TunePlot

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ABSTRACT

TunePlot is an interactive math and music application created to encourage students in their study of mathematics by providing auditory and visual feedback. This project was created with the purpose of increasing engagement and content knowledge in mathematics with the incorporation of music as an enjoyable feature.

1 INTRODUCTION

Much of mathematics is viewed by many school-aged children as a confusing subject filled with esoteric tools that leaves them feeling like computers [1]. Full of abstraction and seemingly random rules, it is sadly a source of boredom and anxiety for many students[5]. Conversely, music is viewed in a much more positive fashion. It forms a core component of several cultures around the globe and has been proven to have significant and positive effects on people [2].

Stemming from a hypothesis that an increase in engagement and content knowledge will come from mixing mathematical studies with something enjoyable such as music, we present TunePlot: an application aimed at teaching and reinforcing functions and their graphs through auditory and visual feedback. Aimed at students ranging from late middle school to high school, TunePlot takes in MIDI files and allows them to be modified according to certain mathematical transformations. For instance, all the notes in a MIDI file are fit to a curve to create a function $f(x)$ that will go through all the points. This function can then be modified by the user by selecting different values for the variables a, b, c, and d which are applied to the function $f(x)$ as shown in equation (1).

$$a * f(b * x + c) + d \tag{1}$$

Displayed are four basic operations that can be applied to a function; a represents a vertical stretch or shrink, b represents a horizontal stretch or shrink, c represents a horizontal shift, and d represents a vertical shift. These functions change initial MIDI data that was sent into the application through transpositions, inversion, retrogrades etc., altering the graph along with sound of the MIDI data.

The remainder of this paper is comprised of a discussion of the literature that has guided and influenced aspects of our research, the objectives of the TunePlot, along with software design, areas of strength, weaknesses, and future growth.

2 DESIGN AND DEVELOPMENT

2.1 Learning Objectives

This project was developed with the intent of increasing engagement and content knowledge with the incorporation of musical features into an educational tool for mathematics. The Glossary of Education Reform refers to engagement as "the degree of attention, curiosity, interest, optimism, and passion that students show when they are learning or being taught, which extends to the level of motivation they have to learn and progress in their education"[7]. Our target audience for this project includes late middle school to high school students with the intent of having them learn more about various functions and the ways they can transform a graph along with musical ideas through the use of those functions. The musical ideas used in this application are the MIDI data, that is transposed, inverted, retrograded, and put through other transformations.
The development of this application used the Common Core State Standards for Mathematics as a guiding point with particular attention being paid to their High School Function standards. This application can be used for educational topics including interpreting and building functions. The specific standards covered so far with this application include, writing functions, finding inverse of functions, relating the domain of a function to its graph, understanding the function from one set to another, and identifying the effects of changing various parameters of a function [4]. These standards are elaborated below.

- **CSS.MATH.CONTENT.HSF.BF.A.1** Write a function that describes a relationship between two quantities.
- **CCSS.MATH.CONTENT.HSF.BF.B.3** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.
- **CCSS.MATH.CONTENT.HSF.BF.B.4** Find inverse functions.
- **CCSS.MATH.CONTENT.HSF.IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
- **CCSS.MATH.CONTENT.HSF.IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

This application also uses the National Association for Music Education Core Music Standards in the composition and theory strand. The specific standards included in our project incorporate the accomplished track of the creating subsection imagine, which is "describe and demonstrate how sounds and musical ideas can be used to represent sonic events, memories, visual images, concepts, texts, or story-lines"[6].

2.2 Educational Philosophy

Social constructivism, cognitive flexibility theory, and humanistic mathematics were the educational philosophies and theories that guided the development and design of this project. Humanistic mathematics is defined as a "philosophy of teaching and learning which attempts to explore the human side of mathematical thought and to guide students to discover this beauty of mathematics"[9].
With the design of the TunePlot application, the transformations connect to the humanistic mathematics. Stemming from one of our central goals of teaching and reinforcing basic coordinate geometry, the idea that mathematics can create art is implied. As the students become more familiar with what each transformation will do to each function mathematically, they also are able to choose what transformation(s) they apply to create what they want musically. This effectively depicts mathematics as a composer's tool.
The cognitive flexibility theory, "advocates skills with multiple representations of concepts to assist with solving unique, complicated problems of the real world as they are confronted"[11]. This theory has been highlighted multiple times as an important educational principle by many researchers including professors Giles and Frego. In their paper *An inventory of music activities used by elementary classroom teachers: An exploratory study*, they mention a study that suggested "when students have an opportunity to connect more than one subject; it becomes more meaningful"[2][3].
The cognitive flexibility theory connects to the TunePlot application through the integration of two visual representations within its layout. By using a graph and score, students have two visual representations of the transformations, which we believe will aid in their understanding of the transformations. The two visual representations also provide a connection between the mathematics and music.

3 LITERATURE REVIEW

3.1 Learning Interventions Involving Music and Math

Some of the applications and curriculum guides that inspired the creation of TunePlot include Make Music Count, Math and Music (An Interdisciplinary Approach to Transformations of Functions), Soundgrid, and the SkiHill Application. Make Music Count is an application where participants solve mathematical problems in order to reveal encrypted songs. According to its creator Mr. Blackwell Jr, "the ultimate goal for the app is that it will eliminate math phobia in students globally. We need more stem field majors and if we can eliminate the bad stigma attached with learning math we will close the gap and have more scientists and students" [5]. Another resource for TunePlot, the curriculum guide Math and Music, was created to have students take part in activities that connect the compositional process music composers take part in to geometric transformations. The use of functions to alter and transform musical ideas within the curriculum guide Math and Music, was a direct influence on TunePlot and helped to inspire the main educational theme for it. Two other important interdisciplinary educational tools were covered in the literature review including Soundgrid and SkiHill. Soundgrid is a graphic tool which provides auditory feedback along with visual representation of graphical information, and the SkiHill application is a graphing tool for teaching musical meters. The design of Soundgrid was a helpful guide in the design of TunePlot especially with its many representations of data within their application.

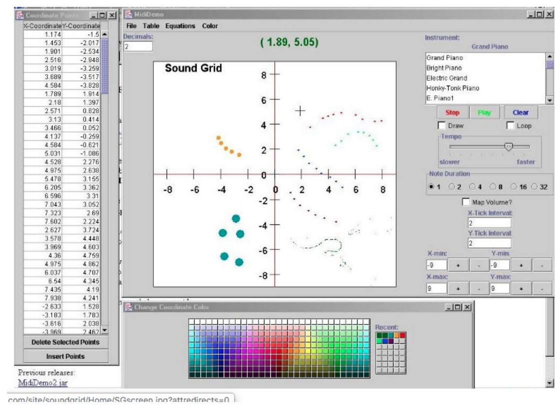

Figure 1: Soundgrid application [12]

The TunePlot application involves the use of mathematical functions in order to transform and alter musical MIDI ideas imported into the application. Although it has drawn inspiration from the many applications mentioned above, TunePlot extends the use of the cognitive flexibility theory by not just providing various visuals of the data altered by functions but also by providing a musical visualization through the music staff on the user interface.

3.2 Mathematics and Music Interdisciplinary Education

During the development of the TunePlot application, many studies incorporating the use of mathematics and music as interdisciplinary educational tools were referenced. These studies involved various grade school student populations and alternative school student populations. Within the interdisciplinary connection of our project design, music takes a subservient role to mathematics. This observation was one of the four styles of Bresler's arts' integration mentioned in Civil's thesis. With this integration style "the arts are used strictly as a vehicle for other academic objectives"[2]. Within An's et al., music is further explored as a passive or active learning process. TunePlot connects to the active role of music with An et al., with music being an "educational resource for contextualizing the teaching and learning of mathematics into a meaningful and relatable medium" [10].

The links between mathematics and music that are emphasized within our literature review include patterns [2][10] along with geometric transformations. Within An et al., the authors mention "for example, repeating patterns have been found in almost all work of music, often contained within small sections or a whole movement, and growing patterns....as well as geometric transformations.... have been massively used by both classical and present-day composers" [10]. In Upson's article, he also mentions similar objectives between mathematics and music education, including organizing meaningful expressions with the use of representative symbols, and problem solving strategies incorporating "comparison, temporal order, and inferring"[11].

3.3 Increasing Mathematical Engagement with Music

Several studies that have offered foundational evidence for the development of TunePlot, mention the presence of "mathematical anxiety", along with negative stereotypes being placed on mathematics [1]. As Bahn-James' article discussed, "these stereotypes cause many students to fail "to realize not only their full potential as students but also the full potential of mathematics as a creative discipline"[1]. An et al., mentions many positive impacts of integrating music and mathematics in education, including "motivating student to undertake more challenging mathematics tasks, engaging students in examining relationships among mathematical concepts..." [10]. This information directly connects to the hypothesis behind TunePlot's creation. In another article, music was also mentioned as helping students to experiment with the other subjects to enhance retention [2]. TunePlot seeks to create an engaging learning environment through the use of music with mathematics. In many ways, like with humanistic mathematics, we seek to incorporate a "philosophy of teaching and learning which attempts to explore the human side of mathematical thought and to guide students to discover this beauty of mathematics"[9].Further helping to create an appreciation for mathematics "as a creative, collaborative, and exciting endeavor"[9].

3.4 Educational Benefits of Music integration into Mathematics

TunePlot's incorporation of music into a mathematical based educational application, provides many benefits for learners and supports the educational philosophies that have helped to develop this project. Spatial Reasoning was cited by several articles [10][2][8] considered for this project as a benefit of musical training. One of the articles that mentioned the testing of spatial reasoning due to musical training includes An et al. In this article, they discussed the controversial Mozart effect study series where participants in the Mozart music group excelled beyond their peers when tested on the Stanford-Binet Intelligence Scale in spatial reasoning skills [10]. An et al., also mentioned research into the correlation between musical training and mathematical achievement, along with this possibly being connected to the stimulation of particular parts of the brain often associated with mathematical reasoning when people participate in musical activities[10]. These studies were guiding points when deciding to incorporate music into a mathematics based application, because they provided insight into other benefits besides increased engagement.

4 PRINCIPLES OF SOFTWARE DESIGN

The principles guiding our software design included the need for audio visual feedback, interactivity, auditory feedback, and multiple representations of data. TunePlot currently has a mapping design of one to one with the MIDI input being converted by the application and outputting a transformed version of the original MIDI input. Even though music currently takes a subservient role in this application, the goal for this project is to have future development geared towards a co-equal cognitive style where we "incorporate objectives that require both cognitive skills as well as aesthetic principles. This style places the arts objective on the same importance level with the other subjects"[2].

With the original design, the layout included a MIDI selector window, a main graph plotting window, a MIDI roll at the top, a music notation window at the bottom, and a functions' window on the left. The evolution of software design for this application went through several stages with the help of the programming languages MaxMSP and JavaScript, along with the digital audio workstation Ableton Live.

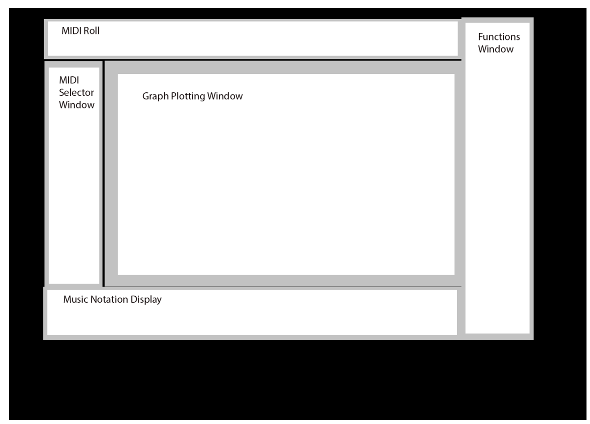

Figure 2: TunePlot original design.

Some of the most pertinent aspects of TunePad's application include the function transformations, the musical representations of the transformations, the visualizations of the MIDI data, and sound design. The transformations for TunePlot were first inspired by the elementary ideas of vertical and horizontal stretches, shifts, and flipping a function over the x-axis and/or y-axis. These then came to fruition as the transformations inversion, retrograde, retrograde-inversion, and transpose up/down. These were originally going to be implemented by performing each transformation on a function f(x) received through a curve fitting algorithm. Significant road blocks were experienced in the implementation of this due to trouble importing necessary packages into JavaScript, so instead the transformations were applied on each of the individual coordinate pairs in a way that would offer the same result.

In an effort to raise the ceiling of the application and give more control to the user, another transformation was added that allows the user to manipulate the values a, b, c, and d as they apply to the function $a * f(b * x + c) + d$. Like the previous transformations, this idea was implemented by performing the transformation on each coordinate pair separately. At the time of writing, functionality exists only for values of a and d due to complications with the Max/MSP object that is used to plot the coordinate pairs. As a result, future work in terms of transformations includes adding transformations that are not linear in nature, adding in functionality for the values b and c, and implementing more transformations that further give control and creative potential to the user.

A musical representation of the transformations was added to the bottom of the user interface and uses the MaxMSP object bach.score. During the transformations, when the new pitches are calculated, they are also converted to cents. The duration of each note is also recorded and both the cents and the durations are sent to bach.roll, which then displays the notes on bach.score. When the user transforms the points on the graph, the bach.score will transform as well and display the new notes after the transformation. We believe the staff is helpful for those with a musical background. The score allows students to understand the transformations musically, while the graph allows students to understand the transformations mathematically.

With the MIDI melodies, the focus was on visualizing MIDI on a 2-Dimensional Graph and sonifying the graphs in real time as the melody progresses. During this aspect of the project many challenges came about with using graphing environments to play audio out in real time. A fair bit of time was spent in implementing and refining MIDI pitches and durations to work as intended. This still has room for refinement in the future.

With the creation of the sound design, multiple directions were explored to achieve this. The cycle object was initially used, a sinusoidal wave synthesizer within MaxMSP, and then we switched to using BEAP synthesizers due to lack of fine grained control over pitch and timbre. The final sound design ended up using Ableton Live's Analog synthesizer with an automated pitch bend to get a continuous audio output.

Below is a visual of the sound design through Ableton Live. The tuner down at the bottom, is a verification method of the correct pitch for the MIDI transformations.

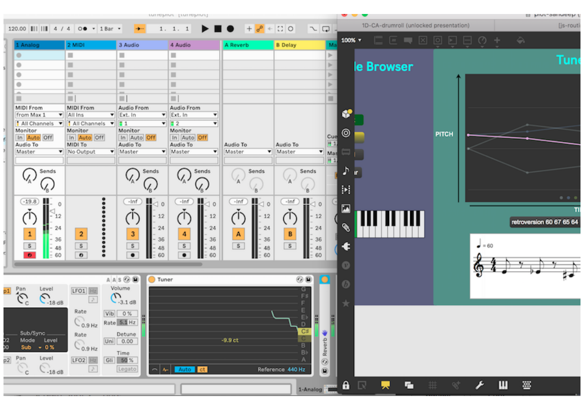

Figure 3: Ableton Live Programming Implementation

Figure 4 shows the prototype design for TunePlot.

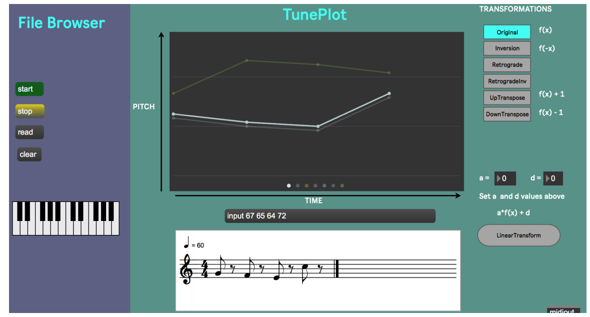

Figure 4: TunePlot prototype design

5 DISCUSSION AND FUTURE WORK

Further research and evaluation into the TunePlot application would involve the creation of a curriculum which would be used to assess participants in the study. The targets that would be evaluated in the study include usability, reliability, and learning outcomes.

Usability will be evaluated with the question, what was the criteria at which someone can pick this application up and understand what is going on? This will be analyzed qualitatively by evaluating how far through concepts and different activities participants in the study can go, along with asking them questions including how long did it take you to feel comfortable with using this application? The last question could also be evaluated quantitatively by evaluating their comfort level with the application on a numerical scale and evaluating how long it takes them to become comfortable with the application on a scale represented by minutes.

Another target that will be evaluated includes reliability. This will be answered through simple yes and no questions, that can be combined on a quantitative scale. These questions include, does sound produced upon hitting a function represent the graph displayed, and how often does the application crash?

One of the main learning outcomes for this application is to teach students about math through making music. With future curriculum development, the learning outcomes can be evaluated by presenting participants with a pre-test assessing them on a particular mathematical topic dealing with a certain aspect of learning about functions. This can be followed by having them participate in a lesson that covers this topic then assessing them with a post-test to see how much they have learned. A qualitative assessment can be added to this inquiring into if the addition of music helped with the learn-ability of the topic along with if it helped to keep their interest in the topic.

Strengths of this project include the various representations of data including the visual graph for students to use and the score underneath the graph. Other strengths include that it is interactive, and it actively sonifies mathematical transformations. Some of the weaknesses of the current application include the need to raise the learning ceiling of the transformations, and that the main graphing window (the plot object) is not labeled so it may be hard to understand how much the functions change the graph.

Some of the areas where further development is being pursued includes polyphonic possibilities, the inclusion of more diverse instrumental timbres, the need to raise the ceiling on mathematical transformations, and developing further expressive capabilities for the TunePlot application to help maintain student engagement. In the future, we also plan to continue working on the sonic design of the application with setup involving one platform instead of our current setup using Max and Ableton Live.

REFERENCES

[1] Tara Bahn-James. 1991. The Relationship Between Mathematics and Music: Secondary School Student Perspectives. In *The Journal of Negro Education (Volume. 60, No. 3)*.

[2] Marie Duplessy Civil. 2007. *Using Music to Improve Learning in Mathematics*.

[3] A. M. Giles and R. J. Frego. 2004. An inventory of music activities used by elementary classroom teachers: An exploratory study. Applications of Research in Music Education, 13–22.

[4] Common Core State Standards Initiative. 2019. Standards for Mathematical Practice. Common Core State Standards Initiaive. http://www.corestandards.org/Math/Practice/

[5] Marcus Blackwell Jr. 2019. Make Music Count Application.

[6] NAfME. 2014. Core Music Standards (Composition-Theory Strand. National Association for Music Education. https://nafme.org/core-music-standards/

[7] Great Partnership. 2016. Student Engagment Definition.

[8] Laia Villadot, Caroline Hilton, Albert Casals, Jo Saunders, Carmen Carrillo, Jennie Henley, Cristina González-Martin, Montserrat Prat and Graham Welch. 2017. The integration of music and mathematics education in Catalonia and England: perspectives on theory and practice. In *Music Education Research 20:1*. Informa UK Limited, trading as Taylor Francis Group, 71–82. https://doi.org/10.1080/14613808.2017.1290595

[9] Raymond F. Tennant. 1999. Interdisciplinary Teaching Strategies in the World of Humanistic Mathematics. https://www.mi.sanu.ac.rs/vismath/tennant1/index.html

[10] Song A. An, Daniel A. Tillman, and Lawrence M. Lesser. 2017. The Hidden Musicality of Math Class: A Transdisciplinary Approach to Mathematics Education. In *Transdiciplinarity in Mathematics Education: Blurring Disciplinary Boundaries*. publisher=Springer International Publishing, isbn=978-3-319-63624-5, 25–45. https://doi.org/10.1007/978-3-319-63624-5_2

[11] Robert Upson. 2002. Educational Sonification Exercises: Pathway for Mathematics and Music Achievement. Georgia Institute of Technology.

[12] Robert Upson. 2013. Soundgrid. https://sites.google.com/site/soundgrid/Home