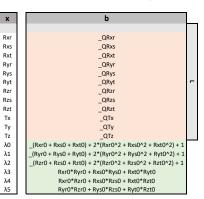
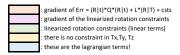
## **Optimization on a manifold:**

 $L(x,y,\lambda) = (R|t)'Q(R|t) + (R|t)L + cst + \lambda[Unit(Rx), Unit(Ry), Unit(Rz), Ortho(Rx,Ry), Ortho(Rx,Rz), Ortho(Rx,Rz)]$ 

Let Ax = b. We solve for x = [R|t,λ] (we do not care about lambdas). We form the rows of A,b so that Grad(L)\_R|t = 0 (Optimality) and Partial(L)/Partial(λ) = 0 (Stationarity), \*

			See the notes on how we form the matrix* next page.																	
											Α									
			Rxr	Rxs	Rxt	Ryr	Rys	Ryt	Rzr	Rzs	Rzt	Tx	Ту	Tz	λ0	λ1	λ2	λ3	λ4	λ5
		d/dRxr													2*Rxr0-1	0	0	Ryr0	Rzr0	0
	(	d/dRxs													2*Rxs0-1	0	0	Rys0	Rzs0	0
	(	d/dRxt	2*Rxt0-1 0 0 Ryt0 Rzt0													Rzt0	0			
_	(	d/dRyr	0 2*Ryr0-1 0 Rxr0 0 Rzr0														Rzr0			
Optimality	(	d/dRys	0 2*Rys0-1 0													0	Rxs0	0	Rzs0	
<u>a</u>	(	d/dRyt		0 (0	umma	tric n	ocitiva	dofin	ita) w	ith 2v	on th	dia	ıσ		0	2*Ryt0-1	0	Rxt0	0	Rzt0
÷	(	d/dRzr	Q (symmetric, positive definite) with 2x on the diag														Rxr0	Ryr0		
8	(	d/dRzs	0													0	2*Rzs0-1	0	Rxs0	Rys0
_	(	d/dRzt													0	0	2*Rzt0-1	0	Rxt0	Ryt0
		d/dTx																		
		d/dTy	0																	
		d/dTz																		
> .		d/dI0	2*Rxr0	2*Rxs0	2*Rxt0	0	0	0	0	0	0									
ri t	Ξ	d/dl1	0	0	0	2*Ryr0	2*Rys0	2*Ryt0	0	0	0									
Ē.	_	d/dl2	0	0	0	0	0	0	2*Rzr0	2*Rzs0	2*Rzt0		0				Λ			
Stationarity	o	d/dl3	Ryr0	Rys0	Ryt0	Rxr0	Rxs0	Rxt0	0	0	0	Ū					U			
Static	Ę	d/dl4	Rzr0	Rzs0	Rzt0	0	0	0	Rxr0	Rxs0	Rxt0									
5	0	d/dl5	0	0	0	Rzr0	Rzs0	Rzt0	Ryr0	Rys0	Ryt0									





Note: Taking the gradient Q becomes linear in R|t, L becomes a cst, and cst are zeroed Note: These do not introduce constant terms since they are functions of lambdas Note: Just the linearization, no magic here...

## \*How do we form the matrix:

Note:

Note:

We want to optimize the quadratic cost:

R\*,t\* = arg min\_{R,t} Err(R,t) subject to R is a rotation

R is (Rxr. Rxs. Rxt.

Ryr, Rys, Ryt, Rzr, Rzs, Rzt)

We are given the quadratic cost Err = (R|t)' Q (R|t) + L (R|t) + cst

| Rx | = | Ry | = | Rz | = 1 and | Rx.Ry = 0, Rx.Rz = 0, Ry.Rz = 0

Let the lagrangian:

Lag(R,t, $\lambda$ ) = Err +  $\lambda$ 0 (|Rx|-1)+  $\lambda$ 1(|Ry|-1) +  $\lambda$ 2(|Rz|-1) +  $\lambda$ 3(Rx\*Ry) +  $\lambda$ 4(Rx\*Rz) +  $\lambda$ 5(Ry\*Rz)

We want to solve for R\*.t\*:

dLag / dR |  $R^*$ ,  $t^* = 0$  (Optimality = we are at a minimum) and dLag / d $\lambda$ i |  $R^*$ ,  $t^* = 0$  (Stationarity = constraints are satisfied)

Note that dLag/dR is linear in R,t since Err is quadratic in and the constraints are quadratic and bilinear in R,t To get a closed form solution, we then need  $dLag/d\lambda i$  to be linear in R,t, this means the constraints

need to be linear.

Then, we linearize, the constraints around R0 = (Rxr0 ... Rzt0), keeping only the first order terms:

Unitary constraints: |Rx| ~ Rxr0 + Rxs0 + Rxt0

- + (Rxr-Rxr0)\*2\*Rxr0

- $\begin{array}{l} + \{ r_{NXS-RXSO} \}^* 2^* RxsO \\ + \{ (Rxt-RxtO)^* 2^* RxtO + O(Rxr, Rxs, Rxt) \\ | Ry | \simeq ... \\ | Rz | \simeq ... \end{array}$

 $\begin{aligned} & \textit{Orthogonality constraints:} \\ & \text{Rx.Ry} & \sim & \text{Rxr0*Ryr0} + \text{Rxs0*Rys0} + \text{Rxt0*Ryt0} \\ & + & \text{(Rxr-Rxr0)Ryr0} + & \text{(Ryr-Ryr0)Rxr0} \end{aligned}$ 

- + (Rxs-Rxs0)Rvs0 + (Rvs-Rvs0)Rxs0
- $+ \left( \mathsf{Rxt}\text{-}\mathsf{Rxt0} \right) \mathsf{Ryt0} + \left( \mathsf{Ryt}\text{-}\mathsf{Ryt0} \right) \mathsf{Rxt0} + \mathsf{O} \left( \mathsf{Rxr}, \mathsf{Rxs}, \mathsf{Rxt}, \mathsf{Ryr}, \mathsf{Rys}, \mathsf{Ryt} \right) \\$

We then write the system of linear equations coming from dLag / dR  $|R^*,t^*| = 0$  (Optimality = we are and dLag / d $\lambda$ i | R\*,t\* = 0 (Stationarity = constraints are satisfied)in a matrix form: Ax = b, that we solve for x = [R,t, $\lambda$ ].