AS5545 Spacecraft dynamics and control-Mini Project

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1. PROBLEM STATEMENT

Based on mission requirements attitude of spaceraft needs to be stabilized either in a inertially pointing or earth pointing sense. This is practically impossible with passive stabilisation techniques and it results in poor set point tracking which in turn can affect the quality of data collected in space for research purposes. Hence active controls are neccessary and we introduce PD control algorithm to stabilze an unstable Astrosat satellite.Astrosat is a zero momentum biased spaceraft. A classical controller developed is based on linearised model of the plant about an equlibrium condition and is tested on a nonlinear model for validation and performance evaluation.

2. EQUATIONS OF MOTION

In our fourmulation the translational dynamics are not taken into consideration. The attitude of the spacecraft is formulated using Euler angles and corresponding rotational dynamics equations are derived.

Kinematic equation can be derived using roational matrices.

$$\frac{d}{dt}\begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix} = \begin{bmatrix}1 & \tan(\theta)\sin(\phi) & \tan(\theta)\cos(\phi)\\ 0 & \cos(\phi) & -\sin(\phi)\\ 0 & \sec(\theta)\sin(\phi) & \sec(\theta)\cos(\phi)\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix}$$

The dynamics are defined by the folowing equations.

$$\frac{d}{dt}\begin{bmatrix}p\\ q\\ r\end{bmatrix} = -I^{-1}\begin{bmatrix}p\\ q\\ r\end{bmatrix}\times I\begin{bmatrix}p\\ q\\ r\end{bmatrix} + I^{-1}\left(\overrightarrow{T_g}+\overrightarrow{T_d}+\overrightarrow{T_c}\right)$$

$$\text{where } I = \begin{bmatrix}I_{xx} & -I_{xy} & -I_{xz}\\ -I_{xy} & I_{yy} & -I_{yz}\\ -I_{xz} & -I_{yz} & I_{zz}\end{bmatrix} \text{ is the moment of inertia matrix}$$

ϕ, θ, ψ are the roll, pitch and yaw anlges of the satellite and p, q, r are the body angular rates.

- $\overrightarrow{T_g}$ gravity gradient torque
- $\overrightarrow{T_d}$ disturbance torque
- $\overrightarrow{T_c}$ control torque

2.1 Limitation

- Singularities
- Computationally expensive compared to other representation

3. LINEARIZED EQUATION OF MOTION

Classical control techinque design apply only to class systems called linear systems and it is in our best interest to linearize the system and design one possible controller for the same about the given equilibrium point. An equilibrium point is defined as follows.

$$\frac{d}{dt}\overrightarrow{X} = \overrightarrow{f}(\overrightarrow{X}, \overrightarrow{U})$$

$$\overrightarrow{f}(\overrightarrow{X}^*, \overrightarrow{U}^*) = \overrightarrow{0}$$

where, $\overrightarrow{X}^*, \overrightarrow{U}^*$ is the required equilibrium point.

Ideally a non-linear equation in has to be solved for obtaining the equilibrium point. For our choice of application we define X^*as

$$X^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The corresponding trim condition is obtained by solving the non-linear equations. The equations of motions will be linearised about the equilibrium point obtained.

$$\delta\dot{\overrightarrow{X}} = A\delta\overrightarrow{X} + B\delta\overrightarrow{U}$$

where A, B are jacobian of $\overrightarrow{f}$ w.r.t to $\overrightarrow{X}$ and $\overrightarrow{U}$ respectively.

$\overrightarrow{T_g}$ depends on the inertial positon of the spacecraft. Since we have a inertial pointing spacecraft the components will be osscilating in nature across the orbital period. For design purpose a position in the orbit was chosen. The gravity gradient torque is periodic in nature for inertial pointing satellites.

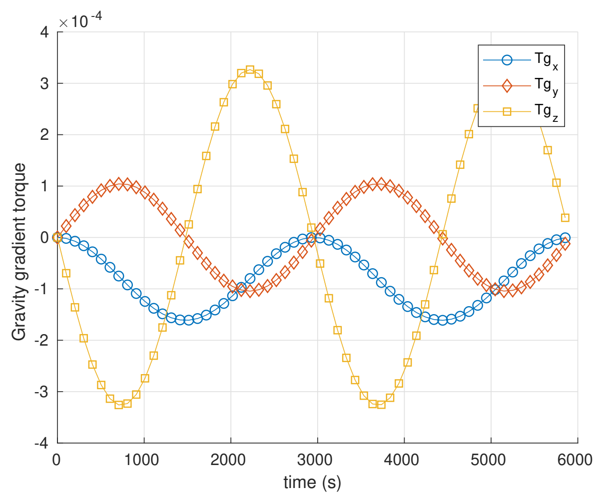

Figure 3.1. Gravity gradient torque over one period

By fixing states and a postion in the orbit the control torque required was found for the equilibrium and joacbians were evaluated at this point. The corresponding A and B matrices are given as follows.

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } B \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 5.67 \times 10^{-4} & 0 & 0 \\ 0 & 6.29 \times 10^{-4} & 0 \\ 0 & 0 & 8.44 \times 10^{-4} \end{bmatrix}$$

We assume off-diagonal terms from the inertia matrix are zero during control design process.

The 6 equation of motion can be reduced to 3 second order differential equation to get the following equation.

$$\begin{bmatrix} \delta\ddot{\phi} \\ \delta\ddot{\theta} \\ \delta\ddot{\psi} \end{bmatrix} = \begin{bmatrix} 5.67 \times 10^{-4} \delta T_{cx} \\ 6.29 \times 10^{-4} \delta T_{cy} \\ 8.44 \times 10^{-4} \delta T_{cz} \end{bmatrix}$$

3.1 Limitations

- Few coupled terms were neglected
- Off diagonal term in inertia matrix were set to zero for design purpose
- For large perturbations this will significantly vary from non-linear model
- Ideally for every position in orbit trim has to be calculated but we work with a fixed a position.

3.2 Need for linearisation

Classical control techniques used so far in the course were meant to be dealt for SISO systems. For our dynamical model we have 6 states and 3 controls and hence its a MIMO system. For earth pointing satellites small angle approximation is used and perturbed motion is analysed. This is equivalent to linearising the the system about state equal to zero. Using this a motivation, the equation motion required for control system design are developed by linearisation of the dynamics.

4. TRANSFER FUNCTION

$$\begin{bmatrix} \phi\left(s\right) = \frac{5.67\times10^{-4}\delta T_{cx}}{s^2} \\ \theta\left(s\right) = \frac{6.29\times10^{-4}\delta T_{cy}}{s^2} \\ \psi\left(s\right) = \frac{8.44\times10^{-4}\delta T_{cz}}{s^2} \end{bmatrix}$$

From the linear model developed, Laplace transform is applied and transfer funtion for the dynamics is found out. All the transfer functions correspond to type 2 systems and they are unstable in nature. A PD control algorithm is proposed for the role of a compensator instead of PID control. This is backed by the fact that addition of integral term will lead to increase in type of the system and complexity.

In order to get initial estimates for the gains, Routh criteria and performance requirements are used. The approximate gains obtained are fine tuned by using root locus method. This is implemented in matlab via the sisotool toolbox.

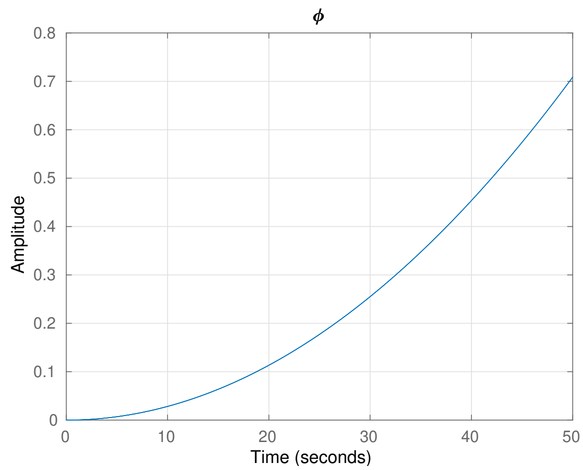

Figure 4.1. ϕ-step response

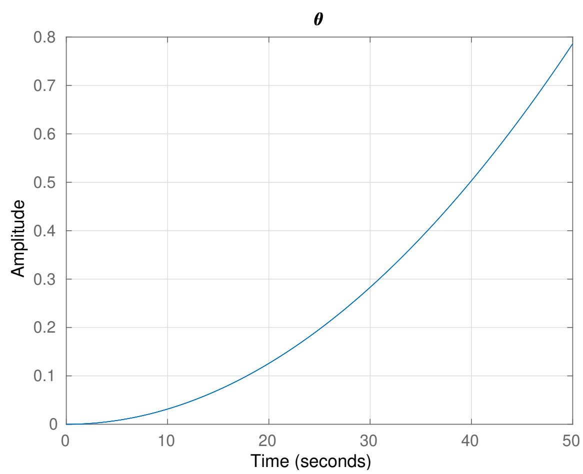

Figure 4.2. θ-step response

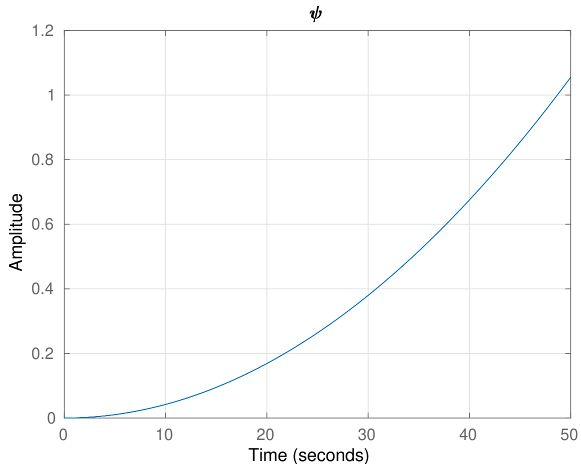

Figure 4.3. ψ-step response

Two poles at origin males the system unstable and this also seen from the step response plots. Hence we need a compensator to stabilize the system and achieve desired performance for the criterias specified.

5. FREE RESPONSE

System simulated by starting from an initial condition in the absence any control.

5.1 Non linear system

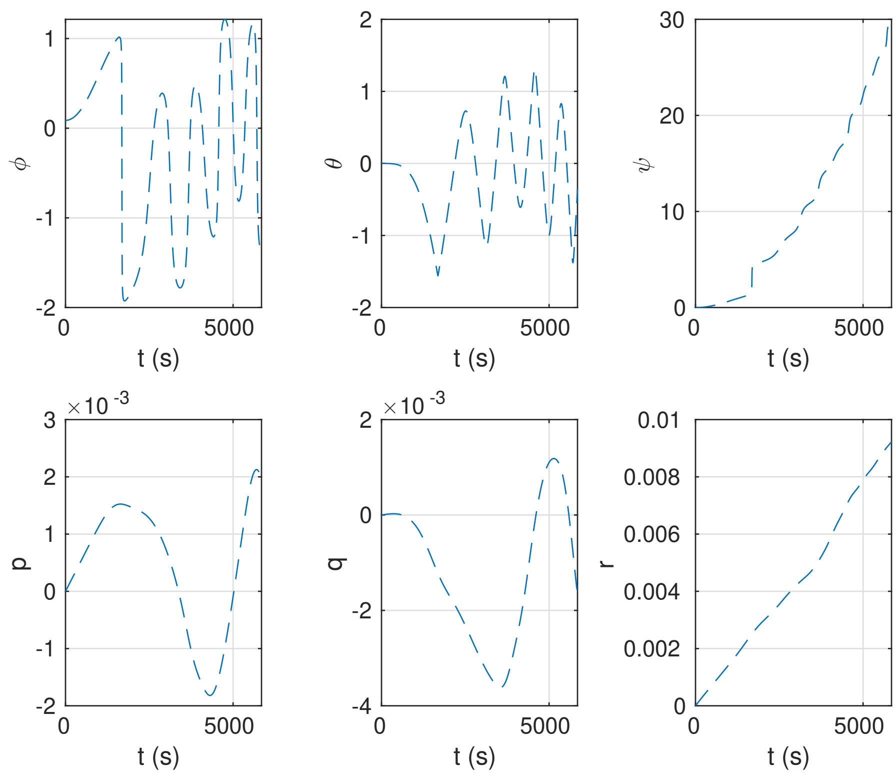

Figure 5.1. Free reponse of non-linear system from a point near equillibrium point in the absence of actuators

The differential equation is integrated by chosing a point near $X^*$with zero actuator control to see response of the system.This results in unstable motion with exponential divergence in ψ .In order to quantify instablity associated we can study the linearised system and use the nature of eigenvalues to conclude on the stabilty associated with the operating point.

5.2 Linearized system

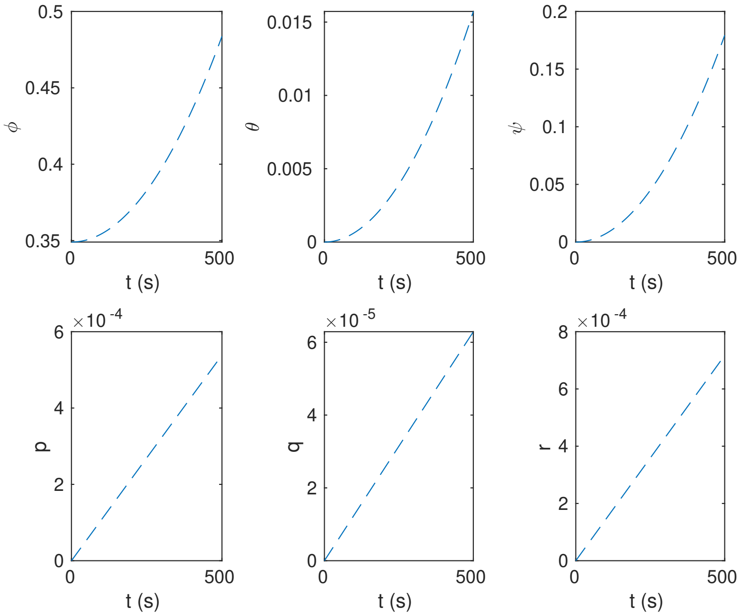

Figure 5.2. Linearized system (Note- Angles and rates are corresponding to the perturbed motion)

Eigenvalue plot for the linearized system has been plotted below. As seen from plot, an eigenvalue lies on the right half plane and this accounts for the equilibrium point being unstable and resulting unstable motion.

For the equilibrium point to be stable, **all the eigenvalues** associated with the jacobian matrix A must lie on the **left half plane**. This is same as the poles criteria for the transfer function to be stable.

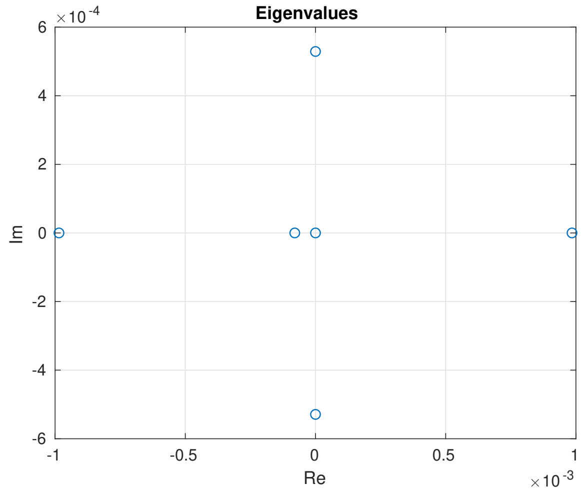

Figure 5.3. Eigenvalues of the jacobian matrix A

There a need for a control system to actively stabilze the system and the work done in realising one such possible control algorthm is presented in the following sections.

6. CONTROL SYSTEM DESIGN

The design of the controller is implemented in the Laplace domain with well known classical techniques.

- Controller transfer function
- Routh criteria
- Performance requirements
- Preliminary gain selection
- Root locus

6.1 Controller transfer function

A PD controller is proposed for the controller implementation. We have a type 2 system and these are unstable as shown from analysis done in previous sections. Integral term inceases the type of system wheras PD will add zero to the system.

$$U_c = K_p(r(t) - y(t)) + K_d(r(\dot{t}) - y(\dot{t}))$$

Modified PD control is used here.

$$U_c = K_p(r(t) - y(t)) + K_d(-y(\dot{t}))$$

This is done because step changes in reference will lead to large error in the slopes and performance will be affected. The **derivative** of the mesurements can be obtained from **dynamics equation**.

The transfer function associated with the controller is given below.

$$G_c(s) = K_p + K_d s$$

Let us define the general plant transfer funciton as follows.

$$G_p(s) = \frac{a}{s^2}$$

where a is constant associated with each axis of the spacecraft.

The closed loop transfer function for unity feedback is as follows.

$$G_{cl}(s) = \frac{G_p G_C}{1+G_p G_C}$$

$$G_{cl}(s) = \frac{aK_p + aK_d s}{s^2 + aK_p + aK_d s}$$

By appropriately tuning the gains K_d and K_p, the system can be made stable and required performance criteria can be met. The natural frequency and damping can be appropriately chosen by varying the gains of the system.

6.2 Routh Criteria

Routh table is constructed as follows.

$$\begin{bmatrix} s^2 & 1 & aK_d \\ s^1 & aK_p & 0 \\ s^0 & aK_d & 0 \end{bmatrix}$$

The basic criteria for gains to lie on the left half plane is that both K_p and K_d must be positive. Any positive gain will work but we need to meet performance requirements. Hence we constraint the LHP and get better gains.

6.3 Performance requirements

From the step response of 2^{nd} order system, rise time, peak time ,M_p, settling time and steady state errors can be used as performance measured to get design gains at preliminary stages.Inertial pointing satellites are used for observatory purposes and hence **stabilisation** needs to be **fast** . For this we use $M_p$and **settling time** to tune our gains.

6.3.1 Maxmimum overshoot percentage

$$M_p = e^{\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

This depends only on the damping ratio of the system. For our requirement we set $M_p = 10\%$ and obtain ζ.

6.3.2 Settling time

$$t_s = \frac{4.4}{\zeta\omega_n}$$

A settling time of 20 s is chosen and ω_n is obtained.

6.3.3 Preliminary Gains

$$K_p = \frac{\omega_n^2}{a}$$

$$K_d = \frac{2\zeta\omega_n}{a}$$

Axis	K_p	K_d
ϕ	244	776
θ	220	700
ψ	164	521

Table 1. Preliminary gains

For the gains obtained ,step input response is plotted and studied.

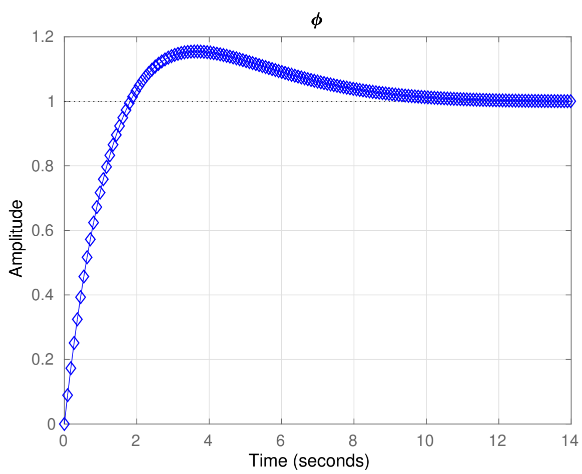

Figure 6.1. ϕ

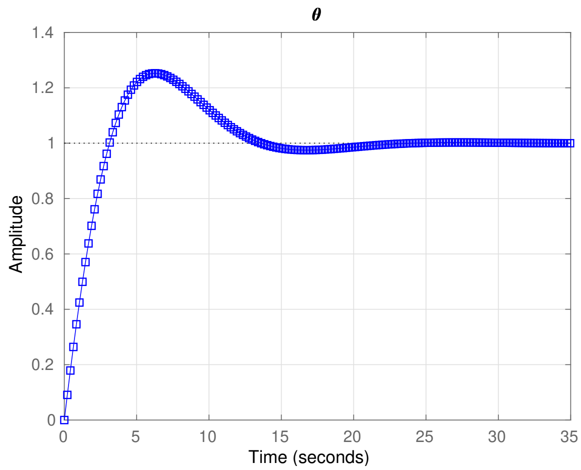

Figure 6.2. θ

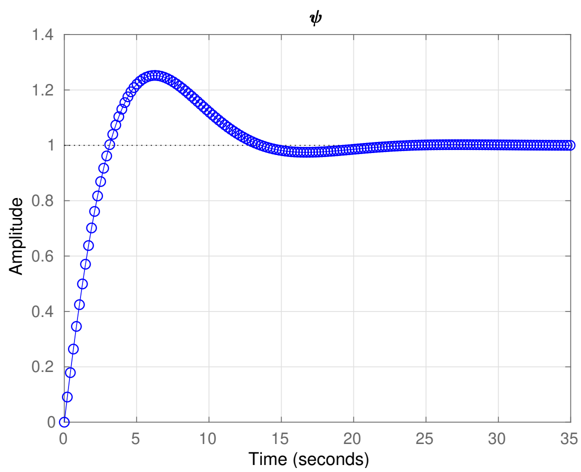

Figure 6.3. ψ

As seen from the plots, $t_s$and $M_p$are different from the gains tuned using performance requirements. This is due to the **transient effect of the pole** in the numerator of the transfer function. We are barely meeting the performance requirements. In order to move forward we will study the root locus of the system and decide the gains from it.

6.3.4 Root locus

By adding a compensator to a type 2 system we added a real zero in the LHP. One of the asymptote is the negative real axis. one of the poles will move towards the zeros and the other will go to $-\infty$. Using sisotool, poles are moved approriately to meet performance requirements.

Since a single gain is to be varied while using root locus we should fix one of the gains and vary the other. A alternative is proposed and used for our analysis.

Assume that K_p, K_d satisfy a linear relationship.

$$K_d = \alpha K_P$$

By doing this there is more flexibity while designing the gains for the systems. Effectively the controller for root locus is as follows.

$$G_p = K(s + \alpha)$$

α can be found from the preliminary gain obtained.

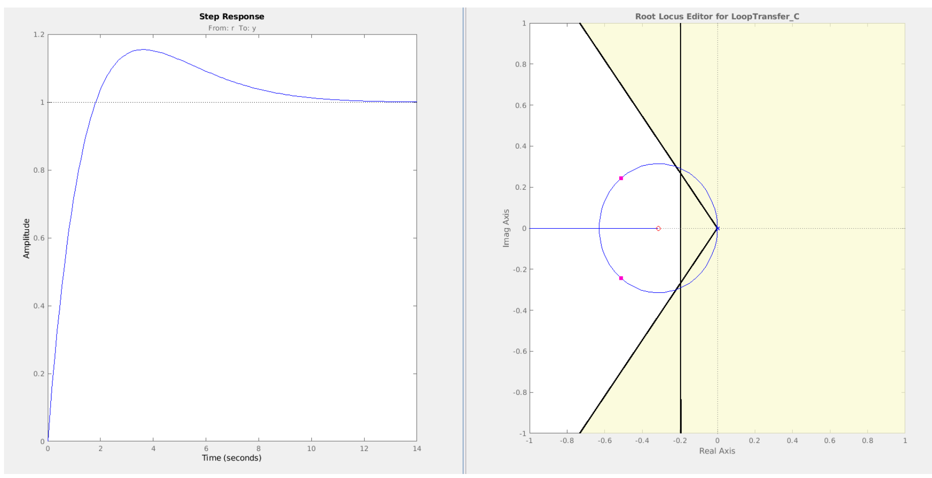

Figure 6.4. ϕ

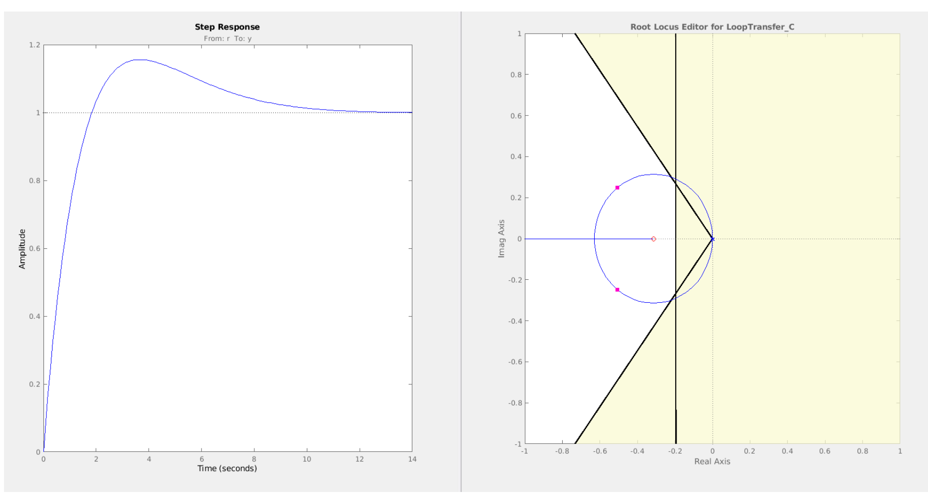

Figure 6.5. θ

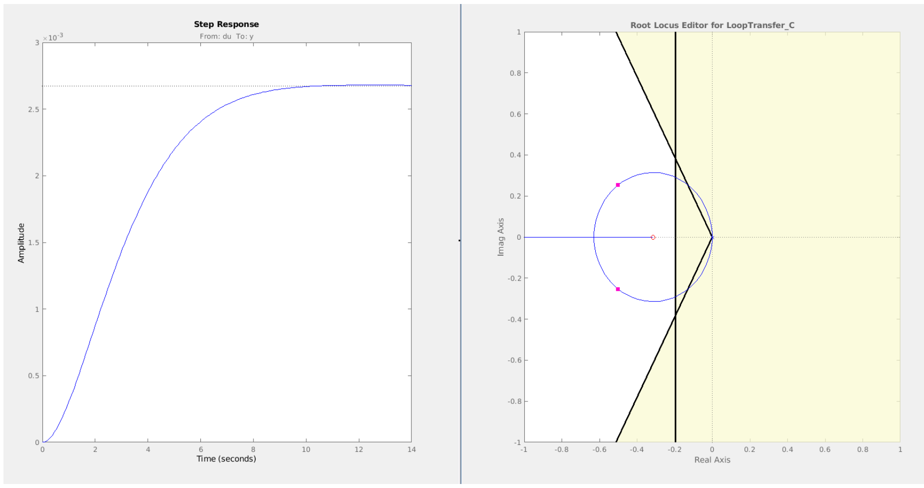

Figure 6.6. ψ

The V shaped region and half plane region correspond to limits on $t_s$andM_p.By only working in the white region gains are tuned and used. Following were the results of the tuning done and have been summarised in the form a table.

Performance	ϕ	θ	ψ	K_p	K_d
$t_s(s)$	9.22	9.25	9.29	591	1812
$t_r(s)$	1.35	1.36	1.37	507.7	1614
$t_p(s)$	3.68	3.63	3.67	374	1188
$M_p($	15.4	15.6	15.7	-	-

Table 2. Final performance requirements and gains

The response of the system is also plotted for a step input.

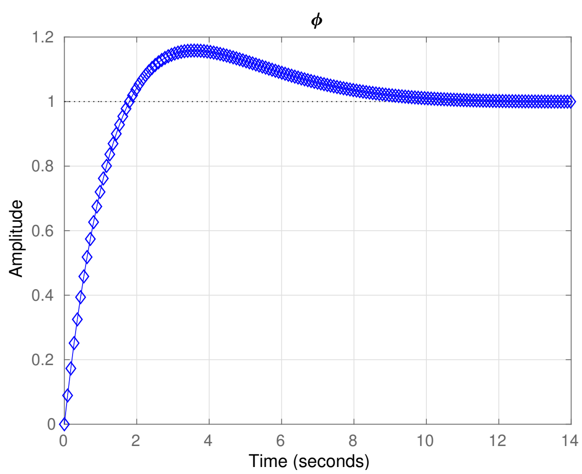

Figure 6.7. ϕ

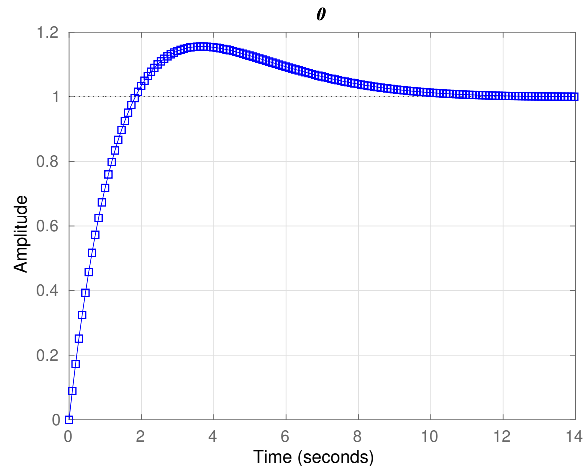

Figure 6.8. θ

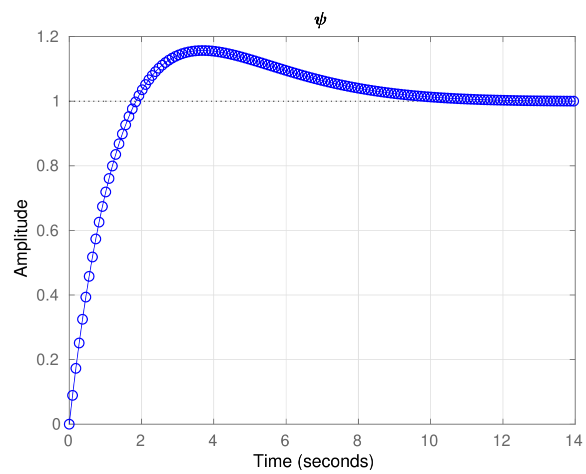

Figure 6.9. ψ

7. NON-LINEAR SIMULATION

We designed control laws by lineazrising the system dynamics about trim condtion and have met the required performance criterias. But in reality these will be deployed on actual systems whose dynamics can be highly non linear. In order to make simulations more relalistic controller is tested with a non-linear dynamical model and its effects on performance are studied.

7.1 Setting up the simulation blocks

The simulator mainly consists of a controller block and a non-linear plant blocks written as MATLAB functions. Controller block calculates the controls to be given based on the gains and error. This is given to the plant and time evolved to get the next states at next instant. This will be fed back to the controller and process continues. This completes the feedback simulation of the system.

State variable	Initial	Set point
$\overset{\circ}{\phi}$	20	0
$\overset{\circ}{\theta}$	0	0
$\overset{\circ}{\psi}$	0	0
$p(rad/s)$	0	0
$q(rad/s)$	0	0
$r(rad/s)$	0	0

Table 3. Simulation parameters

Also at every instant knowledge about the position is also fed to the plant to keep simulations realistic.

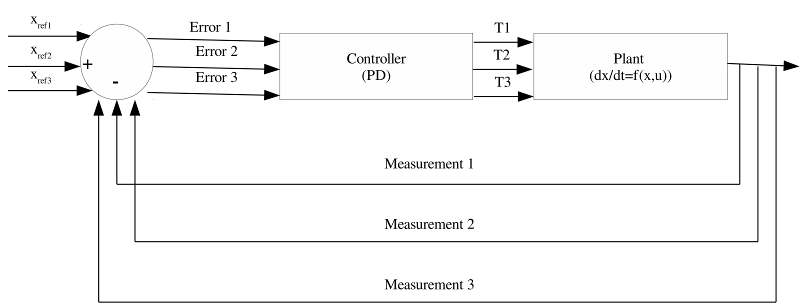

Figure 7.1. Block diagram

7.2 Attitude stabilization

The satellite is to be stabilized about zero degrees in all the axis . To start with we say that the system is kept at some initial configuration and see if controller is able to stablize it.

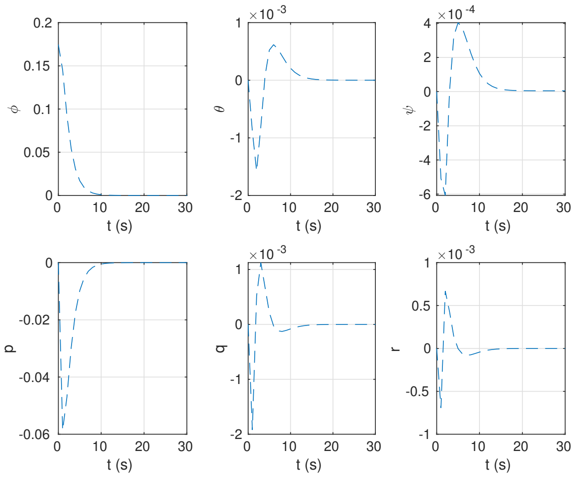

Figure 7.2. States

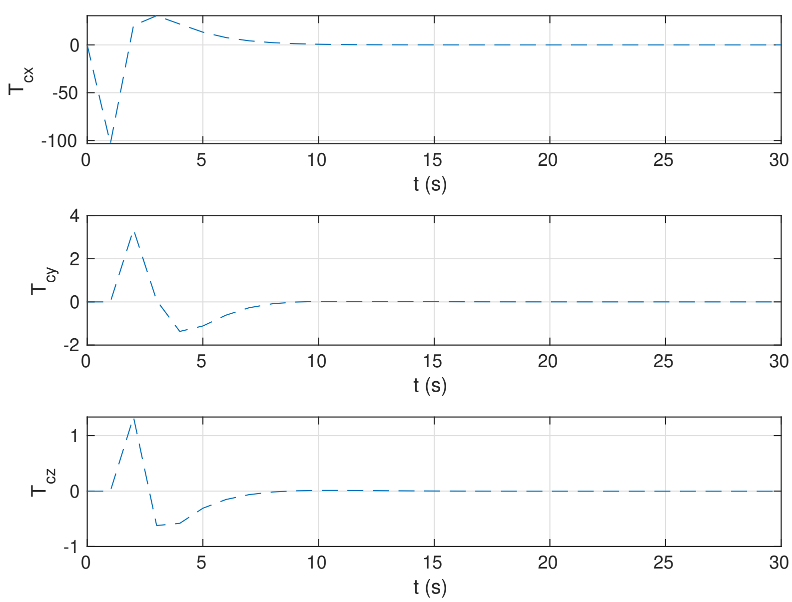

Figure 7.3. Controls

Looking at control response characteristics, we see that a negative moment is applied to the system to reduce the roll error. In doing so, all the coupled states are engaged and corrections for pitch and yaw is also given by the controller. The control moment along roll goes to a large negative value and then begins to decrease. If more moment was added, it would miss the set point.The reduction in moment slows down the satellite and it finally gets stabilized. Key point is that moments to be given were intuitive and similar behaviour is seen in the simulation. The control torques had to be engaged for other axes because of the non-linear coupling in the states. **This however will not be the case with the linearised system all coupling terms won't be there and control torques won't be required.**

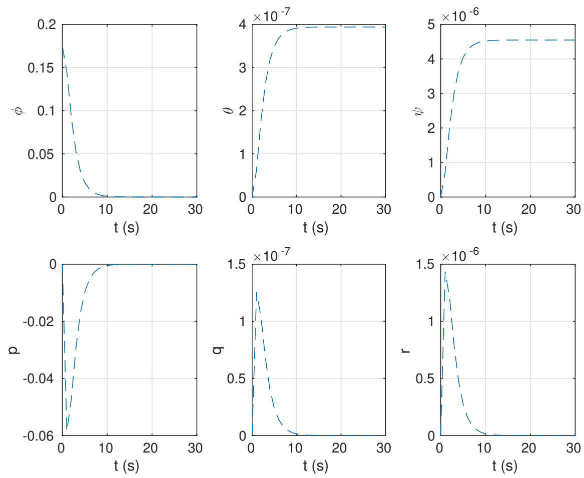

Figure 7.4. States (Linearised system)

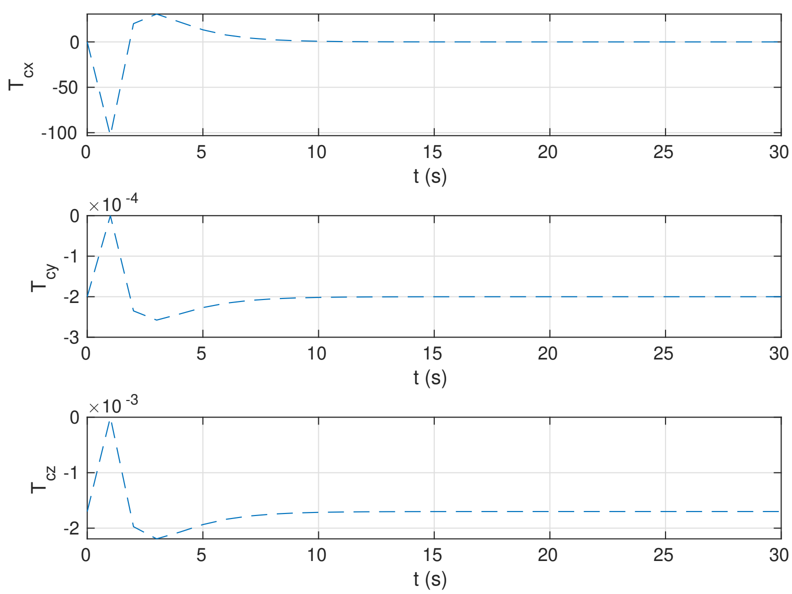

Figure 7.5. Controls (Linearised system)

Here we see that controls generated from pitch and yaw are very small and this is accounted due to decoupled dynamics. On first sight it might seem linearised and non-linearised respond in the same way but it is not.

The **settling time has changed** for the non-linear simulation. Our design requirement was roughly 9 seconds. This behaviour is seen in the linearized system whereas in comparison to non-linear system the settling time got increased to 16 seconds approximately (77% rise) . Hence control system gains need to designed very carefully for deployment.

7.3 Set point tracking

For every 100 seconds we want 2 attitude set point to be tracked alternatively.

State variable	Set point 1	Set point 2
$\mathring{\phi}$	20	0
$\mathring{\theta}$	0	20
$\mathring{\psi}$	0	0
$p(rad/s)$	0	0
$q(rad/s)$	0	0
$r(rad/s)$	0	0

Table 4. Simulation parameters

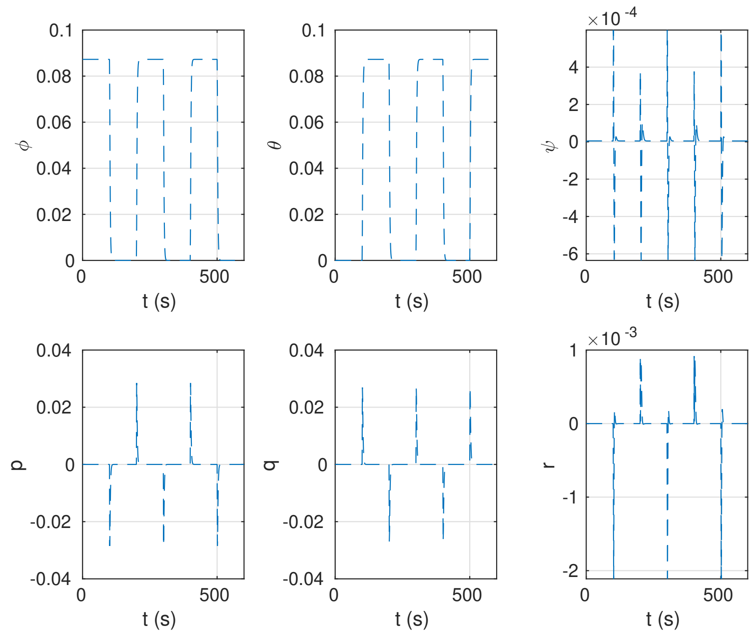

Figure 7.6. States (Change of set points every 100 seconds)

For the above case set points were changed over a time interval of 100 seconds. Lets say we change the set points faster at say every 5 seconds.

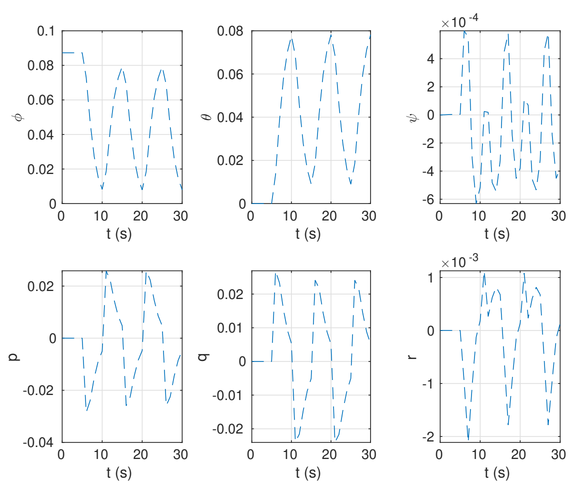

Figure 7.8. States (Change of set points every 5 seconds)

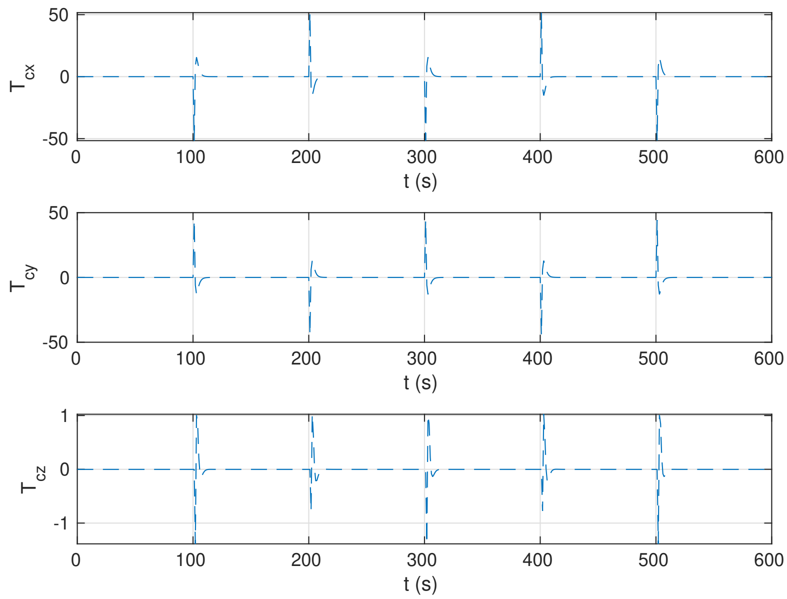

Figure 7.7. Controls (Change of set points every 100 seconds)

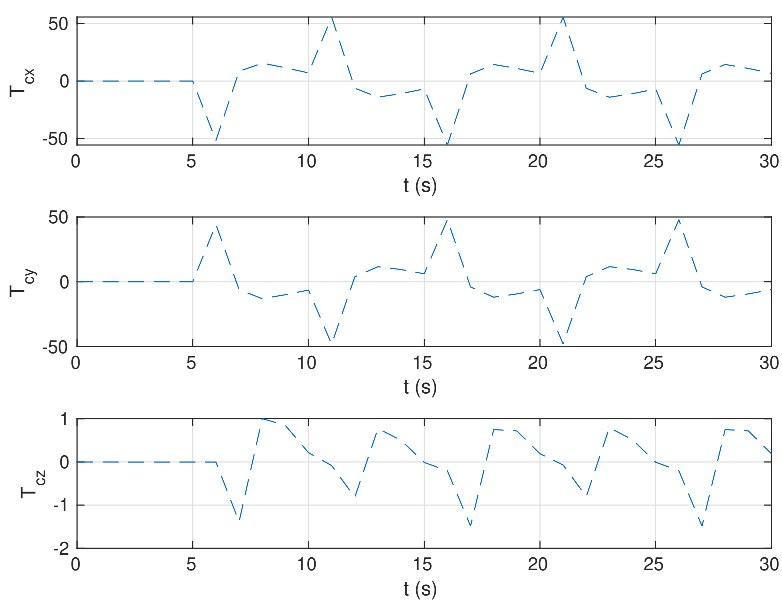

Figure 7.9. Controls (Change of set points every 5 seconds)

Clearly we don't see the required performance as seen from previous case where 100 secs were was used to switch between set points. Before the system can actually go to the set point fixed ,newer set point is given and leads to poor tracking. This exercise is to show that while giving time varying set points to the system its critical to look at the t_s of the system. Note that control histories are same for both the cases. This is evident because controller does not rely on the model unlike model based controller like MPC. The dynamics were slow to respond to the fast changes in the set point.

8. LIMITATIONS

- Actuator dynamics not taken into consideration
- Control torque constraints required for meaning full simulation
- Gain are designed for a fixed equilibrium condition. Gain scheduling will be the next step to be tried out.
- Sensor noise not included
- Position dynamics assumed to be decoupled

9. CONCLUSION

A PD control system for a satellite is developed from classical control techniques using standard methods. To start with preliminary gains are decide based on Routh criteria and performance metrics like t_s. This is further improved upon by using root locus and gains are computed. Computed gains are put to test on a non-linear system and performance is compared. 2 cases for simulation study was used and its outcomes were studied and reasoned.

10. APPENDIX

$$\begin{bmatrix} s^3 & T & aK_d \\ s^2 & 1 & aK_p \\ s^1 & a(K_d - K_p) & 0 \\ s^0 & a(Kp) & 0 \end{bmatrix}$$

$\frac{K_d}{K_p} > T$ for stabilty. Min value for $\frac{K_d}{K_p}$from our design was 3. So $T < 3$ for actuator model chosen if added to the system in future.