PERT-CPM UNIT-4

Project Evaluation and Review Technique

Critical path method

1. **Activity**: It is a part of a total project plan to which a known resource is applied. It consumes time & cost of resource using. It is shown by an arrow diagram

$$(1) \xrightarrow{A} (2)$$

~~and~~ size ~~and~~ shape of arrow doesn't indicate activity time or cost paid for resource.
An activity operates b/n 2 nodes only.
There can be exactly one activity b/n 2 nodes. This is the main logic applicable in network.
An activity is carried out b/n 2 events

Activity relationship:
An activity generally denoted using capital letters like A, B, C . . . is a continous process of some act b/n two nodes.

* **parallel activity**:

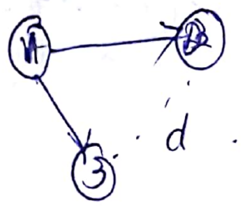
At ~~[illegible]~~ stage in a network you may have more than one activities beginning from one node all such are called parallel activities

When more than one activity coming out from different nodes end up in a common node. All such activities are called concurrent activities

* **Dummy activity**:

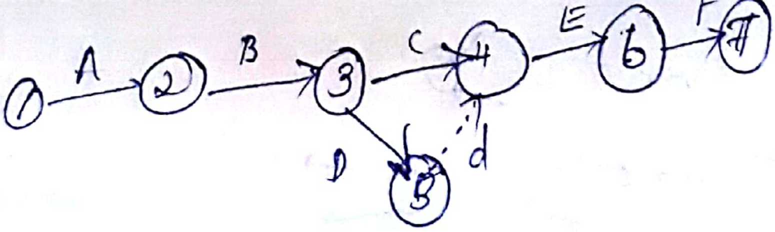
→ An activity from a node can be started only if all the different activities converging ~~onto~~ that node are over

→ There can be one ~~and~~ only activity b/n any two nodes.



Ex: Activity predecessor Activity

Activity	predecessor Activity
A	-
B	A
C	B
D	B
E	C, D
F	E

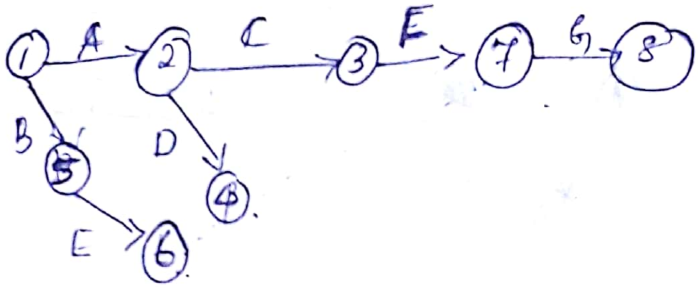
→ 

2. A < C, D

B < E

→

Activity	PA
A	–
B	–
C	A
D	A
E	B
F	C
G	D, E, F



Event:

An event represents a point in time signifying the completion of some activities and beginning of new ones. This is usually represented by a circle in a network which is also called a node or connector. ① - e.g.

The events can be further classified into 3 different categories

1) Merge event 

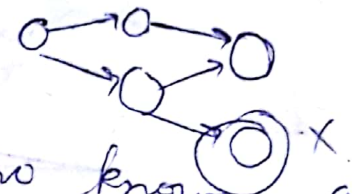
2) Burst event 

3) Merge & Burst event 

* common errors in drawing network.

1) Dangling: To disconnect an activity before

the completion of activities in a network diagram is known as dangling.



2) <u>looping</u>:

Looping error is also known as cycling error in a network. Drawing an endless loop in a network is known as error of looping

3) <u>Redundancy</u>: Unnecessarily inserting dummy activity in a network logic is known as error of redundancy.

* <u>Rules to construct a network</u>

(i) Each activity is ~~(constructed)~~ represented by only one arrow in the network

(ii) No two activities can be identified by the same end events  ×

(iii) In order to ensure the correct precedence in relationship in the arrow diagram, foll ques must be checked whenever any activity is added to netwo

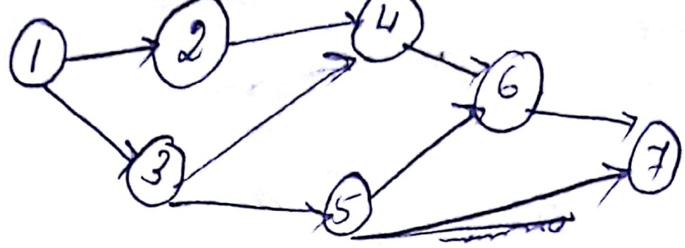
1) what activity must be completed before this activity can start?

2) what activities must follow this activity

3) what activities must occur simultaneously with this activities.

1. Draw an event oriented network

Event no	1	2	3	4	5	6	7
Immediate Predecessor	-	1	1	2,3	3	4,5	5,6

→ 

2) Draw a network for the following project

(a) A is the start event & K is the end event (activity)

(b) B is successors event to A

(c) C & D are successors events to B

(d) D is the preceding event to G

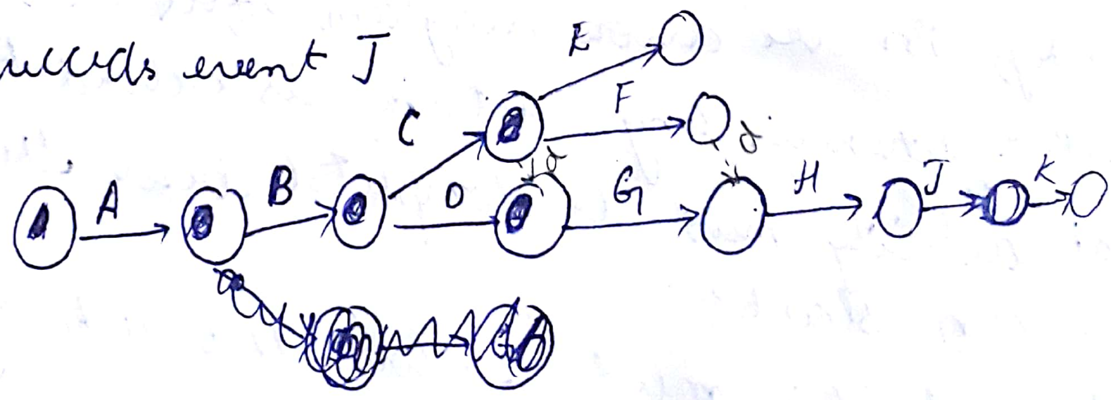
(e) E & F occur after event C.

(f) C restrains the occurrence of G & G precedes H

(g) H precedes J

(h) F restrains the occurrence of H

(i) K succeeds event J

→ 

Activity	preceding activity
A	-
B	A
C	A
D	C
E	B, C
F	D, E
G	E
H	G
I	D, F
J	I, H
K	J

* Labelling : Fulkerson's rules i, j :

1. A start event is the one which has arrows emerging from it but none entering it. Find the start event and number it has 1.

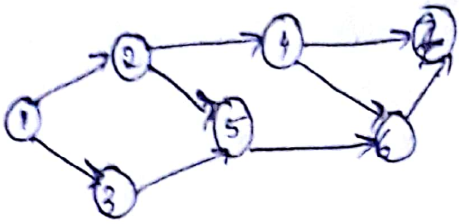
2. Delete all arrows emerging from all numbered events, this will create atleast one new start event out of preceding events

3. Number all new start events 2, 3 . .

4. Go on repeating steps no 2 & 3 until the end is reached.

Ex:

1.



(i, j)

Activity (i, j) from the event i to j,

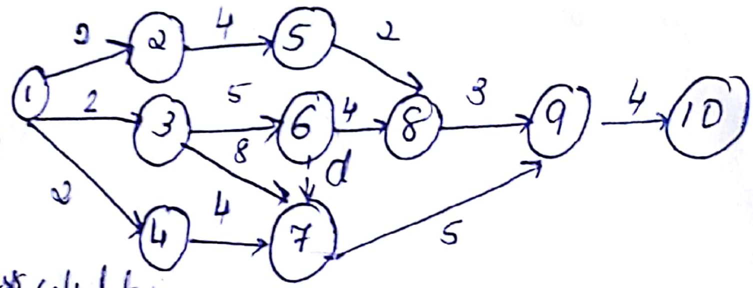
→ E_i (T_E) earliest occurrence time of event i

→ L_j, T_L : latest occurrence time of event j

→ D_{ij} : Estimated completion time of activity (i, j)

1. consider the following network where nodes have been numbered

$E_1 = 0$



(i) Forward pass calculation

$E_2 = 2 \quad E_3 = 2 \quad E_4 = 2$

$E_5 = 2 + 4 = 6 \quad E_6 = 7$

$E_7 = \max(10, 7, 6)$

$= 10$

$E_8 = \max(8, 11) \quad E_9 = \max(14, 15) \quad E_{10} = 19$

$E_8 = 11 \quad E_9 = 15$

(ii) Backward pass calculation:

$L_{10} = 19 \qquad L_7 = 15 - 5 = 10$

$L_9 = 19 - 4 = 15$

$L_8 = 15 - 3 = 12$

$L_6 = (12-4) = 8 \quad 10-0=10$

$\min(8, 10)$

$= 8$

$L_5 = 10$

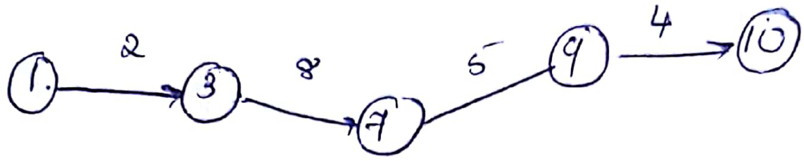
$L_1 = 0$

$L_4 = 10-4 = 6$

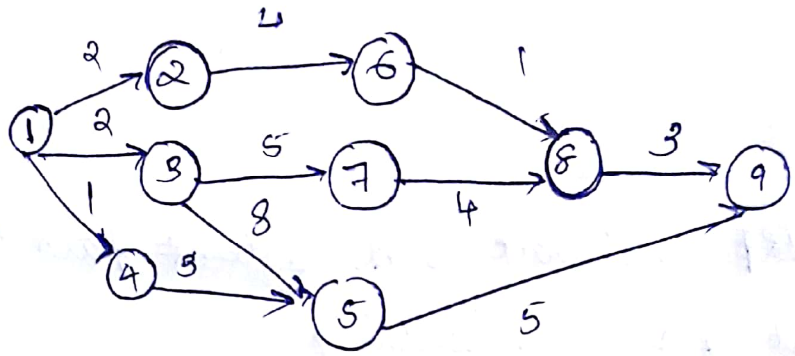
$L_2 = 10-4 = 6$

$L_3 = \min(2, 3)$

$= 2$



2.



→ $E_p = 0 \quad E_2 = 2 \quad E_3 = 2 \quad E_4 = 1$

$E_5 = \max(10, 4) = 10$

$E_6 = 6 \quad E_7 = 7 \quad E_8 = \max(7, 11) = 11$

$E_9 = \max(14, 15) = 15$

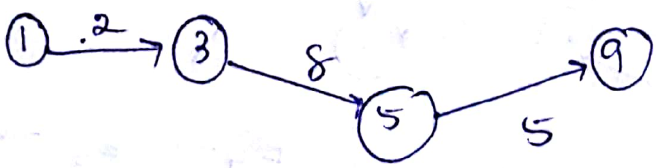
$L_9 = 15 \quad L_8 = 12 \quad L_5 = 10$

$L_6 = 11 \quad L_7 = 8 \quad L_2 = 7$

$L_3 = \min(3, 2) = 2$

$L_4 = 10-3 = 7 \quad L_1 = \min(5, 0, 6)$

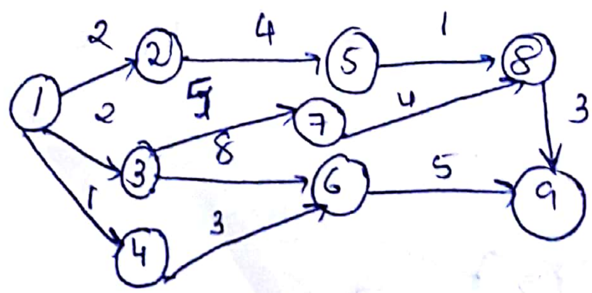
$L_1 = 0$



Activity	Normal Time (D_{ij})	Earliest Time		Latest Time		Flo[illegible]
		Start (E_i)	Finish ($E_i + D_{ij}$)	Start $L_j - D_{ij}$	Finish L_j	L_j
(1, 2)	2	0				
(1, 3)	2					
(1, 4)	1					

3. construct PERT network and ~~critical~~ path compute
(i) Total float for each activity
(ii) critical path & its duration

Activity	Time in months	Activity	Time in months
1 – 2	2	4 – 6	3
1 – 3	2	5 – 8	1
1 – 4	1	6 – 9	5
2 – 5	4	7 – 8	4
3 – 6	8	8 – 9	3
3 – 7	5		



$E_1 = 0$ $E_2 = 2$ $E_3 = 2$ $E_4 = 1$ $E_5 = 6$, $E_6 = ma$[illegible]

$E_7 = 7$ $E_8 = \max(11, 7) = 11$ $10, 4) =$

$E_9 = \max(15, 14) = 15$

(i) $L_9 = 15$ $L_8 = 12$ $L_6 = 10$ $L_7 = 8$ $L_5 = 11$ $L_2 = 7$

$L_1 = 0$

$L_4 = 7$ $L_3 = \min(3, 2) = 2$

Activity	Normal Time D_{ij}	Earliest Time Start (E_i)	Earliest Time Finish $E_i + D_{ij}$	Latest Time Start $L_j - D_{ij}$	Latest Time Finish L_j	Float $(L_j - D_{ij}) - E_i$
(1-2)	2	0	2	5	7	5
(1-3)	2	0	2	0	2	0
(1-4)	1	0	1	6	7	6
(2-5)	4	2	6	7	11	5
(3-6)	8	2	10	2	10	0
(3-7)	5	2	7	3	8	1
(4-6)	3	1	4	7	10	6
(5-8)	1	6	7	11	12	5
(6-9)	5	10	15	10	15	0
(7-8)	4	7	11	8	12	1
(8-9)	3	11	14	12	15	1

(ii) $1 \xrightarrow{2} 3 \xrightarrow{8} 6 \xrightarrow{5} 9$

free float $(F_{f_{ij}}) = (E_i - E_j) - D_{ij}$.

Independant float : $(I_{f_{ij}}) = (E_j - L_i) - D_{ij}$

optimistic t_o

most likely t_m

pesimistic t_p

Expected time $= t_e = (t_o + 4t_m + t_p)/6$

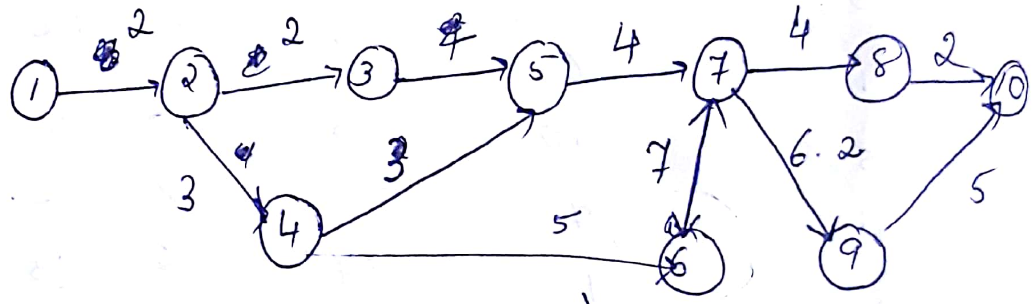
Ex: Find the critical path for following characteristics

Activity	t_o	t_p	t_m	Activity	t_m	t_p	t_o
1-2	1	5	1.5	6-7	6	8	7
2-3	1	3	2	7-8	2	6	4
2-4	1	5	3	7-9	5	8	6
3-5	3	5	4	8-10	1	3	2
4-5	2	4	3	9-10	3	7	5
5-7	4	6	5				
4-6	3	7	5				

construct a PERT network & find CP & Variance for each event

(4,-6) - missing

→



$$t_{e_1} = (t_o + 4t_m + t_p)/6$$

$$= (1 + 4 \times 1.5 + 5)/6$$

$$= 12/6 = 2$$

$$t_{e_2} = (1 + 4 \times 2 + 3)/6$$

$$= 12/6 = 2$$

$$t_{e_3} = (1 + 3 \times \cancel{4} + 5)/6$$

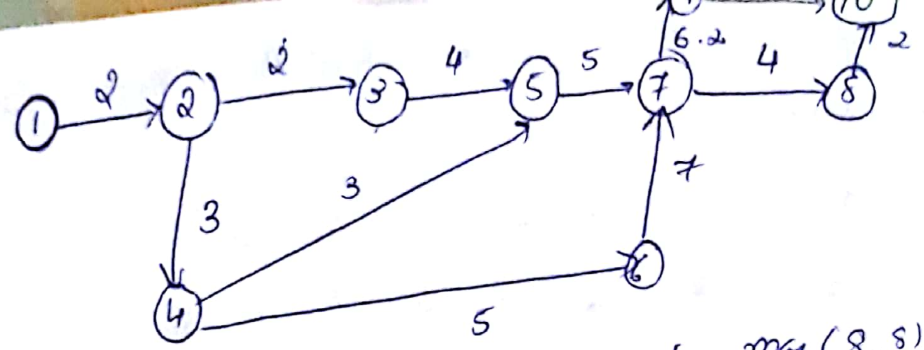
$$= 18/6 = 3$$

$$t_{e_4} = (3 + 4 \times 4 + 5)/6$$

$$= 24/6 = 4$$

$$t_{e_5} = (2 + (3 \times 4) + 4)/6$$

$$= 18/6 = 3$$



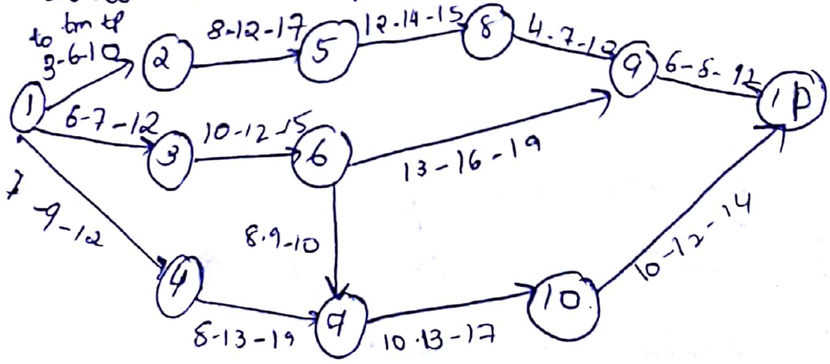
$E_1 = 0$ $E_2 = 2$ $E_3 = 4$ $E_4 = 5$ $E_5 = \max(8, 8) = 8$ $E_6 = 10$

$E_7 = \max(13, 17) = 17$ $E_8 = 21$ $E_9 = 23.2$ $E_{10} = 28.2$

$L_{10} = 28.2$ $L_9 = 23.2$ $L_8 = 26.2$ $L_7 = \min(22.2, 17) = 17$

$L_6 = 10$ $L_5 = 12$ $L_3 = 8$ $L_4 = \min(9,$

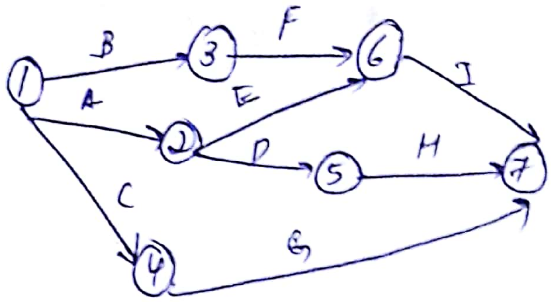
for the activities are indicated number the events according to Fulkerson's rule & calculate the variances & expected time for each activity



Activity	t_e	variance
(1,2)	6.16	1.36
(1,3)	7.66	1
(1,4)	9.16	0.69
(2,5)	12.16	2.25
(3,7)	9	0.11
(4,7)	13.16	3.36
(3,6)	12.16	0.69
(5,8)	13.83	0.25
(6,9)	16	1
(7,10)	13.16	1.36
(8,9)	7	1
(9,10)	8.33	1
(10,11)	12	0.44

* A project is represented by the network shown below & has the following data

TASK	A	B	C	D	E	F	G	H	I
Least time (t_o)	5	18	26	16	15	6	7	7	3
Greatest time (t_p)	10	27	40	20	25	12	12	9	5
Most likely time (t_m)	8	20	33	18	20	9	10	8	4



Determine the following

1) Expected task time & their variance

2) The earliest & latest expected times to reach each node.

3) The critical path and

4) The probability of node occuring at the proposed completion date if the original contract time of completing the project is 41.5 weeks.

→

Task	t_e	Variance	Task	t_e	Variance
(1,2)	7.833	0.69	(2,6)	20	2.77
(1,3)	20.83	2.25	(2,5)	18 ~~[illegible]~~	0.44
(1,4)	33	5.44	(4,7)	9.83	0.694
(3,6)	9	1	(6,7)	4	0.11
			(5,7)	8	0.11

$24/6 = 4$

$t_{e7} = (6 + 7 \times 4 + 8)/6$

$42/6 = 7$

$t_{e8} = (2 + (4 \times 4) + 6)/6$

$24/6 = 4$

$t_{e10} = 2$ $t_{1\to5}$ $D = \frac{ST_i - E_i}{\sqrt{Var_i}}$

$t_{e9} = 6$?

$\sigma^2 = [(t_p - t_o)/6]^2$

$E_2 = 2$ $E_3 = 4$ $E_4 = 5$ $E_5 = 8$ $E_6 = \max(10, 12) = 12$

$E_1 = 0$

$E_7 =$

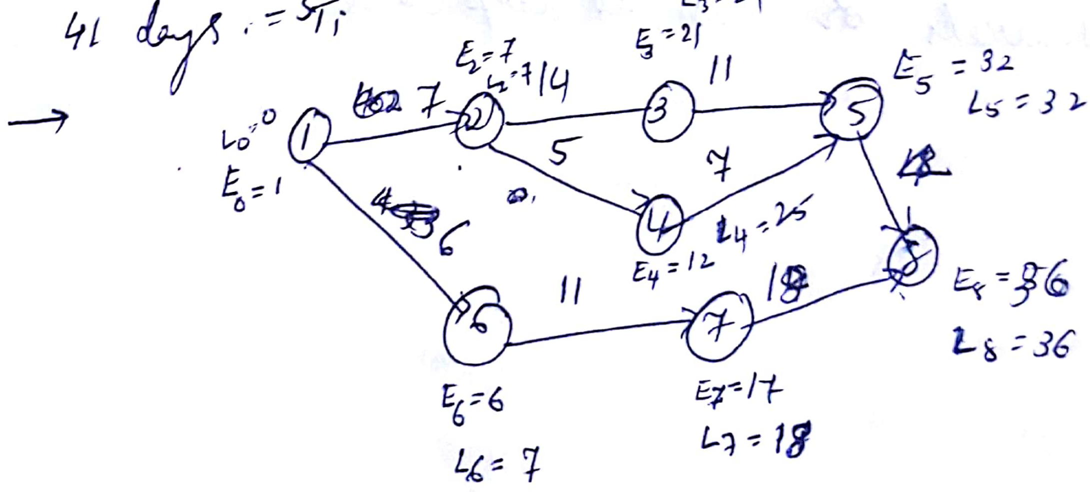
From this it is concluded that if the project is performed 100 times under the same condition there will be chances when this job would take 41.5 weeks or less to complete it

Scanned by CamScanner

* The following table lists the job of a network with their estimates

job (i-j)	t_o	t_m	t_p	t_e	σ^2
1 - 2	3	6	15	~~4~~ 7	4
1 - 6	2	5	14	~~4.33~~ 6	4
2 - 3	6	12	30	14	16
2 - 4	2	5	8	5	1
3 - 5	5	11	17	11	4
4 - 5	3	6	15	7	4
6 - 7	3	9	27	11	16
5 - 8	1	4	7	4	1
7 - 8	4	19	28	18	16

1) Draw the project network calculate the length and variance of the critical path. What is the approximate probability that the jobs on the critical path will be completed within 41 days. = S_{T_i}



critical path : 1, 2, 3, 5, 8

$$D = \frac{41 - 36}{\sqrt{25}}$$ (variance of critical node

$D = 5/5 = 1$

$P(Z \le D_i) = P(Z \le 1)$ (Normal distribution)

$= 0.84$

UNIT-5

Transportation & assignment problem

The TP is used to transport various amounts of a single homogeneous commodities that are initially stored in various sources to different destinations in such a way that total transportation cost is called minimum

*** Tabular representation:**

Suppose there are m factories & n Warehouses

Factory \ Warehouse →	A	B	C	D	Factory capacity
F_1	C_{11}	C_{12}	..	C_{1n}	$100 \to a_1$
F_2	C_{21}				$50 \to a_2$
⋮					⋮
F_m	C_{m1}	C_{m2}	$C_{..}$	C_{mn}	40 a_n
Warehouse requirement	50 $\downarrow b_1$	50 $\downarrow b_2$	50 . . .	40 $\downarrow b_n$	$\Sigma a_i = \Sigma b_i$

The transportation problem is usually represented in tabular form.

*** Methods to find IBFS – Initial basic feasible solution**

1) North-west corner rule
2) Row minima
3) column minima
4) Least cost method
5) Vogel's Approximation method

* Methods of finding an optimal solution of the transportation problem consists of 2 steps
(i) to find an IBFS
(ii) to obtain an optimal solution by making successive improvements to IBFS until no further decrease in the transportation cost is possible

1. Find the IBFS ~~of~~ the following TP

Warehouse→ Factory↓	W_1	W_2	W_3	W_4	factory capacity
F_1	19 (5)	30 (2)	50	10	~~7~~ ~~2~~
F_2	70	30 (6)	40 (3)	60	~~9~~ ~~3~~
F_3	40	8	70 (4)	20 (14)	18 ~~14~~
Warehouse Requirement	~~5~~	~~8~~ ~~6~~	~~7~~ ~~4~~	14	34

minimum $(W_1, F_1) = (7, 5) \Rightarrow 5$

Total ~~minimum~~ cost $= 19\times5 + 30\times2 + 30\times6 + 40\times3 + 70\times4 + 20\times14$

$= \text{Rs } 1015$

Warehouse→ Factory↓	W_1	W_2	W_3	W_4	Factory capacity
F_1	~~19~~	~~30~~	~~50~~	10 (7)	~~7~~
F_2	~~70~~	30 (8)	40 (1)	~~60~~	~~9~~ ~~1~~
F_3	40 (5)	8	70 (6)	20 (7)	18 ~~12~~ ~~5~~
Warehouse Requirement	5	~~8~~ 8	~~7~~ 6	~~14~~ ~~7~~ 7	34

Row minima

Total cost $= 10\times7 + 30\times8 + 40\times1 + 40\times5 + 70\times6 + 20\times7 = \text{Rs } 1110$

→ ↓	W_1	W_2	W_3	W_4	
F_1	19	30	50	10 (7)	7
F_2	70 (2)	30	40 (7)	60	9 2
F_3	40 (3)	8 (8)	70	20 (7)	18 10 3
	$\cancel{5}$ 2	$\cancel{8}$	$\cancel{7}$	14 $\cancel{7}$	34

Least cost Method

Among all the lentries which is less

Total cost = 10×7 + 40×7 + 70×2 + 40×3 + 8×8 + 20×7 = 814

(5) Vogel's Approximation method

Warehouse → Factory ↓	W_1	W_2	W_3	W_4	Factory capacity
F_1	19 (5)	30	50	10 (2)	$\cancel{7}$ 2
F_2	70	30	40 (7)	60 (2)	9 2
F_3	40	$\cancel{8}$ (8)	70	20 (10)	18 10
	$\cancel{5}$ (21)	$\cancel{8}$	$\cancel{7}$ (10)	$\cancel{14}$ 4 (10)	34

(10)
(10) (50)
(40) (60)

Step 1: Penalty = least cost − 2nd least cost in a row

1st row (9) ⇒ 19 − 10

2nd row = (10) ⇒ (40 − 30)

3rd row = 20 − 8 = (12)

1st col = 40 − 19 ⇒ (21)

2nd col ⇒ 30 − 8 ⇒ (22)

3rd col ⇒ 50 − 40 ⇒ (10)

20 − 10 ⇒ (10)

Total cost = 19×5 + 8×8 + 40×7 + 10×2 + 60×2 + 20×10

= 779

80
95
64
20
120
200
779

2. obtain an initial basic S F of the T P.

	I	II	III	IV	Availability				
A	7	3 (6)	9 (11)	5 (17)	34	(2)	(2)	(2)	(2)
B	5 (15)	5	7	6	15	(1)	(1)	x	x
C	8 (6)	6	6 (6)	5	12	(1)	(1)	(1)	(1)
D	6	1 (19)	6	4	19	(3)	x	x	x
Demand	21	25	17	17					
	(1)	(2)	(1)	(1)					
	(2)	(2)	(1)	(1)					
	(1)	(3)	(1)	(0)					
	(1)	x	(1)	(0)					
	(1)	x	(1)	(x)					

Total cost = 3x6 + 5x11 + 5x17 + 6x6 + 8x6 + 1x19 + 5x15

= 336

18
55
85
36
48
19
75
336

* Optionality - U-V Method / MODI Method

i) The initial BFS has m+n-1 allocations in independent positions

(ii) Since u_i ($i = 1, 2, 3 \ldots$) are to be determined
v_j ($j = 1, 2, 3, 4$) in the respective occupied by means of unit cost in the respective cells only assign a u value of any particular amount (conveniently zero), to any particular row (convenient rule is to select the u_i which has the largest number of allocations in its row)

Since all rows contain the same no of allocations, take any of the $u_i = 0$

19(5)			10(2)	u_1
		40(7)	60(2)	u_2
	8(8)		20(10)	u_3
v_1	v_2	v_3	v_4	

v_j

$u_3 = 0$. maxim no of allocation (only row)

$c_{ij} = u_i + v_j$

$8 = 0 + v_2$

$v_2 = 8$

$c_{ij} = 20 = 0 + v_4$

$v_4 = 20$

$c_{ij} = u_i + v_4$

$60 = u_i + 20$

$u_2 = 40$

$c_{ij} = 10 = u_1 + 20$

$u_1 = -10$

$40 = 40 + v_3$

$v_3 = 0$

$19 = -10 + v_1$

$v_1 = 29$

(iii) To compute a matrix of cell evaluation

$d_{ij} = c_{ij} - [u_i + v_j]$

Matrix $[c_{ij}]$ for empty cells.

.	30	50	.
70	30	.	.
40	.	70	.

$u_i + v_j$ for empty cells

	-2	-10	
69	48		
29		0	

$d_{ij} = C_{ij} - [u_i + v_j]$

·	32	60	·
1	-18	·	2-θ
11	8-θ	70	10+θ

⇒ If there are more than 1 negative value - choose most negative value

θ = allocation for

min(2-θ, 8-θ)

2-θ=0 , 8-θ=0

min $[\boxed{\theta = 2}\ (\theta = 8)] \Rightarrow \theta = 2.$

19(5)			10(2)
	30(2)	40(7)	
	8(6)		20(12)

Total cost = 19×5 + 10×2 + 30×2 + 40×7 + 8×6 + 20×12

= 743

(ii)

19(5)			10(2)	u_1
	30(2)	40(7)		u_2
	8(6)		20(12)	$u_3 = 0$
v_1	v_2	v_3	v_4	

$c_{ij} = u_i + v_j$

8 = 0 + v_1

$v_1 = 29$

$u_1 = -10$

$u_2 = 22$

$v_2 = 8$

$v_3 = 18$

$v_4 = 20$

29, 10, 41

22, 29, 51

D_{ij}

(iii)

C_{ij}

·	30	50	·
70	·	4	60
40	·	70	·

⇒

$u_i + v_j$

	-2	8	·
50			42
28		18	

·	32	42	·
49	·	·	1
11	·	52	·

✓

2.

2 (5)	7	4	~~5~~ → (2) ← × × ×
3	9	1 (8)	~~8~~ → (2) (2) ← × ×
5	4 (7)	7	~~7~~ → (1) (1) (1) (1)
1 (2)	6 (2)	2 (5)	~~14~~ ~~6~~ 4 → (1) (1) (1) (5) ←
~~7~~ 2	~~9~~ 2	~~18~~ 8	

(Cross check)

↑ ↑ ↑

(1) (1) (1)

(2) (1) (1)

(4) (2) (5)

(4) (2)

Total cost $= 2(5) + 1 \times 8 + 4 \times 7 + 1 \times 2 + 6(2) + 2 \times 10$

$= 80$

(i)

2 (5)		
		1 (8)
	4 (7)	
1 (2)	6 (2)	2 (10)

$u_4 = 0$

$1 = 0 + v_1$

$v_1 = 1$

$v_3 = 2$

$6 = 0 + v_2$

$\boxed{v_2 = 6}$

$2 = 1 + u_1$

$u_1 = 1$

$1 = 2 + u_2$

$u_2 = -1$

$4 = u_3 + 6$

$u_3 = -2$

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3 Find the initial basic feasible solution for the following TP. solve it

	D_1	D_2	D_3	D_4					
O_1	2 (1)	3 (5)	11	7	~~6~~ 1	→ (1)	(1)	(5) ← x	
O_2	1	0	6	1 (1)	~~1~~	→ (1) x	x	x	x
O_3	5 (6)	8	15 (3)	9 (1)	~~10~~	→ (3)	(3)	(4)	(4)
	~~7~~ 6	~~5~~	~~3~~	~~2~~ 1	17				

↓ ↓ ↓ ↓

→ (1) (3) (5) (6)

(3) (5) (4) (2)

↑

(3) x (4) (2)

↑

(5) x 15 (9)

~~X~~

(5) x 9

Total cost $= 2\times(1) + 5(6) + 15\times(3) + 9(1) + (1)(1) \cdot 3(5)$

$= 2 + 30 + 45 + 9 + 1 + 15$

$= ₹102$

(i)

2 (1)	3 (-5)		
			1 (1)
5 (6)		15 (3)	9 (1)

$m+n-1 = 6$

$u_3 = 0$

Use $5 = 0 + v_1$ $\quad v_3 = 15$ $\quad v_4 = 9$

$v_1 = 5$

Use $2 = 5 + u_1$ $\quad u_2 = 1 - 9$

$= -8$

$u_1 = -3$

3 $-3 + v_2$

3 ~~v_2 (3+8)~~

$v_2 = 6$

Use

(iii)

.	.	11	7
1	0	6	.
.	8	.	9.

$u_i + v_j$

	12	12	6
-3	-2	7	
	6		

d_{ij}

.	.	-1	1
4	2	(θ) -1	(1-θ)
.	2	(3-θ)	(9+θ)

min (1-θ, 3-θ)

= 1-θ = 0 3-θ = 0

θ = 1 θ = 3

2(1)	3(5)	11	7
4	2	6(1)	1
5(6)	8	15 (1)	9 (10)

Total cost = 2 + 15 + 30 + 6 + 30 + 18

= ₹101

2nd step :

2(1)	3(5)	11	7	$u_1 = -3$
1	0	6(1)	1	$u_2 = -9$
5(6)	8	15(2)	9(9)	$u_3 = 0$

$V_1 = 5$ $V_2 = 6$ $V_3 = 15$ $V_4 = 9$

.	.	11	7
1	0	.	1
.	8	.	.

→

		12	6
-4	-3		0
	6		

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d_{ij}

$1-\theta$	.	$-(\theta)$	1
5	3	.	1
$6+\theta$	2	$(2-\theta)$	.

$\min(2-\theta, 1-\theta)$

$= 2-\theta = 0 \quad 1-\theta = 0$

$\theta = 2 \quad \theta = 1$

2	3(5)	11(1)	7
5	0	6(1)	1
5(7)	8	15(1)	9(2)

Total cost $= 15 + 11 + 6 + 35 + 15 + 18$

$= 100$

3rd step

2	3(5)	11(1)	7
5	0	6(1)	1
5(7)	8	15(1)	9(2)

$u_1 = -4$

$u_2 = -9$

$u_3 = 0$

$v_1 = 5 \quad v_2 = 7 \quad v_3 = 15 \quad v_4 = 9$

2	.	.	7
5	0	.	1
.	8	.	.

$u_i + v_j$

1	.	.	5
-4	-2	.	0
.	7	.	.

d_{ij}

1	.	.	2
5	2	.	1
.	5	.	.

* **Degeneracy:**

The degeneracy occurs in TP whenever no of occupied is less than m+n-1

Resolution of degeneracy:

To resolve degeneracy allocate an extremely small amount of goods (close to 0) to 1 or more of empty cells so that a no of occupied cell becomes m+n-1. The cell containing this extremely small allocation is considered to be an occupied cell.

1. A company has 3 plants A, B, C & 3 warehouses X, Y, Z. No of units available at plants is 60, 70 & 80 respectively. Demands at X, Y, & Z are 50, 80 & 80 respectively; Unit cost of transportation are as follows

	X	Y	Z	
A	8	7	3	60
B	3	8	9	70
C	11	3	5	80
	50	80	80	

What would be your tPlan. Give minimum distribution cost

→

8	7	3 (60)	~~60~~ ←	(4)	(4)	(X)	
3 (50)	8	9 (20)	~~70~~ 20 ←	(5)	(X)	X	(1)
11	3 (80)	5	80 ←	(2)	(2)	(2)	(3)
~~50~~	~~80~~	80 ~~20~~					
↑ ~~(5)~~	↑ (4)	↑ (2)					
~~(X)~~	(4)	(2) X					
(8)	(5) ↑	(4)					

$u_1 = -2$

		3(60)	60 - Minimum of unallocated
3(50)		9(20)	70 $u_2 = 4$
	3(80)	5(Δ)	80+Δ $= u_3 = 0$
50	80	80+Δ	

$V_1 = 10$ $V_2 = 3$ $V_3 = 5$

→

u

8	7	.
.	8	.
11	.	.

$u_i + v_j$

-3	11	.
.	7	.
-1	.	.

11	6	.
.	1	.
12	.	.

No -ve values
Hence optimal.

Total cost $= 3 \times (60) + 3 \times (50) + 9(20) + 3(80)$
$= 180 + 150 + 180 + 240$
$= ₹\underline{750}$

* ~~Unbalanced~~ Unbalanced

In TP the sum of all availability quantities is not equal to sum of requirements

ie $\sum_{i=1}^{n} a_i \neq \sum_{j=1}^{n} b_j$

1. A steel company has

	F_1	F_2	F_3	F_4	F_5	
A	4	2	3	2	6	8
B	5	4	5	2	1	12
C	6	5	4	7	3	14
Req	4	4	6	8	8	30 \ 34

A	4 (4)	2 (4)	3	4	6	0	~~8~~ 4 → (2) → (2) (1) (1)
B	5	4	5	2 (8)	1 (4)	0	~~12~~ (4) (1) → (1) (3)
C	6	5	4 (6)	7	3 (4)	0 (4)	14 ~~10~~ 6 (3) (3) (1) (1)
	~~4~~	~~4~~	6	~~8~~	~~8~~ 4	~~4~~	

(1) (2) (1) (5) (2) 0

(1) (2) (1) ✗ (2) (0) ✗

(1) (2) (1) ✗ (2) ✗

(2) (3) (1) ✗ (3) ✗

(2) ✗ (1) ✗ (3) ✗ ✗

(3) ✗ (1)

Total cost = 4(4) + 2(4) + 4(6) + 2(8) + (1)(4) + 3(4) + 0(4)

= 16 + 8 + 24 + 16 + 4 + 12 + 0

= Rs 80