Rxt0*Ryt0
+ (Rxr-Rxr0)Ryr0 + (Ryr-Ryr0)Rxr0
+ (Rxs-Rxs0)Rys0 + (Rys-Rys0)Rxs0
+ (Rxt-Rxt0)Ryt0 + (Ryt-Ryt0)Rxt0 + O(Rxr,Rxs,Rxt,Ryr,Rys,Ryt)

We then write the system of linear equations coming from dLag / dR |R*,t* = 0 (Optimality = we are and dLag / dλi |R*,t* = 0 (Stationarity = constraints are satisfied)in a matrix form:
Ax = b, that we solve for x = [R,t,λ].