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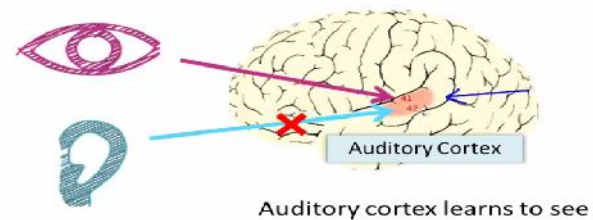
## Machine Learning: Artificial Neural Network

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## About Human Brain

### Human Brain

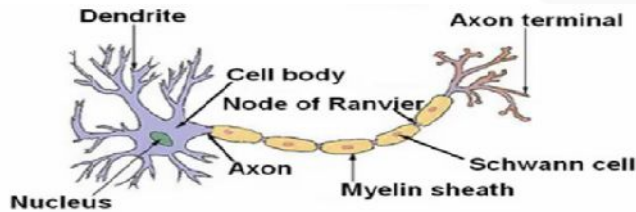
- ✓ Does loads of crazy things
  - Hypothesis is that the brain has a single learning algorithm
- ✓ Evidence for hypothesis
  - Auditory cortex --> takes sound signals
    - If you cut the wiring from the ear to the auditory cortex
    - Re-route optic nerve to the auditory cortex
    - Auditory cortex learns to see



Human echolocation: <https://www.youtube.com/watch?v=A8lzt1tu4o>

## Neural Networks

- ✓ How do we represent neural networks (NNs)?
  - Neural networks were developed as a way to simulate networks of neurons
- ✓ How does a neuron look like



A **neural network** is a set of connected input/output units (neurons) where each connection has a weight associated with it.

In an artificial neural network, a neuron is a logistic unit

- Feed input via input wires
- Logistic unit does computation
- Sends output down output wires that logistic computation is just like our previous logistic regression hypothesis calculation

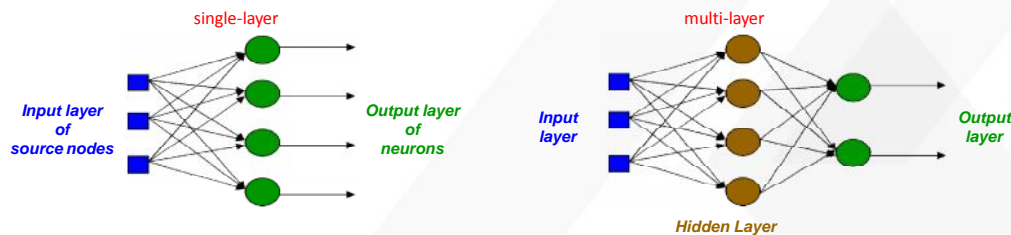
## Neural Networks

- ✓ Why do we need neural networks?
  - Complex Non Linear Hypothesis
  - Good way to build classifiers when N is large
- ✓ **Neural networks (NNs)** were originally motivated by looking at machines which replicate the brain's functionality looked at here as a machine learning technique
- ✓ **Origins**
  - To build learning systems, why not mimic the brain?
  - Used a lot in the 80s and 90s
  - Popularity diminished in late 90s
- ✓ **Recent major resurgence**
  - NNs are computationally expensive, so only recently large scale neural networks became computationally feasible

## Network Architectures

- Three different classes of network architectures
  - single-layer feed-forward
  - multi-layer feed-forward
  - recurrent
- The architecture of a neural network is linked with the learning algorithm used to train

} neurons are organized in acyclic layers



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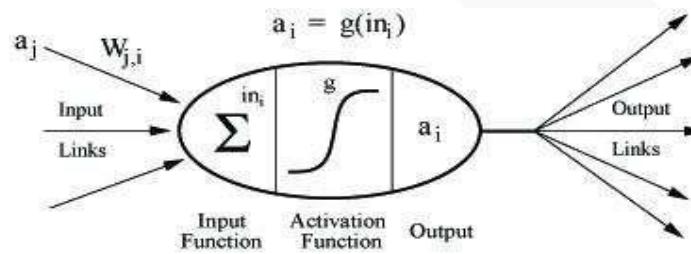
## Neural Networks

- **Advantages**
  - prediction accuracy is generally high
  - robust, works when training examples contain errors or noisy data
  - output may be discrete, real-valued, or a vector of several discrete or real-valued attributes
  - fast evaluation of the learned target function
- **Criticism**
  - parameters are best determined empirically, such as the network topology or structure
  - long training time
  - difficult to understand the learned function (weights)
  - not easy to incorporate domain knowledge

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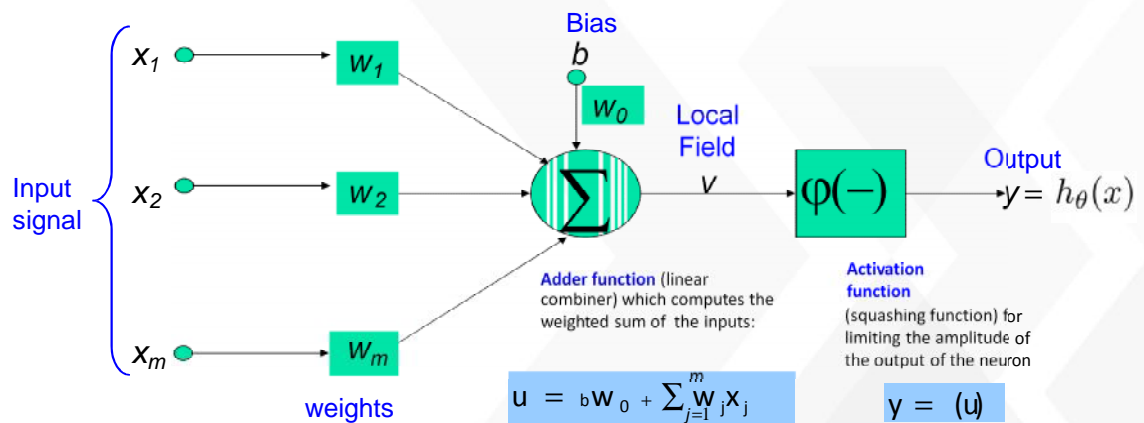
## Neurons

- Neural networks are built out of a densely interconnected set of simple units (**neurons**)
  - Each neuron takes a number of real-valued inputs
  - Produces a single real-valued output
  - Inputs to a neuron may be the outputs of other neurons.
  - A neuron's output may be used as input to many other neurons



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## The Neuron

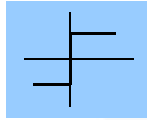


Bias: serves to vary the activity of the unit

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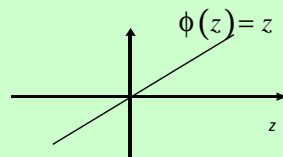
## How does it Works?

- Assign weights to each input-link
- Multiply each weight by the input value (0 or 1)
- Sum all the weight-firing input combinations
- Apply squash function, e.g.:
  - If  $\text{sum} > \text{threshold}$  for the Neuron then
  - Output = +1
  - Else Output = -1

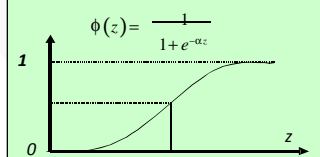


## Popular activation functions

**Linear activation**

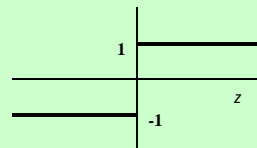


**Logistic activation**



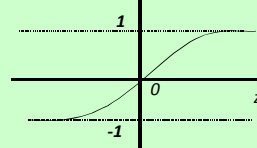
**Threshold activation**

$$\phi(z) = \text{sign}(z) = \begin{cases} 1, & \text{if } z \geq 0, \\ -1, & \text{if } z < 0. \end{cases}$$



**Hyperbolic tangent activation**

$$\phi(u) = \tanh(\gamma u) = \frac{1 - e^{-2\gamma u}}{1 + e^{-2\gamma u}}$$

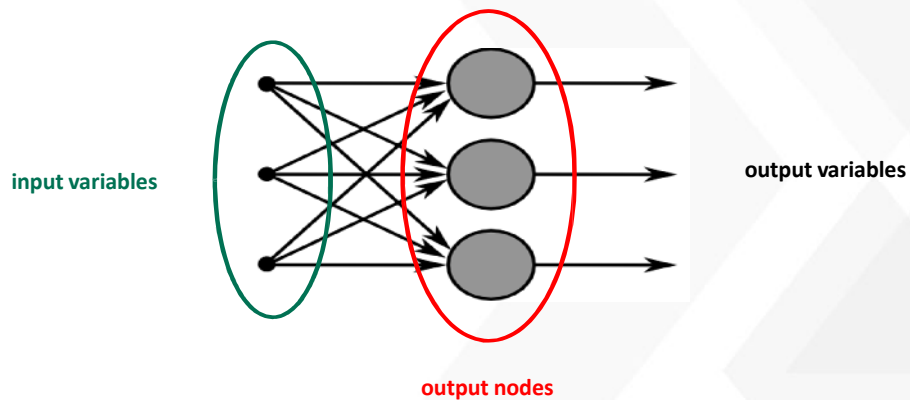


## How Are Neural Networks Trained?

- Initially
  - choose small **random weights** ( $w_i$ )
  - Set **threshold** = 1 (step function)
  - Choose small **learning rate** ( $r$ )
- Apply each member of the **training set** to the neural net model using a **training rule** to adjust the weights
  - For each unit
    - Compute the net input to the unit as a linear combination of all the inputs to the unit
    - Compute the output value using the activation function
    - Compute the error
    - Update the weights and the bias

## Single Layer Perceptron

Are the simplest form of neural networks



## Single layer perceptron: training rule

- Modify the *weights* ( $w_i$ ) according to the *Training Rule*:

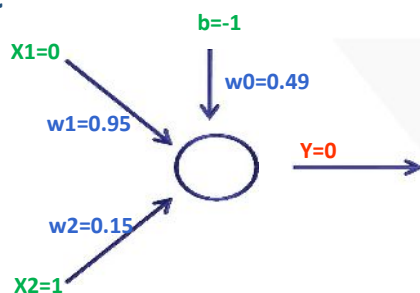
$$w_i = w_i + r \cdot (t - a) \cdot x_i$$

- $r$  is the *learning rate* (eg. 0.2)
- $t$  = target output
- $a$  = actual output
- $x_i$  =  $i$ -th input value

Learning rate: if too small learning occurs at a small pace, if too large it may stuck in local minimum in the decision space

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## Example



x1	x2	Y
0	0	0
1	0	1
0	1	1
1	1	1

threshold = 0.5  
 $r=0.05$

**Compute output for the input**  $u = -1 \times 0.49 + 0 \times 0.95 + 1 \times 0.15 = -0.34 < t$   
thus,  $y=0$

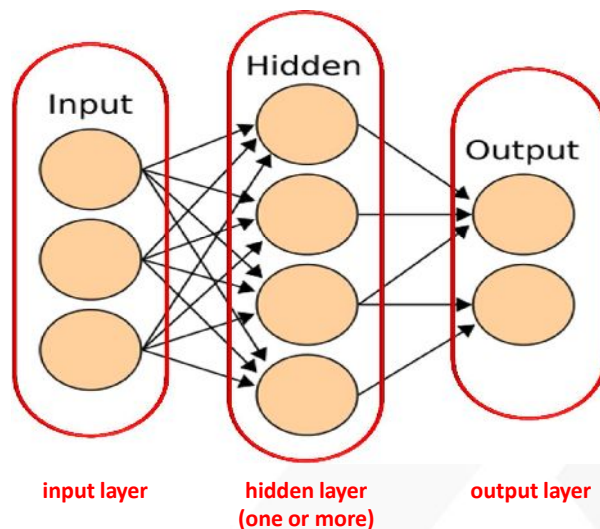
**Compute the error**  
target output = 1  
actual output ( $y$ ) = 0  
error =  $(1-0) = 1$   
correction factor = error  $\times r = 0.05$

**Compute the new weights**  
 $w0 = 0.49 + 0.05 \times (1-0) \times (-1) = 0.44$   
 $w1 = 0.95 + 0.05 \times (1-0) \times 0 = 0.95$   
 $w2 = 0.15 + 0.05 \times (1-0) \times 1 = 0.20$

Repeat the process  
with the new weights  
for a given number of  
iterations

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## Multi layer network

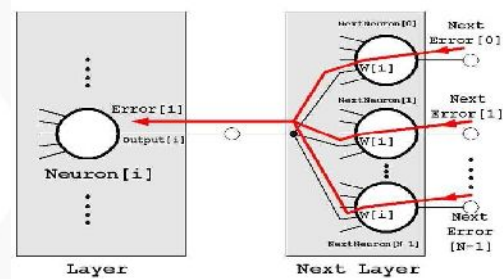


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## Training multi layer networks

### back-propagation algorithm

- **Phase 1: Propagation**
- Forward propagation of a training input
- Back propagation of the propagation's output activations.
- **Phase 2: Weight update**
- For each weight-synapse:
- Multiply its output delta and input activation to get the gradient of the weight.
- Bring the weight in the opposite direction of the gradient by subtracting a ratio of it from the weight.
- This ratio influences the speed and quality of learning. The sign of the gradient of a weight indicates where the error is increasing, this is why the weight must be updated in the opposite direction.
- Repeat the phase 1 and 2 until the performance of the network is good enough.



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## Multi-Layer network of sigmoid units

Problem: what is the desired output for a hidden node? => Backpropagation algorithm

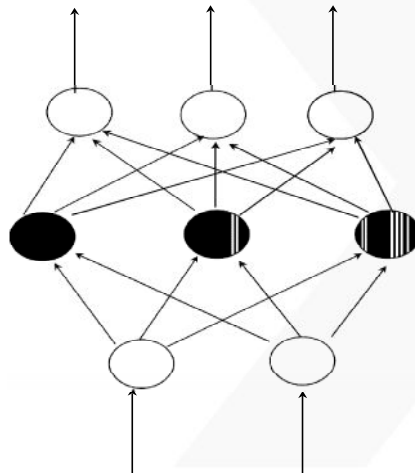
Output vector

Output nodes

Hidden nodes

Input nodes

Input vector:  $x_i$



$$j = j + (r)Err_j$$

to update the bias

$$w_{ij} = w_{ij} + (r)Err_j O_i$$

to update the weights

$$Err_j = O_j(1 - O_j) \sum_k Err_k w_{jk}$$

error for a node in the hidden layer

$$Err_j = O_j(1 - O_j)(T_j - O_j)$$

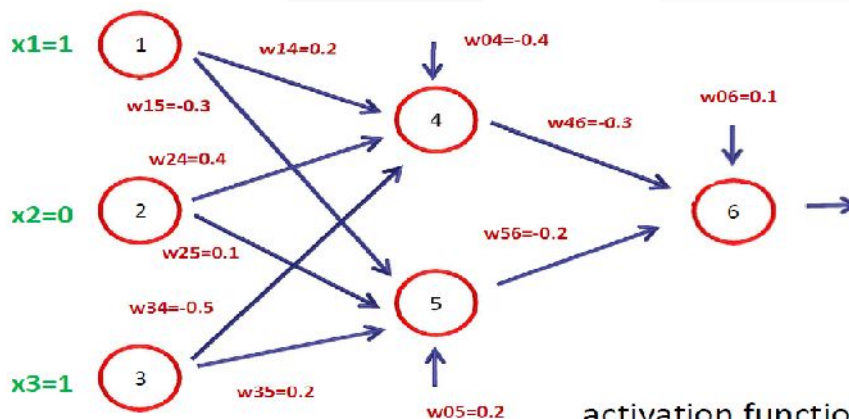
error for a node in the output layer

$$O_j = \frac{1}{1 + e^{-I_j}}$$

$$I_j = \sum_i w_{ij} O_i + j$$

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## Example

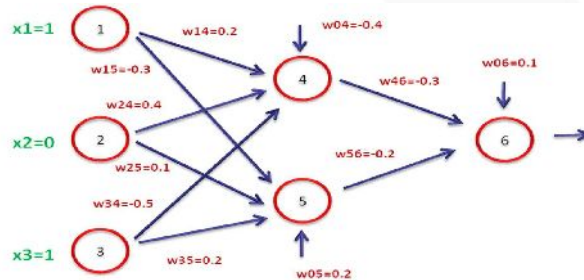


$x_i$  – input variables (1,0,1) whose class is 1  
 $w_{ij}$  – randomly assigned weights

activation function  
 $O_j = 1 / (1 + e^{-I_j})$   
 and  
 learning rate = 0.9

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## Propagation



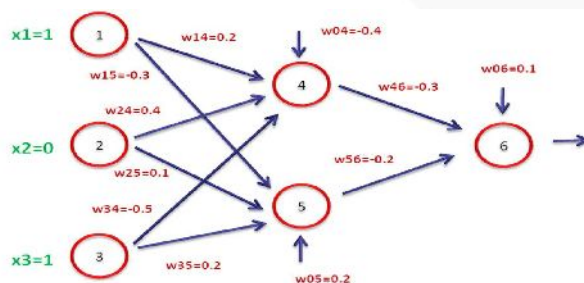
$$I_j = \sum_i w_{ij} O_i + \theta_j$$

$$O_j = \frac{1}{1 + e^{-I_j}}$$

neuron	input	output
4	$0.2 \times 1 + 0.4 \times 0 - 0.5 \times 1 - 0.4 = -0.7$	$1/(1 + e^{0.7}) = 0.332$
5	$-0.3 \times 1 + 0.1 \times 0 + 0.2 \times 1 + 0.2 = 0.1$	$1/(1 + e^{-0.1}) = 0.525$
6	$-0.3 \times 0.332 - 0.2 \times 0.525 + 0.1 = -0.105$	$1/(1 + e^{0.105}) = 0.474$

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## Calculation of the neuron



error for a node in the output layer

$$Err_j = O_j(1 - O_j)(T_j - O_j)$$

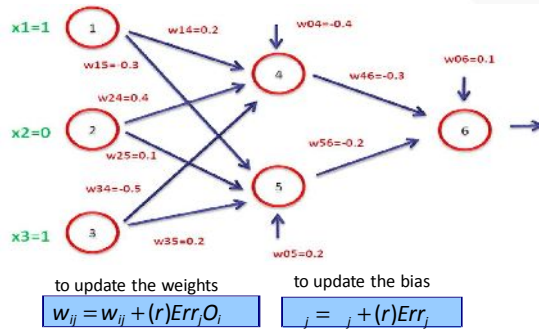
error for a node in the hidden layer

$$Err_j = O_j(1 - O_j) \sum_k Err_k w_{jk}$$

neuron	output	neuron	error
4	0.332	6	$0.474 \times (1 - 0.474) \times (1 - 0.474) = 0.1311$
5	0.525	5	$0.525 \times (1 - 0.525) \times (-0.2) \times 0.1311 = -0.0065$
6	0.474	4	$0.332 \times (1 - 0.332) \times (-0.3) \times 0.1311 = -0.0087$

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## Updating weights

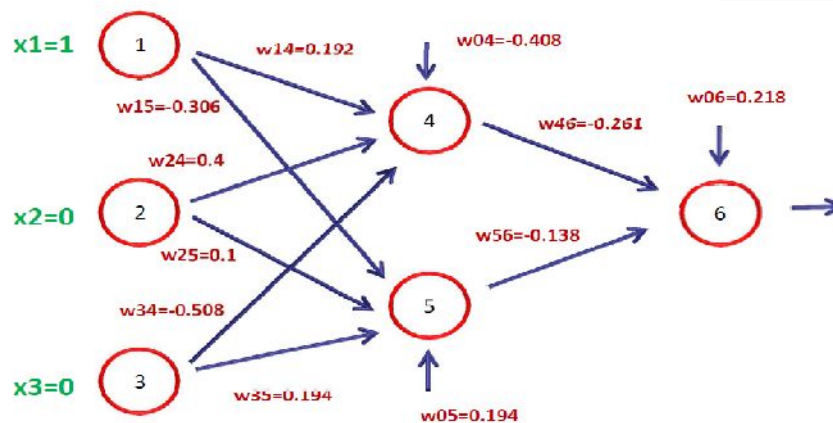


neuron	output	error
4	0.332	-0.0087
5	0.525	-0.0065
6	0.474	0.1311

weight	New value
w46	$-0.3 + 0.9 \times 0.1311 \times 0.332 = -0.261$
w56	$-0.2 + 0.9 \times 0.1311 \times 0.525 = -0.138$
w14	$0.2 + 0.9 \times -0.0087 \times 1 = 0.192$
w15	$-0.3 + 0.9 \times -0.0065 \times 1 = -0.306$
w24	$0.4 + 0.9 \times -0.0087 \times 0 = 0.4$
w25	$0.1 + 0.9 \times -0.0065 \times 0 = 0.1$
w34	$-0.5 + 0.9 \times -0.0087 \times 1 = -0.508$
w35	$0.2 + 0.9 \times -0.0065 \times 1 = 0.194$
w06	$0.1 + 0.9 \times 0.1311 = 0.218$
w05	$0.2 + 0.9 \times -0.0065 = 0.194$
w04	$-0.4 + 0.9 \times -0.0087 = -0.408$

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## Example



This is the resulting network after the first iteration. We now have to process another training example until the overall error is low or we run out of examples.

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## Neural Networks (Cost Function)

The (regularized) logistic regression cost function is as follows;

$$J(\theta) = -\frac{1}{m} \left[ \sum_{i=1}^m y^{(i)} \log h_{\theta}(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)})) \right] + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$$

For neural networks our cost function is a generalization of this equation above, so instead of one output we generate  $k$  outputs

$$h_{\Theta}(x) \in \mathbb{R}^K \quad (h_{\Theta}(x))_i = i^{th} \text{ output}$$

$$J(\Theta) = -\frac{1}{m} \left[ \sum_{i=1}^m \sum_{k=1}^K y_k^{(i)} \log(h_{\Theta}(x^{(i)}))_k + (1 - y_k^{(i)}) \log(1 - (h_{\Theta}(x^{(i)}))_k) \right] + \frac{\lambda}{2m} \sum_{l=1}^{L-1} \sum_{i=1}^{s_l} \sum_{j=1}^{s_{l+1}} (\Theta_{ji}^{(l)})^2$$

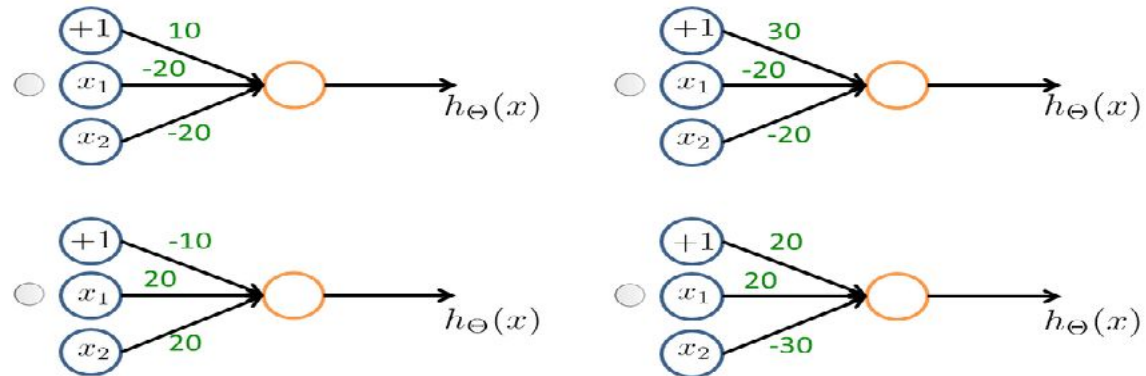
## Minimize Neural Network's Cost Function (Back Propagation)

### Back propagation

- ✓ Back propagation basically takes the output you got from your network, compares it to the real value ( $y$ ) and calculates how wrong the network was (i.e. how wrong the parameters were)
- ✓ It then, using the error you've just calculated, back-calculates the error associated with each unit from the preceding layer (i.e. layer  $L - 1$ )
- ✓ This goes on until you reach the input layer (where obviously there is no error, as the activation is the input)
- ✓ These "error" measurements for each unit can be used to calculate the **partial derivatives**
- ✓ We use the partial derivatives with gradient descent to try minimize the cost function and update all the  $\Theta$  values
- ✓ This repeats until gradient descent reports convergence

## Quiz

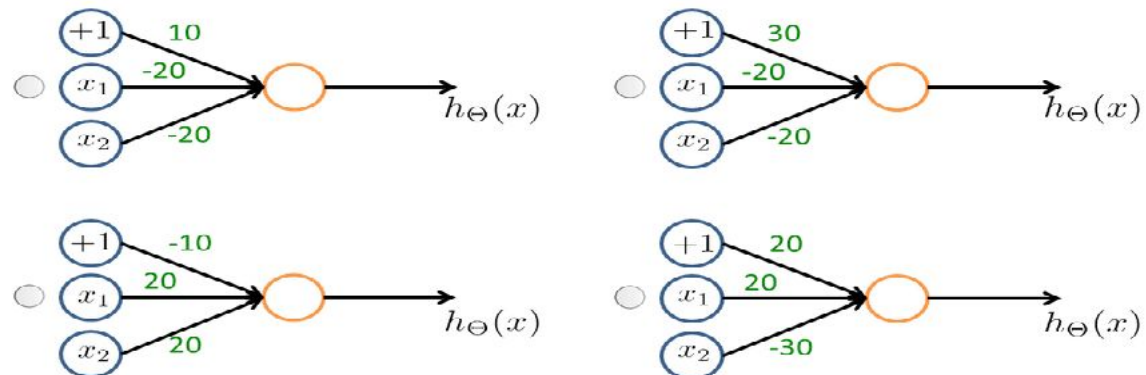
Suppose that  $x_1$  and  $x_2$  are binary valued (0 or 1). Which of the following networks (approximately) computes the boolean function (NOT  $x_1$ ) AND (NOT  $x_2$ )?



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## Quiz

Suppose that  $x_1$  and  $x_2$  are binary valued (0 or 1). Which of the following networks (approximately) computes the boolean function (NOT  $x_1$ ) AND (NOT  $x_2$ )?



The Answer: C

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