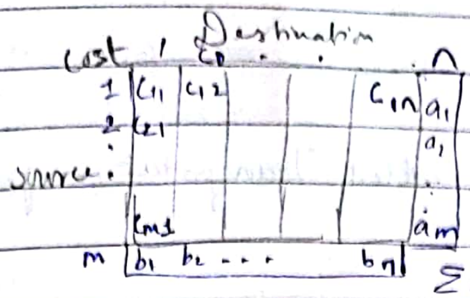
10/12/18

Page No.:
Date

UNIT – V

Transportation Models



$\Sigma a_i = \Sigma b_j$

Transportation problem is to transport various amounts of commodities from various sources to different destinations in such a way That the total transportation cost is minimum.
The availability and requirements are finite. It is assumed that the cost of shipping is linear.

Mathematical formulation.

Let no of origins = m
availability of i th source is given by ~~A~~ a_i.
Let n denote the number of destinations.
Requirement of each desn is denoted by b_j
~~Cost of~~ Let C_{ij} be the cost of shipping 1 unit from source i to destination j.
It is assumed that the total availability = total requirement

i.e $$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

~~The problem now is to determine non-negative~~

Let x_{ij} be the amt to be shipped from ~~X~~ ith source to jth destinate. The problem now is to determine x_{ij} so as to minimise the cost $z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij}$ ~~the~~ (objective function) such that $\sum_{j=1}^{n} x_{ij} = a_i$ and $\sum_{i=1}^{m} x_{ij} = b_j$

$x_{ij} \geq 0$

This is a special case of LPP called Transportation problem.

Tabular Representation

Let m denote the number of factories. Let W denote the no of warehouses

Given

cost	W_1	W_2	$\cdots$	W_n	Availability
F_1	c_{11}	c_{12}	$\cdots$	c_{1n}	a_1
F_2	c_{21}	c_{22}		c_{2n}	a_2
$\vdots$	$\vdots$				$\vdots$
F_m	c_{m1}			c_{mn}	a_m
Requirement	b_1	b_2	$\cdots$	b_n	$\Sigma a_i = \Sigma b_j$

To determine

cost	W_1	W_2	$\cdots$	W_n	Availability
F_1	x_{11}	x_{12}	$\cdots$	x_{1n}	a_1
F_2	x_{21}	x_{22}			a_2
$\vdots$					$\vdots$
F_m				x_{mn}	a_m
Requirement	b_1	b_2	$\cdots$	b_m	$\Sigma a_i = \Sigma b_j$

Feasible Solution

A set of non-negative individual allocations x_{ij} which simultaneously removes deficiencies is called feasible solution

Optimum Solution

A feasible soln is said to be optimal if it minimises the total transportation cost.

Basic Feasible solution

A feasible solution is said to be basic if the no of allocations are $m+n-1$ where m is the no of sources and n is the no of destinations. If no of allocations is less than $m+n-1$ it is called as degenerate BFS, otherwise it is non-degenerate.

Methods for Finding Initial BFS.

1. Northwest corner Rule
2. Row minima method
3. Column minima method
4. Lowest cost entry method (matrix minima method)
5. Vogel's approximation method (unit cost penalty method)

Q1) Find the initial BFS for the follow problem using all the methods

Page No.:
Date:

W→ / F↓	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	34

1) Northwest corner rule

	W_1	W_2	W_3	W_4	
F_1	[19] 5	[30] 2	[50]	[10]	7 / 2 / 0
F_2	[70]	[30] 6	[40] 3	[60]	9 / 3 / 0
F_3	[40]	[8]	[70] 4	[20] 14	18 / 14 / 0
	5	8	7	14	34
	5/0	/6/0	/4	/0	

Transportation cost =

$5\times 19 + 2\times 30 + 6\times 30 + 3\times 40 + 4\times 70 + 14\times 20$

$= 1015.$

2) Row minima method

	W_1	W_2	W_3	W_4	
F_1	[19]	[30]	[50]	[10] 7	7 / 0
F_2	[70]	[30] 8	[40] 1	[60]	9 / 1 / 0
F_3	[40] 5	[8]	[70] 6	[20] 7	18 / 11 / 6 / 0
	5/0	8/0	7/6/0	14/7/0	34

Note: The smallest cost cell in the 1st row of the remaining matrix.

Scanned by CamScanner

transportation cost = $7 \times 10 + 30 \times 8 + 40 + 40 \times 5 + 70 \times 6 + 20 \times 7$

$= 1110$

3) Column minima method.

	W1	W2	W3	W4	
F1	[19] 5	[30] —	[50] —	[10] 2	7 / 2 / 0
F2	[70] —	[30] —	[40] 7	[60] 2	9 / 2 / 0
F3	[40] —	[8] 8	[70] —	[20] 10	18 / 10 / 0
	5/0	8/0	7/0	14/12/10/0	

transportation cost $= 5 \times 19 + 10 \times 2 + 7 \times 40 + 60 \times 2 + 8 \times 8 + 20 \times 10$

$= 799$

4)

	W1	W2	W3	W4	
F1	[19] —	[30] —	[50] —	[10] 7	7 / 0
F2	[70] 2	[30] —	[40] 7	[60] —	9 / 2 / 0
F3	[40] 3	[8] 8	[70] —	[20] 7	18 / 10 / ~~7~~ 3 / 0
	5/2	8/0	7/0	14/7/0	

transportation cost $= 7 \times 10 + 2 \times 70 + 40 \times 7 + 3 \times 40 + 8 \times 8 + 7 \times 20$

$= 814$

Vogel's Approximation Method.

					Penalty	
19 (5)	30	50	10 (2)	7/2/0	9	9
70	30	40 (7)	60 (2)	9/2	10	20
40	8 (8)	70	20 (10)	18/10/0	12	20
5/0	8/0	7/0	14/4/2			
Penalty 21	22	10	10			
21	—	10	10			

→ Penalty :- It is the difference b/w the smallest 2 values in every row & column.

→ To select the largest penalty and choose the corresponding row or column. There make allocation to the cell which has the least cost.

⇒ Again calculate new penalties repeat till all allocation are made.

—	—	10	10
—	—	10	50
—	—	40	60

$$T.C = 19\times5 + 20 + 40\times7 + 60\times2 + 8\times8 + 200$$
$$= \underline{\underline{779}}$$

Scanned by CamScanner

2) Find the transportation cost for the initial (CBFS) using all the methods

	I	II	III	IV	
A	21	16	15	13	11
B	17	18	14	23	13
C	32	27	18	41	19
	6	10	12	15	43

Warehouse

40 40 —

20 20 20

50 — —

21 — 6	16 — 5	15	13	11/5/0
17	18 — 5	14 — 8	23	13/8/0
32	27	18 — 4	41 — 15	19/15
6/0	10/5/0	12/4/0	15	

N-W method. = T. C = 1095

Row Minima Method.

21	16	15	13 — 11	11/0
17	18	14 — 12	23 — ~~8~~	13/1
32 — 5	27 — 10	18	41 — 4	19/9/4
6/5	10/0	12/0	15/4	

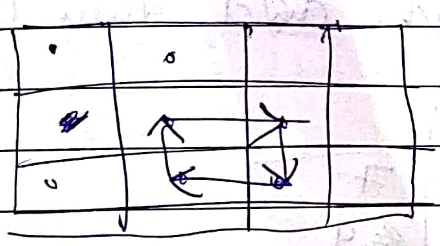
A BFS is said to be non-degenerate if it has the foll properties.

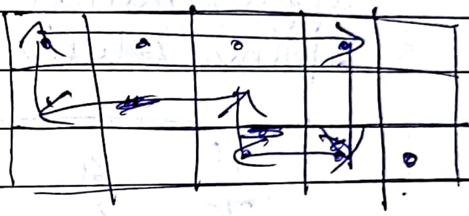
1) Initial B.F.S must contain exactly $(m+n-1)$ no of individual allocations.

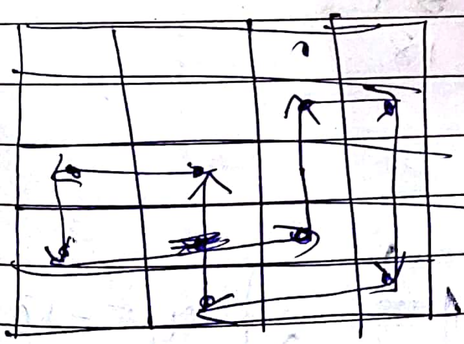
2) These allocations must be in independent positions.

Independent positions of a set of allocations means that it is always impossible to form any closed loop through these allocations.

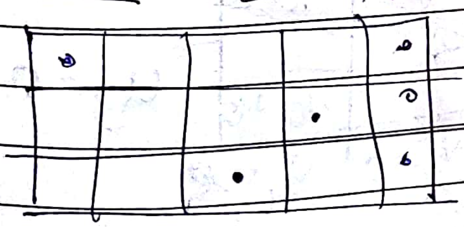
Non-independent allocations







Independent allocations.



Finding optimal solution.

Step 1) examine initial BFS for non-degeneracy if it is degenerate some modification is required which will be discussed later.

Step 2) i) Determine net evaluations for empty cells.
ii) Optimality test of current solution.
iii) Select the entering variable.
iv) Select the leaving variable.
v) Repeat the steps till optimal solution is reached.

Transportation Algorithm for

(MODI Method)

⇒ Consider the foll. transportation ~~the~~ problem. Determine the transpⁿ. cost using ~~Bap~~ Vogel's approximation method and then find the optimal solution

19	30	50	10	7
70	30	40	60	9
40	8	70	20	18
5	8	7	14	

VAM

19 / 5	30 / 32	50 / 60	10 / 2	$u_1 = -10$
70 / 1	30 / -18, θ	40 / 7	60 / 2−θ	$u_2 = 40$
40 / 11	8 / 8−θ	70	20 / 10+θ	$u_3 = 0$
$v_1 = 29$	$v_2 = 8$	$v_3 = 0$	$v_4 = 20$	

Min (8−θ, 2−θ)

θ = 2

19 5	30	50	10 2	$u_1 = -10$
70 +	30 2	40 7	60 +	$u_2 = 22$
40 +	8 6	70 +	20 12	$u_3 = 0$

$v_1 = 29$ $v_2 = 8$ $v_3 = 18$ $v_4 = 20$

∴ As all $d_{ij} > 0$

T.C = 5×19 + . . .

=

Find B.F.S using LCM and obtain the optimum solution for the foll problem.

50	30 1	220	1/0
90 2	45 1	170	3/2/0
250 2	200	50 2	4/2
4/2	2/1	2/0	

30×1 + 90×2 + 45×1 + 500
+ 100
= 855

Date:

Optimum solution for Transportation Model

Find the initial BFS using ~~L~~ L, CM and obtain an optimum solution for the following problem

	To			Available
from	50	30	220	1
	90	45	170	3
	250	200	50	4
Required	4	2	2	

$\boxed{50}$ —	$\boxed{30}$ 1	$\boxed{220}$ —	1/0
$\boxed{90}$ 2	$\boxed{45}$ 1	$\boxed{170}$	3/2 /0
$\boxed{250}$ 2	$\boxed{200}$	$\boxed{50}$ 2	4/2
4/2/0	2/1/0	2/0	

T.C = 855

Optimum solution

$d_{ij} > 0$

I

			u_i
$\boxed{50}$ -25 θ	$\boxed{30}$ $1-\theta$	$\boxed{220}$ +	$u_1 = -15$
$\boxed{90}$ $2-\theta$	$\boxed{45}$ $1+\theta$	$\boxed{170}$ +	$u_2 = 0$
$\boxed{250}$ 2	$\boxed{200}$ -5	$\boxed{50}$ 2	$u_3 = 160$

$v_1 = 90$ $\quad v_2 = 45$ $\quad v_3 = -110$

Scanned by CamScanner

$d_{ij} = c_{ij} - (u_i + v_j)$

50) $= 50 - (90 - 15)$
$= -25$

$\text{Min}(1-\theta, 2-\theta)$
$\theta = 1$

if $\theta = 1$

II

50 1	30 +	220 +	$u_1 = -40$
90 $1+\theta$	45 $2-\theta$	170 +	$u_2 = 0$
250 $2-\theta$	200 -5 θ	50 2	$u_3 = 160$
$v_1 = 90$	$v_2 = 45$	$v_3 = -110$	

Degeneracy

Cost $= 830$

$\theta = 2$

III

To

From				
	50 1	30	220	$u_1 = -40$
	90 3	45 0	170	u_2 0
	250	200 2	50 2	$u_3 = 155$

~~Cost = 820~~

$v_1 = 90$ $v_2 = 45$ $v_3 = -105$

Min Cost $= 820$

$d_{ij} = c_{ij} - (u_i + v_j)$

$50) \quad = 50 - (90 - 15)$

$= -25$

$\text{Min}(1-\theta, 2-\theta)$

$\theta = 1$

if $\theta = 1$

II

$\boxed{50}$ 1	$\boxed{30}$ +	$\boxed{220}$ +	$u_1 = -40$
$\boxed{90}$ $1+\theta$	$\boxed{45}$ $2-\theta$	$\boxed{170}$ +	$u_2 = 0$
$\boxed{250}$ $2-\theta$	$\boxed{200}$ θ -5	$\boxed{50}$ 2	$u_3 = 160$
$v_1 = 90$	$v_2 = 45$	$v_3 = -110$	

Degeneracy

Cost = 830

$\theta = 2$

III

To

From				
	$\boxed{50}$ 1	$\boxed{30}$	$\boxed{220}$	$u_1 = -40$
	$\boxed{90}$ 3	$\boxed{45}$ 0	$\boxed{170}$	$u_2 = 0$
	$\boxed{250}$	$\boxed{200}$ 2	$\boxed{50}$ 2	$u_3 = 155$

~~Cost = 820~~

$v_1 = 90 \quad v_2 = 45 \quad v_3 = -105$

Min Cost = 820

Scanned by CamScanner

Page No.:
Date:

Q3 Is $x_{13} = 50$, $x_{14} = 20$, $x_{21} = 55$, $x_{31} = 30$, $x_{32} = 35$, $x_{34} = 25$ and optimal soln to the following transportation problem if not find the optimal solution

6	1 35	9	3 35	70/35/0
11 5	5	2 50	8	55/5/0
10 80	12	4	7 10	90/80
85/5/0	35/0	50/0	45/10/0	

TrC = 2010

6	1	9 50	3 20	$U_1 = -4$
11 55	5	2	8	$U_2 = -7$
10 30	12 35	4	7 25	$U_3 = 0$
$V_1 = 10$	$V_2 = 12$	$V_3 = 11$	$V_4 = 7$	

Scanned by CamScanner

1) Find the optimum solution of the problem using VAM as the initial B.F.S.

					Availability
4	2	3	2	6	8
5	4	5	2	1	12
6	5	4	7	3	14
Request 4	4	6	8	8	30 / 34

The problem is unbalanced.

						Penalty
4	2 (4)	3	2 (4)	6	0	8/4/0 2/0/1/1
5	4	5	2 (4)	1 (8)	0	12/4/0 1/1/1/3
6 (4)	5	4 (6)	7	3	0 (4)	14/10/6/3/4/0 3 —
4	4/0	6	8/4/0	8/0	4/0	
penalty 1	2	1	0	2	0	
1	2	1	0	2	—	
1	—	1	0	2	—	
1	—	1	0	—	—	

17/12/18

Assignment Models

In assignment problems, the objective is to assign a number of jobs to equal number of people at a minimum cost or maximum profit. If there are n jobs to be performed and n persons available and if c_{ij} is the cost for assigning ith person to the jth job then the assignment problem is to find an assignment so that the total cost is the minimum.

Mathematical formulation

Cost matrix

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & & & \\ c_{n1} & c_{2n} & \cdots & c_{mn} \end{bmatrix}$$

$x_{ij} = \begin{cases} 1 & \text{is ith person in assigned jobs} \\ 0, & \text{if not} \end{cases}$

objective fn

Min cost, $z = \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij}$

$\sum_{j=1}^{n} x_{ij} = 1$ (one job to ith person)

$\sum_{i=1}^{n} x_{ij} = 1$ (one person to jth job)

Page No.:
Date:

Algorithm for assignment problem (Hungarian method)

Step 1: Subtract the min of each row from all the elements of the respective rows, to obtain row reduced matrix.

Step 2: From the row reduced matrix subtract the min of each column from all the elements of the respective columns.

Step 4: draw the min num of horizontal & vertical lines cover ~~the~~ all the 0's in the resulting matrix.

Step Let the minimum number of lines be N. Now there are two possibilities

1) If N = n then an optimal assignment can be made.

2) If N < n ~~then~~ proceed to step 5.

Step 3: Test whether it is possible to make 0 assignment as follows.

1) Starting with row 1 examine the rows one by one until a row containing exactly ~~zero~~ single zero element is found, mark it with □.
Now cross all other zeroes in that column.

2) when the rows are examined identical procedure is applied to columns. Continue these successive operations until all zeroes are either assigned or crossed. If no unmarked zero is left then process ends otherwise go to step 4.

Step 5: Determine the smallest element in the matrix not covered by n lines. Subtract this element from all the uncovered elements and add this element at the intersection of horizontal and vertical lines.

Scanned by CamScanner

Page No :
Date:

Repeat steps 3, 4 and 5 until each row & each column has exactly one zero assignment.

A department has 4 subordinates and 4 tasks to be performed. Time taken is given in the matrix. How should the tasks be allocated to each person in order to minimise the total cost.

Subordinates

Tasks		I	II	III	IV
	A	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

Step 1: obtain row reduced matrix

0	18	9	3
9	24	0	20
23	4	3	0
9	16	14	0

Step 2: obtain the column reduced matrix

$\boxed{0}$	14	9	3
9	20	$\boxed{0}$	22
23	$\boxed{0}$	3	$\cancel{0}$
9	12	14	$\boxed{0}$

Soln is

A → I, B → III, C → IV

Cost = 8 + 4 + 19 + 10 = 41

(P2) A car hire company has 1 car at each of 5 depots a, b, c, d, e. A customer requires a car in each town namely A, B, C, D, E. Distance in km is given. make assignment to minimise the distance.

	Depots a	b	c	d	e
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

Step 1: ~~60 30 45 30 40~~

30	0	45	60	70
15	0	10	40	55
30	0	25	60	75
0	0	30	30	60
20	0	35	45	70

105
35
0

Step 2.

30	$\boxed{0}$	35	30	75
15	0	$\boxed{0}$	10	0
30	0	35	30	20
$\boxed{0}$	0	20	0	5
20	0	25	15	15

15	~~0~~	20	15	$\boxed{5}$
15	15	$\boxed{0}$	10	~~0~~
15	$\boxed{0}$	~~20~~	15	5
$\boxed{0}$	15	20	~~0~~	5
5	~~0~~	10	$\boxed{0}$	0

solution is

A → e

B → c

C → b

D → a

E – d

cost: 200 + 130 + 110 + ~~140~~ 50 + 80

= 570

Scanned by CamScanner

Page No.:
Date:

Q3) in Transportation problem

	A	B	C	D	Available
F_1	6	1	9	3	70
F_2	11	5	2	8	55
F_3	10	12	4	7	90
	85	35	50	45	

6	1 35	9	3 35	70/35/0
11 5	5	2 50	8	55/5/0
10 80	12	4	7 10	90/80
85/5/0	35/0	50/0	45/10/0	

T.C = 2010

45
30
10