$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \theta \\ 0 & 1 & -\phi \\ 0 & \phi & 1 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} p + r\theta \\ q - \phi r \\ q\phi + r \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & +\phi \\ 0 & -\phi & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -\theta \\ 0 & 1 & 0 \\ \theta & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \phi \\ 0 & -\phi & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \phi \\ 0 & -\phi & 1 \end{bmatrix} \begin{bmatrix} -\theta\dot{\psi} \\ 0 \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} \dot{\phi} + 0 \\ \dot{\theta} \\ -\phi\dot{\theta} \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} -\theta\dot{\psi} + \dot{\phi} \\ \phi\dot{\psi} + \dot{\theta} \\ \dot{\psi} - \phi\dot{\theta} \end{bmatrix}$$

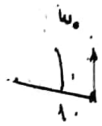
$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} +\ddot{\phi} - \boxed{\dot{\theta}\dot{\psi}} - \theta\ddot{\psi} \\ \boxed{\dot{\phi}\dot{\psi}} + \phi\ddot{\psi} + \ddot{\theta} \\ \ddot{\psi} - \boxed{\dot{\phi}\dot{\theta}} - \phi\ddot{\theta} \end{bmatrix}$$

$$\boxed{} \approx 0 \qquad \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\theta \\ 0 & 1 & \phi \\ 0 & -\phi & 1 \end{bmatrix} \begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} p \\ q \\ r \end{bmatrix} + [p \ q \ r] \times \left(I [p \ q \ r]^T \right) = \underbrace{T_d}_{\text{(constant)}} + T_e + T_g$$

$r^2 w^2 \quad \frac{mv^2}{r} = \frac{GmM}{r^2}$

$w^2 = \frac{(GM)}{r^3}$



$$\bar{T_g} = 3 \frac{\mu}{(r^5)} (Cr)^T I (Cr)$$

$$\bar{r_b} = (C \bar{r_i})$$

$$C = \left(R_\psi R_\theta R_\phi \right)^T$$

$$C = \left(R_\phi R_\theta R_\psi \right)$$

$$C = \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} \quad \text{(small angle)}$$

$$C\bar{r} = \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - y\psi + z\theta \\ x\psi + y - z\phi \\ -x\theta + \phi y + z \end{bmatrix}$$

$$I = \begin{bmatrix} i_1 & i_2 & i_3 \\ i_4 & i_5 & i_6 \\ i_7 & i_8 & i_9 \end{bmatrix}$$

Assume ~~that~~ I as diagonal $\Rightarrow$ 0 (transformation).

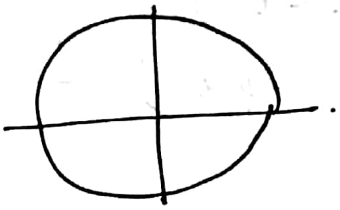
$\downarrow$ 0 (neglect i_3, i_6, i_2)

Which to chose?

$$I = \begin{bmatrix} i_1 & 0 & 0 \\ 0 & i_5 & 0 \\ 0 & 0 & i_9 \end{bmatrix}$$

$$\bar{a} = r_b^{\times} I r_b = \begin{bmatrix} -(i_3 - i_2)(y - \phi z + \psi x)(z + \phi y - \theta x) \\ (i_1 - i_3)(z + \phi y - \theta x)(x - \psi y + \theta z) \\ -(i_1 - i_3)(y - \phi z + \psi x)(x - \psi y + \theta z) \end{bmatrix}.$$

$$\left(\frac{\mu}{r^5}\right) = \left(\frac{\mu}{r^3}\right)\left(\frac{1}{r^2}\right) = \overbrace{\left(\frac{\mu}{r^3}\right)}^{\approx \omega_0^2}\left(\frac{1}{r^2}\right) = \left(\frac{\omega_0^2}{r^2}\right)$$



$z = 0.$

$x = r\cos(\omega_0 t)$

$y = r\sin(\omega_0 t)$

$t \in [0,\ 365 \times 24 \times 60 \times 60]$

plot a_1, a_2, a_3 of $\bar{a}$ for some r.

ϕ, θ, ψ } small angles $\Rightarrow$ $\max\{\phi, \theta, \psi\} = 10^\circ$
$= \frac{10\pi}{180}$ rad.

$10^{-4} - 10^{-3}$ } sinusoids } $\vec{\tau}_g(t) = a\sin(bt + c)$.

$$\begin{bmatrix} 1 & 0 & -\theta \\ 0 & 1 & \phi \\ 0 & -\phi & 1 \end{bmatrix} \Rightarrow$$

ϕ, θ in this matrix causes coupling. Assume $-\theta\dot{\psi}$, or other terms to zero.

[

$$\Downarrow$$

$$\begin{bmatrix} \delta p \\ \delta q \\ \delta r \end{bmatrix} = \begin{bmatrix} \delta\dot{\phi} \\ \delta\dot{\theta} \\ \delta\dot{\psi} \end{bmatrix}$$

$$\begin{bmatrix} \delta\ddot{\phi} \\ \delta\ddot{\theta} \\ \delta\ddot{\psi} \end{bmatrix} + [\bar{0}] = \delta\bar{\tau}_g + \delta\bar{\tau}_c + \delta\bar{\tau}_d$$

$$\delta\bar{\tau}_g = f(t)$$

$$\bar{a} = \begin{bmatrix} -(i_5 - i_4)(yz + \phi y^2 - \theta xy - \phi z^2 + \psi xz) \\ (i_1 - i_4)(xz - \psi xy + \theta xz + \phi xy - \theta x^2) \\ -(i_1 - i_5)(xy - \psi y^2 + \theta xz - \phi xz + \psi x^2) \end{bmatrix}$$

$$[\psi^2, \theta^2, \phi^2 \approx 0]$$

$$x = \lambda_0 \cos(\omega_0 t) \qquad \frac{2\pi}{\omega_0} = T_p$$

$$\bar{a} = \begin{bmatrix} -(i_5 - i_4)(\lambda^2)(0 + \phi s^2 - \theta cs + 0 + 0 \\ (i_1 - i_4)(\lambda^2)(0 - \psi cs + 0 + \phi cs - \theta c^2 \\ -(i_1 - i_5)(\lambda^2)(cs - \psi s^2 + 0 + 0 + \psi c^2 \end{bmatrix}$$

$$\bar{a} = \lambda^2 \begin{bmatrix} (i_4 - i_5)(-\theta cs + \phi s^2) \\ (i_1 - i_4)((-\psi cs + \phi cs) - \theta c^2) \\ (i_5 - i_1)(cs + \psi(c^2 - s^2)) \end{bmatrix}$$

$$\langle \bar{a} \rangle = \frac{\int_0^{T_p} \bar{a}\, dt}{T_p} = (\lambda^2) \begin{bmatrix} (i_4 - i_5)(0 + (\phi)(1/2)) \\ (i_1 - i_4)(0 - \theta(1/2)) \\ 0 + 0 \end{bmatrix}$$

$$\bar{a} = (\lambda^2)\begin{bmatrix} (i_q - i_5)(\phi/2) \\ (i_q - i_1)(\theta)/2 \\ 0 \end{bmatrix}$$

$\cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$

$c2 = c^2 - s^2$

$c2 = 1 - 2s^2$

$= 2c^2 - 1$

$$\bar{\tau}_g = \frac{3(\mu)}{(\lambda 5)} (\lambda^2) \begin{bmatrix} ((I_z) - I_y)\ \phi/2 \\ (I_z - I_x)\ \theta/2 \\ 0 \end{bmatrix}$$

$\int \sin(2x)dx$

$2x = t$

$dx = \frac{dt}{2}$

$$\frac{3}{2}(w_0)^2 \begin{bmatrix} (I_z - I_y)\ \phi \\ (I_z - I_x)\theta \\ 0 \end{bmatrix}$$

$\int \sin(t)\left(\frac{dt}{2}\right)$

$\underline{\cos t}$

$\frac{1 - c2}{2}$

$c^2 = \frac{(c2+1)}{2}$

$$\begin{bmatrix} \delta\ddot{\phi} \\ \delta\ddot{\theta} \\ \delta\ddot{\psi} \end{bmatrix} = \left[\frac{3w_0^2}{2}\right] \begin{bmatrix} (I_z - I_y)(\phi) \\ (I_z - I_x)\ \theta \\ 0 \end{bmatrix} + \begin{bmatrix} 2\times10^{-3} \\ 1\times10^{-4} \\ 2\times10^{-3} \end{bmatrix} + \bar{T}_c$$

$$(\delta\ddot{\phi}) = \left(\frac{3w_0^2}{2}\right)(I_z - I_y)(\delta\phi) + 2\times10^{-3} + \delta\bar{T}_{cx}$$

$$\delta\ddot{\theta} = \left(\frac{3w_0^2}{2}\right)(I_z - I_x)(\delta\theta) + 1\times10^{-4} + \delta T_{cy}$$

$$\delta\ddot{\psi} = \left(\frac{3w_0^2}{2}\right)(0) + (2\times10^{-3}) + \cancel{2x}\ \delta T_{cz}$$

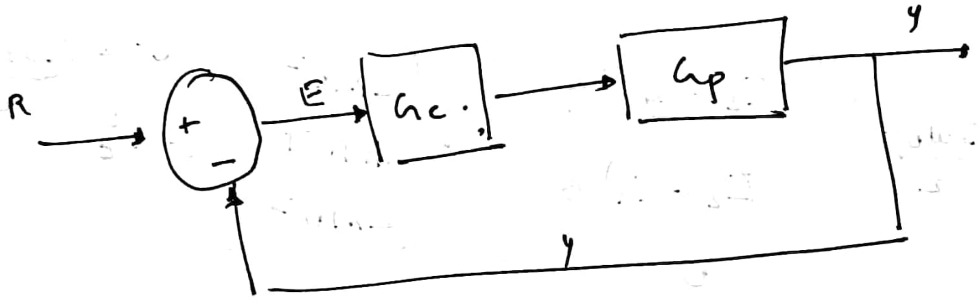
$$\left.\begin{array}{lll} \phi(0), & \dot{\phi}(0) & = 0 \\ \theta & \dot{\theta} & = 0 \\ \psi & \dot{\psi} & = 0 \end{array}\right\}(\text{All cases})$$

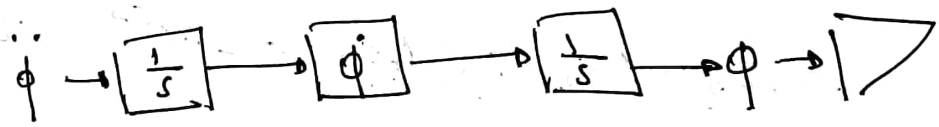
$$\left[s^2 - \frac{3\omega_0^2}{2}(I_z - I_y) \right][\delta\phi(s)] = \delta T_{cx}(s)$$

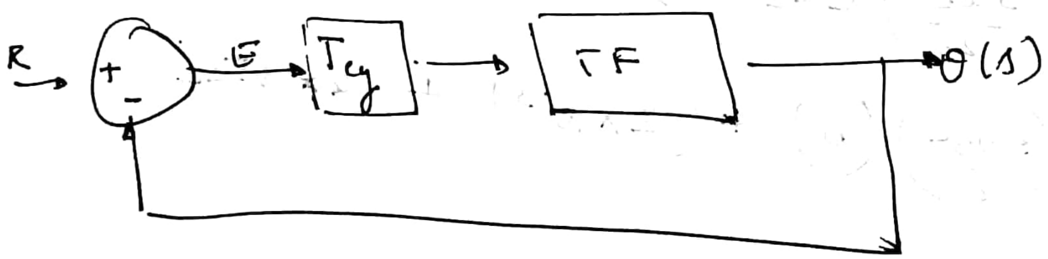
$$\frac{\phi(s)}{T_{cx}(s)} = \frac{1}{\left[s^2 - \left(\frac{3\omega_0^2}{2}\right)(I_z - I_y) \right]}$$

$$\frac{\theta(s)}{T_{cy}(s)} = \frac{1}{\left[s^2 - \frac{3\omega_0^2}{2}(I_z - I_x) \right]}$$

$$\frac{\psi(s)}{T_{cz}(s)} = \frac{1}{(s^2)}$$







$$\omega_0 = 1.0741 \times 10^{-3}$$

$$I_x = 1763$$

$$I_y = 1591$$

$$I_z = 1185$$

$$\frac{\phi(s)}{T_{cx}(s)} = \frac{1}{s^2 + 7.026 \times 10^{-4}}$$

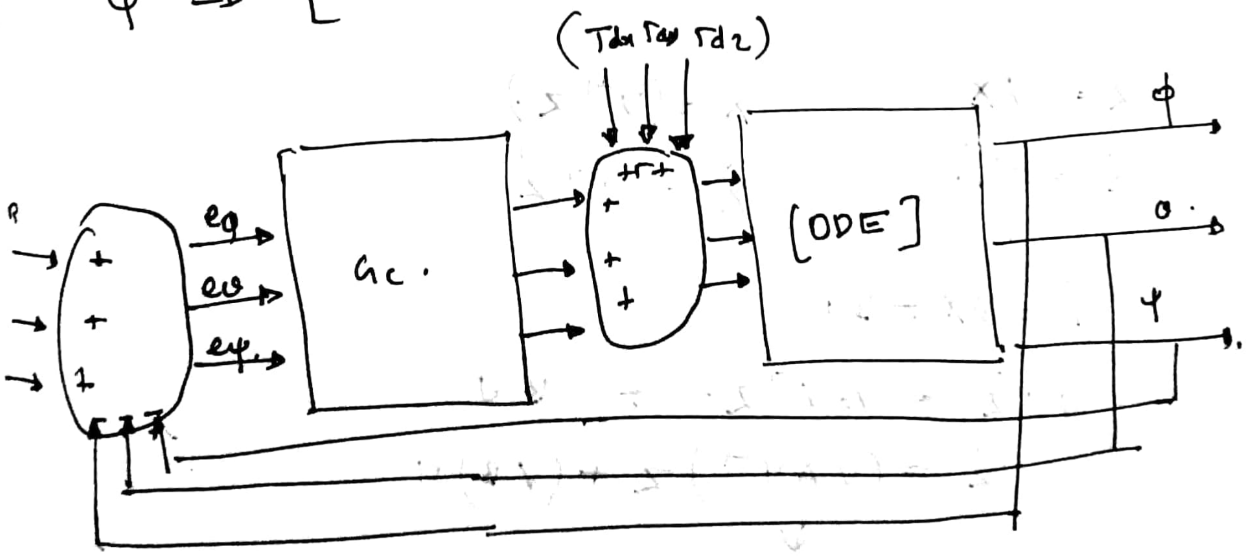
$$\frac{\theta(s)}{T_{cy}(s)} = \frac{1}{(s^2 + 10^{-3})}$$

$$\frac{\psi(s)}{T_{cz}(s)} = \frac{1}{s^2}$$

$$\phi \Rightarrow \begin{bmatrix} 212 & 158 & 63.6 \end{bmatrix}$$
p i d

$$\theta \Rightarrow \begin{bmatrix} 148 & 244 & 22.5 \end{bmatrix}$$
p i d

$$\psi \Rightarrow \begin{bmatrix} 290.3 & 251.9 & 74.42 \end{bmatrix}$$



$u = [\quad]$

$$\begin{bmatrix} \phi \\ \theta \\ \psi \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

$[\phi, \theta, \psi, \omega_1, \omega_2, \omega_3]$

$[\phi, \theta, \psi, \cancel{\omega_1}, \cancel{\omega_2}, \cancel{\omega_3}]$, $[\dot\phi, \dot\theta, \dot\psi, \dot\omega_1, \dot\omega_2, \dot\omega_3]$ (t)

$\int (\ ) dt \Rightarrow [\Sigma \phi_i \quad \Sigma \theta_i \quad \Sigma \psi_i]$

initially

(For large N use trapz)

$x_{sim} = (\text{ode45})(@f(t, x, u), [t_0 \ t_f], x_0)$

$$I\dot{\omega} + \bar{\omega} \times (I\bar{\omega}) = \bar{\tau}_g + \bar{\tau}_d + \bar{\tau}_c$$

$z = VX$ $\qquad X = (V^{-1} z)$

$\dot z = V\dot X$ $\qquad \dot X = V^{-1} \dot z$

$\dot X = AX + BU$

$V^{-1}\dot z = AV^{-1}z + BU$

$\dot z = (VAV^{-1})z + (VB)U$

(diagonal)

$$\frac{d}{dt}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{bmatrix} = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_6 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_6 \end{bmatrix} + \underbrace{\underset{6\times 3}{\begin{bmatrix} - & \cdot \\ - - & \cdot \\ - - & \end{bmatrix}} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}}_{(6\times 1)}$$

$$\underset{B'}{\begin{bmatrix} B & \bar{0} \\ \bar{0} & \bar{I} \end{bmatrix}} \begin{bmatrix} T_c \\ T_d \end{bmatrix}$$

$$\underset{6}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ T_{dx} \\ T_{dy} \\ T_{dz} \end{bmatrix}} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ T_{cx} \\ T_{cy} \\ T_{cz} \end{bmatrix}.$$

$$\dot{x} = A\bar{x} + B'\bar{U}$$

$$\dot{\bar{z}} = \Lambda \bar{Z} + (VB')\bar{U}$$

T_{dx}
T_{dy}
T_{dz}
T_{cx}
T_{cy}
T_{cz}.

$\cancel{0_{3\times}}$ $(0_{3\times 6})$

$$\begin{matrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{matrix}$$

$$B' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_{dx} \\ T_{dy} \\ T_{dz} \\ T_{cx} \\ T_{cy} \\ T_{cz} \end{bmatrix}$$

[Pseudo-rate generator]

Optimal ... PPT

$$\Rightarrow \begin{bmatrix} \delta \dot p \\ \delta \dot q \\ \delta \dot r \end{bmatrix} = \begin{bmatrix} \delta \phi \\ \delta \ddot\theta \\ \delta \ddot\psi \end{bmatrix}$$

Eqn. point.

$(\phi, \theta, \psi), (p, q, r) = 0$

$$\begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \checkmark$$

$$0 + 0 = (\vec{T_d} + \vec{T_g}) + \vec{T_c}$$

$$(\vec{T_c})^* = -(\vec{T_d} + \vec{T_g})$$

$$\left[\frac{(\mu)}{(r^5)}\right] (\overline{f(x,y,z)}) = \vec{T_g}$$

$G = 6.674 \times 10^{-11}$ $\quad m = 5.972 \times 10^{24}$

$\mu = GM = (3.98571) \times 10^{14}$

$$\Rightarrow \begin{bmatrix} 0 & -x & y \\ x & 0 & -x \\ -y & x & 0 \end{bmatrix} \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 0 & -(I_y)x & (I_z)y \\ xI_x & 0 & -xI_z \\ -yI_x & xI_y & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} -I_y(xy) + I_z(yz) \\ I_x x^2 - xz I_z \\ -xy I_x + xy I_y \end{bmatrix}$$

(Derive from scratch & simulate)

Non linear eqn. of motion

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta \sin\phi & \tan\theta \cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sec\theta \sin\phi & \sec\theta \cos\phi \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}.$$

$$\frac{d}{dt}\begin{bmatrix} I_x p \\ I_y q \\ I_z r \end{bmatrix} + \bar{\omega} \times (I\omega) = \bar{\tau}_g + \bar{\tau}_c + \bar{\tau}_d.$$

$$\bar{\omega} \times I\bar{\omega} = \begin{vmatrix} i & j & k \\ p & q & r \\ I_x p & I_y q & I_z r \end{vmatrix}$$

$$= (I_z - I_y) qr - (pr)(I_z - I_x) + (pq)(I_y - I_x).$$

$$\bar{\omega} \times I\bar{\omega} = \begin{bmatrix} (I_z - I_y) qr \\ (I_x - I_z) pr \\ (I_y - I_x) pq \end{bmatrix}$$

$$r_b = (R_\phi \, R_\psi)(x^-)$$

$$X = \begin{bmatrix} \phi \\ \theta \\ \psi \\ p \\ q \\ r \end{bmatrix} \quad u = \begin{bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{bmatrix}.$$

$$\bar{\tau}_g = \left(\frac{(\mu)}{|\lambda^-|^5}\right) r_b{}^{\times} I \lambda_b.$$

$$\bar{\tau}_g = \begin{bmatrix} 2e^{-3} \\ 1e^{-4} \\ 2e^{-3} \end{bmatrix} \qquad p = \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

$$A = \left(\frac{\partial \bar{f}}{\partial X}\right)_* \cancel{\delta x} \qquad B = \frac{\partial \bar{f}}{\partial \bar{u}}$$

$$\delta \dot{X} = A_* \delta \bar{x} + B_* \delta \bar{u}$$

$$f(X^*, u^*, p) = 0.$$

$\Rightarrow$ say $p, q, r, \phi, \theta, \psi = 0$,

$$(\bar{\tau}_c{}^*) = -(\bar{\tau}_g + \bar{\tau}_d) \quad \text{for } (p = p^*)$$

$$\begin{bmatrix}\delta\ddot{\phi}\\ \delta\ddot{\theta}\\ \delta\ddot{\psi}\end{bmatrix} = \begin{bmatrix}\alpha_1 & 0 & 0\\ 0 & \alpha_2 & 0\\ 0 & 0 & \alpha_3\end{bmatrix}\begin{bmatrix}\delta u\\ \delta u_2\\ \delta u_3\end{bmatrix}$$

$$\begin{bmatrix}\delta\dot{\phi} = \delta p\\ \delta\dot{\theta} = \delta q\\ \delta\psi = \delta r.\end{bmatrix}$$

$$\begin{bmatrix}\delta\dot{p} = \alpha_1\,\delta u_1\\ \delta\dot{q} = \alpha_2\,\delta\,\delta u_2\\ \delta\dot{r} = \alpha_3\,\delta u_3\end{bmatrix}.$$

$$(s^2)\,\phi(s) = (\alpha_1)(U(s))$$

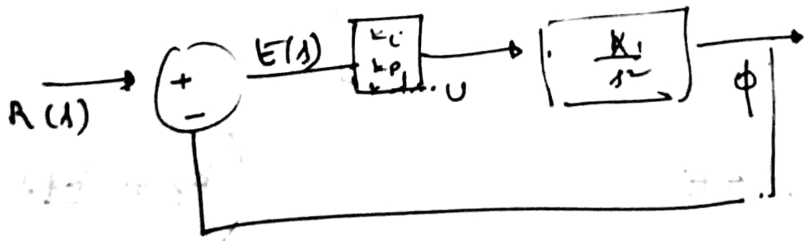
$$\frac{\phi(s)}{U(s)} = \left(\frac{\alpha_1}{s^2}\right)$$

$$\frac{\theta(s)}{U(s)} = \left(\frac{\alpha_2}{s^2}\right)$$

$$\frac{\psi(s)}{U(s)} = \left(\frac{\alpha_3}{s^2}\right)$$

$$\delta\ddot{\phi} = \alpha\left(k_p e + k_i \int e\,dt + k_d \dot{e}\right).$$

$$s^2\,\phi(s) = (\alpha_1)\left(k_p\,\phi(s) + \frac{k_i}{s}\,\phi(s) + s\,\phi(s)\,k_d\right).$$



$$\Rightarrow (\phi, \theta, \psi, p, q, r) = (\bar{0})$$

$$(x, y, z) = (3.6371$$

$$\delta\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\\ \dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}O_{3\times3} & I_{3\times3}\\ O_{3\times3} & O_{3\times3}\end{bmatrix}\delta\begin{bmatrix}\phi\\ \theta\\ \psi\\ p\\ q\\ r\end{bmatrix} + \begin{bmatrix}O_{3\times3}\\ \begin{bmatrix}5.672 & & \\ & 6.29 & \\ & & 8.44\end{bmatrix}e^{-4}\end{bmatrix}\delta\begin{bmatrix}m_1\\ m_2\\ m_3\end{bmatrix}$$

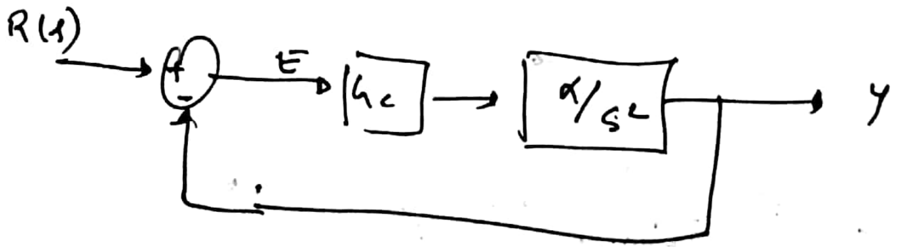
$$\frac{d}{dt}\begin{bmatrix} \delta\phi \\ \delta\theta \\ \delta\psi \end{bmatrix} = \begin{bmatrix} \delta p \\ \delta q \\ \delta r \end{bmatrix}.$$

$$\begin{bmatrix} \delta\ddot{\phi} \\ \delta\ddot{\theta} \\ \delta\ddot{\psi} \end{bmatrix} = 10^{-4}\begin{bmatrix} 5.67 & 0 & 0 \\ 0 & 8.29 & 0 \\ 0 & 0 & 8.44 \end{bmatrix}\begin{bmatrix} \delta m_1 \\ \delta m_2 \\ \delta m_3 \end{bmatrix}.$$

$$\ddot{x} = (\alpha)\, u$$

$$s^2 X(s) = \alpha\, U(s)$$

$$\frac{X(s)}{U(s)} = \left(\frac{\alpha}{s^2}\right).$$



$$(R - Y) = E \qquad \left(k_p + k_d s + \frac{k_i}{s}\right)$$

$$(G_c E)(\alpha / s^2) = Y$$

$$\frac{(\alpha)}{s^2}(G_c)(R - Y) = Y$$

$$G_c = \left(k_p + k_d s + \frac{k_i}{s}\right)$$

$$\frac{(\alpha)}{(s^2)}\left(k_p + k_d s + \frac{k_i}{s}\right)(R - Y) = Y.$$

$$\left(\frac{(\alpha)}{s^2}\right)\left(k_p + k_d s + \frac{k_i}{s}\right)(R) = Y\left(1 + \frac{\alpha}{s^2}\left(\quad\right)\right]$$

$$\frac{\frac{(\alpha)}{(s^2)}\left(k_p + k_d s + \frac{k_i}{s}\right)}{\left[1 + \left(\frac{\alpha}{s^2}\right)\left(k_p + k_d s + \frac{k_i}{s}\right)\right]} = \left(Y/R\right).$$

$$\frac{(\alpha)(k_p s + k_d s^2 + k_i)}{\left((s^3) + \alpha(k_p s + k_d s^2 + k_i)\right)} = \frac{Y}{R}.$$

$$\frac{(\alpha)(k_i + k_p s + k_d s^2)}{(s^3 + \alpha k_d s^2 + \alpha k_p s + \alpha k_i)} = \left(\frac{Y}{R}\right).$$

$\boxed{k_i = 0} \Rightarrow \underline{PD}$

$$(\alpha)(k_p + k_d s) = \left(Y\right.$$

$$\frac{d}{dt}\begin{bmatrix}\phi\\ \theta\\ \psi\\ \omega_1\\ \omega_2\\ \omega_2\end{bmatrix} = f(\bar{x},\bar{u})$$

kinematics

dynamics

$$\frac{d}{dt}(\bar{x}) = \bar{f}(\bar{x},\bar{u})$$

$$\delta(\dot{\bar{x}}) = \left.\frac{\partial \bar{f}}{\partial x}\right|_{x^*} \delta\bar{x} + \left.\frac{\partial \bar{f}}{\partial \bar{u}}\right|_{u^*} \delta\bar{u}$$

where $(x^*, u^*) =$ Eqn. point

$\dot{\bar{x}} = 0 \Rightarrow f(x^*, u^*) = 0$

[for some fixed position]

$$A = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ 0_{3\times3} & 0_{3\times3}\end{bmatrix} \qquad B = \begin{bmatrix} & 0_{3\times3} & \\ 5.67 & 0 & 0 \\ 0 & 6.29 & 0 \\ 0 & 0 & 8.44\end{bmatrix} \times 10^{-4}$$

$$\dot{\delta}\begin{bmatrix}\phi\\ \theta\\ \psi\\ \delta\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} = \begin{bmatrix} A \end{bmatrix}\begin{bmatrix}\delta\phi\\ \delta\theta\\ \delta\psi\\ \delta\omega_1\\ \delta\omega_2\\ \delta\omega_3\end{bmatrix} + B\begin{bmatrix}\delta T_{cx}\\ \delta T_{cy}\\ \delta T_{cz}\end{bmatrix}$$

$$\begin{bmatrix} \delta\ddot{\phi} \\ \delta\ddot{\theta} \\ \delta\ddot{\psi} \end{bmatrix} = \begin{bmatrix} 5.67\times10^{-4}\,\delta T_{cx} \\ 6.29\times10^{-4}\,\delta T_{cy} \\ 8.44\times10^{-7}\,\delta T_{cz} \end{bmatrix}$$ (Type 2 system).

↓

(PD would suffice)

ϕ	402	23.31	1042.35	from PID tuner
θ	362.12	21.012	1568.81	
ψ	570.6	15.65	1169.17	

$T_s \approx 12-15\,s$

$M_p \approx 15\%$.

$t_r = 2-4\,s$

$$G_p = \frac{5.67 \times 10^{-4}}{s^2} \qquad G_c = k_d s + \frac{k_i}{s} + k_p.$$

$$\frac{G_p G_c}{1 + G_p G_c} = G_{CL}$$

$$\cancel{G_{CL}} \quad (5.67 \times 10^{-4}) = \alpha.$$

$$G_{cl} = \quad)(\alpha/s^2)$$

$$(a) \frac{\cancel{(k_i + k_d s^2 + k_p s^2)}}{a k_i + s^3 + a k_d s^2 + a k_p s^2}$$

$$(a) \frac{(k_d s^2 + k_p s + k_i)}{s^3 + a k_d s^2 + a k_p s + a k_i}$$

$$\frac{(a)(k_d s^2 + k_p)}{s^2 + a k_d s + a k_p.}$$

$$t_r = \frac{\pi}{2\omega_n}.$$

s^3	1	ak_p
s^2	ak_d	ak_i
s^1	$\left(ak_p - \frac{k_i}{k_d}\right)$	0
s^0	$(a k_i)$.	0

$$-\frac{(ak_i - a^2 k_p k_d)}{a k_d}.$$

$$-\left(\frac{k_i}{k_d} - a k_p\right)$$

$$+\frac{\left(+(ak_i)\cancel{\left(ak_p - \frac{k_i}{k_d}\right)}\right)}{\cancel{\left(ak_p - \frac{k_i}{k_d}\right)}}$$

$$a k_d > 0$$

$$ak_p > \frac{k_i}{k_d}$$

$$a k_i > 0$$

$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ $t_r = \frac{\pi}{2\omega_n} = 1.49$

$t_s = \frac{4.4}{\zeta\omega_n}$.

$2\zeta\omega_n = a\,k_d$

& $\omega_n^2 = a\,k_p$.

$\omega_n = \frac{\pi}{(2)(1.49)}$

$\Rightarrow k_p = \frac{(\omega_n^2)}{(a)}$.

$\omega_n = \sqrt{(402 \times 5.67 \times 10^{-4})}$

$k_p = 1968$

$= \underline{\underline{0.4774}}$

$\frac{4.4}{\zeta\omega_n}$

$\frac{3.14}{\omega_n\sqrt{1-\zeta^2}}$

$\sqrt{(740.35)(5.67 \times 10^{-4})}$

$\frac{3.14}{(0.244)(0.43)}$

$\zeta = 1.0335$.

$\frac{4.4}{0.9\,(0.244)}$

$t_s = \frac{4.4}{(1.0335)(0.47}$

$t_s = 8.9178$

$t_r = 3.2866$.

$M_p = e^{-\zeta\omega_n/\sqrt{1-\zeta^2}}$

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$$\frac{4.4}{\zeta \omega_n} = 20 \qquad 20 = \frac{(4.4)}{2\zeta\omega_n}$$

$$10 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$t_p = 10 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} =$$

$$\frac{8.8}{(aled)} =$$

$$t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)}{\omega_n\sqrt{1-\zeta^2}}.$$

Say $= \zeta = 0.9$.

$$\cancel{\omega_n = \frac{4.4}{20} = 0.22}$$

$$\cancel{\zeta = 1}$$

$$\cancel{a_{lcp} = \omega_n^2}$$

$$\cancel{k_p = \frac{(0.22)^2}{a}}$$

$$\cancel{= 85.36}$$

$$t_s = \frac{4.4}{(0.9)(\omega_n)} = 20$$

$$\left(\frac{3.14}{0.244 \times 20}\right) < \sqrt{1-\zeta^2}$$

$$\omega_n = \frac{20^{-1} \times 4.4}{0.9} = 0.244$$

$$t_p = \frac{3.14}{(0.244)(\sqrt{1-0.9^2})} \leq 20 \qquad 0.6435 < \sqrt{1-\zeta^2}$$

$$\left[a k_p > \frac{k_i}{k_d}\right] \qquad s^2 + \underbrace{a k_d}_{2\zeta\omega_n} s + a k_p .$$

$$t_s = \frac{4.4}{\left(\frac{k_d}{2}\right)} = \qquad \zeta\omega_n = \left(\frac{k_d}{2}\right).$$

$$\omega_n = \sqrt{a k_p} .$$

$$t_r = \pi - \tan^{-1}\left(\frac{\sqrt{1-\left(\frac{k_d}{2k_p}\right)^2}}{\left(\frac{k_d}{2k_p}\right)}\right) \qquad \zeta = \frac{k_d}{2\sqrt{a k_p}}$$

$$\frac{\qquad\qquad}{(\sqrt{k_p})\sqrt{1-\left(\frac{k_d}{2\sqrt{k_p}}\right)^2}} ,$$

$$t_r = \pi - \tan^{-1}\left(\frac{\sqrt{1 - k_d^2/4k_p}}{\frac{k_d}{2\sqrt{k_p}}}\right)$$

$$\frac{\qquad\qquad}{\sqrt{k_p}\sqrt{k_p - \frac{k_d^2}{4k_p}}}$$

$$t_s = \frac{4.4}{\frac{k_d}{2}} \qquad k_p$$

$$M_p = 0.15 = e^{\frac{-\zeta\omega_n}{\sqrt{1-\zeta^2}}}$$

$$M_p = \exp\left(\frac{\cancel{\frac{-k_d}{2\sqrt{k_p}}}\ -\frac{k_d}{2}}{\sqrt{1-\left(\frac{k_d}{2\sqrt{k_p}}\right)^2}}\right)$$

$kp = 1666$

$kd = 0.722$

$$t_r = \frac{\pi - \tan^{-1}\left(\dfrac{\sqrt{1 - \left(\dfrac{kd^2}{4a\,kp}\right)}}{\dfrac{kd}{2\sqrt{akp}}}\right)}{\sqrt{akp}\,\sqrt{1 - \dfrac{kd^2}{4\,akp}}}$$

$$t_r = \frac{\pi - \tan^{-1}\left(\dfrac{\sqrt{1 - kd^2/(4akp)}}{kd/(2\sqrt{akp})}\right)}{\sqrt{akp - \dfrac{kd^2}{4}}}$$