UNIT - I

Definitions of OR.

1.) OR is a scientific approach to problem solving for executive management.

2) OR is a scientific knowledge through interdisciplinary team effort for purpose of determining best utilization of limited resources.

3) OR is concerned with scientifically deciding how to design and operate man machine system in the best maner usually under condition requiring the allocations of limiting resources.

OR Models:

1.) Linear Programming model.
2.) Transportation model.
3.) Assignment model.
4) network model.
5) Game theory / decision model
6) Queuing model.

7) Simulation model.

8) Goal Programming.

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Phases of OR Model.

1) Definition of the problem.
2) Construction of the model (LPP)
3) Solution of the model.
4) Validation of the model.
5) Implementation of the model.

Linear Programming.

1) Decision variables:
2) Objective function.
3) Maximize / Minimize.

*1) Q) Company A,B,C produces both exterior & interior paints from 2 raw materials. M1 & M2. Following table provides the basic data of the problem.

	Tons of Raw material per ton of exterior paint	interior paint	Max daily available
Raw material M1	6	4.	24
Raw material M2	1	2.	6.
profit per ton ($1,000)	5	4.	

ABC wants to determine the optimum (best) product mix of interior & exterior paints that maximises the total daily profits.

* A market survey indicates that the daily demand for interior paint does not exceed ext paint by more than 1 ton. Also max. daily demand for int paint is 2 ton.

Soln →

x_1 = tons produced daily of exterior paints.

x_2 = tons produced daily of interior paints.

profit from exterior paint = $5x_1$ (thousand \$)

profit from interior paint = $4x_2$ (")

$\boxed{\max Z = 5x_1 + 4x_2}$ (objective function)

Subjected to constraints,

$6x_1 + 4x_2 \leq 24$ (M1)

$x_1 + 2x_2 \leq 6$ (M2)

$x_2 - x_1 \leq 1$ (Market limit)

$x_2 \leq 2$ (Demand limit)

ability $x_1 \geq 0$ $x_2 \geq 0$ (Non-negetivity constraint)

Steps

Mathematical formulaⁿ of LPP.

General form

Find n decision variables $x_1, x_2 \dots x_n$ to maximize/ minimize the objective function

$$Z = c_1 x_1 + c_2 x_2 \dots\dots c_n x_n$$

Satisfying m constraints.

$$a_{11} x_1 + a_{12} x_2 \dots\dots a_{1n} x_n \ (\leq, =, \geq) \ b_1$$

$$a_{21} x_1 + a_{22} x_2 \dots\dots a_{2n} x_n \ (\leq, =, \geq) \ b_2.$$

$\vdots$

$$a_{m1} x_1 + a_{m2} x_2 \dots\dots a_{mn} x_n \ (\leq, =, \geq) \ b_n.$$

$\vdots$

$$x_1, x_2 \dots\dots x_n \geq 0$$

Q) company manufactures FM radios and calculator. The radios contribute ₹ 100 / unit & calci's ₹ 150/ unit as profit. Each radio requires 4 diodes and 4 ~~registers~~ sistors while each calci req. 10 diodes & 2 resistors. A radio takes 12 mins & calci takes 9.6 mins of time on the company electronic testing machine & product manager estimates that 160 hrs of test time is available. The firm has 8000 diodes & 3000 resistors in the stock. Formulate the problem as LPP.

<u>solⁿ</u> →

$x_1 \to$ no. of FM radio

$x_2 \to$ no. of calculator

$$\text{Max } Z = 100x_1 + 150x_2.$$

subjected to constraints.

$4x_1 + 10x_2 \leq 8000$ (diodes)

$4x_2 + 2x_2 \leq 3000$ (resistors).

$12x_1 + 9.6x_2 \leq 9600$ (time)

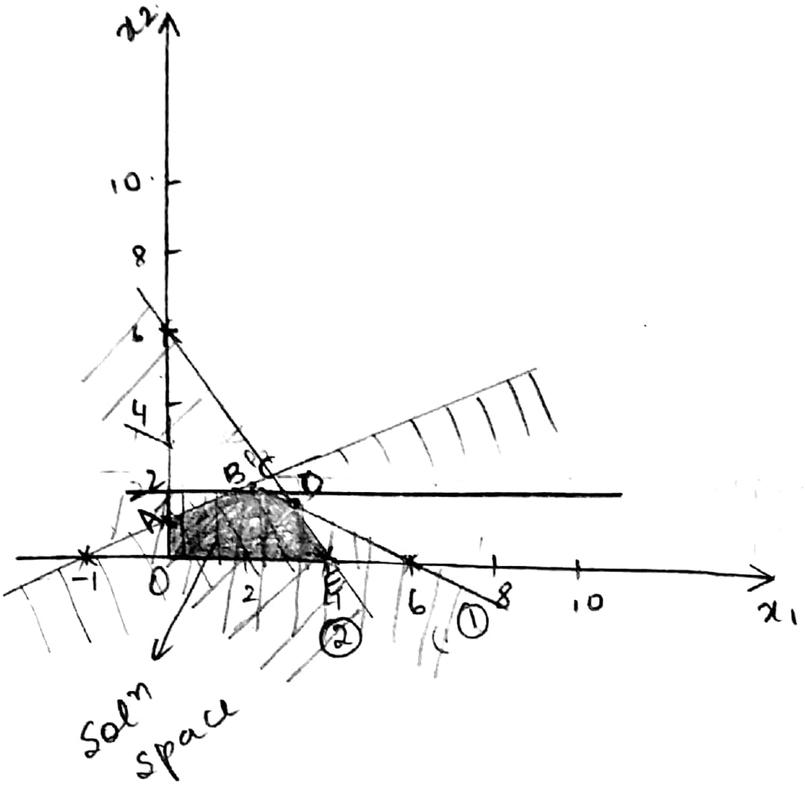
$x_1 \geq 0$, $x_2 \geq 0$ (non-neg constraint).

Q) A person requires 10, 12 & 12 units of chemicals A, B & C resp. for his garden. A liquid product contains 5, 2 & 1 units of A, B & C resp. A dry product contains 1, 2 & 4 units of A, B & C per carton. If the liquid product sells for ₹ 3/jar & the dry product sells for ₹ 2 per carton. How many of each should be purchased in order to ~~maximise~~ minimise the cost & meet the requirements.

* Tutorial

Solⁿ →

Graphical soln method for *)



① $6x_1 + 4x_2 = 24$

$x_1 = 0, \quad x_2 = 6$

$x_1 = 4, \quad x_2 = 0$

② $x_1 + 2x_2 = 6$

$x_1 = 0 \qquad x_2 = 3$

$x_1 = 6 \qquad x_2 = 0$

③ $x_2 - x_1 = 1$

$x_1 = 0, \quad x_2 = 1$

$x_1 = -1 \qquad x_2 = 0$

④ $x_2 \leq 2$

⑤ $x_1, x_2 \geq 0$

A - (0, 1)

B - (1, 2)

C - (2, 2)

D - (3, 1.5)

E - (4, 0)

Substituting the above values in Z

$Z = 5x_1 + 4x_2$

1) A(0, 1)
$Z = 4$

2) B(1, 2)
$Z = 13$

3) (2, 2)
$Z = 18$

4) (3, 1.5)
$Z = 21$

5) E(4, 0)
$Z = 20$

$\therefore$ Max value is \$21,000

Graphical solution for two variable LPP :

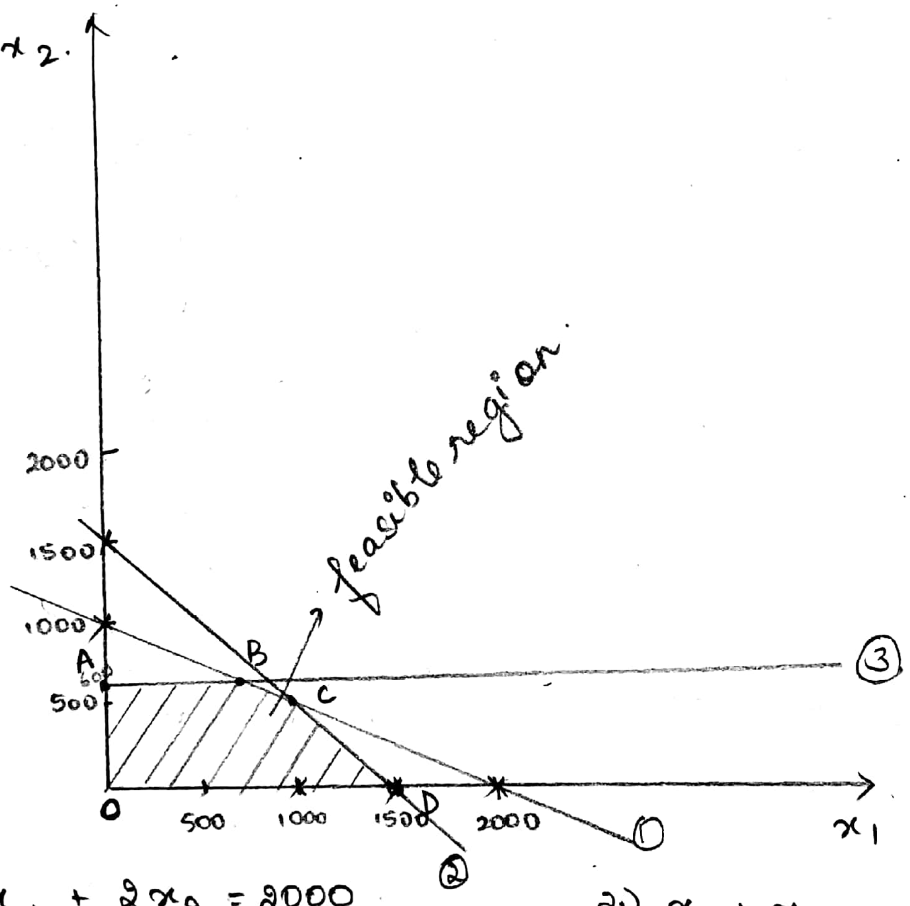
$\text{Max } Z = 3x_1 + 5x_2$.

Sub to constraints,

$x_1 + 2x_2 \le 2000$

$x_1 + x_2 \le 1500$

$x_2 \le 600 \qquad x_1 \ge 0 \quad x_2 \ge 0$



1) $x_1 + 2x_2 = 2000$

$x_1 = 0, \quad x_2 = 1000$

$x_1 = 2000 \quad x_2 = 0$

2) $x_1 + x_2 = 1500$

$x_1 = 0, \quad x_2 = 1500$

$x_1 = 1500, \quad x_2 = 0$

3) $x_2 = 600$

$A(0, 600)$

$B(800, 600)$

$C(1000, 500)$

$D(1500, 0)$

1) $Z = 3x_1 + 5x_2$ for $A(0, 600)$

$Z = 3000$

2) Z for $B(800, 600)$

$Z = 5400$.

3) Z for $C(1000, 500)$ } max

$Z = 5500$

4) Z for $D(1500, 0)$

$Z = 4500$.

8) Maximise $Z = 8000x_1 + 7000x_2$.

s.t.c, $3x_1 + x_2 \le 66$.

$x_1 + x_2 \le 45$

$x_1 \le 20$, $x_2 \le 40$.

$x_1, x_2 \ge 0$.

1) $3x_1 + x_2 = 66$.

$x_1 = 0$, $x_2 = 66$.

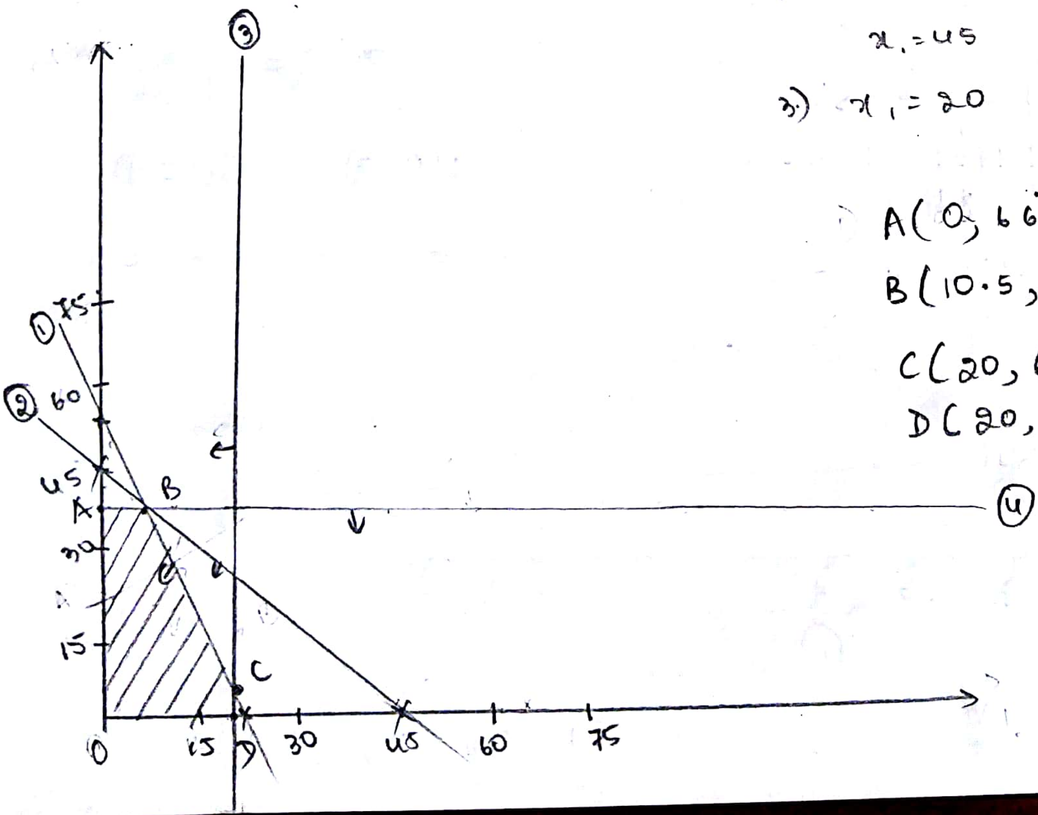
$x_1 = 22$. $x_2 = 0$.

2) $x_1 + x_2 = 45$

$x_1 = 0$, $x_2 = 45$

$x_1 = 45$ $x_2 = 0$.

3) $x_1 = 20$ 4) $x_2 = 40$



$A(0, 66)$

$B(10.5, 34.5)$

$C(20, 6)$.

$D(20, 0)$

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$Z = 8000x_1 + 7000x_2$

$A(0, 66)$ $B(10.5, 34.5)$ $C(20, 6)$

$Z = 4,62,000$ (max) $Z = 3,25,500$ $Z = 2,02,000$

$D(20, 0)$

$Z = 1,60,000$

2) Max $Z = 3x_1 + 2x_2$

s.t,

$x_1 - x_2 \le 1$

$x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0$

1) $x_1 - x_2 = 1$

$x_1 = 0, \; x_2 = -1$

$x_1 = 1 \quad x_2 = 0$

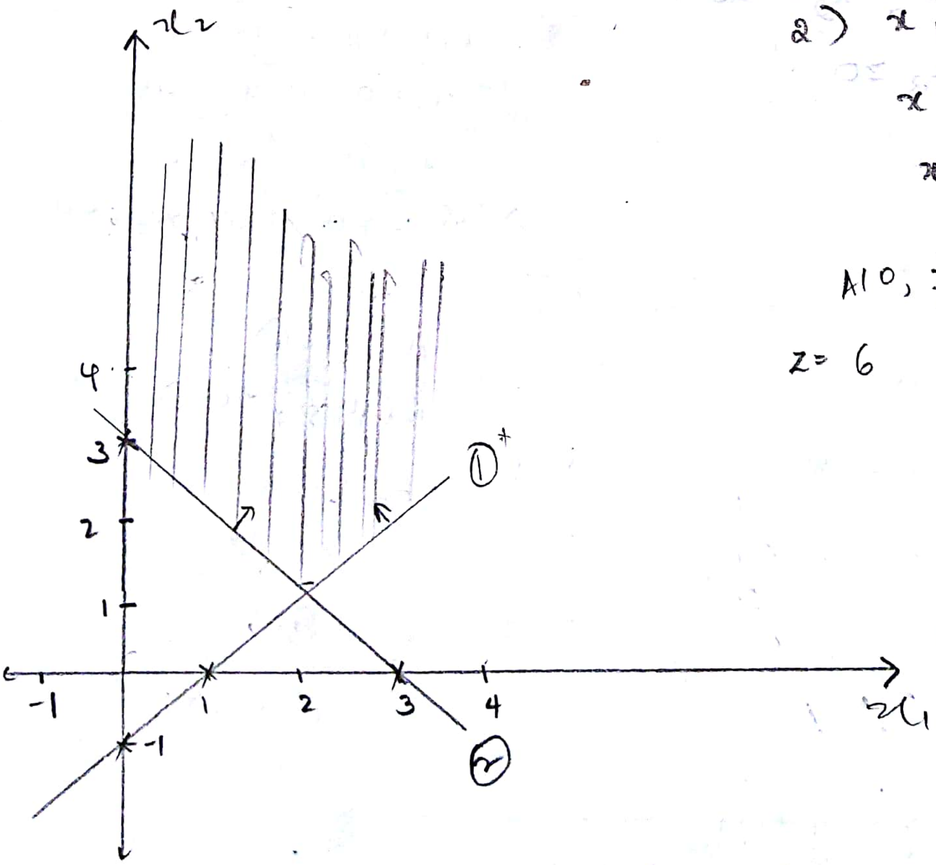
2) $x_1 + x_2 = 3$

$x_1 = 0 \quad x_2 = 3$

$x_1 = 3 \quad x_2 = 0$

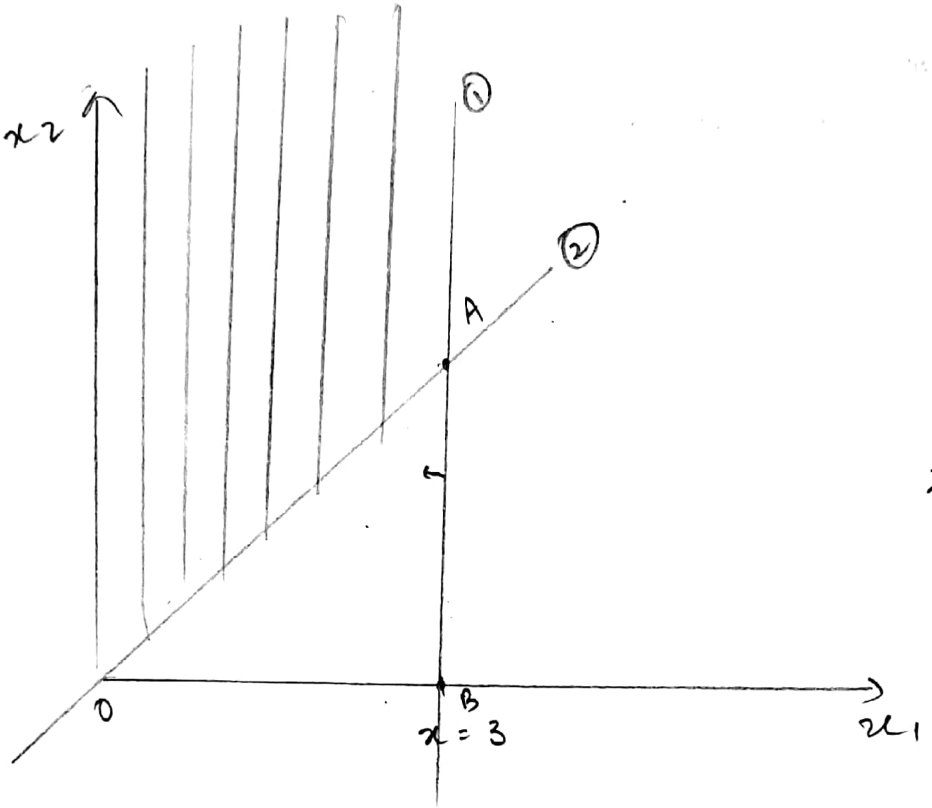
$A(0, 3)$ $B(2, 1)$

$Z = 6$ $Z = 8$



Q) $Z_{max} = -3x_1 + 2x_2$.

$x_1 \le 3$, $x_1 - x_2 \le 0$.



1) $x_1 = 3$

2) $x_1 - x_2 = 0$

$x_1 = x_2$.

A(3, 3) B(3, 0)

$Z = -3$ $Z = -9$

Q) Min $Z = 5x_1 + 8x_2$

$x_1 \le 4$, $x_2 \ge 2$.

$x_1 + x_2 = 5$.

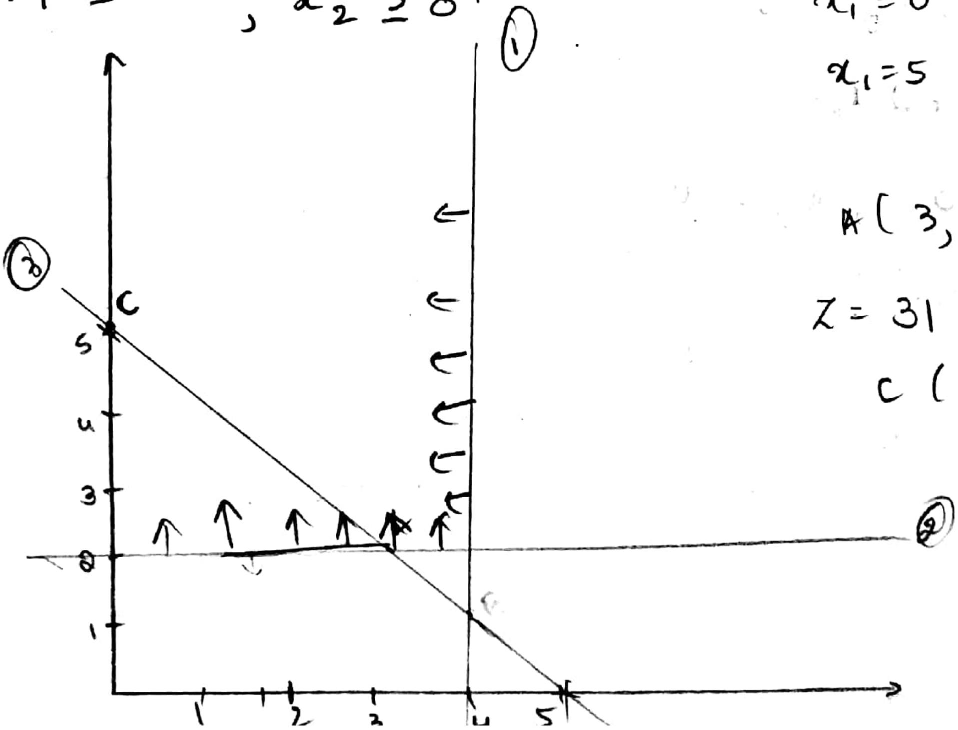
$x_1 \ge 0$, $x_2 \ge 0$.

1) $x_1 = 4$ 2) $x_2 = 2$.

3) $x_1 + x_2 = 5$

$x_1 = 0$ $x_2 = 5$

$x_1 = 5$ $x_2 = 0$



A(3, 2)

$Z = 31$

C(0, 5)

$Z = 35$.

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1) Max $z = 3x_1 - 2x_2$.

s.t,

$x_1 + x_2 \le 1$

$2x_1 + 2x_2 \ge 4$.

1) $x_1 + x_2 = 1$

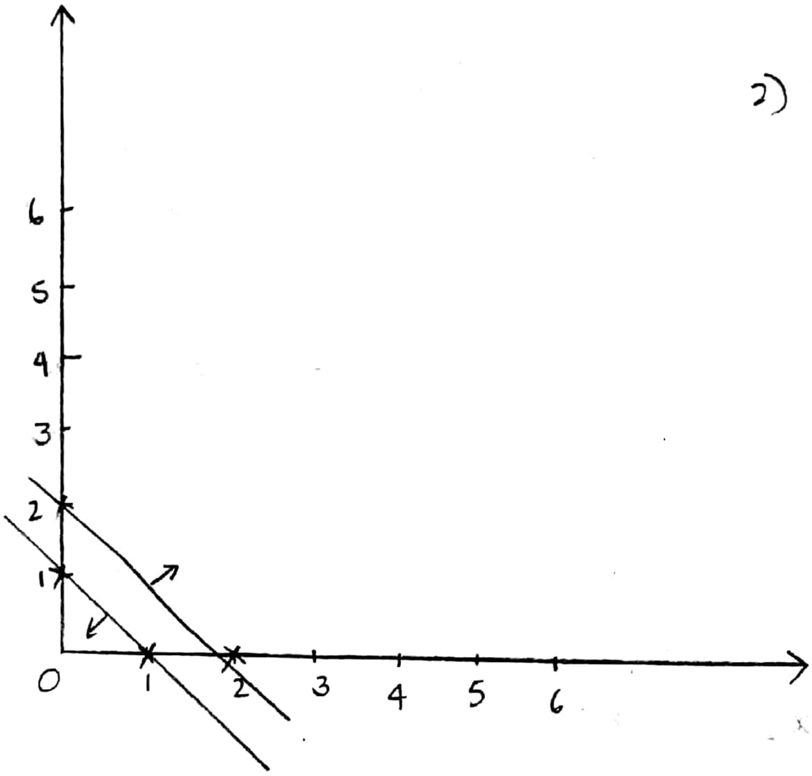
$x_1 = 0$, $x_2 = 1$

$x_1 = 1$, $x_2 = 0$

2) $2x_1 + 2x_2 = 4$

$x_1 = 0$, $x_2 = 2$

$x_2 = 0$, $x_1 = 2$



Q.) Max $z = 5x_1 + 3x_2$.

s.t

$(3x_1 + 5x_2 = 15) \times 5$

$(5x_1 + 2x_2 = 10) \times 3$

$x_1 \ge 0$, $x_2 \ge 0$

1) $3x_1 + 5x_2 = 15$

$x_1 = 0$, $x_2 = 3$

$x_1 = 5$ $x_2 = 0$

2) $5x_1 + 2x_2 = 10$

$x_1 = 0$, $x_2 = 5$

$x_1 = 2$, $x_2 = 0$

$15x_1 + 25x_2 = 75$

$\cancel{15x_1} + 6x_2 = 30$

$19x_2 = 45$

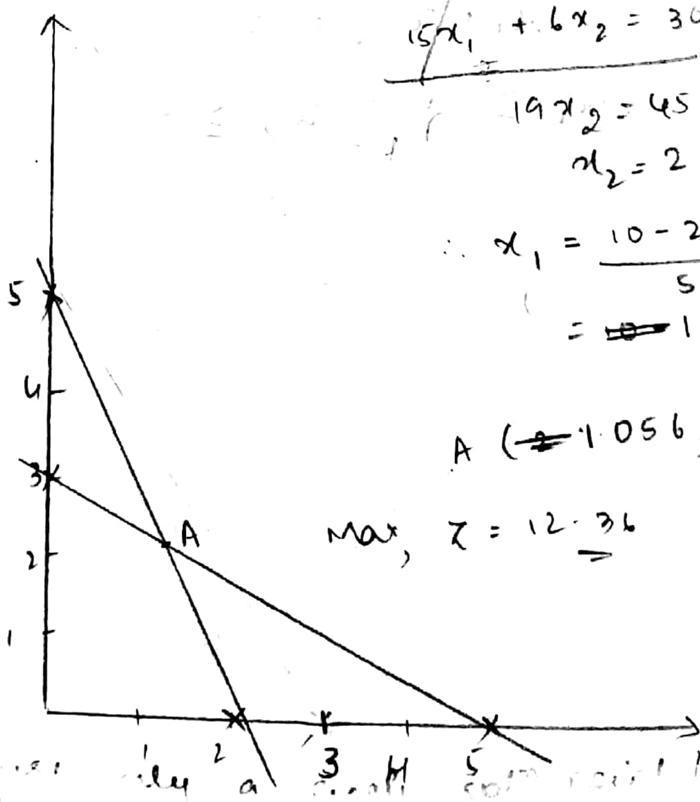
$x_2 = 2.36$

$\therefore x_1 = \frac{10 - 2x_2}{5}$

$= 1.056$

A $(1.056, 2.36)$

Max, $z = 12.36$



3) Min $z = -x_1 + 2x_2$.

s.t,

$-x_1 + 3x_2 \le 10$.

$x_1 + x_2 \le 6$.

$x_1 - x_2 \le 2$.

$x_1, x_2 \ge 0$.

1) $-x_1 + 3x_2 = 10$.

$x_1 = 0, \quad x_2 = 10/3 = 3.3$

$x_1 = -10 \quad x_2 = 0$.

2) $x_1 + x_2 = 6$

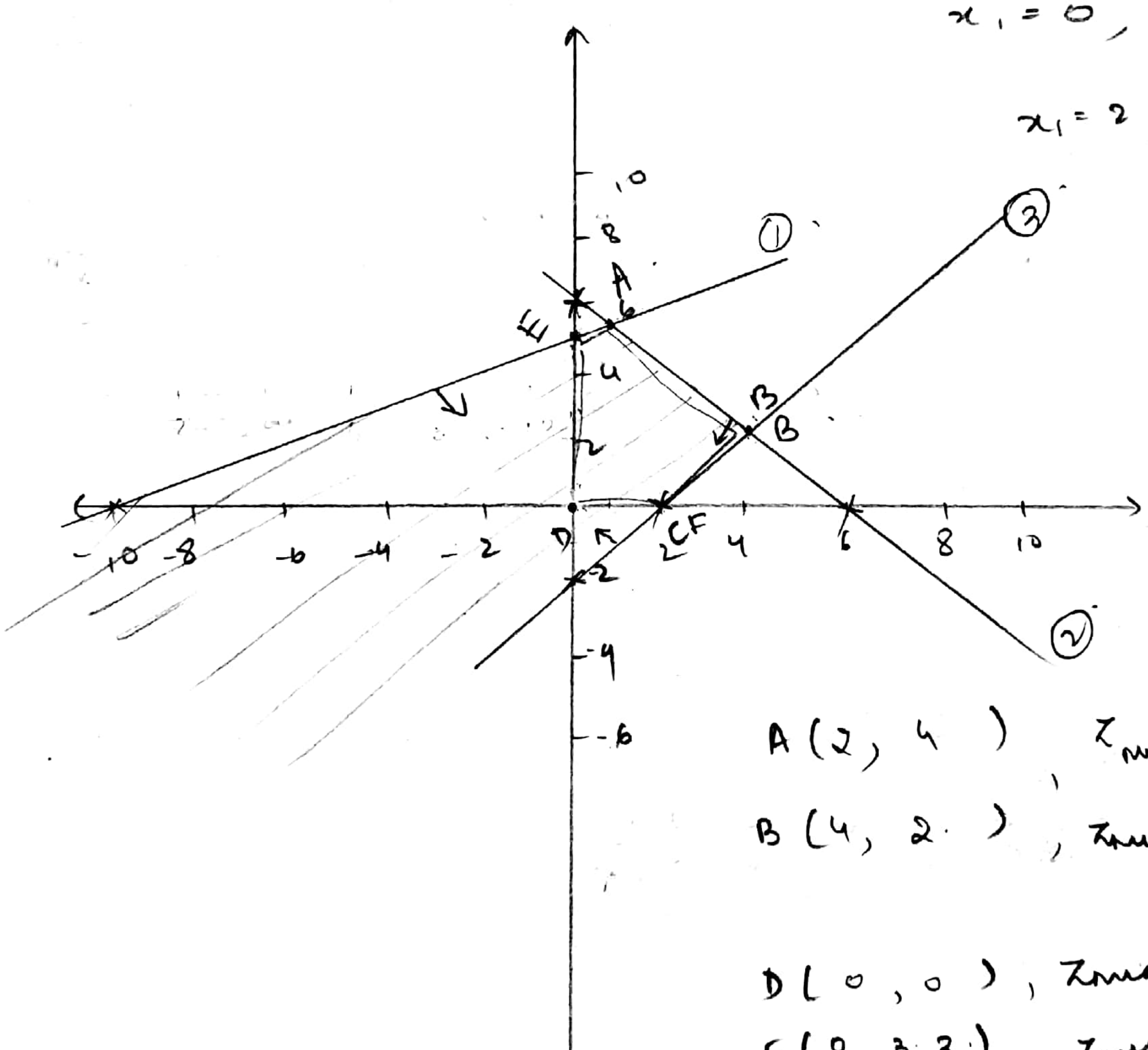
$x_1 = 0, \quad x_2 = 6$.

$x_1 = 6, \quad x_2 = 0$

3) $x_1 - x_2 = 2$

$x_1 = 0, \quad x_2 = -2$

$x_1 = 2, \quad x_2 = 0$.



A (2, 4), $z_{min} = 6$

B (4, 2), $z_{min} = 0$

D (0, 0), $z_{min} = 0$

E (0, 3.3), $z_{min} = 6.6$

F (2, 0), $z_{min} = -2$

4) ~~Max~~ Min $z = 20x_1 + 10x_2$.

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s.t,

$x_1 + 2x_2 \le 40$ — ①

$3x_1 + x_2 \ge 30$ — ②

$4x_1 + 3x_2 \ge 60$ — ③

$x_1, x_2 \ge 0$.

1) $x_1 + 2x_2 = 40$

$x_1 = 0, \; x_2 = 20$

$x_1 = 40, \; x_2 = 0$

2) $3x_1 + x_2 = 30$

$x_1 = 0, \; x_2 = 30$

$x_1 = 10, \; x_2 = 0$

3) $4x_1 + 3x_2 = 60$

$x_1 = 0, \; x_2 = 20$

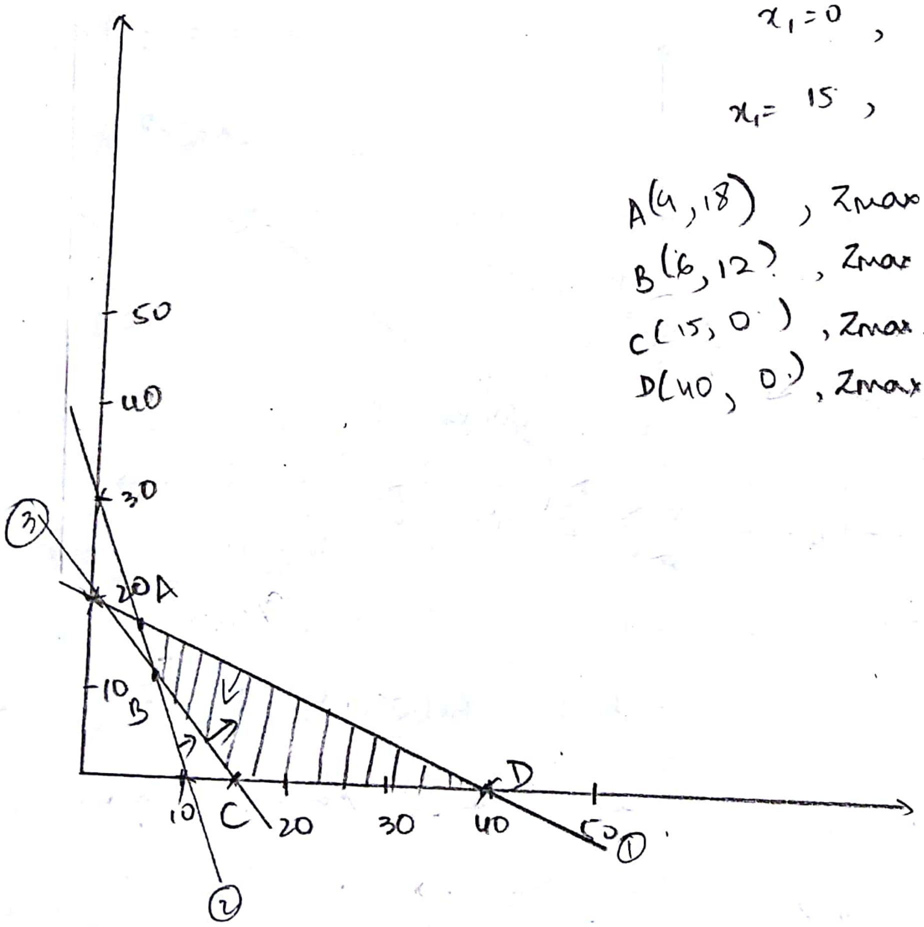
$x_1 = 15, \; x_2 = 0$

A(4, 18), $Z_{max} = 260$

B(6, 12), $Z_{max} = 240$ (Min)

C(15, 0), $Z_{max} = 300$

D(40, 0), $Z_{max} = 800$



8) Ozal farm uses atleast 80lb of special feed daily. The special feed is a minute of corn and soybean meal with the following compositions.

lb per lb of feedstuff.

Feed shift	Protein	fibre	Cost ($lb)
corn	.09	.02	.30
soybean	.60	.06	.90

The dietary requirements of the special feed are at least 30% protein and atmost 5% fibre. The goal is to determine the daily minimum cost feed mix.

sol^{n}** → x_1 = cost of corn x_2 = cost of soybean.

$$Z_{min} = 0.30x_1 + 0.90x_2$$

$$0.09x_1 + 0.06x_2 \geq (x_1 + x_2)\, 30/100$$

$$9x_1 + 60x_2 \geq 30(x_1 + x_2)$$

$$0.02x_1 + 0.06x_2 \leq (x_1 + x_2)(15/100)$$

$$2x_1 + 6x_2 \leq 5(x_1 + x_2)$$

St. C,

$$-7x_1 + 10x_2 \geq 0.$$

$$-3x_1 + x_2 \leq 0.$$

$$x_1, x_2 \geq 0.$$

$x_1 + x_2 \geq 800$

F.P) A $(200, 600)$

B$(470.58, 329.41)$

$Z_{min} = 437.643$

Q) Express the following LPP in standard form

Min $Z = x_1 - 2x_2 + x_3$ (to maximise multiply with -1)

Sub to,

$(2x_1 + 3x_2 + 4x_3 \geq -4) \times -1$

$3x_1 + 5x_2 + 2x_3 \geq 7$

$x_1, x_2 \geq 0$, x_3 is unrestricted.

Sol^n

$\max(-z) = -x_1 + 2x_2 - x_3$

$\max(z') = -x_1 + 2x_2 - (x_3' - x_3'')$

S.t,

$-2x_1 - 3x_2 - 4(x_3' - x_3'') \leq 4$ ⟶ (1)

$3x_1 + 5x_2 + 2x_3 \geq 7$ ⟶ (2)

$x_1, x_2, x_3', x_3'' \geq 0$

⟶

Slack variable, Surplus variable.

Add s_1 to LHS of (1) & $-s_2$ to (2)

$$-2x_1 - 3x_2 - 4(x_3' - x_3'') + s_1 = 4$$

$$3x_1 + 5x_2 + 2(x_3' - x_3'') - s_2 = 7$$

$$x_1, x_2, x_3', x_3'', s_1, s_2 \geq 0$$

Q.) Max $z = 2x_1 + 3x_2$

sub. t.c,

$$2x_1 + x_2 \leq 4$$

$$x_1 + 2x_2 \leq 5$$

$$x_1 + x_2 \geq 0$$

soln →

$$2x_1 + x_2 + s_1 = 4$$

$$x_1 + 2x_2 + s_2 = 5$$

$$x_1, x_2, s_1, s_2 \geq 0$$

Algebraic solun method:

Non-basic variables	Basic variables	Basic sol^n	Associated corner points	Feasible	obj. value of z
(x_1, x_2)	(s_1, s_2)	$(4, 5)$		Yes	0
$(x_1, \cancel{x}s_1)$	(x_2, s_2)	$(4, -3)$		NO	—
(x_1, s_2)	(x_2, s_1)	$(5/2, 3/2)$		Yes	7.5
(x_2, s_1)	(x_1, s_2)	$(2, 3)$		Yes	4
(x_2, s_2)	(x_1, s_1)	$(5, -6)$		NO	—
(s_1, s_2)	(x_1, x_2)	$(1, 2)$		Yes	8 (max)

1) $z = 2\times0 + 3\times0 + 0\times4 + 0\times5$

$= 0$

1) $2x_1 + x_2 = 4$

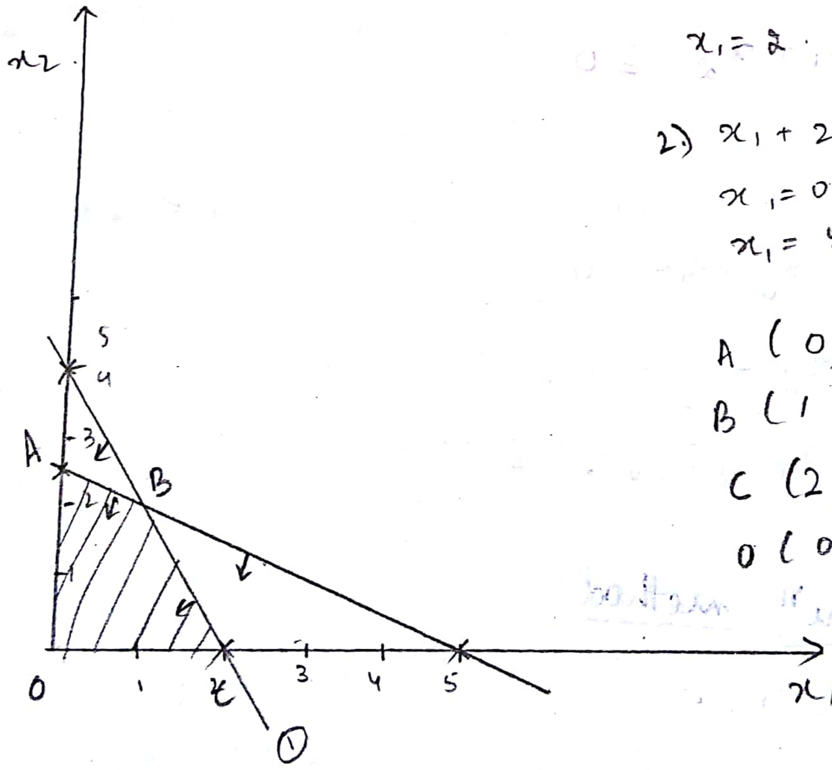
$x_1 = 0, \quad x_2 = 4$

$x_1 = 2, \quad x_2 = 0$

2) $x_1 + 2x_2 = 5$

$x_1 = 0, \quad x_2 = 5/2 = 2.5$

$x_1 = 5, \quad x_2 = 0$



A $(0, 2.5)$

B $(1, 2)$

C $(2, 0)$

O $(0, 0)$

$(2x_1 + x_2 = 4) \times 2 = 4x_1 + 2x_2 = 8$

$x_1 + 2x_2 = 5 \qquad x_1 + 2x_2 = 5$

$3x_1 = 3$

$x_1 = 1$

$\Rightarrow x_2 = 2$

Q) Discuss how to translate the graphical sol^n method to algebraic method.

Q.) Consider the following LPP to max. $z = 2x_1 + 3x_2$.

Sub to,

$x_1 + 3x_2 \leq 6$.

$3x_1 + 2x_2 \leq 6$.

$x_1, x_2 \geq 0$.

a) express the problem in eq^n form.

b) Determine all the basic solution of the problem, and classify them as feasible and infeasible.

c) Use direct substitution in the objective function to determine the optimum basic feasible solution.

(d) Verify graphically that the solu^n obtained in (c) is the optimum LP solution.

(e) Show how the infeasible basic solutions are represented on the graphical solution space.

Sol^n →

$Z = 2x_1 + 3x_2$.

Sub to, $x_1 + 3x_2 \leq 6$.

$3x_1 + 2x_2 \leq 6$

$Z = 2x_1 + 3x_2 + 0S_1 + 0S_2$.

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Now $x_1 + 3x_2 + S_1 = 6$

$3x_1 + 2x_2 + S_2 = 6$

$x_1, x_2, S_1, S_2 \geq 0$

Non basic variables	Basic variable	Basic soln	Associated corner point	Feasible	obj value of Z
(x_1, x_2)	(S_1, S_2)	$(6, 6)$		Y.	0
(x_1, S_1)	(x_2, S_2)	$(2, 2)$		Y.	6
(x_1, S_2)	(x_2, S_1)	$(3, -3)$		N	—
(x_2, S_1)	(x_1, S_2)	$(6, -12)$		N.	—
(x_2, S_2)	(x_1, S_1)	$(2, 4)$		Y.	4
(S_1, S_2)	(x_1, x_2)	$(6/7, 12/7)$		Y.	48/7.

Max, $Z = 2x_1 + 3x_2$

$x_1 + 3x_2 = 6.$

$3x_1 + 2x_2 = 6.$

1) $x_1 + 3x_2 = 6.$

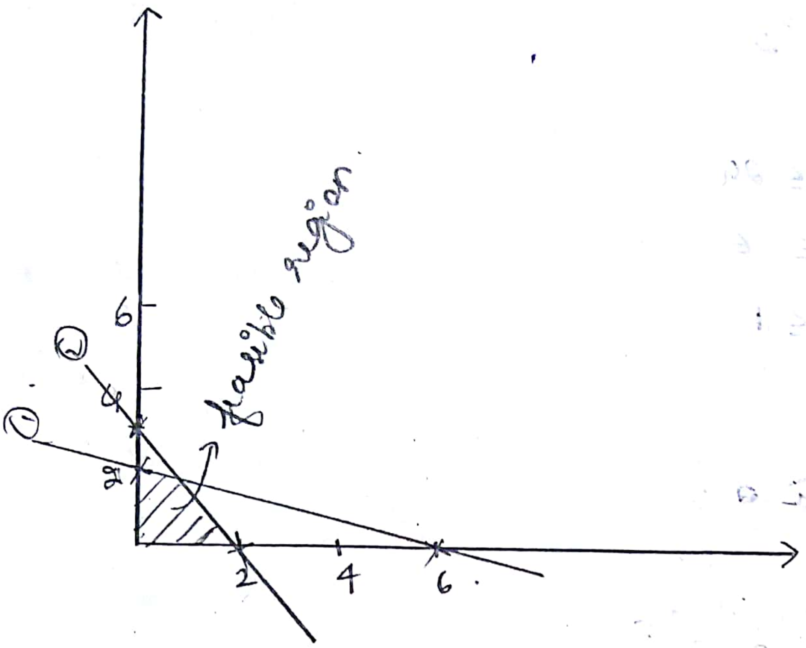
$x_1 = 0, \quad x_2 = 2.$

$x_1 = 6. \quad x_2 = 0.$

2) $3x_1 + 2x_2 = 6.$

$x_1 = 0 \quad x_2 = 3.$

$x_1 = 2. \quad x_2 = 0$



A-W

Q) Show algebraically that all the basic ~~function~~ soln of the following LP are infeasible.

Max $Z = x_1 + x_2$

s.t.,

$$x_1 + 2x_2 \leq 6$$

$$2x_1 + x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

soln →

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Max $Z = 5x_1 + 4x_2$.

s.t,

$6x_1 + 4x_2 \le 24$.

$x_1 + 2x_2 \le 6$.

$-x_1 + x_2 \le 1$.

$x_2 \le 2$.

$x_1, x_2 \ge 0$

Soln

Max $Z = 5x_1 + 4x_2$

s.t,

$6x_1 + 4x_2 + s_1 = 24$.

$x_1 + 2x_2 + s_2 = 6$.

$-x_1 + x_2 + s_3 = 1$

$x_2 + s_4 = 2$.

$x_1, x_2, s_1, s_2, s_3, s_4 \ge 0$.

Basic	Z	x_1	x_2	s_1	s_2	s_3	s_4	Solution
Z_1	1	-5	-4	0	0	0	0	0
s_1	0	6	4	1	0	0	0	24
s_2	0	1	2	0	1	0	0	6
s_3	0	-1	1	0	0	1	0	1
s_4	0	0	1	0	0	0	1	2

Continue in the same table

Now, $z = 5x_1 + 4x_2$.

$z - 5x_1 - 4x_2 =$

~~Simplex method~~:

$\cancel{Max,\ z = 5x_1 + 4x_2}$.

~~sub to,~~

$\cancel{6x_1 + 4x_2 \leq 24}$.

$\cancel{x_1 + 2x_2 \leq 6}$.

$\cancel{-x_1 + x_2 \leq 1}$

$\cancel{x_2 \leq 2}$.

$\cancel{x_1, x_2 \geq 0}$.

Basic	Z	x_1	x_2	s_1	s_2	s_3	s_4	Solution	M.R.
Z		0	$-2/3$	$5/6$	0	0	0	20.	
x_1	0	1	$4/6$	$1/6$	0	0	0	4.	6
s_2		0	$8/6$	$-1/6$	1	0	0	2.	$3/2$
s_3		0	$10/6$	$1/6$	0	1	0	5	5
s_4.		0,	1	0	0	0	1	2.	2

(ii) new s_2 row = old s_2 row − new x_1 row

$= 1 - 1 = 0$.

$= 2 - 4/6$

$= 0 - 1/6$.

new s_3 row = old s_3 − new x_1

new Z row = old Z row + 5 (new x_1 row)

$= 1 + 5(0) = 1$

$= -5 + 5(1) = 0$

$= -4 + 5(4/6) = -4 + 20/6 = -2/3$

$= 1 + 1 = 0$

$= 1 + 4/6$

$= 0 + 1/6$.

$= 0 + 0$

$= 1 + 0$

$= 0 + 0$.

		x_1	x_2	s_1	s_2	s_3	s_4	solution
Z	1	0	0	3/4	1/2	0	0	21
x_1	0	1	0	1/4	-1/2	0	0	3
x_2	0	0	1	-1/8	3/4	0	0	3/2
s_3	0	0	0	3/8	-5/4	1	0	5/2
s_4	0	0	0	1/8	-3/4	0	1	1/2

2)

new x_1 row = old x_1 row $-\frac{4}{6}$(new x_2 row)

new s_3 row = old s_3 row $-\frac{10}{6}$(new x_2 row)

$= \frac{10}{6} - \frac{10}{6}(1) = 0$

$= \frac{1}{6} - \frac{10}{6}\left(-\frac{1}{8}\right)$

$= \frac{1}{6} + \frac{10}{48} = \frac{48+40}{48}$

The optimal solution:

$x_1 = 3,\ x_2 = 3/2$

Max $Z = 21$

Use Simplex method to solve the following LPP.

Max $Z = 4x_1 + 10x_2$

sub to.

$2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_1 + 3x_2 \leq 90$

$x_1, x_2 \geq 0$

Soln. $\rightarrow$ Standard form.

Max $Z = 4x_1 + 10x_2$.

$Z - 4x_1 - 10x_2 = 0$.

Sub to,

$2x_1 + x_2 + S_1 = 50$

$2x_1 + 5x_2 + S_2 = 100$

$2x_1 + 3x_2 + S_3 = 90$

$x_1, x_2, S_1, S_2, S_3 \geq 0$

Basis	Z	x_1	x_2	S_1	S_2	S_3	solution	Min. Ratio
Z	1	-4	-10	0	0	0	0	
S_1	0	2	1	1	0	0	50	50/1
S_2	0	2	5	0	1	0	100	100/5
S_3	0	2	3	0	0	1	90	90/3
Z	1	0	0	0	2	0	200	
S_1	0	8/5	0	1	-1/5	0	20	
x_2	0	2/5	1	0	1/5	0	20	
S_3	0	4/5	0	0	-3/5	1	30	

1) new S_1- row = old S_1 -row − x_2 row

new S_3 row = old S_3 row − $3(x_2$ row$)$.

$2 - 3 \cdot 2/5$

new Z row = old Z row $+10(x_2$ row$)$.

Max $Z = 200$, $x_1 = 0$, $x_2 = 20$.

Q.) Max $Z = 3x_1 + 2x_2$

s.t.

$x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

Sol^n →

Max $Z - 3x_1 - 2x_2 = 0$

s.t

$x_1 + x_2 + s_1 = 4$

$x_1 - x_2 + s_2 = 2$

Basic	Z	x_1	x_2	s_1	s_2	Solution	Max Ratio
Z	1	-3	-2	0	0	0	0
s_1	0	1	1	1	0	4	4
s_2	0	+1	-1	0	1	2	2
Z	1	0	-5	0	3	6	–
s_1	0	0	2	1	-1	2	1
x_1	0	1	-1	0	1	2	–
Z	0	0	0	5/2	1/2	11	
x_2	0	0	1	1/2	-1/2	1	
x_1	0	1	0	1/2	1/2	3	

Max = 11 , $x_1 = 3$, $x_2 = 1$

1.) new s_1 = old s_1 − new x_1

new z = old z + 3 new x_1

2) new x_1 = old x_1 + new x_2.

new z = old z + 5 new x_2.

Q) Max $z = x_1 + 3x_2$.

s.t.

$x_1 + 2x_2 \le 10$.

$x_1 \le 5$.

$x_2 \le 4$.

$x_1, x_2 \ge 0$

Ans: $x_1 = 2$

$x_2 = 4$.

max $z = 14$.

sol^n →

max $z - x_1 - 3x_2 = 0$.

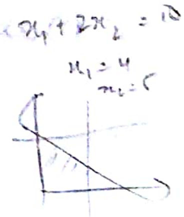
s.t,

$x_1 + 2x_2 + s_1 = 10$

$x_1 + s_2 = 5$.

$x_2 + s_3 = 4$.

Basic	z	x_1	x_2	s_1	s_2	s_3	sol^n	Min Ratio.
z	1	-1	-3	0	0	0	0	
s_1	0	2	1	1	0	0	10	10
s_2	0	1	0	0	1	0	5	5
s_3	0	0	1	0	0	1	4	4



Basic	Z	x_1	x_2	s_1	s_2	s_3	sol^n	min ratio
Z	1	-1	0	0	0	3	12	~~-12~~
s_1	0	2	0	1	0	-1	6	3
s_2	0	1	0	0	1	0	5	5
x_2	0	0	1	0	0	1	4	4/0

Z	1	0	0	1/2	0	5/2	15
x_1	0	1	0	1/2	0	-1/2	3
s_2	0	0	0	-1/2	1	1/2	1
x_2	0	0	1	0	0	1	4

* Simplex Method - Minimization problem :

1) Solve the following LPP by simplex method :

$$Z_{min} = x_1 - 3x_2 + 2x_3$$

sub. to,

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$-2x_1 + 4x_2 \leq 12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

Sol^n

$$\max(-Z) = -x_1 + 3x_2 - 2x_3$$

$$Z' + x_1 - 3x_2 + 2x_3 = 0$$

Sub to,

$$3x_1 - x_2 + 2x_3 + s_1 = 7$$

$$-2x_1 + 4x_2 + s_2 = 12$$

$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Basics	Z'	x_1	x_2	x_3	s_1	s_2	s_3	Sol^n	min ratio.
Z'	1	1	-3	2	0	0	0	0	
s_1	0	3	-1	2	1	0	0	7	-7 X
s_2	0	-2	4	0	0	1	0	12	$12/4 = 3$
s_3	0	-4	3	8	0	0	1	10	$10/3 = 3.33$
Z'	1	$-1/2$	0	2	0	$3/4$	0	9	$\to 9/_{-1/2} = -18$ X
s_1	0	$5/2$	0	2	1	$1/4$	0	10	$\to 10/5/2 = 20/5 = 4$
x_2	0	$-1/2$	1	0	0	$1/4$	0	3	$\to 3/_{-1/2} = -6$
s_3	0	$-5/2$	0	8	0	$-3/4$	1	1	$\to -3/4/-5/2 = 6/20$
Z'	0	0	0	$12/5$	$1/5$	$16/20$	0	11	
x_1	0	1	0	$4/5$	$2/5$	$1/10$	0	4	
x_2	0	0	1	$2/5$	$1/5$	$6/20$	0	5	
s_3	0	0	0	10	1	$-2/4$	1	11	

Max$(-Z) = 11$, $x_1 = 4$ $x_2 = 5$ $x_3 = 0$

2) Min $Z = 2x_1 + 3x_2 + x_3$.

sub to,

$3x_1 + 2x_2 + x_3 \leq 3$

$2x_1 + x_2 + x_3 \leq 2$.

$x_1, x_2, x_3 \geq 0$

Sol^n

$$Max(-z) = -2x_1 - 3x_2 - x_3$$

$$z' + 2x_1 + 3x_2 + x_3 = 0$$

$$3x_1 + 2x_2 + x_3 + S_1 = 3$$

$$2x_1 + x_2 + x_3 + S_2 = 2.$$

$$x_1, x_2, S_1, S_2 \geq 0.$$

Basic	z'	x_1	x_2	x_3	S_1	S_2	sol^n.
z'	1	2	3	1	0	0	0
S_1	0	3	2	1	1	0	3.
S_2	0	2	1	1	0	1	2.

Co-efficients of z are all positive

optimum $z = 0$.

$x_1 = 0$, $x_2 = 0$, $x_3 = 0$.

* Special cases in the simplex method.

(i) Alternative Optima.

(ii) Degeneracy.

(iii) Unbound solution

(iv) non-existing solution

1) Max $Z = 2x_1 + 4x_2$

subject,

$x_1 + 2x_2 \leq 5$

$x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

Sol^n →

$x_1 + 2x_2 + s_1 = 5$

$x_1 + x_2 + s_2 = 4$

$s_1, s_2, x_1, x_2 \geq 0$

Basic	Z	x_1	x_2	s_1	s_2	Sol^n	min ratio
Z	1	-2	-4	0	0	0	
s_1	0	1	2	1	0	5	$5/2$
s_2	0	1	1	0	1	4	$4/1$
Z	1	0	0	2	0	10	
x_2	0	$1/2$	1	$1/2$	0	$5/2$	$5/2 / 1/2 = 5$
s_2	0	$1/2$	0	$-1/2$	1	$3/2$	3
Z	1	0	0	2	0	10	
x_2	0	0	1	1	-1	1	
x_1	0	1	0	-1	2	3	

$x_1 = 3$ $x_2 = 1$

$Z_{max} = 10$.

* Intervals →

~~Alternative optimum solution~~.

~~non basic variable = 0~~.

#

~~Region~~

Degeneracy:

1) Finding Degeneracy.

2) Resolving Degeneracy.

*

Ex!- Max $Z = 3x_1 + 9x_2$

sub. to,

$x_1 + 4x_2 \leq 8$.

$x_1 + 2x_2 \leq 4$.

$x_1, x_2 \geq 0$.

Soln:

standard form,

$Z - 3x_1 - 9x_2 = 0$.

sub to,

$x_1 + 4x_2 + s_1 = 8$.

$x_1 + 2x_2 + s_2 = 4$.

$x_1, x_2, s_1, s_2 \geq 0$

Basis	Z	x_1	x_2	s_1	s_2	soln	min ratio
Z	1	-3	-9	0	0	0	$0/-9 = 0$
s_1	0	1	4	1	0	8	$8/4 = 2$
s_2	0	1	2	0	1	4	$4/2 = 2$

Degeneracy

1) Pick up the rows for which the min. non-~~one~~ ratio is the same.

2) Rearrange the columns of the usual simplex table so that the columns forming the original unit matrix come first in proper order.

3) To find the min. ratio, we use the formula.

$$\frac{[\text{elements of the 1st column of the unit matrix}]}{[\text{corresponding elements of the key element}]}$$

only for the rows for which the min. ratio was not ~~is~~ unique is for the rows 1st, 3rd etc. as picked up in step 1. If this min. is obtained for 3rd row, then this row will determine the key element by intersecting key column.

If this min. is still not unit then go to next step.

4) Now compute the min. of the ratio

$$\frac{[\text{elements of 2nd column of unit matrix}]}{[\text{corresponding elements of key element}]}$$

*3. Max $Z = 3x_1 + 2x_2$

subjected to

$4x_1 + 3x_2 \le 12$

$4x_1 + x_2 \le 8$

$4x_1 - x_2 \le 8$

$x_1, x_2 \ge 0$

→

Basis	Z	x_1	x_2	S_1	S_2	S_3	sol
Z	1	-3	2	0	0	0	0
S_1	0	4	3	1	0	0	12
S_2	0	4	1	0	1	0	8
S_3	0	4	-1	0	0	1	8

	S_1	S_2	S_3	x_1	x_2	MR	MR S_2/x_1
	1	0	0	4	3		
S_2	0	1	0	4	1	0/4	1/4
S_3	0	0	1	4	-1	0/4	0/4 ← 0.

Z	1	0	-1/4	0	0	3/4	6	× 2 +3/4
S_1	0	0	4	1	0	-1	4	4/4 = 1 2 - (-3/4)
S_2	0	0	2	0	1	-1	0	0/2 = 0
x_1	0	1	-1/4	0	0	1/4	2	×

-1/8 +1/4

Z	1	0	0	0	11/8	-5/8	6	Min	-11/8 + 6/4
S_1	0	0	0	1	-2	1	4	4/1	16/8
x_2	0	0	1	0	1/2	-1/2	0	×	
x_1	0	1	0	0	1/8	1/8	2	16	

Z	1	0	0	0	1/8	0	17/2	
S_3	0	0	0	1	-2	1	4	2 - (4/8) 11/8 + (-2×5/8)
x_2	0	0	1	1/2	-1/2	0	2	6 + 5/2 11/8 - 10/8
x_1	0	1	0	-1/8	3/8	0	3/2	

$Z = 17/2 \quad x_1 = 3/2, \; x_2 = 2$

* unbounded solution:

1. Max $z = 2x_1 + x_2$

st

$x_1 - x_2 \le 10$

$2x_1 - x_2 \le 40$

$x_1 \ge 0, x_2 \ge 0$

→

Basis	z	x_1	x_2	s_1	s_2	solution
z	1	-2	-1	0	0	0
s_1	0	1	-1	1	0	10
s_2	0	2	-1	0	1	40

coefficients of constraints, under any of the non-basic variable column are ≤ 0 meaning of the statement, non-basic variable column values can be increased indefinitly without violating any of the constraints. This situation is taken as unbounded solution

2. Max $z = 2x_1 + x_2$

st

$x_1 - x_2 \le 10$

$-2x_2 \le 40$

$x_1, x_2 \ge 0$

→

Basis	z	x_1	x_2	s_1	s_2	solution
z	1	-2	-1	0	0	0
s_1	0	1	-1	1	0	10
s_2	0	0	-2	0	1	40

∴ unbounded.

Sensitivity analysis:

In LPP the parameters (input data) of the model can change within certain limits without causing changes in the optima, this is referred to as sensitivity analysis.

one of the sensitivity analysis is Graphical sensitivity analysis.

Two different cases:

i) sensitivity of the optimum solution to changes in the availability of the resources.
(Right hand side constraints)

ii) sensitivity of the optimum sol? to changes in unit profit or unit cost
(change in the coefficients of objective fn?)

A, B, C manufactures two products on 2 machines. A unit of product 1 requires 2 hrs on machine one and one hour on machine 2. For product 2 one unit requires 1 hr on machine 1 and 3 hrs on mac 2. The revenues per unit of products 1 & 2 $30 & $20 resp'tly. The total daily processing time available for each machine is 8 hrs.

→

x_1 = no of units of product 1

x_2 = no of units of product 2

$Z_{max} = 30x_1 + 20x_2$

subjected to

$2x_1 + x_2 \le 8$

$x_1 + 3x_2 \le 8$

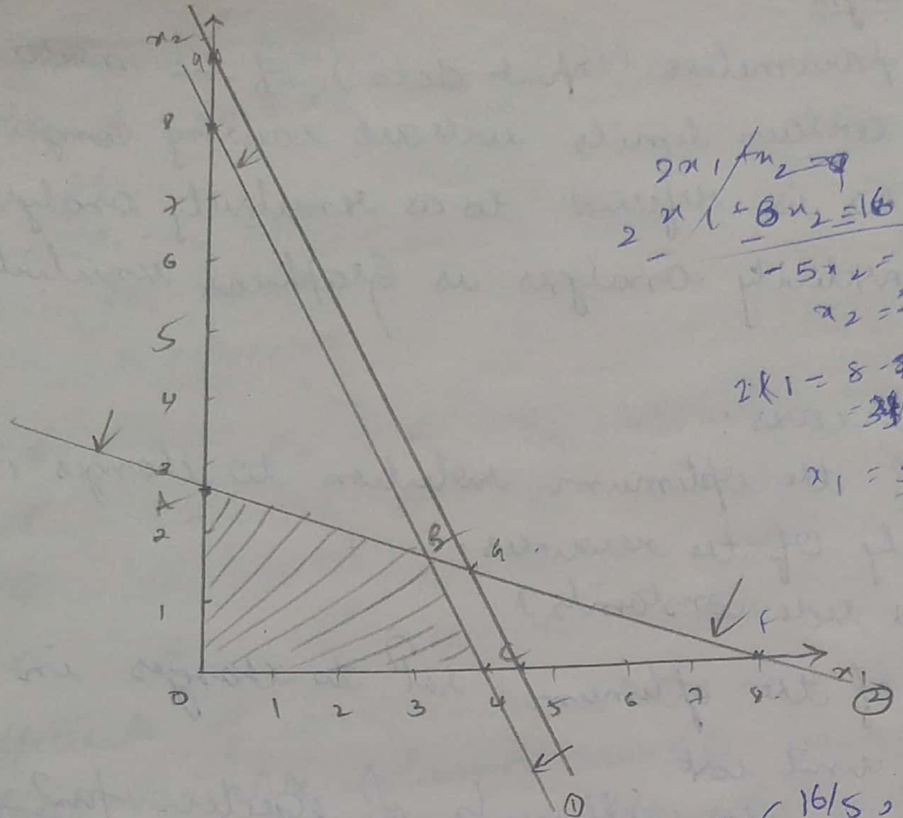
put $x_1 = 0$

$x_2 = 8$

put $x_2 = 0$, $x_1 = 4$

put $x_1 = 0$ $x_2 = 8/3$

$x_2 = 0$ $x_1 = 8$



$2x_1 + x_2 = 9$
$2x_1 + 8x_2 = 16$
$-5x_2 = 7$
$x_2 = 7/5$
$2x_1 = 8 - 7/5$
$= 33/5$
$x_1 = \frac{33}{10}$

corner points are $(0, 8/3)$, $(4, 0)$ $(16/5, 8/5)$

$Z = 30(16/5) + 20(8/5)$

$= 96 + 32$

$\boxed{Z_{max} = 128}$

Now increase right hand side of machine 1 by 1hr,

$2x_1 + x_2 \leq 9$,
put $x_1 = 0$, $x_2 = 9$
$x_1 = 9/2$ $x_2 = 0$

Fig 1: illustrates the change in the optimum solution when the changes are made in the capacity of machine 1. If the daily capacity from 8 hrs to 9 hr the new optima will move to point G. The rate of change in optimum Z resulting from change in machine 1 capacity from 8hrs to 9 hrs can be computed as

Rate of revenue change resulting from increasing m/c 1 capacity by 1 hr (point B to G) =

$= \dfrac{Z_G - Z_C}{(\text{capacity change})}$ $\quad Z_G = 30 \times (\frac{33}{10})^{3} + 20 \times (7/5)^{4}$

$= 99 + 28$

$= \dfrac{142 - 128}{9 - 8} = \$ 14/hr$ $\quad = 142$

The computed rate provides a direct link b/w the model I/P & the O/P. It says that unit increase/dc in m/c 1 capacity will increase (decrease) revenue $14 hr. This price is known dual price / shadow price.

We can see that the dual price of $14/hr remains valid for changes (increases/ decreases) in m/c 1 capacity that move its constraint parallel to itself to any point on the line segment AF. We compute m/c 1 capacities at A & F as follows.

Minimum m/c 1 capacity at B $= (0, 2.6)$
$= 2 \times 0 + 1 \times 2.6$
$= 2.6 hr$

Max m/c 1 capacity
[at $F = (8, 0)$] $= 2 \times 8 + 0$
$= 16 hr$

$2.6 hrs \leq m/c\ 1\ capacity \leq 16\ hrs.$

Rate of revenue change resulting from increasing m/c 2 capacity by 1 hr $= \dfrac{Z_A - Z_B}{9 - 8}$ $\quad E = (3, 2)$

$= \dfrac{130 - 128}{9 - 8}$

$= \$ 2/hr.$

Minimum m/c 2 capacity at

$4 hr \leq m/c\ 2\ capacity \leq 24 hr$

i. If ABC can increase the capacity of both machines which machine should receive priority

(ii) A suggestion is made to increase the capacity of machine 1 & machine 2 at the additional cost of \$10/hr for each machine. Is this advisable?

(iii) If the capacity of machine 1 is increased from 8hrs to 13 hrs, how will this increase impact the optimum revenue.

→ i) Based on the real prices of m/c 1 & m/c 2, m/c 1 ~~profit~~ increases revenue by \$14/hr, when compared to m/c 2.

So m/c 1 should receive priority.

(ii) m/c can be increased, because $(14-10) = 4/hr$

for m/c 2 $(2-10) = -8$ (negative value)

* **Artificial variable technique:**

1. Solve by using Big-M method for the following LPP.

Max $Z = -2x_1 - x_2$

Sub to

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

→ Standard form

Max $Z = -2x_1 - x_2 - Ma_1 - Ma_2$ — (1)

Sub to

$3x_1 + x_2 + a_1 = 3$ $\times M$ ⇒ $3Mx_1 + Mx_2 + Ma_1 = 3M$

$4x_1 + 3x_2 - s_1 + a_2 = 6$ $\times M$ + $4Mx_1 + 3Mx_2 - Ms_1 + Ma_2 = 6M$

$x_1 + 2x_2 + s_2 = 4$

$x_1, x_2, s_1, s_2, a_1, a_2 \geq 0$

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After add
$\Rightarrow 7Mx_1 + 4Mx_2 + Ma_1 - Ms_1 + \ldots$

① - ②

$Z + 2x_1 + x_2 - 7Mx_1 - 4Mx_2 + Ms_1 = -9M$

$Z + (2-7M)x_1 + (1-4M)x_2 + Ms_1 = -9M$

Basis	Z	x_1	x_2	s_1	s_2	a_1	a_2	solution	
Z	1	$2-7M$	$1-4M$	M	0	0	0	$-9M$	
a_1	0	3	1	0	0	1	0	3	= 1
a_2	0	4	3	−1	0	0	1	6	= 3/2
s_2	0	1	2	0	1	0	0	4	= 4

Consider M = larger Values

Basis	Z	x_1	x_2	s_1	s_2	a_1	a_2	solution	
Z	1	0	$(1-5M)/3$	M	0	$-\frac{(2-7M)}{3}$	0	$-2M-2$	
x_1	0	1	1/3	0	0	1/3	0	1	$= 1/1/3 = 3$
a_2	0	0	5/3	−1	0	−4/3	1	2	$2/5/3 = 6/5$
s_2	0	0	5/3	0	1	−1/3	0	3	$3/5/3 = 9/5$

$NZ = OZ - (2-7M)(Nx_1)$

Basis	Z	x_1	x_2	s_1	s_2	a_1	a_2	solution
Z	1	0	0	3/15	0	$M-1/5$		−12/5
x_1	0	1	0	1/5	0	9/15	−1/5	9/15
x_2	0	0	1	−3/5	0	−4/5	3/5	6/5
s_2	0	0	0	1	1	1	1	1

$1-4M - \frac{(2-7M)}{3}$

$\frac{3-12M-2+7M}{3}$

$M - \frac{(2-7M)}{3}$

$\frac{1-5M}{3}$

$2/5/3$

$-9M-2+7M$

$-4/3 \times 3/5$

$-4/3 + 4/3$

$6/5 \times 5/3$

$-3/5 \times 5/3$

2

$-6/15$

$\frac{1+4/15}{3}$

$M - \frac{(1-5M)}{3}(-3/5)$

$\frac{M + 3 - 15M}{15}$

$\frac{7M-2}{3}$

$Z_{max} = -12/5$ $x_1 = 3/5$

$x_2 = 6/5$

$-1/3$

2. Max $Z = 3x_1 - x_2$

s.t

$2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0$

→ Z Max $-3x_1 + x_2 + Ma_1 = 0$ ①

$(2x_1 + x_2 - S_1 + a_1 = 2)M \Rightarrow 2Mx_1 + Mx_2 - MS_1 + Ma_1 = 2M$ → ②

$x_1 + 3x_2 + S_2 = 3$

$x_2 + S_3 = 4$

$x_1, x_2, S_1, S_2, a_1 \geq 0$

① − ②

$Z - 3x_1 + x_2 + Ma_1 = 0$

$2Mx_1 + Mx_2 - MS_1 + Ma_1 = 2M$

$Z - (3+2M)x_1 + (1-M)x_2 - MS_1 = -2M$

Basis	Z	x_1	x_2	S_1	S_2	S_3	a_1	solution	
Z	1	$-3-2M$	$(1-M)$	$-M$	0	0	0	$-2M$	
a_1	0	2	1	−1	0	0	1	2	$2/2 = 1$
S_2	0	1	3	0	1	0	0	3	$3/1 = 3$
S_3	0	0	1	0	0	1	0	4	

Basis	Z	x_1	x_2	S_1	S_2	S_3	a_1	solution	
Z	1	0	$5/2$	$-3/2$	0	0	$(3+2M)/2$	3	
x_1	0	1	$1/2$	$-1/2$	0	0	$1/2$	1	
S_2	0	0	$5/2$	$1/2$	1	0	$-1/2$	2	$=4$
S_3	0	0	1	0	0	1	0	4	

$(1-M) - (-3-2M)\frac{1}{2}$

$2-2M+3+2M)/2$

$-M + (-3-2M)/2$

$-2M-3-2M$

$-4M-3$

$(3+2M) - 2M + 3 + 2M$

$-M + (-3-2M) \cdot -1/2$

$\frac{-2M + 3 - 2M}{2}$

$-(3+2M)$

$(3+2M)/2$

$Z_{max} = 3$

$x_1 = 1 \quad x_2 = 0$

3) Max $Z = 2x_1 + 3x_2$

s. to

$x_1 + 2x_2 \le 4$

$x_1 + x_2 = 3$

$x_1, x_2 \geq 0$

$\Rightarrow$ $Z_{max} - 2x_1 - 3x_2 + Ma_1 = 0$

$x_1 + 2x_2 + s_1 = 4$

$(x_1 + x_2 + a_1 = 3)M$

$x_1, x_2, s_1, a_1 \geq 0$

$Z_{max} - 2x_1 - 3x_2 + \cancel{Ma_1} = 0$

$\underline{\underset{(-)}{M x_1} + \underset{(-)}{M x_2} + \underset{(-)}{\cancel{Ma_1}} = \underset{(-)}{3M}}$

$Z_{max} - 2x_1 - Mx_1 - 3x_2 - Mx_2 = -3M$

$\Rightarrow Z - (2+M)x_1 - (3+M)x_2 = -3M$

Basis	Z	x_1	x_2	s_1	a_1	solution	
Z	1	$-(2+M)$	$-(3+M)$	0	0	$-3M$	
s_1	0	1	2	1	0	4	$= 4/2 = 2$
a_1	0	1	1	0	1	3	$= 3/1$
Z	1	$-(1+M)/2$	0	$(3+M)/2$	0	$6-M$	
x_2	0	$1/2$	1	$1/2$	0	2	$= 2/{}_{1/2} = 4$
a_1	0	0	0	$-1/2$	1	1	
Z	1	0	$(1+M)/2$	$(2+M)$	0	$M+8$	
x_1	0	1	2	1	0	4	
a_1	0	0	0	$-1/2$	1	1	

$Z_{max} = 7$ X

$x_1 = 2$

$-3M$

$\frac{-4}{2} + (3+M)\frac{2}{2}$

$\frac{-4-2M+3+M}{2} =$

$-1-M$

$-2M + 6 + 2M$

$\frac{(1+M)}{2} + \frac{(3+M)}{2}$

$\frac{4+2M}{2}$

$\frac{(1+M)\times 4}{2} + \frac{6-M}{2}$

$2+2M+6-M$

$-(1+M)/2 +$

$2+2M+6-M.$

(3) Min $z = 2x_1 + 3x_2$

sub to

$x_1 + x_2 \geq 5$

$x_1 + 2x_2 \geq 6$

$x_1, x_2 \geq 0$

$\rightarrow$

$((-z_{max}) = -2x_1 - 3x_2)$

$(z' + 2x_1 + 3x_2 = 0)$

$(x_1 + x_2 - s_1 + a_1 = 5)\, M$

$x_1 + 2x_2 - s_2 + a_2 = 6)\, M$

$x_1, x_2, s_1, s_2, a_1, a_2 \geq 0$

$z' + 2x_1 + 3x_2 + Ma_1 + Ma_2 = 0$

$Mx_1 + Mx_2 - Ms_1 + Ma_1 = 5M$

$Mx_1 + 2Mx_2 - Ms_2 + Ma_2 = 6M$

$2Mx_1 + 3Mx_2 + Ma_1 + Ma_2 - Ms_1 - Ms_2 = 11M$

$z' + 2x_1 + 3x_2 + Ma_1 + Ma_2 = 0$

$z' + 2x_1 + 3x_2 + Ms_1 + Ms_2 - 2Mx_1 - 3Mx_2 = -11M$

~~$z' - (2M + 3M)x_2$~~

$z' + (2-2M)x_1 + (3-3M)x_2 + Ms_1 + Ms_2 = -11M$

basis	z	x_1	x_2	s_1	s_2	a_1	a_2	solution	
z	0	$(2-2M)$	$(3-3M)$	M	M	0	0	$-11M$	
a_1	0	1	1	-1	0	1	0	5	$=5$
a_2	0	1	2	0	-1	0	1	6	$6/2=3$
z	1	$(1-M)/2$	0	M	$(3-M)/2$	0	$\frac{(3M-3)}{2}$	$\frac{(-9-2M)}{2}$	
a_1	0	$1/2$	0	-1	$1/2$	1	$-1/2$	2	$=4$
x_2	0	$1/2$	1	0	$-1/2$	0	$1/2$	3	$=6$

$(3-3M)\frac{1}{2} + (\ldots)M$ $\quad \frac{3 - 3M + 4M}{2}$ $\quad (1-M)/2 = \frac{3-M}{2}$ $\quad \frac{(-3+3M)}{2}$ $\quad \frac{(-9+9M)-2M}{2}$ $\quad \frac{-9-13M}{2}$

z	1	0	0	1	1	$M-1$	$M-1$	-11
x_1	0	1	0	-2	1	2	-1	4
x_2	0	0	1	1	-1	-1	$3/2$	1

$z_{min} = +11$

$-\frac{(1-M)}{2}\times 4 \cancel{+ \frac{(9+2M)}{2}}$ $\quad (1-M)\frac{(1-M)}{2} + \frac{3M-3}{2}$

$\cancel{\frac{-4+4M-9}{2}}$ $\quad 2M$

$-\frac{(1-M)}{2} + \frac{(3-M)}{2}$

$\frac{-1+M+3-M}{2}$

$\frac{(3M-3)}{2} - \frac{1+M}{2}$

$\frac{4M-4}{2}$

$\frac{(-9-2M)}{2} + (-2)\frac{(1-M)}{2}$

$\frac{-9-2M-4+4M}{2}$

$-13M + 2M/2$

(4) $\text{Min } z = 4x_1 + x_2$

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0.$

$\rightarrow$ $(z_{min}) = -4x_1 - x_2 \Rightarrow z' + 4x_1 + x_2 = 0$

$(3x_1 + x_2 + a_1 = 3)M$

$(4x_1 + 3x_2 - s_1 + a_2 = 6)M$

$x_1 + 2x_2 + s_2 = 4$

$z' + 4x_1 + x_2 + Ma_1 + Ma_2 = 0.$

$7Mx_1 + 4Mx_2 + Ma_1 + Ma_2 - Ms_1 = 9M$

$z' + 4x_1 + x_2 - 7Mx_1 - 4Mx_2 = -9M$

$z' + (4-7M)x_1 + (1-4M)x_2 = -9M$

$-(4-7M)\frac{1}{3} + (1-4M)$

$\frac{-4+7M+3-12M}{3}$

$\frac{-1-5M}{3}$

Basis	z	x_1	x_2	s_1	s_2	a_1	a_2	solut	
z	1	$(4-7M)$	$(1-4M)$	0	0	0	0	$-9M$	
a_1	0	3	1	0	0	1	0	3	$= 3/3 =$
a_2	0	4	3	-1	0	0	1	6	$= 6/4 = 1.$
s_2	0	1	2	0	1	0	0	4	$= 4/1 = 4$
z	1	0	$-(1+5M)/3$	0	0	$-(4-7M)/3$	0	$-4+7M$	
x_1	0	1	$1/3$	0	0	$1/3$	0	1	$= 3$
a_2	0	0	$5/3$	-1	0	$-4/3$	1	2	
s_2	0	0	$5/3$	0	1	$-1/3$	0	3	

z

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* In two-phase method, phase 1 attempts to find starting feasible solution & if ~~repeat the problem in eqn form & add~~
1 is ~~forward~~ found, phase 2 is invoked to solve the original problem.

Phase-1:

* Put the problem in eq^n form & add necessary artificial variable to the constraints to secure a starting basic solution. Next find basic solution of the resulting equations that always minimize the sum of the artificial variables regardless of whether the LP is maximization or minimization. If the minimum value of the sum is +ve, the LP problem has no feasible solution, otherwise proceed to phase 2.

Phase-2

Use the feasible solution from phase-1 as the starting basic feasible solution for the original problem.

Ex:1. Max $Z = 20x_1 + 10x_2$

sub to

$x_1 + x_2 = 150$

$x_1 \leq 40$ $\quad x_2 \geq 20$

$x_2, x_1 \geq 0$

$\rightarrow$

$x_1 + x_2 + a_1 = 150$

$x_1 + s_1 = 40$

$x_2 - s_2 + a_2 = 20$

$x_1, x_2, s_1, s_2, a_1, a_2 \geq 0$

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Phase I :

$Max(-z) = -a_1 - a_2$

$z' = -a_1 - a_2$

$z' + a_1 + a_2 = 0 \rightarrow (1)$

$z' + a_1 + a_2 = 0$

$x_1 + 2x_2 + a_1 + a_2 = 170$

$z' - x_1 - 2x_2 + s_2 = -170$

Basis	z'	x_1	x_2	s_1	s_2	a_1	a_2	solution
z	1	-1	-2	0	1	0	0	-170
a_1	0	1	1	0	0	1	0	150
s_1	0	1	0	1	0	0	0	40
a_2	0	0	1	0	-1	0	1	20
z	1	-1	0	0	-1	0	2	-130
a_1	0	1	0	0	1	1	-1	130 =130
s_1	0	1	0	1	0	0	0	40 40
x_2	0	0	1	0	-1	0	1	20 0
z	1	0	0	1	-1	0	2	-90
a_1	0	0	0	-1	1	1	-1	90, 90
x_1	0	1	0	1	0	0	0	40
x_2	0	0	1	0	-1	0	1	20
z	1	0	0	0	0	1	1	0
s_2	0	0	0	-1	1	1	-1	90
x_1	0	1	0	1	0	0	0	40
x_2	0	0	1	-1	0	1	0	110

Since $z = 0$, LP has feasible solution

Phase-2

$z = 20x_1 + 10x_2$.

$z - 20x_1 - 10x_2 = 0 \quad — ⓪$

$-s_1 + s_2 = 90 \quad — ①$

$x_1 + s_1 = 40 \quad — ②$

$x_2 - s_1 = 110 \quad \rightarrow ③$

$⓪ + 20(2) + 10③$

$z + 20s_1 - 10s_1 = (800 + 1100)$

$z + 10s_1 = 1900$

Basic	z	x_1	x_2	s_1	s_2	sol
z	1	0	0	10	0	1900
s_2	0	0	0	-1	1	90
x_1	0	1	0	1	0	40
x_2	0	0	1	-1	0	110

$x_1 = 40,\ x_2 = 110$

$z_{max} = 1900$

2. Min $z = 2x_1 + x_2$

sub to

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

$$3x_1 + x_2 + a_1 = 3$$
$$4x_1 + 3x_2 - s_1 + a_2 = 6$$
$$x_1 + 2x_2 + s_2 \cancel{+a_3} = 3$$

2-3+4
3/4
1-9/4
2-1/3
3-4/3
7/3-1/2
7/3
-9+21

Phase-I

$$Z_{min} = a_1 + a_2$$
$$Z' + a_1 - a_2 = 0$$
$$Max\,(-Z_m) = -a_1 - a_2$$
$$Z' + a_1 + a_2 = 0 \rightarrow (1)$$
$$7x_1 + 4x_2 - s_1 + a_1 + a_2 = 9 \rightarrow (2)$$
$$(1) - (2)$$
$$Z' - 7x_1 - 4x_2 + s_1 = -9$$

Basis	Z'	x_1	x_2	s_1	s_2	a_1	a_2	solution	
Z'	1	-7	-4	1	0	0	0	-9	
a_1	0	3	1	0	0	1	0	3	3/3=1
a_2	0	4	3	-1	0	0	1	6	6/4=3/2
s_2	0	1	2	0	1	0	0	3	3/1=3

3 - 4/3 - 1

Z'								
~~x_1~~	0	0	-5/4	3	0	0	1/4	3/2
a_2	0	1	3/4	-1/4	1	0	-1	-3
~~s_2~~	0	0	5/4	1			0	-2
Z'	1	0	-5/3	1	0	7/3	0	1
x_1	0	1	1/3	0	0	1/3	0	1
a_2	0	0	5/3	-1	0	-4/3	1	2
s_2	0	0	5/3	0	1	-1/3	0	2

Remove artificial variables

6/5

z'	1	0	0	1	1	2	0	4	
x_1	0	1	0	0	$-1/5$	$6/15$	0	$3/5$	
x_2	0	0	0	-1	-1	$-5/3$	1	30	
x_2	0	0	1	0	$3/5$	$-1/5$	0	$6/5$	

z'	1	0	0	0	0	1	1	0
x_1	0	1	0	$1/5$	0	$3/5$	$-1/5$	$3/5$
x_2	0	0	1	$-3/5$	0	$-4/5$	$3/5$	$6/5$
S_2	0	0	0	1	1	1	4	0

$$\text{Max } z = 2x_1 + x_2$$

$$\text{Max}(-z) = -2x_1 - x_2)$$

$$z' + 2x_1 + x_2 = 0 \quad (1)$$

$$2\left(x_1 + \frac{1}{5}S_1 = 3/5\right)$$

$$x_2 - 3/5\, s_1 = 6/5 \quad (3)$$

$$(1) - (2) - (3) \quad \cancel{S_1 + S_2 = 0} \qquad 2x_1 + \frac{2}{5}S_1 = 6/5 \quad (2)$$

$$z' + 1/5\, s_1 = -12/5$$

	z'	x_1	x_2	s_1	s_2	solution
z'	1	0	0	$1/5$	0	$-12/5$
x_1	0	1	0	$1/5$	0	$3/5$
x_2	0	0	1	$-3/5$	0	$6/5$

$$x_1 = 3/5 \quad x_2 = 6/5$$

$$z_{min} = 12/5$$

3. Max $Z = 3x_1 + 2x_2$ Pseudo optimal

sub to

$2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

→

* <u>Remarks</u>:

Removal of artificial variables and their columns at the end of phase 1 can take place only when they are all non-basic

If one or more artificial variables are basic at the end of phase 1, then their removal requires the following additional step.

<u>step 1</u>: select zero artificial variable to leave the basic solution & designate its row as pivot row. The entering variable can be any non-basic or (non-artificial variable) with non-zero (positive / negative) coefficient in the pivot row perform the associated simplex iteration

<u>Step 2</u>: Remove the column of (just leaving) artificial variable from the table if all the zero artificial variable have been removed go to phase 2 otherwise go back to step 1.

The logic behind step 1 is that the feasibility of remaining basic variable will not be affected when a zero artificial variable is made non-basic regardless of whether the pivot element is -ve/+ve

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Min $Z = x_1 - 2x_2 - 3x_3$

sub to

$-2x_1 + x_2 + 3x_3 = 2$

$2x_1 + 3x_2 + 4x_3 = 1$

$x_1, x_2, x_3 \geq 0$

$-2x_1 + x_2 + 3x_3 + a_1 = 2$

$2x_1 + 3x_2 + 4x_3 + a_2 = 1$

$x_1, x_2, x_3, a_1, a_2 \geq 0$

$Z_{min} = a_1 + a_2$

$Z' + a_1 + a_2 = 0$

$Z' + a_1 + a_2 = 0$

$4x_2 + 7x_3 \pm a_1 \pm a_2 = 3$

$Z' - 4x_2 - 7x_3 = -3$

Basis	Z'	x_1	x_2	x_3	a_1	a_2	solution	
Z'	1	0	-4	-7	0	0	-3	0.6
a_1	0	-2	1	3	1	0	2	$= 2/3$
a_2	0	2	3	4	0	1	1	$= 1/4 = 0.2$

Z'	1	$7/2$	$5/4$	0	0	$7/4$	$-5/4$
a_1	0	$-7/2$	$-5/4$	0	1	$-3/4$	$5/4$
x_3	0	$1/2$	$3/4$	1	0	$1/4$	$1/4$

$-2 - 3/2$ $1 - 9$

$21/4 - 4$ $2 - 3/4$ $-3 + 7/4$

$21 - 16$ / 4 $-3 + 7/4$ $1 - 9/4$

$-12 +$

Since Basic column contains artificial variable and $Z < 0$, therefore no feasible solution.

* General rules for converting any primal into its dual :

(iii)

1) convert the OF function to maximization form if not (iv)

2) If a constraint has inequality sign $\geq$ then multiply both the sides by (-1) & make the inequality sign $\leq$ Ex 1

3) If a constraint has an equality sign then it is replaced by 2 constraints involving the inequalities going in opposite directions simultaneously

4) Every unrestricted variable is replaced by the difference of two non-negative variables.

5) We get the standard primal form of given LPP in which all the constraints <= sign where the objective function is of maximization form or all the constraints >= sign where the objective function is of minimization form

6) Finally the dual of the given problem is obtained by

(i) Transposing the rows & columns of constraints coefficients

(ii) Transposing the coefficient of the objective f^n as the right side constants of the dual problem.

(iii) changing the inequalities from $\leq$ to $\geq$ sign

(iv) Minimizing the objective f^n instead of maximizing it.

Ex 1: Find the dual of the following given Primal problem

$$\text{Min } Z_x = 2x_2 + 5x_3$$

Sub to

$$x_1 + x_2 \geq 2$$

$$2x_1 + x_2 + 6x_3 \leq 6$$

$$x_1 - x_2 + 3x_3 = 4$$

$$x_1, x_2, x_3 \geq 0$$

→ Standard primal form.

$$\text{Max } (-Z_x) = -2x_2 - 5x_3$$

$$\text{Max } Z_x = -2x_2 - 5x_3$$

Sub to

$$-x_1 - x_2 \leq -2$$

$$2x_1 + x_2 + 6x_3 \leq 6$$

$$x_1 - x_2 + 3x_3 \leq 4$$

$$-x_1 + x_2 - 3x_3 \leq -4$$

$$x_1, x_2, x_3 \geq 0$$

Dual form.

$$-w_1 + 2w_2 + w_3 - w_4 \geq 0 \quad \text{(point 6 (iii))}$$

$$-w_1 + w_2 - w_3 + w_4 \geq -2$$

$$w_2 + 3w_3 - w_4 + 0w_1 \geq -5$$

$$\text{Min } Z_w = -2w_1 + 6w_2 + 4w_3 - 4w_4$$

$$w_1, w_2, w_3, w_4 \geq 0$$

2) Min $Z = 3x_1 - 2x_2 + 4x_3$

s to

$3x_1 + 5x_2 + 4x_3 \geq 7$

$6x_1 + x_2 + 3x_3 \geq 4$

$7x_1 - 2x_2 - x_3 \leq 10$

$x_1 - 2x_2 + 5x_3 \geq 3$

$4x_1 + 7x_2 - 2x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$

→ Max $(-Z_x) = -3x_1 + 2x_2 - 4x_3$

s to

$-3x_1 - 5x_2 - 4x_3 \leq -7$

$-6x_1 - x_2 - 3x_3 \leq -4$

$7x_1 - 2x_2 - x_3 \leq 10$

$-x_1 + 2x_2 - 5x_3 \leq -3$

$-4x_1 - 7x_2 + 2x_3 \leq -2$

Dual form

$-3w_1 - 6w_2 + 7w_3 - w_4 - 4w_5 \geq -3$

$-5w_1 - w_2 - 2w_3 + 2w_4 - 7w_5 \geq 2$

$-4w_1 - 3w_2 - w_3 - 5w_4 + 2w_5 \geq -4$

Min $(Zw) = -7w_1 - 4w_2 + 10w_3 - 3w_4 - 2w_5$

3. Max $Z = 2x_1 + 3x_2 + x_3$

st

$4x_1 + 3x_2 + x_3 = 6$

$x_1 + 2x_2 + 5x_3 = 4$

$x_1, x_2, x_3 \geq 0$

→ Max $(z) = 2x_1 + 3x_2 + x_3$

st

$4x_1 + 3x_2 + x_3 \le 6$ w_1

$-4x_1 - 3x_2 - x_3 \le -6$ w_2

$x_1 + 2x_2 + 5x_3 \le 4$ w_3

$-x_1 - 2x_2 - 5x_3 \ge -4$

~~x_1, x_2, x_3~~

$4w_1 - 4w_2 + w_3 - w_4 \ge 2$

$3w_1 - 3w_2 + 2w_3 - 2w_4 \ge 3$

$w_1 - w_2 + 5w_3 - 5w_4 \ge 1$

Min $(z_{max}) = 6w_1 - 6w_2 + 4w_3 - 4w_4$

$w_1, w_2, w_3, w_4 \ge 0$

(4) Min $z = x_1 + x_2 + x_3$

sub to

$x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \le 3$

$2x_2 - x_3 \ge 4$

$x_1, x_2 \ge 0$ is unrestricted

→ $x_1 - 3x_2 + 4x_3 \le 5$

$-x_1 + 3x_2 - 4x_3 \le -5$

$x_1 - 2x_2 \le 3$

$-2x_2 + x_3 \le -4$

~~$w_1 - w_2 + w_3 - 2w_4$~~

~~-3~~

Max $(z') = -x_1 - x_2 - x_3$

$= -x_2 - x_2 - x_3' + x_3''$

$x_1 - 3x_2 + 4(x_3' - x_3'') \le 5$

$-x_1 + 3x_2 - 4(x_3' - x_3'') \le -5$

$x_1 - 2x_2 \le 3$

$-2x_2 + (x_3' - x_3'') \le -4$

$x_1, x_2, x_3', x_3'' \ge 0$

$x_1 - 3x_2 + 4x_3' - 4x_3'' \leq 5$

$-x_1 + 3x_2 - 4x_3' + 4x_3'' \leq -5$

$x_1 - 2x_2 \leq 3$

~~$2x_2 - x_3' + x_3''$~~

$-2x_2 + x_3' - x_3'' \leq 4$

$w_1 - w_2 + w_3 - 0w_4 \geq -1$

$-3w_1 + 3w_2 - 2w_3 + 2w_4 \geq -1$

$4w_1 - 4w_2 + w_4 \geq 1$

$-4w_1 + 4w_2 - w_4 \geq 1$

$w_1, w_2, w_3, w_4 \geq 0$

$\text{Min } z_w' = 5w_1 - 5w_2 + 3w_3 + 4w_4$

$= 5(w_1 - w_2) + 3w_3 + 4w_4$

1. Find ~~an infeasible but~~ an optimal basic solution for the LLP. then solve ~~them~~ by dual simplex method

$\text{Min. } z = 2x_1 + x_2$

sub to

$3x_1 + x_2 \geq 3 \quad \rightarrow \quad -3x_1 - x_2 \leq -3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

$\text{Max}(-z) = -2x_1 - x_2$

$z' + 2x_1 + x_2 = 0$

Sub to

$-3x_1 + x_2 + s_1 = -3$

$-4x_1 - 3x_2 + s_2 = -6$

$x_1 + 2x_2 + s_3 = -3$

Basis	z	x_1	x_2	s_1	s_2	s_3	solution
z	1	2	1	0	0	0	0
s_1	0	-3	1	1	0	0	-3
s_2	0	-4	-3	0	1	0	-6
s_3	0	1	2	0	0	1	-3

$\leftarrow$ min value x_{B_2}

To find leaving vector denoted as β_r for which r is obtained by $x_{B_r} = \min [x_{B_i}, x_{B_i} < 0]$

To find entering vector denoted as a_k

$$\frac{\Delta_k}{x_{rk}} = \max \left[\frac{\Delta_j}{x_{rj}}, x_{rj} < 0 \right]$$

$$\frac{\Delta_2}{x_{22}} = \max \left[\frac{2}{-4}, \frac{1}{-3} \right]$$

$= 1/-3$

$k = 2$,

Basis	z	x_1	x_2	s_1	s_2	s_3	sol
z	1	2/3	0	0	1/3	0	-2
s_1	0	-5/3	0	1	-1/3	0	-5
x_2	0	4/3	1	0	1/-3	0	2
s_3	0	-5/3	0	0	-2/3	1	1

-3

$-3 + 4/3$

$-9 + 4/3$

$2 - 4/3$

$2 \quad 4$

$1 - \frac{8}{3}$

z'	1	0	0	2/5	1/5	0	-12/5
x_1	0	1	0	-3/5	1/5	0	3/5
x_2	0	0	1	4/5	-3/5	0	6/5
s_3	0	0	0	1	-1	1	0

$$z' = -12/5$$

~~Zmax~~

$Z_{min} = 12/5$ at $x_1 = 3/5$ $x_2 = 6/5$

2. Max $z = -2x_1 - x_3$

sub to

$x_1 + x_2 - x_3 \geq 5$

$x_1 - 2x_2 + 4x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

→ $z_{max} + 2x_1 + x_3 = 0$

$-x_1 - x_2 + x_3 + s_1 = -5$

$-x_1 + 2x_2 - 4x_3 + s_2 = -8$

$x_1, x_2, x_3, s_1, s_2 \geq 0$

Basics	z	x_1	x_2	x_3	s_1	s_2	sol
z	1	2	0	1	0	0	0
s_1	0	-1	-1	1	1	0	-5
s_2	0	-1	2	-4	0	1	-8
z	1	7/4	1/2	0	0	1/4	-2
s_1	0	-5/4	-1/2	0	1	1/4	-3 ←
x_3	0	1/4	-1/2	1	0	-1/4	+2

$\frac{2}{1}$, $0/2$, $1/4$

$-1 + 1/2$

$-1 - 1/4$

$-5 + 8$

$-5 - 2$

$7/4 \times \frac{4}{5}$ 1/2

$-7/5$ -1

$-1/7$

$-5/4 x^2$ $\frac{1}{4}$ -2

-5/2

1/2

$\frac{1}{4} \times 1/2$

$-1/2 + 1/2$

+4x1

7 +2

1/4 5/4

-5/4 1/4 - 1/2

-7

Z	1	1/2	0	0	1	1/2	-9
x_2	0	5/2	1	0	-2	-1/2	14
x_3	0	6/4	0	1	-1	-1/2	9

$Z_{max} = -9 \quad x_1 = 0 \quad x_2 = 14 \quad x_3 = 9$

* Theory of games:

Game: Game is defined as an activity between two or more persons involving activities by each person according to a set of rules at the end of which each person receives some benefit or satisfaction or suffers loss.

The set of rules defines the game going through the set of rules once by the participants defines a play

characteristics of game theory:

There can be various types of games that can be classified on the basis of following characteristics

i) chance of strategy: If in a game activities are determined by skill it is said to be a game of strategy. If they are determined by chance, it is a game of chance. In general a game may involve, game of strategy as well as game of chance

ii) No of persons: A game is called an n-person game, if the no of persons playing is n. The person means an individual or a group aiming at a particular objective.

(iii) no of activities : There may be finite or infinite.

(iv) No of alternatives or choices available to each person in a particular activity may also be finite or infinite

(v) Information to the players about the past activities of other players is completely available partly available or not available at all.

(vi) Pay off A Quantitative measure of satisfaction a person gets at the end of each play is called pay off. It is a real valued function of variables in the game. Let V_i be the payoff to the player P_i $1 \le i \le n$ in an n-person game. If $\sum_{i=1}^{n} v_i = 0$, then the game is said to be zero-sum game

* Basic definitions:

1) competitive game 2) Zero-sum game non-zero sum game 4 properties 5. Payoff matrix

3) Strategy (Mixed strategy, Pure strategy) 4. 2-person 0-sum game / rectangular game

A strategy of a player has been loosely defined as a rule for decision making in advance of the place by which P decides the activities he should adapt.

In other words a strategy for given player is set of rules that specifies which of the available course of action he should make at each play.

This strategy may be of 2 kinds

1) M S

2) P S

Mixed: If a player is guessing as to which activity is to be by other on any particular probabilistic situation the objective function is to maximize the expected game.

Pure: If a player knows exactly what the other player is going to do. A deterministic situation is obtained & objective fun is to maximize the game. Therefore the pure strategy is decision rule always to select a particular course of action.

1. consider (2-person, zero sum) game matrix which represents payoff to the player A. Find the optimal strategy, if any

A \ B				Row min
	-3	-2	$\boxed{6}$	-3
	2	$\boxed{0}$	2	0 → saddle point
	$\boxed{5}$	-2	-4	-4
column max	5	0	6	

→ optimal strategy at saddle point

Player A's selection is Maximin Strategy and his corresponding gain is called maximin value or (Rowmin) lower value ($\underline{V}$)

The player B's selection is called min max value or upper value ($\bar{V}$)

Maximin = minMax, The corresponding pure strategies are called optimal strategies and the game is said to have a saddle point.

2. consider following game

$i \backslash j$	1	2	3
1	3	-4	8
2	-8	5	-6
3	6	-7	6

3. player A can choose his strategies from $\{A_1, A_2, A_3\}$ only, while B can choose from the set $\{B_1, B_2\}$ only. The rules of the game state that the payments should be made in accordance with the selection strategies.

strategy pair selected	payments to be made
(A_1, B_1)	player A plays Rs 1 to player B
(A_1, B_2)	" B " Rs 6 " " A
(A_2, B_1)	" B " Rs 2 " " A
(A_2, B_2)	" B " Rs 4 " " A
(A_3, B_1)	" A " Rs 2 " " B
(A_3, B_2)	" A " Rs 6 " " B

2. Ans :

$i \backslash j$	$\boxed{1}$	2	3
1	3	$\boxed{-4}$	8
2	$\boxed{-8}$	5	-6
3	6	$\boxed{-7}$	6

maxmin & minmax

$\underline{V} \neq \bar{V}$, such games are known as without saddle point game. This game has no optimal strategy

~~3.~~

A_i / B_j	1	2
1	$\boxed{-1}$	6
2	$\boxed{2}$	4 → saddle point
3	-2	-6

maximin = minimax.

The optimal strategy for player A is A_2 and player B is B_1 at $(2,1)$ & the value of game is Rs 2 for A & Rs -2 for B.

This game has optimal strategy

Also $B \neq 0$. The game is not fair

He it is strictly determinable

4. The payoff matrix of a game is given. Find the solution of the game for player A & B.

$$
\begin{array}{cc}
 & \begin{array}{ccccc} I & II & III & IV & V \end{array} \\
A \begin{array}{c} I \\ II \\ III \\ IV \end{array} & \begin{bmatrix} -2 & 0 & 0 & 5 & 3 \\ 3 & 2 & 1 & 2 & 2 \\ -4 & -3 & 0 & -2 & 6 \\ 5 & 3 & -4 & 2 & -6 \end{bmatrix}
\end{array}
$$

→

$$
\begin{array}{c}
\begin{array}{c} I \\ II \\ III \\ IV \end{array}
\begin{bmatrix} \boxed{-2} & 0 & 0 & 5 & 3 \\ 3 & 2 & \boxed{1} & 2 & 2 \\ \boxed{-4} & -3 & 0 & -2 & 6 \\ 5 & 3 & -4 & 2 & \boxed{-6} \end{bmatrix}
\end{array}
\rightarrow \text{saddle point.}
$$

The best strategy for Player A = 2 & player B is 3

The value of game for player A = 1 & for B = -1

5.
$$\begin{array}{c|c} & \begin{array}{ccc} I & II & III \end{array} \\ \begin{array}{c} I \\ II \\ III \end{array} & \begin{bmatrix} \boxed{-2} & 15 & \boxed{-2} \\ -5 & -6 & -4 \\ -5 & \boxed{20} & -8 \end{bmatrix} \end{array}$$

Two Saddle Points

The best strategy for A is 1 & player B is 1 & 3

The value of game for player A = -2

* Rectangular games without Saddle point

Let player A choose a particular activity i, such that i lies b/n 1 and m $(1 \le i \le m)$ with probability. This can also be interpreted as the relatively frequency for which A chooses activity i from no of activities of the game, then set $x = \{x_i, 1 \le i \le m\}$ of probabilities constitute the strategy of A. Similarly $y = \{y_j; 1 \le j \le n\}$ defines the strategy of the player B. Thus the vector $x = (x_1, x_2 \ldots x_m)$ of non-negative number Satisfying $x_1 + x_2 + \ldots x_m = 1$ is called mixed Strategy of player A.

Vector $Y = (y_1 + y_2 \ldots y_n)$ of non-negative number Satisfying $y_1 + y_2 + \ldots + y_n = 1$ is called mixed strategy of player B.

Consider the symbol S_m which denotes the set of ordered tuples of non-negative numbers whose sum is unity and $x \in S_m$ similarly y belongs S_n unless otherwise stated assume that $x \in S_m$ and $y \in S_n$ where x & y are mixed strategies of player A & B respectively

The mathematical expectation of the payoff function $E(X,Y)$ in a game whose pay off matrix is $\{v_{ij}\}$ is defined by

$$E(X,Y) = \sum_{i=1}^{m} \sum_{j=1}^{n} (x_i, v_{ij}) y_j = X^T V Y$$

~~where X, Y are mix~~

Strategic Saddle point:

If $\min \max E(X,Y) = E(X_0, Y_0) = \max \min E(X,Y)$
(X_0, Y_0) is called strategic saddle point of the game where (X_0, Y_0) define the optimal strategies

$V = E(X_0, Y_0)$