Degeneracy :- A tie for the minimum ratio may occur and can be broken arbitarily. Atleast one basic variable will be zero in the next iteration and the new solution is said to degenarate. This can cause the simplex iterations to cycle indefinitly. Thus never terminating the algorithm. This situation arises when the model has atleast one redundant constraint.

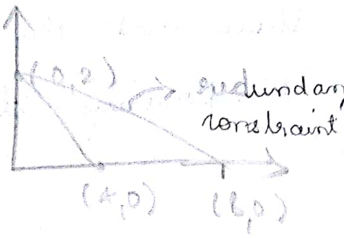
Max $z = 3x_1 + 9x_2$

S.T

$x_1 + 4x_2 \le 8$

$x_1 + 2x_2 \le 4$

$x_1, x_2 \ge 0$



In equation form :

$z - 3x_1 - 9x_2 - 0s_1 - 0s_2 = 0$

S.T

$x_1 + 4x_2 + s_1 = 8$

$x_1 + 2x_2 + s_2 = 4$

$x_1, x_2, s_1, s_2 \ge 0$

Iteration		z	x_1	x_2	s_1	s_2	solution	min ratio
0 (x_2 enters, s_1 leaves)	z	1	-3	-9	0	0	0	
	s_1	0	1	4	1	0	8	8/4 = 2
	s_2	0	1	2	0	1	4	4/2 = 2
1	z	1	-3/4	0	9/4	0	18	
	x_2	0	1/4	1	1/4	0	2	2x4 = 8
	s_2	0	1/2	0	-1/2	1	0	0
2	z	1	0	0	3/2	3/2	18	
	x_2	0	0	1	1/2	-1/2	2	
	x_1	0	1	0	-1	2	0	

new z row

$= (1, -3, -9, 0, 0, 0) - (-9)(0, 1/4, 1, 1/4, 0, 2)$

new z row

$= (1, -3/4, 0, 9/4, 0, 18) - (-3/4)(0, 1, 0, -1, 2, 0)$

$= \frac{9}{4} - \frac{3}{4}$

$\frac{6}{4} = \frac{3}{2}$.

$0 + \frac{3}{4} \times 2$

18

$(0, 1/4, 1, 1/4, 0, 2)$

$- (1/4)(0, 1, 0, -1, 2, 0)$

$\frac{1}{4}$

In iteration 0, s_1 and s_2 tie for the leaving variable leading to degeneracy in iteration 1 where basic variable s_2 obtains a zero value. Degenaracy can lead to cycling i.e in ~~to~~ following iterations the objective value of the problem does not improve and thus it is possible for the simplex method to entire a repetitive sequence of iterations never improving the objective function.

Max $z = 3x_1 + 2x_2$. Show that the simplex situation temporarily degenerates.

S.T

$4x_1 - x_2 \le 8$

$4x_1 + 3x_2 \le 12$

$4x_1 + x_2 \le 8$

$x_1, x_2 \ge 0$

$= (1, -3, -2, 0, 0, 0, 0)$

$- (-3)(0, 1, -1/4, 1/4, 0, 0,$

$(0, 4, 3, 0, 1, 0, 12) - (4)(0, 1, -1$

$0, 0,$

$(0, 4, 1, 0, 0, 1, 8) - (4)(0, 1, -1/4,$

-2

$-2 - \frac{3}{4}$

$0 + \frac{3}{4}$

In equation form:

$z - 3x_1 - 2x_2 - 0s_1 - 0s_2 - 0s_3 = 0$

S.T

$4x_1 - x_2 + s_1 = 8$

$4x_1 + 3x_2 + s_2 = 12$

$4x_1 + x_2 + s_3 = 8$

$x_1, x_2, s_1, s_2, s_3 \ge 0$

Iteration	Basic variables	z	x_1	x_2	s_1	s_2	s_3	solution	min ratio
x_1 leaves	z	1	-3	-2	0	0	0	0	
0	s_1	0	4	-1	1	0	0	8	8/4
s_1 enters	s_2	0	4	3	0	1	0	12	12/4
	s_3	0	4	1	0	0	1	8	8/4
	z	1	0	-11/4	3/4	0	0	6	
1	x_1	0	1	-1/4	1/4	0	0	2	
	s_2	0	0	4	-1	1	0	4	
	s_3	0	0	2	-1	0	1	0	

$3 + \frac{1}{4}$

2	x_1	1	0	0					
	s_2			0					
	x_2	0	0	1	$-1/2$	0	$1/2$	0	
				0					

$\frac{3}{4} + \frac{11}{4}\left(-\frac{1}{2}\right)$

$\frac{3}{4} - \frac{11}{8}$

$\frac{24-44}{32}$

$(1, 0, -11/4, 3/4, 0, 0, 6) - (-11/4)(0, 0, 1, -1/2, 0, 1/2, 0)$

= write explanation after 2nd same solutions

For some LP problems the solution space is unbounded in atleast 1 variable i.e the variable may be increased indefinitly without violating any of the constraints. The associative objective value may also be unbounded. Unbounded solution space is the sign that the problem is poorly constructed.

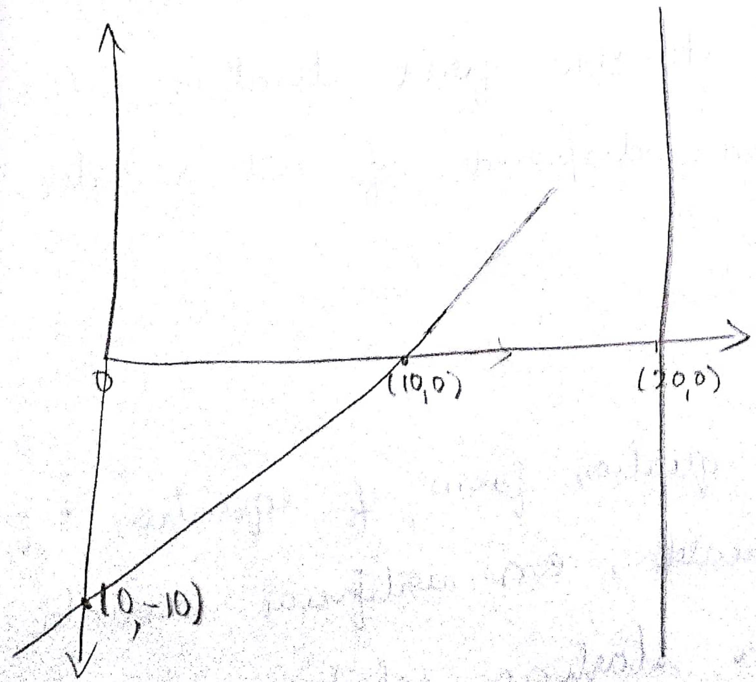
Max $z = 2x_1 + x_2$

S.T

$x_1 - x_2 \leq 10$

$2x_1 \leq 40$

$x_1, x_2 \geq 0$



Max $z - 2x_1 - x_2 = 0$

$x_1 - x_2 + S_1 = 10$

$2x_1 + S_2 = 40$

$x_1, x_2, S_1, S_2 \geq 0.$

Iteration	Basic variables	z	x_1	x_2	S_1	S_2	Solution	Min ratio
0	z	1	-2	-1	0	0	0	
	S_1	0	1	-1	1	0	10	
	S_2	0	2	0	0	1	40	

all the co-efficients of constraints are $-ve$ or 0
hence it is unbounded solution.

<u>Artificial starting solution</u> < m-method

Procedure for starting "ill-behaved LP's" with (=) or ($\geq$) constraints is to use artificial variables that play the role of slacks at the first iteration. The artificial variables are then disposed of at a later iteration.

M- Method:

Starts with an LP in equation form, if equation 'i' does not have a slack variable, an artificial variable 'R_i' is added to form a starting solution similar to the all slack basic solution. The desired goal is achieved by penalizing these variables in the objective function using the following rule i.e penalty rule.

penalty rule for artificial variables :

Given M, a sufficiently large +ve number the objective co-efficient of an artificial variable represents an appropriate penalty of artificial variable objective co-efficient is equal to

$$\begin{cases} -M & \text{in maximization problem} \\ +M & \text{in minimization problem} \end{cases}$$

Q) Max $z = -2x_1 - x_2$

S.T

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

Standard form

Penalty

Max $z = -2x_1 - x_2 + 0S_1 + 0S_2 - Ma_1 - Ma_2$

S.T

artificial variables

$3x_1 + x_2 + a_1 = 3$

$4x_1 + 3x_2 - S_1 + a_2 = 6$ → surplus variable

$x_1 + 2x_2 + S_2 = 4$ → (iii)

→ slack variable

Before going to simplex approach, artificial variables are to be removed, so remove artificial variables i.e a_1 and a_2.

$$Z + 2x_1 + x_2 + M\cancel{a_1} + Ma_2 = 0$$

$$\text{(i)} \times M \Rightarrow 3Mx_1 + Mx_2 + \cancel{Ma_1} = 3M$$

$$Z + (2-3M)x_1 + (1-M)x_2 + \cancel{Ma_2} = -3M$$

$$\text{(ii)} \times M \Rightarrow 4Mx_1 + 3Mx_2 - MS_1 + \cancel{Ma_2} = 6M$$

$$Z + (2-3M-4M)x_1 + (1-M-3M)x_2 + MS_1 = -3M-6M$$

$$\Rightarrow Z + (2-7M)x_1 + (1-4M)x_2 + MS_1 = -9M \Rightarrow \text{objective function}$$

Non - basic variables $\Rightarrow x_1, x_2, S_1 = 0$

$\Rightarrow$ Basic variables $= S_2, a_1, a_2$

In the first row to get biggest -ve values give M values as 100 and check $\Rightarrow$

$$x_1 = 2-7M = 2-700 = -698$$

$$x_2 = 1-4M = 1-400 = -399$$

$\rightarrow$ not there in equation

Iteration	Basic variables	Z	x_1	x_2	S_1	S_2	a_1	a_2	Solution	
0	Z	1	$2-7M$	$1-4M$	M	0	0	0	$-9M$	
	S_2	0	1	2	0	1	0	0	4	4/1
	a_1	0	3	1	0	0	1	0	3	3/3
	a_2	0	4	3	-1	0	0	1	6	6/4
1	Z	1	0	$\frac{1-5M}{3}$	M	0	$\frac{-2+7M}{3}$	0	$-2M-2$	
	S_2	0	0	5/3	0	1	$-1/3$	0	3	
	x_1	0	1	1/3	0	0	1/3	0	1	
	a_2	0	0	5/3	-1	0	$-4/3$	1	2	
2	Z	1	0	0	$\frac{8M-1}{3}$	0	$\frac{15M-6}{15}$		$-12/5$	
	S_2	0	0	0	1	1	1		1	
	x_1	0	1	0	1/5	0	3/5		8/5	
	x_2	0	0	1	$-3/5$	0	$-4/5$	3/5	6/5	

Two - phase method:

phase 1: Put the problem in equation form and add the nesscessary artifical variables to the constraints to secure a starting basic solution.

Find basic solution of the resulting equations that minimizes the sum of artificial variables regardless of whether the LP is max or min.

If the min value of the sum is +ve the ~~the~~ LP problem has no feasible solution otherwise proceed to phase II.

Phase 2:- Use the feasible solution from phase 1 as the starting basic feasible solution for original problem.

1) Max $z = -2x_1 - x_2$

S.T

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

phase 1

min $R = a_1 + a_2$

S.T

$$3x_1 + x_2 + a_1 = 3$$
$$4x_1 + 3x_2 - S_1 + a_2 = 6$$
$$x_1 + 2x_2 + S_2 = 4$$
$$x_1, x_2, S_1, S_2, a_1, a_2 \geq 0$$

Max

$-R = -a_1 - a_2$

$R' = -a_1 - a_2$

Eliminate artificial variables from the objective function

$R' + \cancel{a_1} + a_2 = 0$

$(-)\ 3x_1 \pm x_2 + \cancel{a_1} = 3$

$R' - 3x_1 - x_2 + \cancel{a_2} = -3$

$4x_1 + 3x_2 - s_1 - \cancel{a_2} = -6$

$R' - 7x_1 - 4x_2 + s_1 = -9$

Iteration	Basic variables	R'	x_1	x_2	s_1	s_2	a_1	a_2	Solution	min. ratio
0	R'	1	-7	-4	1	0	0	0	-9	
	a_1	0	3	1	0	0	1	0	3	3/3 = 1
	a_2	0	4	3	-1	0	0	1	6	6/4 = 1.5
	s_2	0	1	2	0	~~0~~1	0	0	4	4/1 = 4
1	R'	1	0	-5/3	1	[illegible]/3	7/3	0	-2	
	x_1	0	1	1/3	0	0	1/3	0	1	3
	a_2	0	0	5/3	-1	0	-4/3	1	2	6/5 = 1.2
	s_2	0	0	5/3	0	1	-1/3	0	3	5
2	R'	1	0	0	0	0	1	1	0	
	x_1	0	1	0	1/5	0	3/5	-1/5	3/5	
	x_2	0	0	1	-3/5	0	-4/5	3/5	1.2	
	s_2	0	0	0	1	1	1	-1	1	

and new $R' = (1, -7, -4, 1, 0, 0, 0, -9) - (-7)(0, 1, 1/3, 0, 0, 1/3, 0, 1)$

$-9+7$

$-\frac{12+7}{3} =$

$-4 + 7 \times \frac{1}{3}$

new $a_2 = (0, 4, 3, -1, 0, 0, 1, 6) - (4)(0, 1, 1/3, 0, 0, 1/3, 0, 1)$

$=$

$3 - \frac{4}{3}$

As minimum $R = 0$, phase I produces the basic feasible solution & artificial variables a_1 & a_2 columns are eliminated.

min. ratio

$3/3 = 1$

$4 = 1.5$

$= 4$

1.2

Phase II :

Max $z = -2x_1 - x_2$

S.T

$$s_1 + s_2 = 1$$
$$x_1 + 1/5\, s_1 = 3/5$$
$$x_2 - \frac{3}{5} s_1 = 1.2$$

form equations from the last table & eliminate artificial variables

$$z + 2x_1 + x_2 = 0$$
$$-2x_1 - 2/5\, s_1 = 6/5$$

$$z + \cancel{x_2} - 2/5\, s_1 = -6/5$$
$$-\cancel{x_2} + \frac{3}{5} s_1 = -6/5$$

$$z + 1/5\, s_1 + \cancel{\frac{4}{5} s_1} = \cancel{3/5}\ 0\ -12/5$$

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	z	x_1	x_2	s_1	s_2	solution
z	1	0	0	1/5	0	-12/5
x_1	0	1	0	1/5	0	3/5
x_2	0	0	1	-3/5	0	6/5
s_2	0	0	0	1	1	1

no negative values in z row. Hence optimal solution with $z = -12/5$ $x_1 = 3/5$ $x_2 = 6/5$ $s_1 = 0$, $s_2 = 1$

Disadvantages of M-method over 2 phase method:

→ computationaly inconvinient because of the manupulation of the constant M. 2 phase method ~~meth~~ eliminates the constant M from calculations.

→ To compute ~~in~~ LP using computer, M must be assigned to a numerical value larger than all the co-efficients used in the objective function.

→ Max $z = 3x_1 + 2x_2 - Ma_1$

S.T

$2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

In eq form

$2x_1 + x_2 + S_1 = 2$

$3x_1 + 4x_2 - S_2 + a_1 = 12$

$x_1, x_2, S_1, S_2 \geq 0$

use Big-M method:

$z - 3x_1 - 2x_2 - 0S_1 + 0S_2 + Ma_1 = 0$

Scanned by CamScanner

$$Z - 3x_1 - 2x_2 + Ma_1 = 0$$
$$-3Mx_1 \mp 4Mx_2 \overset{(+)}{-} MS_2 \mp Ma_1 = 12M$$

$$Z - 3x_1 - 2x_2 - 3Mx_1 - 4Mx_2 + MS_2 = -12M$$

$$Z - (3+3M)x_1 - (2+4M)x_2 + MS_2 = -12M$$

Iteration	Basic variable	Z	x_1	x_2	S_1	S_2	a_1	solution	min ratio
x_2 enters	Z	1	$-(3+3M)$	$-(2+4M)$	0	M	0	$-12M$	
0	S_1	0	2	$\boxed{1}$	1	0	0	2	
S_1 leaves	a_1	0	3	$\boxed{4}$	0	-1	1	12	
	Z	1	$1+5M$	0	$2+4M$	M	0	$-4M+4$	
1	x_2	0	2	1	1	0	0	2	
pseudo optimum	a_1	0	-5	0	-4	-1	1	4	→ value is not zero

we have reached the optimum solution &

new Z row = $(1, -(3+3M), -(2+4M), 0, M, 0, -12M)$ artificial
$\quad - (-(2+4M))(0, 2, 1, 1, 0, 0, 2)$. solution variable $\neq 0$.

$= (-3-3M) + (2+4M)(2)$

$= -3 - 3M + 4 + 8M$

$= 1 + 5M$

Optimum iteration 1 shows that the artificial variable a_1 is +ve meaning that LP is infeasible. The result is what we call pseudo optimum.

① Toolco produces 3 types of tools. T_1, T_2, T_3 the tools use raw ~~mate~~ materials M_1 & M_2. According to the data given the table. The available daily quantities of raw materials M_1 & M_2 are 1000, 1200. The marketing research, shows that the daily demand of all the 3 tools must be atleast 500 units can the manufacturing department satisfy the demand.

Raw Material	Number of units of raw material / tool		
	T_1	T_2	T_3
M_1	3	5	6
M_2	5	3	4

Big-M.

$3x_1 + 5x_2 + 6x_3 \leq 1000$
$5x_1 + 3x_2 + 4x_3 \leq 1200$ (infeasible)
$x_1 + x_2 + x_3 \geq 500$
$x_1, x_2, x_3 \geq 0$.

In equation form,

$z = x_1 + x_2 + x_3 - Ma$

→ Use 2-phase simplex method,

min $z = x_1 - 2x_2 - 3x_3$ S.T

$-2x_1 + x_2 + 3x_3 = 2$

$2x_1 + 3x_2 + 4x_3 = 1$ & $x_1, x_2, x_3 \geq 0$.

Max $(-z) = -x_1 + 2x_2 + 3x_3$

phase 1: Min $R = a_1 + a_2$
S.T

$-2x_1 + x_2 + 3x_3 + a_1 = 2$

$2x_1 + 3x_2 + 4x_3 + a_2 = 1$

$x_1, x_2, x_3, a_1, a_2 \geq 0$

Max $\quad -R = -a_1 - a_2$

$R^1 = -a_1 - a_2$

$+2x_1 \overset{(-)}{+} x_2 \overset{(-)}{+} 3x_3 \overset{(-)}{+} \cancel{a_1} = 2$

$R^1 + \cancel{a_1} + a_2 = 0$

$R^1 + \cancel{a_2} + 2x_1 - x_2 - 3x_3 = -2$

$-2x_1 \overset{(-)}{+} 3x_2 \overset{(-)}{+} 4x_3 \overset{(-)}{+} \cancel{a_2} = -1$

$R^1 + \cancel{2x_1} - \cancel{x_2} - 3x_3 - \cancel{2x_1} - \cancel{3x_2} - 4x_3 = -3$

$R^1 - \cancel{4x_1} - 4x_2 - 7x_3 = -3$

Iteration	Basic variables	R^1	x_1	x_2	x_3	a_1	a_2	solution	ratio
0	R^1	1	0	-4	-7	0	0	-3	
	x_1								
	a_1								
	a_2								

15/10/18

Unit - II

Sensitivity Analysis :- In linear programming the parameters (input data) of the model can change within certain limits without causing a change in the optimum solution. This is referred to as sensitivity analysis.

Two cases are considered for sensitivity analysis:

1> Sensitivity of optimal solution to changes in the availability of the resources. (RHS of the constraints)

2> Sensitivity of optimum solution to changes in the unit profit or unit cost (co-efficients of the objective function).

1> Jobco manufactures 2 products on 2 machines. a unit of product 1 requires 2 hrs on machine 1 & one hr on machine 2. For product 2 one unit requires 1 hr on machine 1 and 3 hrs on machine 2. The revenues of ~~ur~~ per unit of product 1 & product 2 are \$ 30 & \$20. The total daily processing time available for each m/c is 8 hrs.

Max $z = 30x_1 + 20x_2$

x_1: no of products 1

x_2: no of product 2.

S.T $2x_1 + 1x_2 \leq 8$

$x_1 + 3x_2 \leq 8$

$x_1, x_2 \geq 0$

① $2x_1 + x_2 = 8$

$x_2 = 0 \quad x_1 = 4$

$x_1 = 0 \quad x_2 = 8$

② $x_1 + 3x_2 = 8$

$x_1 = 0 \quad x_2 = 2.66$

$x_2 = 0 \quad x_1 = 8$

$2x_1 + x_2 \le 9$

$x_1 = 0 \quad x_2 = 9$

$x_2 = 0 \quad x_1 = 4.5$

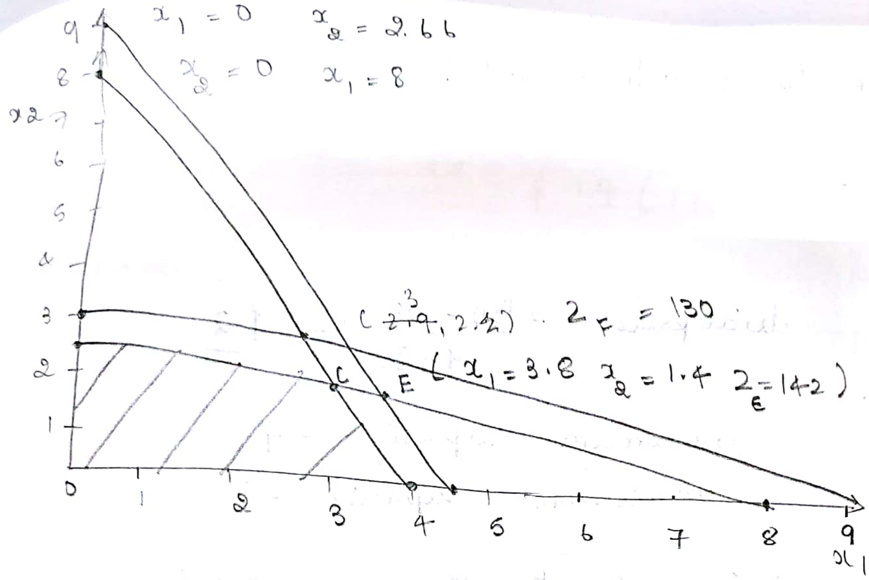
$2x_1 + x_2 = 8$

$(-)\, 2x_1 + 6x_2 = 18$

$-5x_2 = -10$

$x_2 = 2$

$x_1 = \frac{2.9}{3}$



Z	point
0	(0,0)
52	(0,2.66)
120	(4,0)
$Z_C = 128$	(3.2, 1.6)

$(0,8) \rightarrow 24$

$(4,0) \rightarrow 4$

Machine I

profit (dual price) (shadow price) $= \dfrac{Z_E - Z_C}{\text{capacity change}} = \dfrac{142 - 128}{9 - 8} = \$14$

If the daily capacity is ~~increased~~ is increased from 8 hrs to 9 hrs the new optimum will move to point E.

The rate of revenue change resulting from increasing machine 1 capacity $= \dfrac{Z_E - Z_C}{\text{capacity change}}$ (Z_E → new optimum, Z_C → previous optimum)

The computed rate provides a direct link b/w the model i/p and its o/p.

1. Minimum capacity → 2.67 hrs (0, 2.66)

Maximum capacity → 16 hrs (8, 0)

2 ~~4~~ 8 + 0 = 16

2.6 hrs ≤ capacity of m/c ≤ 16 hrs

Machine II

$x_1 + 3x_2 \leq 9$

$x_1 + 3x_2 = 9$

$x_1 = 0 \quad x_2 = 3$

$x_2 = 0 \quad x_1 = 9$

dual price $= \frac{130 - 128}{9 - 8} = \$2$

minimum capacity = 4

maximum capacity = 24

4 ≤ capacity of m/c ≤ 24 hrs.

feasibility limits

(a) If jobco can increase the capacity of both machines which m/c would be given priority.

machine 1, dual price = \$14.

(b) If a suggestion is made to increase the capabilities of m/c 1 & 2 at the additional cost of \$10 per hr for each m/c. Is this advisible.

In m/c 1 the additional net revenue per hr is = 14-10 = \$4 & for m/c 2 the net is 2-10 = -\$8

hence only m/c 1 should be considered for capacity increase.

(c) If the capacity of m/c 1 is increased 8 hrs to 13 hrs how will this increase impact ~~on~~ the optimal revenue.

the dual price for m/c 1 is \$14 and is applicable in the range 2.67 to 16 hrs. ~~Inc~~ Hence the increase in the revenue is

$$\$14(13-8) = 70.$$

(d) Suppose the capacity of M1 is ↑ to 20 hrs, how will this increase affect the optimum revenue.

The proposed change is outside the feasibility range of 2.67, 16 hrs. Thus we can only make an immediate conclusion regarding an increase upto 16 hrs. The available information is not sufficient to make a complete decision

(2) change in the co-efficients of the objective function.

Max $z = 30x_1 + 20x_2$

S.T

$$2x_1 + x_2 \le 8 \qquad \frac{c_1}{c_2} = 2$$

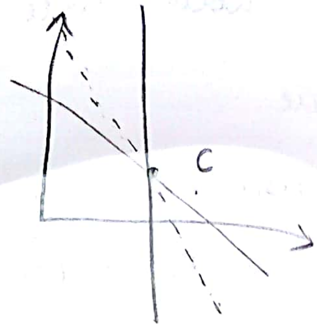
$$x_1 + 3x_2 \le 8$$

$$x_1, x_2 \ge 0 \qquad \frac{c_1}{c_2} = \frac{1}{3}.$$

(a) Suppose that the unit revenue for products 1 & 2 are changed to \$35 and \$25 respectively will the current optimum remain the same.

$\frac{1}{3} \leq \frac{c_1}{c_2} \leq 2$

$0.333 \leq \frac{c_1}{c_2} \leq 2$ ⟶ optimality range



new objective function :

$z = 35x_1 + 25x_2$

$\frac{c_1}{c_2} = \frac{35}{25} = 1.4$

Solution at c will remain optimal because $\frac{c_1}{c_2} = 1.4$ which remains in the optimality range $(0.333, 2)$.

$x_1 = 3.2 \qquad x_2 = 1.6$

$z = 35 \times 3.2 + 25 \times 1.6$

$= 152$

(b) The unit revenue of product 2 is fixed at its current value $c_2 = \$20$. What is the associated optimality range for the unit revenue of product I c_1 that will keep the optimum unchanged.

$\frac{1}{3} \leq \frac{c_1}{c_2} \leq 2$

$\frac{1}{3} \leq \frac{c_1}{20} \leq 2$

$\frac{20}{3} \leq c_1 \leq 40$

$6.66 \leq c_1 \leq 40$

6.6

3 20
18
020

c) at $c_1 = \$30$

$$\frac{1}{3} \le \frac{c_1}{c_2} \le 2$$

$$\frac{1}{3} \le \frac{30}{c_2} \le 2$$

$$\frac{1}{90} \le \frac{1}{c_2} \le \frac{2}{30}$$

$$90 \le c_2 \le 15$$

$$15 \le c_2 \le 90 .$$

2) A company manufactures 2 products A & B, and revenues are \$2 & \$3. Two raw materials M_1 & M_2 used in the manufacture of 2 products have daily availabilities of 8 & 18 units. One unit of A uses two units of M_1 & 2 units of M_2 & 1 unit of B uses 3 units of M_1 & 6 units of M_2.

(a) Determine the dual prices of M_1 & M_2 and their feasibility ranges.

(b) suppose that 4 additional units of M_1 can be acquired at the cost of 30 cents/unit, would you recommend an additional purchase.

(c) What is the most the company should pay for unit of M_2.

(d) If M_2 availability is increased by 5 units. Determine the associated optimum revenue.

(e) Determine the optimality condition for $\frac{c_a}{c_b}$ that will keep the optimum unchanged.

(f) If the unit revenues C_A & C_B are changed simultaneously to \$5 and \$4. Determine the new optimum solution.

The dual problem is defined systematically from the original LP model. Optimal solution of one problem automatically provides the optimal solution to the other.

Key ideas for constructing the dual from the primary.

1) Assign a new variable for each primal constraint ~~with~~ (equality) (dual variable).

2) Construct a dual constraint for each primal variable.

3) The constraint co-efficients and the objective co-efficients of the j^{th} primal variable ~~defined~~ the LHS & RHS of the j^{th} dual constraint.

4) The dual objective co-efficients equal the RHS of the primal constraints equation.

5) The sense of optimization, direction of inequalities and the size of the variables in the dual are shown as follows:

Maximization Problem	Minimization Problem
constraints	variables
$\geq$	≤ 0
$\leq$	≥ 0
$=$	unrestricted
variables	constraints
≥ 0	$\leq$
≤ 0	$\geq$
unrestricted	variables

General rules for converting any primal into its dual :

Step 1: First convert the objective function to maximization form, if not.

Step 2: If the constraint has the inequality sign ('$\geq$') then multiply both sides by (-1) and make the inequality sign into ($\leq$).

Step 3: If the constraint has the equality sign then it is replaced by 2 constraints involving the inequalities going in the opposite directions simultaneously

eg: $x_1 + 3x_2 = 8$

$x_1 + 3x_2 \leq 8$

Step 4: Every unrestricted variable is replaced by the difference of 2 non-negative variables.

Step 5: We get the standard primal form of the given LPP in which

(i) all the constraints have less than or = sign where the objective function is of the maximization form.

(ii) All constraints have $\geq$ sign where the objective function is in the minimization form.

Step 6: Finally the dual of the given problem is obtained by

(i) Transposing the rows & columns of the constraint co-efficients.

(2) Transposing the ~~co-efficing~~ co-efficients of the objective function and the RHS of the constraints.

(3) Changing the inequalities from $\leq$ to $\geq$ sign

(4) Minimizing the objective function instead of maximizing it.

Convert primal to dual:

Max $z = 5x_1 + 12x_2 + 4x_3$

S.T

$x_1 + 2x_2 + x_3 \leq 10$

$$2x_1 - x_2 + 3x_3 = 8$$

$$x_1, x_2, x_3 \geq 0$$

~~In dual~~ form:

$$
\left.
\begin{aligned}
Z &= 5x_1 + 12x_2 + 4x_3 \\
\text{S.T} \\
& x_1 + 2x_2 + x_3 \leq 10 \quad - y_1 \\
& 2x_1 - x_2 + 3x_3 \leq 8 \quad - y_2 \\
& -2x_1 + x_2 - 3x_3 \leq -8 \quad \to y_3 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
\right\}
$$

standard primal form.

$$[\,5 \quad 12 \quad 4\,]$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 3 \\ -2 & 1 & -3 \end{bmatrix} \leq \begin{bmatrix} 10 \\ 8 \\ -8 \end{bmatrix}$$

Dual form:

$$\min \quad w = 10y_1 + 8y_2 - 8y_3$$

S.T

$$y_1 + 2y_2 - 2y_3 \geq 5$$

$$2y_1 - y_2 + y_3 \geq 12$$

$$y_1 + 3y_2 - 3y_3 \geq 4$$

$$y_1, y_2, y_3 \geq 0$$

$$[\,10 \quad 8 \quad -8\,]$$

$$\begin{bmatrix} 1 & 2 & -2 \\ 2 & -1 & 1 \\ 1 & 3 & -3 \end{bmatrix} \geq \begin{bmatrix} 5 \\ 12 \\ 4 \end{bmatrix}$$

2) min $z_1 = 2x_2 + 5x_3$

S.T

$x_1 + x_2 \geq 2$

$2x_1 + x_2 + 6x_3 \leq 6$

$x_1 - x_2 + 3x_3 = 4$

& $x_1, x_2, x_3 \geq 0$

$$\left.\begin{array}{ll} \max \quad z' = -2x_2 - 5x_3 & \\ -x_1 - x_2 \leq -2 & y_1 \\ 2x_1 + x_2 + 6x_3 \leq 6 & y_2 \\ x_1 - x_2 + 3x_3 \leq 4 & y_3 \\ -x_1 + x_2 - 3x_3 \leq -4 & y_4 \\ x_1, x_2, x_3 \geq 0 & \end{array}\right\} \text{standard primal form}$$

$w_x = -2y_1 + 6y_2 + 4y_3 - 4y_4$ (or)

S.T

$-y_1 + 2y_2 + y_3 \cancel{+ 4y_4} \geq \cancel{-2}\ 0$

$-y_1 + y_2 - y_3 + \cancel{4}y_4 \geq -2$

$0y_1 + 6y_2 + 3y_3 + \cancel{3}y_4 \geq -5$

~~$0y_1 + 0y_2 + 0y_3$~~

$y_1, y_2, y_3 \geq 0$.

y_4

min $= w_x = -2y_1 + 6y_2 + 4p$

S.T

$-y_1 + 2y_2 + p \geq 0$

$-y_1 + y_2 - p \geq -2$

$6y_2 + 3p \geq -5$

$y_1, y_2 \geq 0$, p is unrestricted.

where $p = y_3 - y_4$

3 Max $z = 2x_1 + 3x_2 + x_3$

S.T

$4x_1 + 3x_2 + x_3 = 6$

$x_1 + 2x_2 + 5x_3 = 4$

$x_1, x_2, x_3 \geq 0$.

In standard primal form:

Max $z = 2x_1 + 3x_2 + x_3$

S.T

$4x_1 + 3x_2 + x_3 \leq 6 \quad - y_1$

$-4x_1 - 3x_2 - x_3 \leq -6 \quad - y_2$

$x_1 + 2x_2 + 5x_3 \leq 4 \quad - y_3$

$-x_1 - 2x_2 - 5x_3 \leq -4 \quad - y_4$.

$x_1, x_2, x_3 \geq 0$

$y_1 - y_2 = P_1$

$y_3 - y_4 = P_2$

In Dual form:

min $w = 6y_1 - 6y_2 + 4y_3 - 4y_4$

S.T

$4y_1 - 4y_2 + y_3 - y_4 \geq 2$

$3y_1 - 3y_2 + 2y_3 - 2y_4 \geq 3$

$y_1 - y_2 + 5y_3 - 5y_4 \geq 1$

$\rightarrow$ here P_1, P_2 are unrestricted

$4P_1 + P_2 \geq 2$

$3P_1 + 2P_2 \geq 3$

$P_1 + 5P_2 \geq 1$

$y_1, y_2, y_3, y_4 \geq 0$

(4) min $z = 2x_1 + 3x_2 + 4x_3$

S.T

$2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 = 3$

$x_1 + 4x_2 + 6x_3 \leq 4$

$x_1, x_2 \geq 0$ & x_3 is unrestricted

min $z = 2x_1 + 3x_2 + 4(x_3 - x_3')$

S.T

$2x_1 + 3x_2 + 5(x_3 - x_3') \geq 2$

$3x_1 + x_2 + 7(x_3 - x_3') = 3$

$x_1 + 4x_2 + 6(x_3 - x_3') \leq 4$

$x_1, x_2, x_3, x_3' \geq 0.$

Max $z' = -2x_1 - 3x_2 - 4(x_3 - x_3')$

$-2x_1 - 3x_2 - 5(x_3 - x_3') \leq -2$ y_1

$3x_1 + x_2 + 7(x_3 - x_3') \leq 3$ y_2

$-3x_1 - x_2 - 7(x_3 - x_3') \leq -3$ y_3

$x_1 + 4x_2 + 6(x_3 - x_3') \leq 4$ y_4.

$w = -2y_1 + 3y_2 - 3y_3 + 4y_4$.

S.T

$-2y_1 + 3y_2 - 3y_3 + y_4 \geq -2$

$-3y_1 + y_2 - y_3 + 4y_4 \geq -3$

$-5y_1 + 7y_2 - 7y_3 + 6y_4 \geq -4$

$5y_1 - 7y_2 + 7y_3 - 6y_4 \geq 4$

$y_1, y_2, y_3, y_4 \geq 0.$

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Dual Simplex method:

① min $z = Z = 2x_1 + x_2$

S.T

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

& $x_1, x_2 \geq 0$

max $z' = -2x_1 - x_2$ (Standard LPP)

S.T

$-3x_1 + x_2 \leq -3$

$-4x_1 - 3x_2 \leq -6$

$-x_1 - 2x_2 \leq -3$

$x_1, x_2 \geq 0.$

Max $z' + 2x_1 + x_2 = 0$ (Equation form)

$-3x_1 - x_2 + S_1 = -3$

$-4x_1 - 3x_2 + S_2 = -6$

$-x_1 - 2x_2 + S_3 = -3$

$x_1, x_2, S_1, S_2, S_3 \geq 0$

	z'	x_1	x_2	S_1	S_2	S_3	Solution	Iteration
z'	1	2	1	0	0	0	0	0
S_1	0	-3	-1	1	0	0	-3	leaving var
S_2	0	-4	-3	0	1	0	-6	= min (-3, -6, -3, 0)
S_3	0	-1	-2	0	0	1	-3	S_2 leaves

max $\left(\frac{2}{-4}, \frac{1}{-3}\right)$

$\hookrightarrow$ denominator < 0.

						0		-2
z'	1	$2/3$	0	0	$1/3$			
s_1	0	$-5/3$	0	1	$-1/3$	0		-1
x_2	0	$4/3$	1	0	$-1/3$	0		2
s_3	0	$5/3$	0	0	$-2/3$	1		1
z'	1	0	0	-1	$2/3$	0		-1
x_1	0	$+1$	0	$-3/5$	$1/5$	0		$3/5$
x_2	0	0	1	$4/5$	$-9/15$	0		$6/5$
s_3		0						

min $(-1, 2, 1)$
s_1 leaves

max $\left(\frac{2/3}{-5/3}, \frac{1/3}{-1/3}\right)$

max $\left(\frac{2}{-5}, -1\right)$

solve till solution space becomes +ve.

new $z' = (1, 2, 1, 0, 0, 0, 0) - (1)(0, 4/3, 1, 0, -1/3, 0, 2)$

$= 2 - \frac{4}{3}$

new $s_1 = (0, -3, -1, 1, 0, 0, -3) - (-1)(0, 4/3, 1, 0, -1/3, 0, 2)$

$= -3 + \frac{4}{3}$

new $s_3 = (0, -1, -2, 0, 0, 1, -3) - (-2)(0, 4/3, 1, 0, -1/3, 0, 2)$

$= -1 + 2 \times \frac{4}{3}$

$= -1 + \frac{8}{3}$

$-\frac{1}{3} - \frac{4}{15}$

$\frac{1}{3}\left[-1 - \frac{4}{5}\right]$

new $z' = \left(1, \frac{2}{3}, 0, 0, \frac{1}{3}, 0, -2\right) - (-5/3)\left(0, 1, 0, -\frac{3}{5}, \frac{1}{5}, 0, \frac{3}{5}\right)$

$= \frac{\cancel{5}}{\cancel{3}} \times -\frac{\cancel{3}}{\cancel{5}}$ $\quad \frac{1}{3} + \frac{\cancel{5}}{3} \times \frac{1}{\cancel{5}} = \frac{2}{3}$ $\quad -2 + \frac{\cancel{5}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{5}}$

$\frac{1}{3}\left[\frac{-9}{5}\right]$

$2 - \frac{4}{5}$

$\frac{6}{5}$

new $x_2 = \left(0, \frac{4}{3}, 1, 0, -1/3, 0, 2\right) - \left(\frac{4}{3}\right)\left(0, 1, 0, -\frac{3}{5}, \frac{1}{5}, 0, \frac{3}{5}\right)$

$=$ $\quad \frac{4}{\cancel{3}} \times \frac{\cancel{3}}{5}$ $\quad -\frac{1}{3} - \frac{4}{3} \times \frac{1}{5}$ $\quad 2 - \frac{4}{\cancel{3}} \times \frac{\cancel{3}}{5}$

② max $z = -2x_1 - x_3$

S.T

$x_1 + x_2 - x_3 \geq 5$

$x_1 - 2x_2 + 4x_3 \geq 8$

$x_1, x_2, x_3 \geq 0.$

new z

$= (1, 2, 0, 1, 0, 0, 0)$
$- (1)(0, \frac{1}{4}, \frac{-2}{4}, 1, 0, \frac{-1}{4}, -2)$

$2 - \frac{1}{4}$

$0 + \frac{2}{4}$

In standard LPP

$z = -2x_1 - x_3$

$-x_1 - x_2 + x_3 \leq -5$

$-x_1 + 2x_2 - 4x_3 \leq -8$

$x_1, x_2, x_3 \geq 0.$

new s_1

$= (0, -1, -1, 1, 1, 0, -5)$
$- (1)(0, \frac{1}{4}, \frac{-2}{4}, 1, 0, \frac{-1}{4}, -2)$

$-1 - \frac{1}{4}$

$-1 + \frac{2}{4}$

$-5 + 2$

In equation form,

$z + 2x_1 + x_3 = 0$

S.T

$-x_1 - x_2 + x_3 + s_1 = -5$

$-x_1 + 2x_2 - 4x_3 + s_2 = -8$

$x_1, x_2, x_3, s_1, s_2 \geq 0.$

	z	x_1	x_2	x_3	s_1	s_2	Solution	iteration
z	1	2	0	1	0	0	0	0
s_1	0	-1	-1	1	1	0	-5	min (-8, -5)
s_2	0	-1	2	-4	0	1	-8	max
								$(\frac{2}{-1}, \frac{1}{-4})$
z	1	7/4	2/4	0	0	1/4	2	(-2, -0.25)
s_1	0	-5/4	-2/4	0	1	1/4	-3	
x_3	0	$\frac{1}{4}$	$\frac{-2}{4}$	1	0	$-\frac{1}{4}$	-2	
z								
x_1	0	1	2/5	0	$-\frac{4}{5}$	-1/5	$\frac{12}{5}$	
x_3								

3. Prove that a dual of a dual is primal.

Max $z + 3x_1 + x_2 + x_3 - x_4 = 0$

S.T

$$x_1 + 5x_2 + 3x_3 + 4x_4 \le 5$$

$$x_1 + x_2 = -1$$

$$x_3 - x_4 \le -5$$

$$x_1, x_2, x_3, x_4 \ge 0.$$

max $z + 3x_1 + x_2 + x_3 - x_4 = 0$

S.T $z = -3x_1 - x_2 - x_3 + x_4$

$$x_1 + 5x_2 + 3x_3 + 4x_4 \le 5 \qquad y_1$$

$$x_1 + x_2 \le -1 \qquad y_2$$

$$-x_1 - x_2 \le 1 \qquad y_3$$

$$x_3 - x_4 \le -5 \qquad y_4$$

$w = 5y_1 - y_2 + y_3 - 5y_4$

S.T

$$y_1 + y_2 - y_3 + 0y_4 \ge -3$$

$$5y_1 + y_2 - y_3 + 0y_4 \ge -1$$

$$3y_1 + 0y_2 + 0y_3 + y_4 \ge -1$$

$$4y_1 + 0y_2 + 0y_3 - y_4 \ge 1$$

$w = 5y_1 - y_2 + y_3 - 5y_4$

S.T

$$-y_1 - y_2 + y_3 \le 3 \qquad x_1$$

$$-5y_1 - y_2 + y_3 \le 1 \qquad x_2$$

$$-3y_1 - y_4 \le 1 \qquad x_3$$

$$-4y_1 + y_4 \le -1 \qquad x_4$$

$$2 = 3x_1 + x_2 + x_3 - x_4$$

$$-x_1 - 5x_2 - 3x_3 - 4x_4 \geq 5$$

$$-x_1 - x_2 - x_3(0) + x_4(0) \geq -1$$

$$x_1 + x_2 \geq 1$$

$$\cancel{y} - x_3 + x_4 \geq -5$$

Game theory - It is a type of decision theory in which one's choice of action is determined after taking into account all possible alternatives available to an opponent playing the same game, rather than just by possibilities of several outcomes. The mathematical analysis of complicated problems is fundamentally based on the minimax criterion. This criterion implies the assumption of rationality from which it is argued that each player will act so as to maximize is minimum ~~game~~ gain or minimize is maximum loss.

Characteristics of game theory :-

1) Chance of strategy - If in a game activities are determined by skill and it is called game of strategy. If they are determined by chance, it is chance. In general a game a game of ~~strateg~~

can include game of strategy as well as game of chance.

2> No of persons :- A game is called an n-person game if the no of persons playing it is n. The person means an individual or group aiming at a particular objective.

3> No of activities :- There can be finite or infinite activities.

4> No of alternatives or choices :- available to each person in a particular activity they can also be finite or infinite.

5> Information to the players about the past activities of other players is completely available, or partially available or not available at all.

6> Pay off :- A quantitative measure of satisfaction a person gets at the end of each play is cal[led] a pay off. It is a real valued function of varia[bles] in the game. Let v_i be the pay off to the player P_i lies $1 \leq i \leq n$ in a n-person game

If $\sum_{i=1}^{n} v_i = 0$ then the game is said to be a zero sum game.

Pure strategy :- If a player knows exactly what the other player is going to do, a deterministic situation is obtained and the objective function is to maximize the gain. Therefore the pure strategy is a decision rule always used to select a particular course of action.

Mixed strategy :- If a player is guessing as to which activity is to be selected by the other on a particular situation a probabilistic situation is obtained and the objective function is to maximize the ~~exep~~ expected gain.

Two - person - zero - sum game [rectangular games] -
A game with only 2 players is called a 2-person zero sum game if the losses of one player are equivalent to the gain's of the other. So that the sum of their net gains is zero

Pay off matrix :- suppose the player A has m activities and the player B has n activities then the pay-off matrix can be formed by adopting the following rules :

→ Row designations for each matrix are activities available to player A.

→ Column designations for each matrix are activities available to player B.

→ ~~Sell~~ cell entry v_{ij} is a payment of player A's pay off matrix when A chooses the activity i & B chooses the activity j.

→ With the zero-sum 2 person game the cell entry in the player B's ~~paff~~ pay off matrix will be negative of the corresponding cell entry v_{ij} in the player A's pay off matrix so that the sum of the pay off matrices for player A & B is ultimatly zero.

→ Consider a coin matching game involving 2 players only. Each player selects either a head or a tail if the outcomes match A wins ₤1 from B otherwise B wins ₤1 from player A obtain the pay off matrix.

A' ~~pay off~~ pay off matrix :

$$A \begin{array}{c} \\ H \\ T \end{array} \begin{array}{cc} H & T \\ \hline \end{array}$$

$$\begin{array}{cc} & \begin{array}{cc} H & T \end{array} \\ A \begin{array}{c} H \\ T \end{array} & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}$$

B's pay off matrix

$$\begin{array}{cc} & \begin{array}{cc} & A \\ H & T \end{array} \\ B \begin{array}{c} H \\ T \end{array} & \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \end{array}$$

minimax criterion :- Minimax criteria of optimality states that if a player lists the worse possible outcomes of all potential strategies he will choose that strategy to be more suitable for him which corresponds to the best of these worse outcomes. Such a strategy is called an optimal strategy.

Consider zero-sum 2 person ~~game~~ pay off matrix of A. Find optimal strategy if any.

max {-3, 0, -4} → 0

min {5, 0, 6} = 0

A \ B	I	II	III	
I	-3	-2	6	-3
II	2	$\boxed{0}$	2	0
III	5	-2	-4	-4
	5	$\boxed{0}$	6	

→ saddle point

→ max

0 → among the min the max is 0.

min → among the max the min is 0.

player A wishes to obtain the largest possible v_{ij} by choosing one of his activities while the player B is determined in minimizing the game of player A. Player A is called ~~it~~ a maximizing player & player B is called the minimizing player.

player A is minmax strategy

player B is maximization ~~strategy~~

maximum strategy

player A → the maximizing player

The corresponding pure strategies are called optimal strategies & the game is said to have saddle point.

player B → the minimizing player

$\max_{i} \min_{j} = 0$

$\min_{i} \max_{j} = 0$

→ Give the optimal strategies for each player for the following 2-person zero sum game:

$$\begin{array}{c|ccc|c}
i \backslash j & 1 & 2 & 3 & \\
\hline
1 & 3 & -4 & \boxed{8} & -4 \\
2 & -8 & \boxed{5} & -6 & -6 \\
3 & \boxed{6} & -7 & 6 & -7 \\
\hline
 & 6 & 5 & 8 &
\end{array}$$

no saddle point: $\min_{i} \max_{j} = -4$

$\max_{i} \min_{j} = 5$

$$\begin{array}{cc|c}
 & B & \\
A & \begin{bmatrix} 1 & 1 \\ 4 & -3 \end{bmatrix} & \begin{array}{c} 1 \\ -3 \end{array}
\end{array}$$

→ Find the range of p and q which will render an entry (2,2) as saddle point for the game

$$\begin{array}{c} \begin{pmatrix} 2 & 4 & 5 \\ \boxed{10} & \boxed{7} & q \\ 4 & p & 6 \end{pmatrix} \begin{array}{c} 2 \\ 7 \\ 4/p \end{array} \\ \begin{array}{ccc} 10 & 7 & q \end{array} \end{array}$$

$4 \leq p \leq 7$

$q \geq 7$

$V \rightarrow$ value of the game

$\underset{i}{\min}\ \underset{j}{\max} = V_{ij} = 7$ maximin $= \underline{V}$

$\underset{j}{\min}\ \underset{i}{\max} = V_{ij} = 7$ minimax $= \overline{V}$.

→ A saddle point of a pay-off matrix is a position of such an element in the pay off matrix which is minimum in its row & maximum in its column

$$\begin{cases} \underset{j}{\max}\ \underset{i}{\min}\ V_{ij} \\ \underset{j}{\min}\ \underset{i}{\max}\ V_{ij} \end{cases}$$

mathe

optimal strategies :- If a pay off matrix has ~~a~~ the saddle ~~p~~ (r, s) then the players A and B are said to have the r^{th} & s^{th} optimal strategy..

value of game :- The pay off $V_{r,s}$ at saddle point at saddle point (r, s) is called value of game and is equal to maximin ($\underline{V}$) and minimax ($\overline{V}$) of the game.

The game is a fair game if $\bar{V} = \underline{V} = 0$

A game is said to be strictly determinable if
$\bar{V} = V = \underline{V}$
$\hookrightarrow$ value of the game.

→ Determine which of the following 2 person zero sum are strictly determinable & fair give the optimal strategies for each of the player in the case of strictly determinable games.

(a)

player B

$$\text{player A} \begin{bmatrix} 1 & 1 \\ \boxed{4} & -3 \end{bmatrix} \begin{matrix} 1 \\ -3 \end{matrix}$$
$$\quad 4 \quad 1$$

1. Saddle point (1, 2)
2. optimal strategy for player A → 1
   optimal strategy for player B → 2

It is not a fair game, but strictly determinable

$$\begin{bmatrix} -5 & +2 \\ -7 & -4 \end{bmatrix}$$

→ player A can choose his strategy from A_1, A_2, A_3 only while B can choose from B,
only.

the rules of the game state that the payments must be made in accordance with selection of strategies.

Strategy Pair selected	Payments to be made
$(A_1 B_1)$	player A pays ₹1 to B.
$(A_{\cancel{1}} B_2)$	player B pays ₹6 to A.
$(A_2 B_1)$	B pays ₹2 to A.
$(A_2 B_2)$	B pays ₹4 to A.
$(A_3 B_1)$	A pays ₹2 to B.
$(A_3 B_2)$	player A pays ₹6 to B.

what strategies should A & B play in order to get the optimum benefit of the game. Give the value of the game & indicate whether the strategies are fair & strictly determinable.

$$\begin{array}{c|cc|c} & B_1 & B_2 & \\ A_1 & -1 & 6 & -1 \\ A_2 & \boxed{2} & 4 & 2 \\ A_3 & -2 & -6 & -6 \\ \hline & 2 & 6 & \end{array}$$

pay off of A

max min ② = the strategies are fair & strictly determinable.

2

not fair strictly determinable

$A \rightarrow A_2$
$B \rightarrow B_1$

$$\begin{array}{c|cc|cc} & & & & \\ A_1 & 1 & -6 & -6 & \\ A_2 & -2 & -4 & -4 & -4 \\ A_3 & \boxed{2} & +6 & +2 & \\ \hline & 2 & -4 & & \end{array}$$

pay off of B

-4

$A \rightarrow A_3$
$B \rightarrow B_1$

not fair strictly determinable

$$\begin{array}{c|ccccc} & \mathrm{I} & \mathrm{II} & \mathrm{III} & \mathrm{IV} & \mathrm{V} \\ \hline \mathrm{I} & -2 & 0 & 10 & \boxed{5} & 3 \\ \mathrm{II} & 3 & 2 & \boxed{1} & 2 & 2 \\ \mathrm{III} & -4 & -3 & 0 & -2 & \boxed{6} \\ \mathrm{IV} & \boxed{5} & \boxed{3} & -4 & 2 & -6 \end{array}$$

$\max\{-2, 1, -4, -6\} = 1$

$\min\{5, 3, 1, 5, +6\} = 1$

$$\begin{array}{c} \\ \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \\ \\ \end{array} \begin{array}{ccc} \mathrm{I} & \mathrm{II} & \mathrm{III} \\ \end{array}$$

$$\begin{array}{c|ccc|c} & \mathrm{I} & \mathrm{II} & \mathrm{III} & \\ \hline \mathrm{I} & -2 & 15 & \boxed{-2} & -2 \\ \mathrm{II} & -5 & -6 & -4 & -6 \quad -2 \\ \mathrm{III} & -5 & \boxed{20} & -8 & -8 \\ \hline & -2 & 20 & -2 & \end{array}$$

$\max\{-2, -5, -8\} = -2$

$\min\{-2, 20, -2\} = -2$

~~A → (1, 1)~~ B (1

$A \to 1$, $B \to 1$ or 3

The optimal strategy of A

→ Two players A & B without showing each other put on a table a coin with H or T. A wins ₹ 8 when both the coins show head and ₹ 1 when both are tails. B wins ₹ 3 when the coins do not match. Given the choice of being the matching player A or non-matching player B. Which one would you choose & what would be your strategy? ~~to~~ solve using arithmetic 2x2 method:

$$
\begin{array}{c|cc|c}
 & & B & \\
 & H & T & \\
\hline
A\ H & 8 & -3 & 4 \\
T & -3 & 1 & 11 \\
\hline
 & 4 & 11 &
\end{array}
$$

column values

A's pay off matrix

this method

we add column 1

& place value

near column 2

& vice versa

ignore negative sign

optimum stragy for player A $\rightarrow$

if H $\rightarrow \frac{4}{11+4} = \frac{4}{15}$ (consider the column values)

if T $\rightarrow \frac{11}{11+4} = \frac{11}{15}$

optimum strategy for player B $\rightarrow \left(\frac{4}{15}, \frac{11}{15}\right)$.

Using A's oddments:

Value of game for A with head (H) -

$$= \frac{8 \times 4 + -3 \times 11}{11+4}$$ (consider rows) 1st one.

$$= ₹\left(-\frac{1}{15}\right)$$

Value of game for B with T

$$= \frac{-3\,(4) + 1 \times \cancel{11}}{4+11}$$

using B's oddments.

value for H

$$= \frac{5 \times 4 + (-3) \times 11}{11+4}$$

value for T

$$V = \frac{-3 \times 4 + 1 \times 11}{11+4}$$

value of the game $= ₹\left(-\frac{1}{15}\right)$ for player A (loss)

$-\left(+\frac{1}{15}\right)$ for player B (gain)

→ Solve the following using the arithmetic method for the given pay-off matrix :

(a)

$$\begin{array}{cccc} & & B & \\ A & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} & & \begin{array}{c} + \\ 4 \end{array} \\ & \begin{array}{cc} +3 & 2 \end{array} & & \end{array}$$

→ for A's

optimum strategy for

$A \rightarrow \frac{1}{1+4}$

optimum strategy for

$A \rightarrow \frac{4}{1+4}$.

opti $B \rightarrow \frac{3}{5}$

$B \rightarrow \frac{2}{5}$

b >

$$\begin{array}{cccc} & & B & \\ A & \begin{bmatrix} -4 & 6 \\ 2 & -3 \end{bmatrix} & & \begin{array}{c} 5 \\ 10 \end{array} \\ & \begin{array}{cc} 9 & 6 \end{array} & & \end{array}$$

For A $\rightarrow \left(\frac{5}{15}, \frac{10}{15}\right)$

For B $\rightarrow \left(\frac{9}{15}, \frac{6}{15}\right)$

$$= \frac{-4(5)+6(10)}{15} = \frac{40}{15} = \frac{-4(9)+\ldots}{15}$$

$$= \frac{2(5)-3(10)}{15} = \frac{-20}{15} = \frac{6 \times 9 \ldots - 3(6\ldots}{1\ldots}$$

$\frac{-36+1\ldots}{15}$

$$= \frac{1 \times 3 + 4 \times 2}{5} = \frac{11}{5}$$

using A's oddments

B's

I

$$= \frac{5 \times 3 + 1 \times 2}{5} = \frac{15+2}{5} = \frac{17}{5}$$

I $= \frac{5 \times 1 + 3 \times 4}{1+4}$

II

$$= \frac{3 \times 3 + 4 \times 2}{3+2} = \frac{9+8}{5} = \frac{17}{5}$$

II $= \frac{1 \times 1 + 4 \times 4}{1+4}$.

$= \frac{-24}{15}$

$= \frac{36}{16}$

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$$= \frac{-4 \times 9 + 6 \times 6}{9+6} = 0$$

$$= \frac{2 \times 9 + (-3) \times 6}{9+6} = 0$$

$$= \frac{-4(5) + 2 \times 10}{15}$$ for B's

$$= \frac{6 \times 5 + (-3) \times 10}{15}$$

value of the game = 0

no gain & no loss.

→ A & B each take out 1 or 2 matches and guess how many matches the opponent has taken. If one of the players guess correctly then the loser has to pay as many rupess as the sum of the no held by ~~the~~ both players. otherwise the ~~play o~~ pay out is zero. write down the ~~play of~~ pay off matrix & optimum strategy for both players.

$$\begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ A \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \end{array}$$

Unit - IV

PERT | CPM

PERT - project evaluation and review technique

CPM - critical path method.

PERT | CPM are time - oriented methods that is they both lead to the determination of time schedule for project.

Applications of PERT | CPM techniques :-

1. These methods have been applied to a wide variety of problems in industries, these include construction of the dam or ~~canal~~ canal system in a region.
2. construction of a building or highway.
3. Defining the prototype of a machine
4. maintenance or overhaul of aeroplane or oil machine
5. cost control of a project.
6. Development of supersonic planes.

Basic steps in PERT/CPM technique:-

Planning: The planning phase is started by splitting the total project into smaller projects, smaller projects into activities and are analysed by the department or section. The relationships of the activities are defined and established.

Scheduling:- The ultimate objective of this phase is to prepare a time chart showing the start and finish times for each activity as well as its relationships to other activities of the project. The schedule must pin point the critical path activities which require special attention if the project is to be completed in time. For non-critical activities the schedule must show the slack or ~~and~~ float which can be used advantageously when such activities are delayed when limited resources are to be utilized effectively.

Allocation of resources:- It is performed to achieve the desired objective. A resource is a physical variable such as labour, finance, equipment and space that will impose a limitation time for the project.

controlling - critical path methods facilitate the application of principle of managment by exception to identify areas that are critical to completion of the project.

In a network representation the following basic definitions are used:

a) activity - Any individual operation which utilizes the resources and has an end and an beginning is called an activity. An arrow is commonly used to represent an activity with its head indicating the direction of the object.

(a) predecessor activity - Activities that must be completed immediately prior to the start of another activity is called predecessor activity.

(b) successor activity - Activities that cannot be started until one or more of other activities are completed, ~~no~~ but immediatly succeed them.

(c) concurrent activity :- Activities which can be accompanied concurrently are known as this.

(d) Dummy activity :- Activity which does not consume any amount of resource but is mainly depicted for ~~techno~~ technological dependance is called a dummy activity. →

event :- An event represents a point in time signifying the completion of some activities & beginning of new activities. It is denoted by O node / connector

1. merge event :- When more than one activity comes and joins an event, such event is known as the merge event.

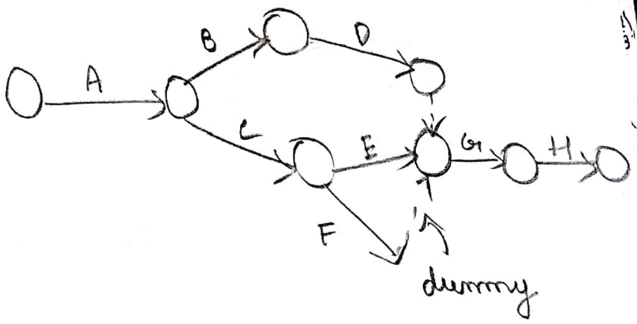
2. burst event :- When more than one activity leaves an event, such event is known as the burst event.

3. merge & burst event :- An activity can be merge & burst event at the same time.

Sequencing :- The first pre-requiste in the development of the network is to maintain the precedence relationship. In order to make a network the following points needed to be considered :

1. what jobs | job precede it?
2. what job | jobs run concurrently?
3. " " " follow it?
4. what controls the start & finish of a job.

jobs	predecessor
A	—
B	A
C	A
D	B
E	C
F	C
G	D, E, F
H	G



only 1st start event & one end event

A	—
B	—
C	A
D	A
E	B
F	D
G	B, C