Mathematical Aspects of Biomedical Electronic System Design
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Lecture 04
Fourier Series

Hello everyone, welcome to the second lecture of the course Mathematical Aspects of Biomedical Electronic System Design. We are studying the module signals and systems and today in this second lecture, we will learn about Fourier series.

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To elaborate, we will learn about continuous-time periodic signals, continuous-time Fourier series, properties of Fourier series, and similar things same concepts for discrete-time signals as well. So, let us begin with today's lecture.

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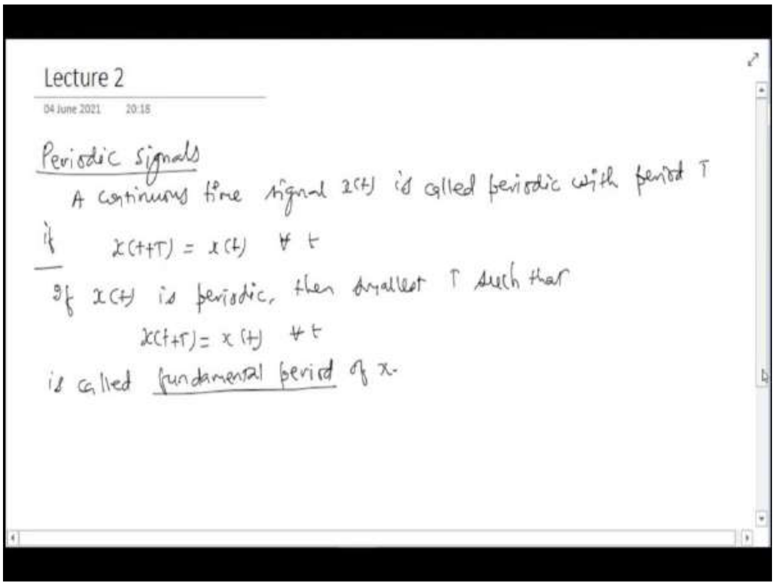

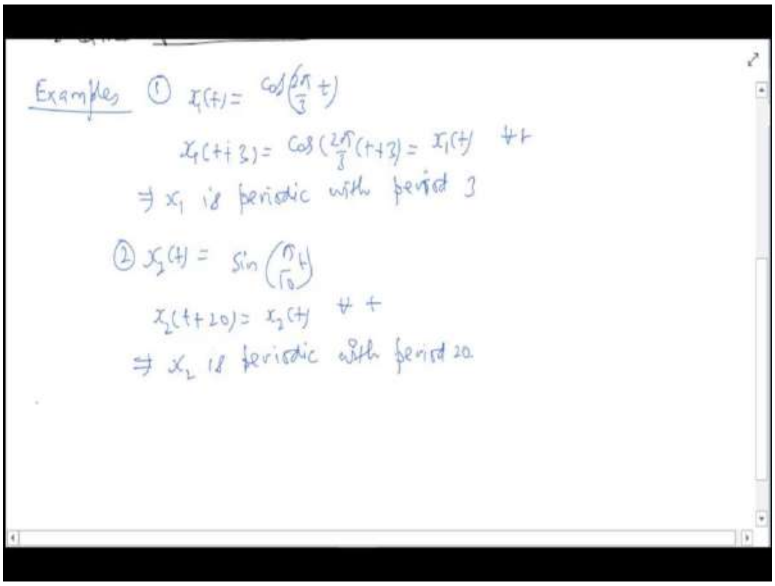

So, let us begin with looking at what are periodic signals. A continuous-time signal x(t) is called periodic with period T if x(t+T) = x(t) ∀ t. Moreover, if x(t) is periodic then smallest T, smallest capital T such that the above equation holds that is x(t+T) = xt ∀ t is called fundamental period of x. Notice that fundamental period is a period of x(t) and any integer multiple of fundamental period is also a period of x.

Here are examples, let us consider $x_1(t) = \cos\left(\frac{2\pi}{3}t\right)$, observe that $x_1(t)$ is periodic with period, so let us see, if I compute $x_1(t+3)$, it becomes $\cos\left(\frac{2\pi}{3}(t+3)\right)$, which is $x_1(t)$. So, x_1 is periodic with period 3, 3 is fundamental period of x_1 in this case. Let us look at another example, say $x_2(t) = \sin\left(\frac{\pi}{10}t\right)$, as in case of x_1 we can verify that $x_2(t+20)$ is $x_2(t)$ ∀ t, x_2 is periodic with period 20. So, here is a question support $x_1(t)$ and $x_2(t)$ are periodic is the sum of periodic signals also periodic.

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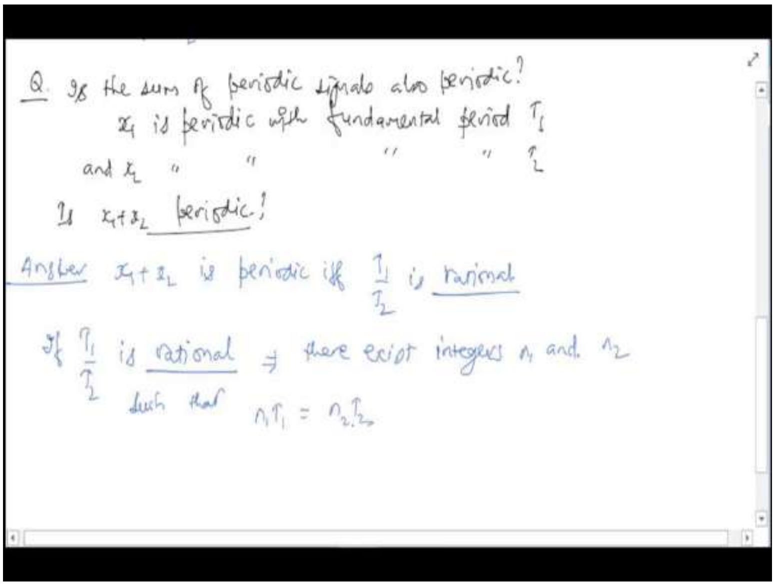

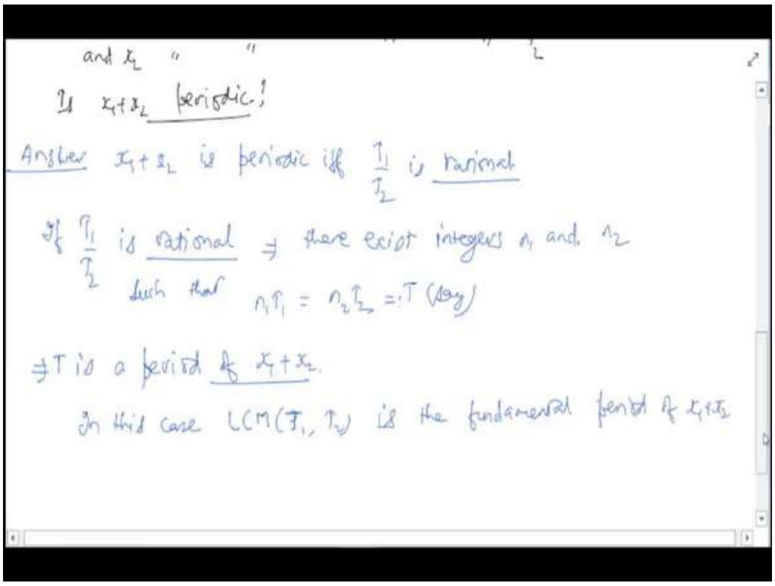

Question is the sum of periodic signal also periodic, more precisely if x_1 is periodic with fundamental period T_1 and x_2 is periodic with fundamental period T_2, is $x_1 + x_2$ periodic? Here is the answer, $x_1 + x_2$ each periodic if and only if $\frac{T_1}{T_2}$ is rational. Notice that if $\frac{T_1}{T_2}$ is rational then there exists integers n_1 and n_2 such that $n_1T_1 = n_2T_2$ in this case, say this is equal to T, then T is a period of $x_1 + x_2$. In fact, in this case, least common multiple LCM of T_1 and T_2 is the fundamental period of $x_1 + x_2$.

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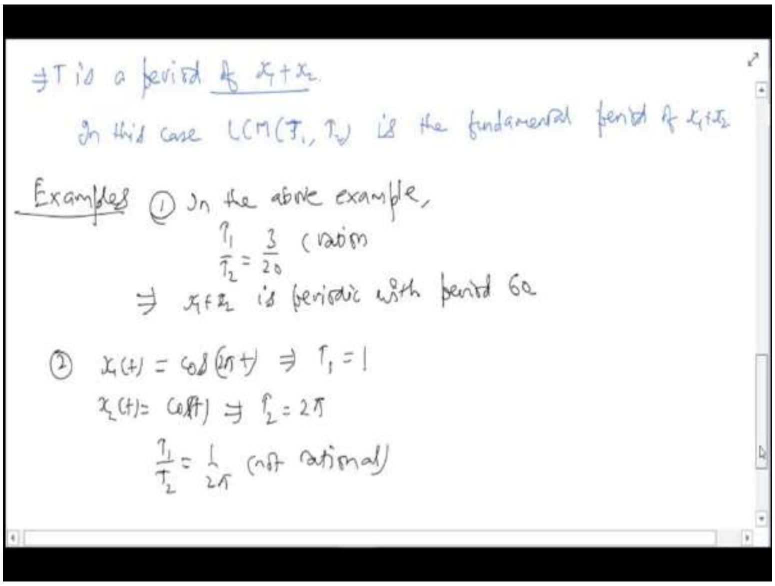

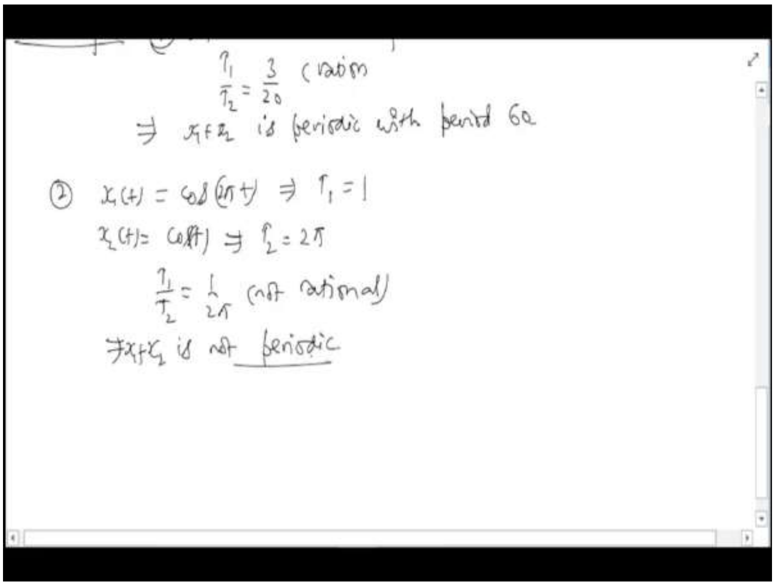

Let us see examples. In the above example, we saw that x_1 was periodic with period 3 and x_2 was periodic with period 20, then the least common multiple that is 60 which period of $x_1 + x_2$ and $x_1 + x_2$ is periodic, let us see in the above example, $\frac{T_1}{T_2} = \frac{3}{20}$ which is rational. So, $x_1 + x_2$ is periodic with fundamental period 60.

However, let us see another example, where $x_1(t) = \cos(2\pi t)$. So, its fundamental period is 1, $x_2(t) = \cos(t)$. What is it fundamental period? It is fundamental period is 2π, $\frac{T_1}{T_2}$ is not rational. So, this time $x_1 + x_2$ is not periodic. Having seen periodic signals, let us now also recall the notion of complex exponentials.

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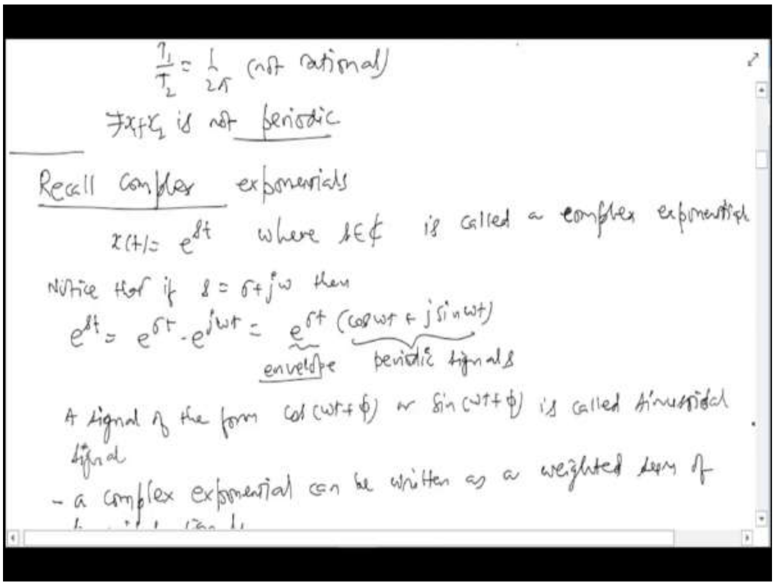

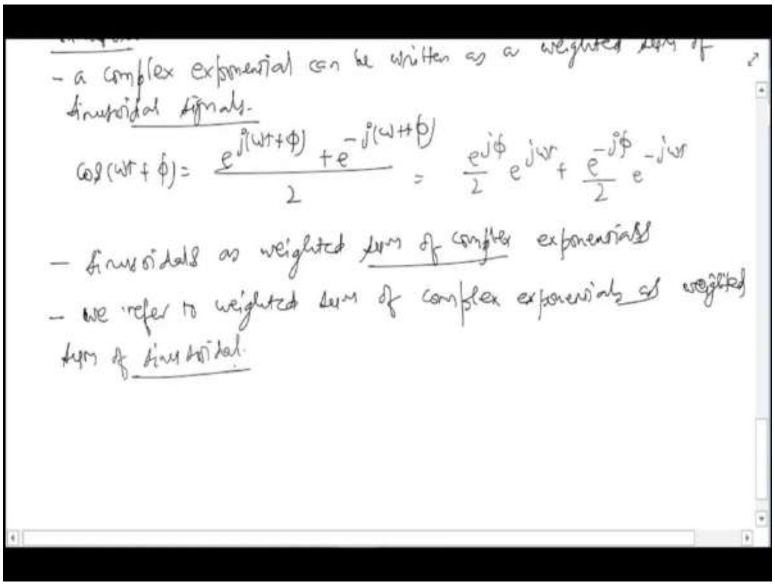

Let us recall complex exponentials. We call that x(t) = e^{st}, where s is a complex number is called a complex exponential, so these are complex exponentials. Notice that, if s the complex number is $\sigma + j\omega$, then

$$e^{st} = e^{\sigma t} e^{j\omega t} \ ,$$

which in turn is

$$e^{\sigma t}(\cos \omega t + \mathrm{j}\sin \omega t).$$

Here, this, sorry jsin ωt, this sin and cos functions they constitute the periodic part. So, this is a periodic signal, these are, whereas this is envelope of e complex exponential. Now, this periodic signal is called a sinusoid. So, formally a signal of the form cos $(\omega t + \varphi)$ or say sin $(\omega t + \varphi)$ is called a sinusoidal signal.

So, what we see that in exponential, complex exponential can be written as a weighted sum of sinusoidal signals. A complex exponential can we written as a weighted sum of sinusoidal signals we can say. On the other hand, if I take any sinusoidal signal let us say cos $(\omega t + \varphi)$ this can be written as

$$\frac{e^{j(\omega t+\varphi)}+e^{-j(\omega t+\varphi)}}{2}$$

which in turn can be written as

$$\frac{e^{j\varphi}}{2}e^{j\omega t} + \frac{e^{-j\varphi}}{2}e^{-j\omega t}\ .$$

So, now, we see that we can write a sinusoidal as a convex aggerated sum of complex exponential. So, we see that interchangeably we can write complex exponentials as weighted sum of sinusoidal and vice versa. So, for this region, so now, we have sinusoidal signals, sinusoidal as weighted sum of complex exponentials.

So, one can be represented in terms of others. For this region, we also refer to a weighted sum of, we refer to a weighted sum of complex exponentials as weighted sum of sinusoidals. With this notion in mind, let us now look at what a Fourier series is.

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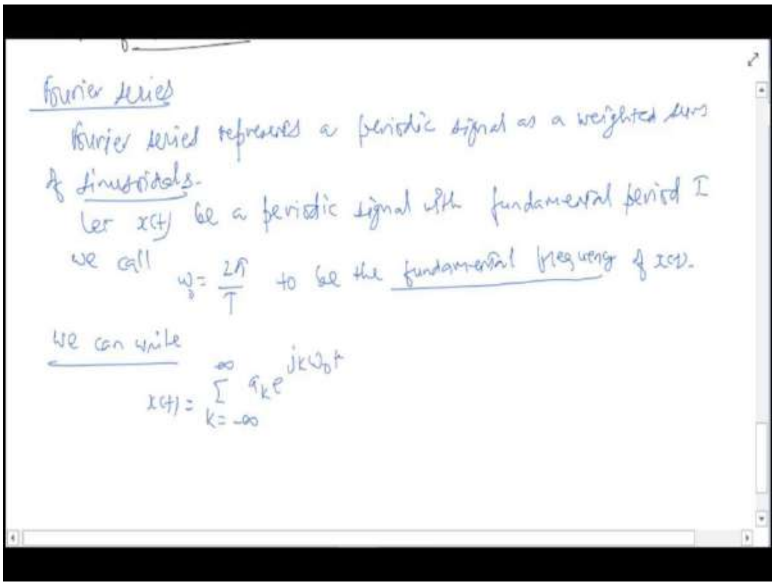

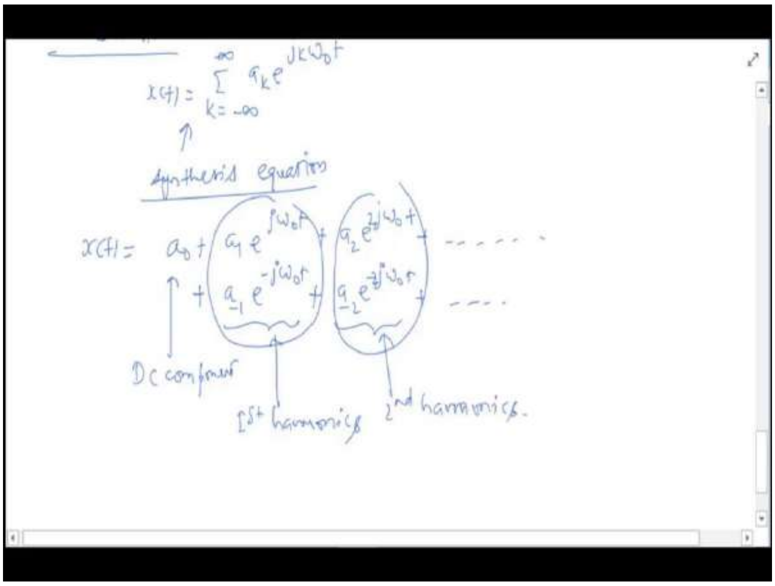

Fourier series represents a periodic signal as a weighted sum of sinusoidal, a periodic signal as a weighted sum of sinusoidals. In particular, let us consider x(t) a periodic signal with fundamental period T, fundamental period capital T in this case we call $\omega_0 = \frac{2\pi}{T}$ to be the fundamental frequency of x(t), to be the fundamental frequency of x(t).

For x(t) we can represent it as

x(t) $= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$.

This series on the right is called Fourier series. This equation itself is called synthesis equation. It is called synthesis equation because it synthesizes a periodic signal from sinusoidal components.

Notice that

$$x(t) = a_0 + a_1 e^{j\omega_0 t} + a_2 e^{2j\omega_0 t} + \cdots$$

and then

$$+a_{-1} e^{-j\omega_0 t} + a_{-2} e^{-2j\omega_0 t} + \cdots$$

In this expansion, the first term which is a constant, this is called DC term, DC component.

These two terms are called first harmonics; they are called first harmonics since they oscillate at fundamental frequency omega naught. Similarly, these terms are called second harmonics and so on. So, these two are first harmonics, these two are second harmonics.

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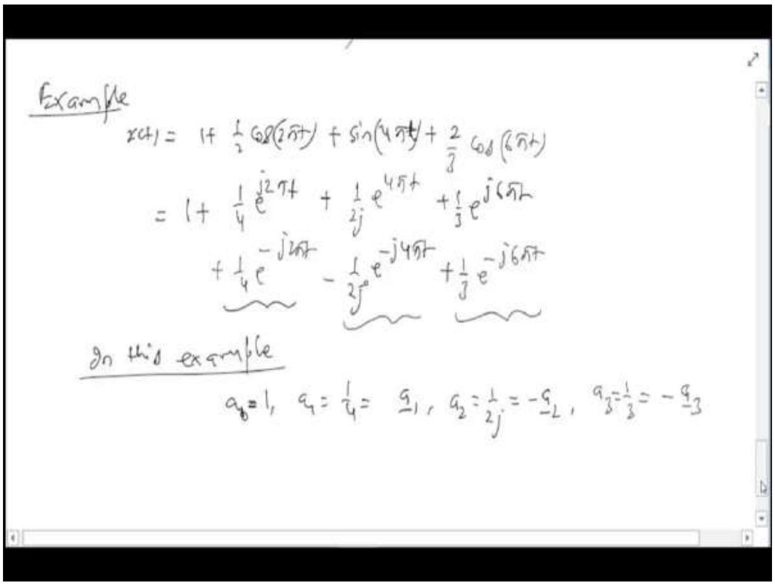

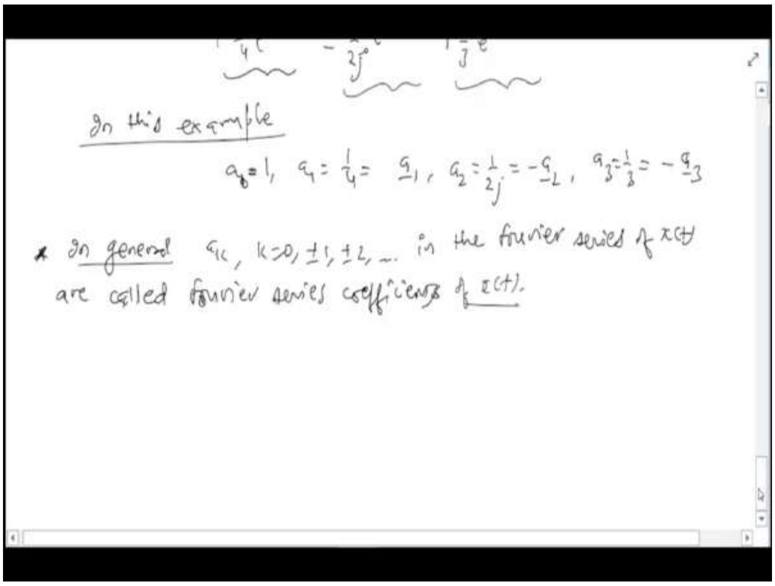

Let us take an example.

Let $x(t) = 1 + \frac{1}{2}cos2\pi t + sin4\pi t + \frac{2}{3}cos6\pi t.$

We can write these sinusoidals as weighted sum of complex exponentials and can write the whole thing $1 + \frac{1}{4}e^{j2\pi t}$ and the corresponding other term will be $e^{-j2\pi t}$. Here, we will have$\frac{1}{2j}e^{4\pi t} - \frac{1}{2j}e^{-j4\pi t} + \frac{2}{3}$, this will become $\frac{1}{3}e^{j6\pi t}$ and then there will be a one more term $\frac{1}{3}e^{-j6\pi t}$.

So, we see that the signal x(t) has a DC term these are first harmonics, second harmonics, and third harmonics. In this example, $a_0 = \frac{1}{2}$, sorry 1, $a_1 = \frac{1}{4} = a_{-1}$, $a_2 = \frac{1}{2j} = a_{-2}$, $a_3 = \frac{1}{3} = a_{-3}$.

In general, this terms a_k are called Fourier series coefficient of x(t), in general, a_k is for k=0, ±1, ±2, etc in the expansion, in the Fourier series of x(t) are called Fourier series coefficients of x(t). Before we proceed let us see a couple of important properties of Fourier series.

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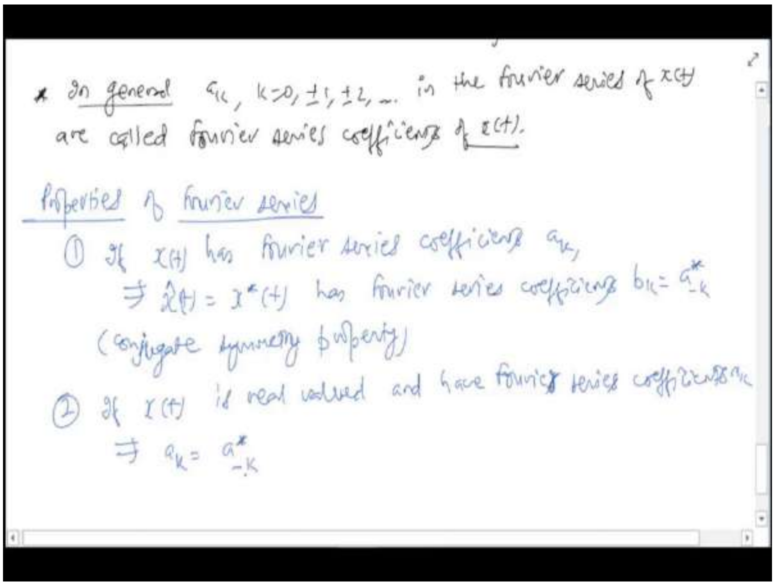

Properties for Fourier series. We will see more properties as we go along. So, the first property is, if x(t) has Fourier series coefficients a_k then $x^*(t)$ which is complex conjugate of x(t) has Fourier series coefficients a^*_{-k}, so let me write it, if x(t) has Fourier coefficients a_k then let us say $x^{\wedge}(t) = x^*(t)$ conjugate of x(t) has Fourier series coefficients $b_k = a^*_{-k}$ complex conjugate.

So, this property is called conjugate symmetry property. The other property is and it follows from the first one, it says that if x is real valued, if x(t) is real valued and Fourier series coefficients a_k, then a_k is equal to a^*_{-k}. In particular, this says that if x(t) is real value, we only need to compute a_k for non-negative k and we can recover or we can retrieve a_k for negative k from these values. So, next point is given a periodic signal x(t), how do we find Fourier series coefficients a_k for x(t).

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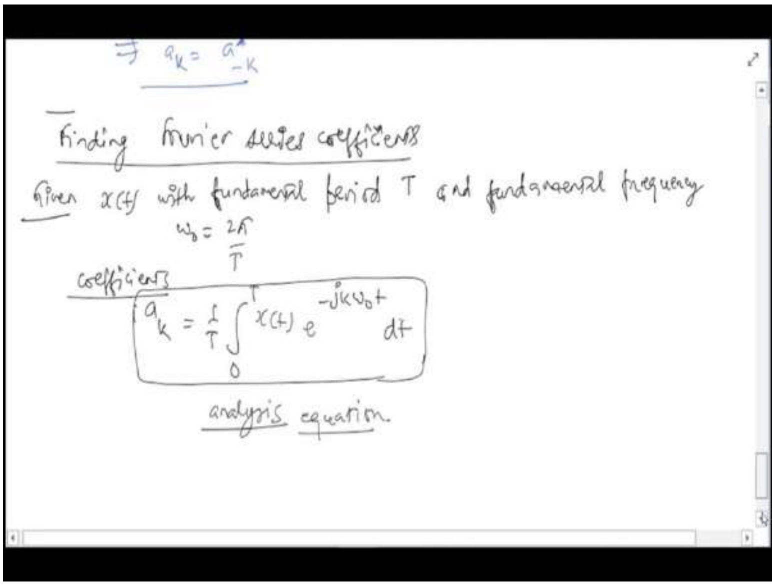

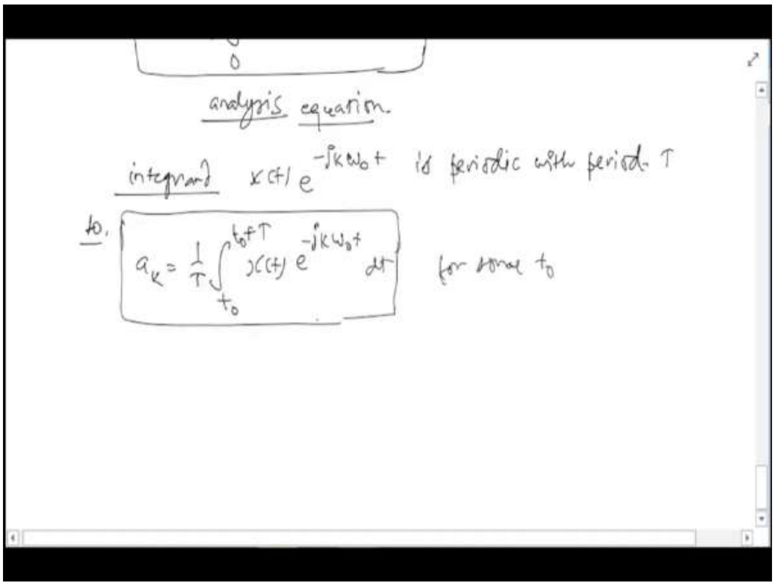

Finding Fourier series coefficients. So, given x(t) with period T, that the fundamental period T, let me write it, fundamental period T and fundamental frequency ω_0, $\omega_0 = \frac{2\pi}{T}$. The Fourier series coefficients are obtained using, coefficients are obtained using the formula,

$$ak = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t} dt\ .$$

Now, this equation that gets Fourier series coefficients from the signal itself this is called analysis equation. Notice that the integrand in this equation, integrand that is $x(t)\ e^{-jk\omega_0 t}$ is periodic with period t. So, rather than integrating it over 0 to T, we can integrate it over any interval of with T and we will get the same coefficient, same outcome. So, we can also define $a_k = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\ e^{-jk\omega_0 t} dt$

for some t_0. So, this is alternate definition of Fourier series coefficients. Let us see an example.

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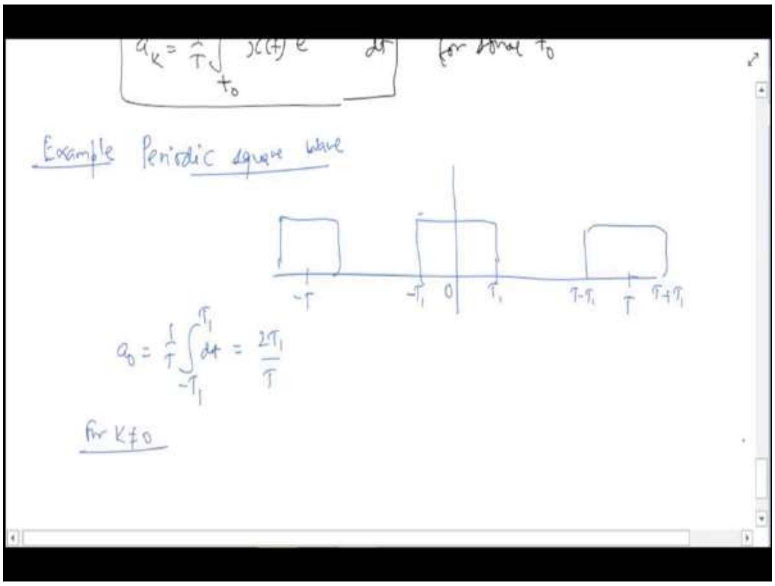

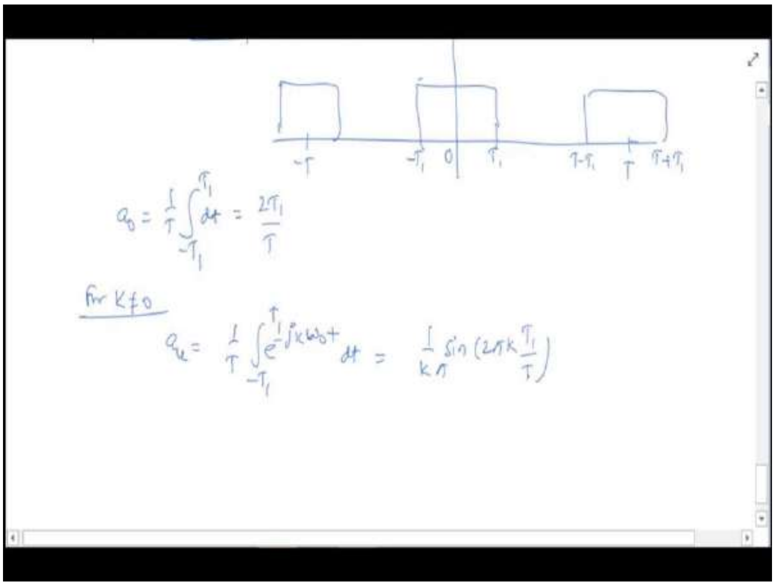

So, example that we take is of periodic square wave. Here is the example, so it is a square wave with period T, so this is 0, T, -T and the square pulse width is to T1. So, this -T1 to T1 will be T-T1 to T+T1 and so on. So, clearly, this is a periodic signal with period T and the coefficients can be computed as follows,

$a_0 = \frac{1}{T}\int_{-T_1}^{T_1} dt = \frac{2T_1}{T}$.

Similarly, for K ≠ 0,

$a_k = \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk\omega_0 t} dt$, which if we work out it turns out to be $\frac{1}{k\pi}\sin(2\pi k\frac{T_1}{T})$. Let us see another example.

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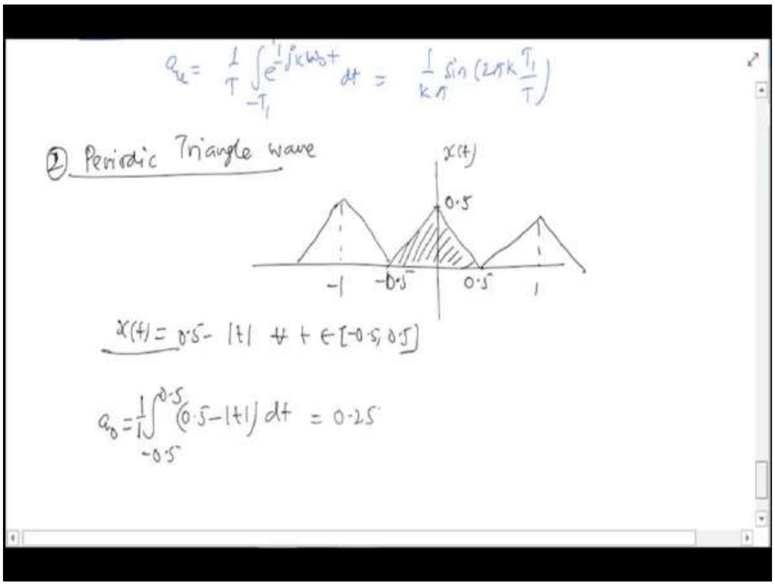

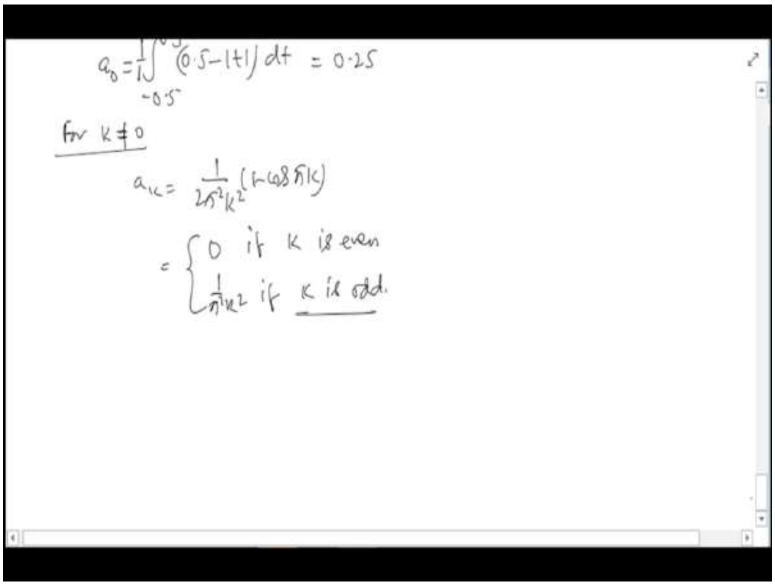

And this time, we look at triangular weight surveys consisting of triangular pulsation. This is called Periodic Triangle wave. It looks as follows. The height of the triangular pulse is 0.5, on the other hand, its period is 1. We can compute the Fourier series coefficients for this way following the similar procedure as for the square pulse above, and it turns out that for instance, a0 will be simply the area under this, the average value of this, so -0.5 to 0.5.

And before I compute it, let us also see that this mathematically, this wave can be written as follows x(t) = (0.5 - $|t|$) $\forall$ t in -0.5 and 0.5, and then it repeats with period 1. So, with this in mind, with this in view, I can write a_0 as -0.5 to 0.5, 1 by 1 which is the period, and here it will be (0.5-$|t|$)dt.

And this turns out to be equal to 0.25. If we compute coefficients for a_k for $k \neq 0$, we find that $a_k = \frac{1}{2\pi^2 k^2}(1 - cos\pi k)$ which is equal to since $1 - cos\pi k$ is 0, when K is even and 2 when K is odd, it turns out to be 0 is K even and $\frac{1}{2\pi^2 k^2}$, if K is odd. So, this is how we compute Fourier series coefficients. Now, let us look at a more fundamental question.

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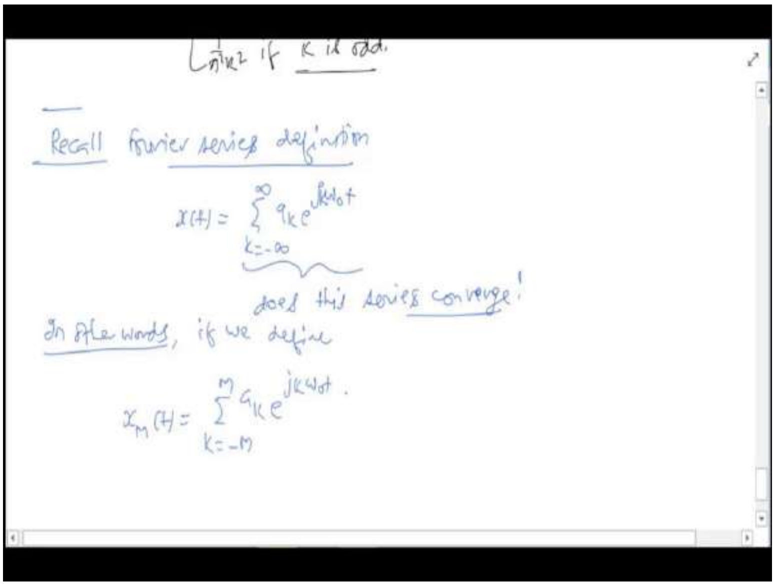

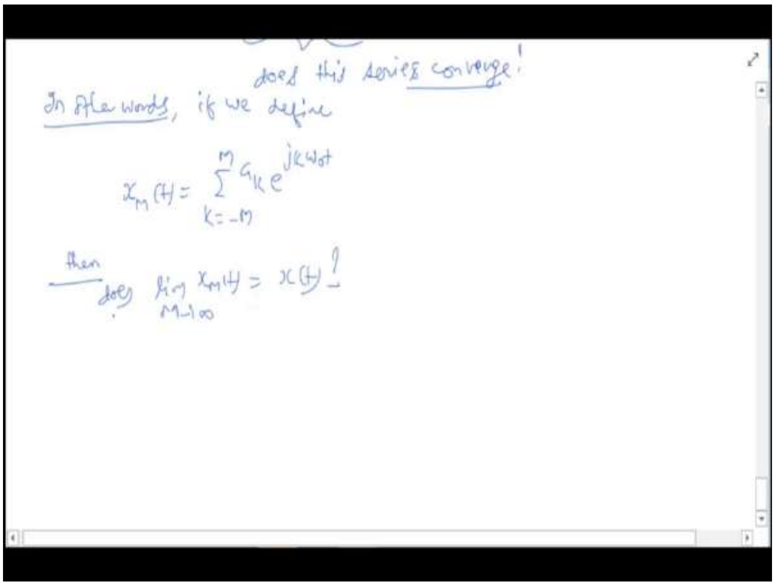

In particular, let us recall the definition of Fourier series. Recall Fourier series definition, we defined Fourier series for a signal

$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$.

In writing this, we assume that the series on the right-hand side converges well but the question is does the series converge?

In particular, what we I mean that, if I define a signal $x_M(t)$ to be in other words, if we define $x_M(t)$ to be a finite sum where I only take terms between -M and M. So, $a_k e^{jk\omega_0 t}$, then as I take M to ∞ does $x_M(t)$ approach x(t). So, this is the question that we would like to answer because the very existence of, very definition of Fourier series relies on convergence of this. We answered this question by the following theorem.

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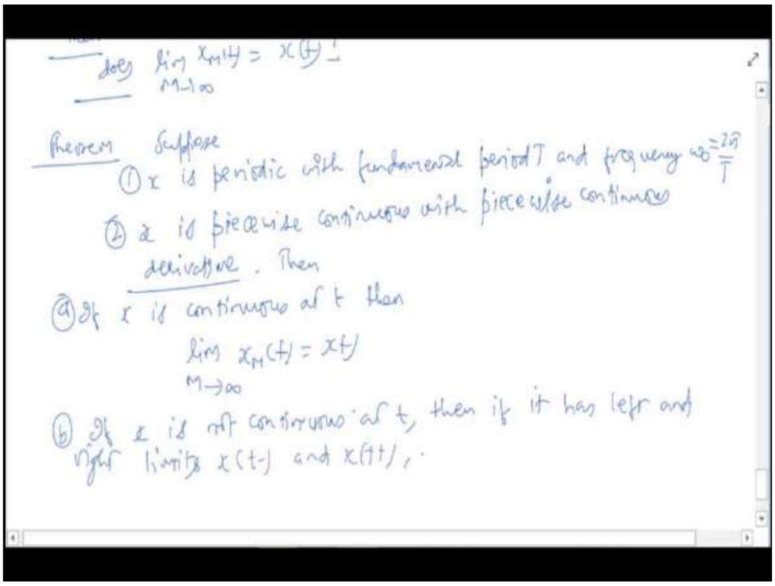

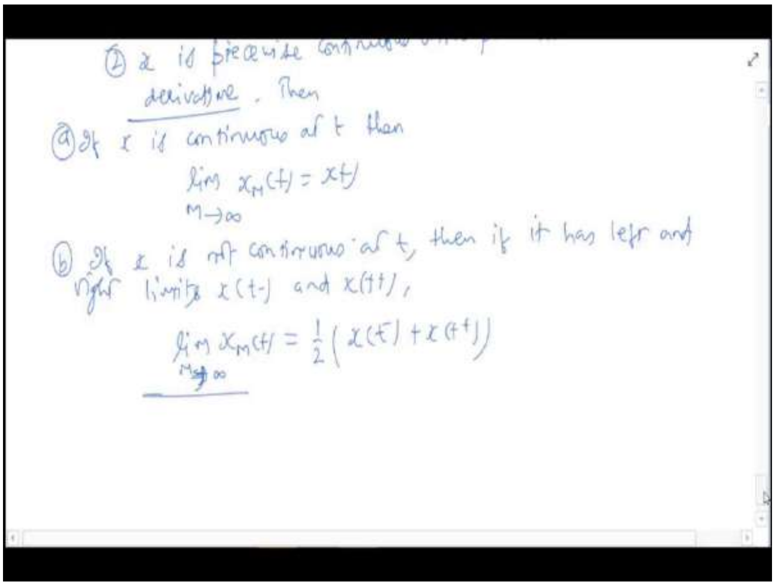

So, here is the theorem. Suppose we make two hypotheses, suppose x is periodic with fundamental period T and fundamental frequency $\omega_0 = \frac{2\pi}{T}$. This has been the standing assumption all through and the second hypothesis is, x is piecewise linear, it is a piecewise continuous with piecewise continuous derivative. Under these hypotheses, if x is continuous at a point t, if x is continuous at t then the limit that we would have liked to adjust indeed exists, so limit M tends to infinity $x_M(t)$ is equal to x(t).

On the other hand, then on the other hand, if x is not continuous at t then also not everything is lost, then if it has left hand and right-hand limits, left and right limits x(t-) and x(t +) limit M tends to infinity, still the sequence $x_M(t)$ converges, but it converges to the mean value of this left and right limits, x(t-) + x(t+). So, let us see a couple of examples.

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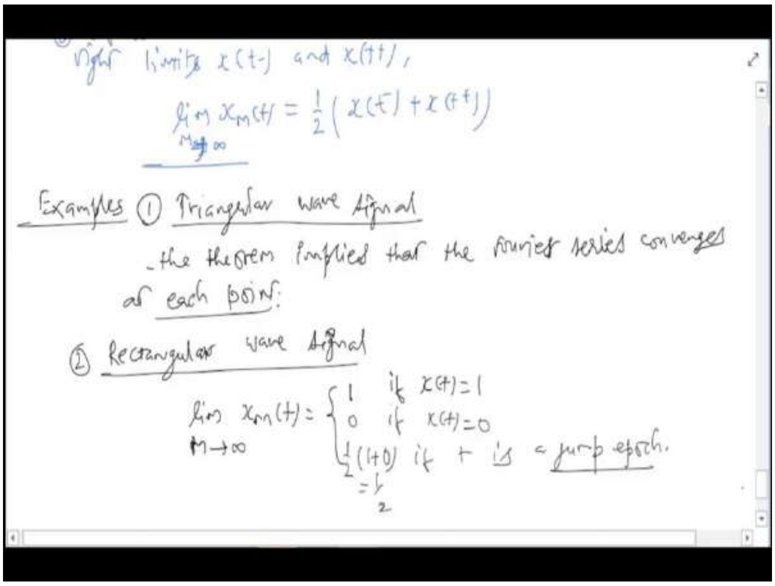

Let us again go back to that triangular pulse, triangular wave example that we saw just a while ago, triangular wave signal. Notice that for this signal x(t) is continuous everywhere. So, the above theorem implies that the theorem implies that Fourier series converges at each point, the theorem implies that the Fourier series converges at each point.

On the other hand, if we see the rectangular wave signal, here we see that x(t) is either 1 or 0 or there is a jump, then the above theorem implies that limit t, sorry M tends to infinity $x_M(t)$ it will be same as x(t) that is 1 if x(t) i= 1, 0 if x(t) = 0, and it will be $\frac{1}{2}$(1+0), that is $\frac{1}{2}$ if t is a jump instead, if t is a what we call jump epoch. So, this is about convergence of Fourier series. Let us now see a few properties of, few more properties of Fourier series.

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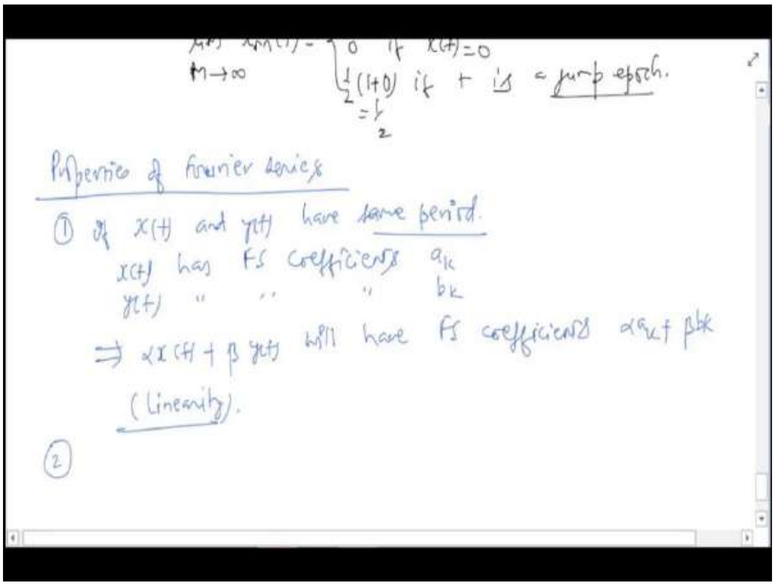

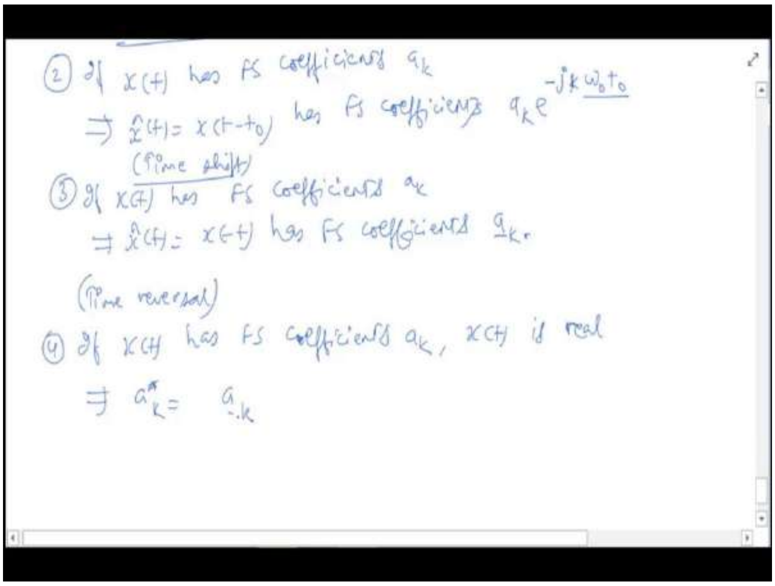

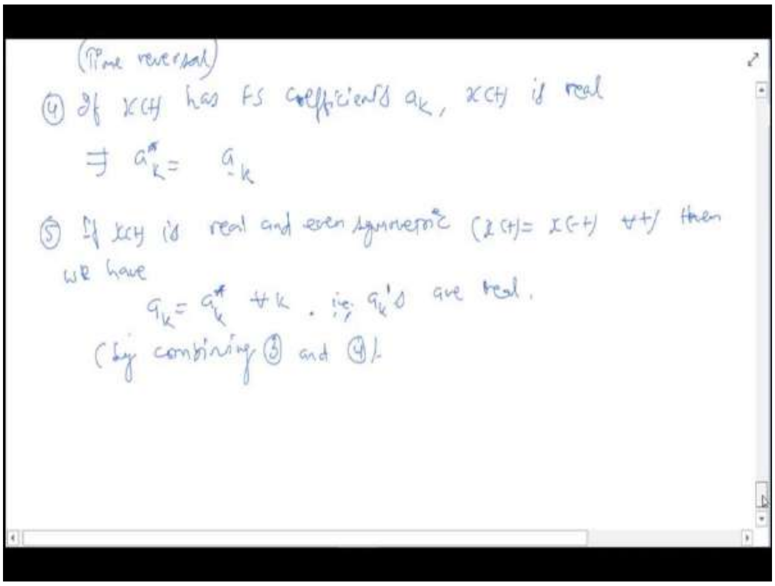

We have already seen a few properties of Fourier series. So, the first property says that if there are two signals x and y with identical periods, and their Fourier coefficients a_k and b_k, then Fourier coefficient of a linear combination of x and y can be written as a linear combination of Fourier coefficients a_k and b_k. To be precise let us see, if x(t) and y(t) have same period, I would say the same fundamental period x(t) has Fourier series coefficients a_k, y(t) has Fourier series coefficients b_k, then

αx(t) + βy(t) will have Fourier series coefficients $\alpha a_k + \beta b_k$.

This property as we can see it, it is referred to as linearity. Let us look at the next property, if x(t) has Fourier series coefficients a_k, then the time-shifted version of x(t) that is

x^(t)=x(t) - t_0

has Fourier series coefficients

$a_k e^{-jk\omega_0 t_0}$.

So, see, there will be $jk\omega_0 t_0$. So, we see that the time shift reflects in the exponential here. Let us look at another property that is called time reversal.

So, this was time shift. The next property that we will see it is called time reversal. So, what does it say? It says that if x(t) has Fourier series coefficients a_k then x^t = x(-t) has Fourier series coefficients a_{-k}, has Fourier series coefficients a_{-k}. To be precise if I denote Fourier series coefficients of x^t by b_k, then $b_k = a_{-k}$ ∀ k. So, this is called time reversal.

The next property, well this is a property that we have already seen, but let me put it again for recollection if x(t) has a Fourier series coefficients a_k and x(t) is real then a^*_k will be same as minus, sorry a_{-k}. So, this is a property that we had seen earlier also but I have put it here so that all the properties are there at one place. Finally, real and even symmetric. So, what I mean by even symmetric is x(t) =x(-t) ∀ t then we have a_k=a^*_k ∀ k, that is a_k's are real. This is obtained by combining 3 and 4. We now turn our attention to discrete-time signals.

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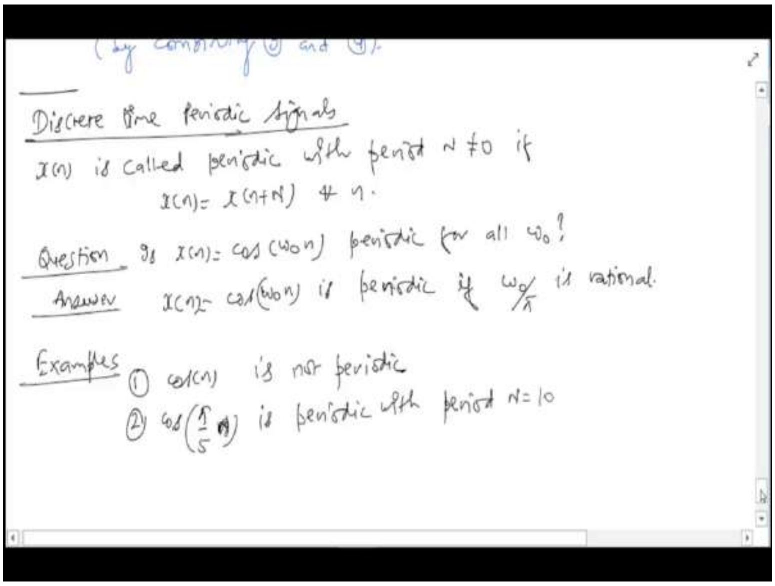

So, to begin with, let us look at discrete-time periodic signals. So, x(n) is called the discrete-time periodic with period N and N≠0 if

x(n) = x(n+N) ∀ n.

So, here is a question, is x(n) = cos(ω_0n) periodic for all ω_0? Answer is, x(n) = cos(ω_0n) is periodic if ω_0 is rational, sorry if $\frac{\omega_0}{\pi}$ is rational. Here are the few examples cos(n), clearly $\frac{1}{\pi}$ is not rational, so this is not periodic cos($\frac{\pi}{5}$n), what about this, this is periodic with period N=10.

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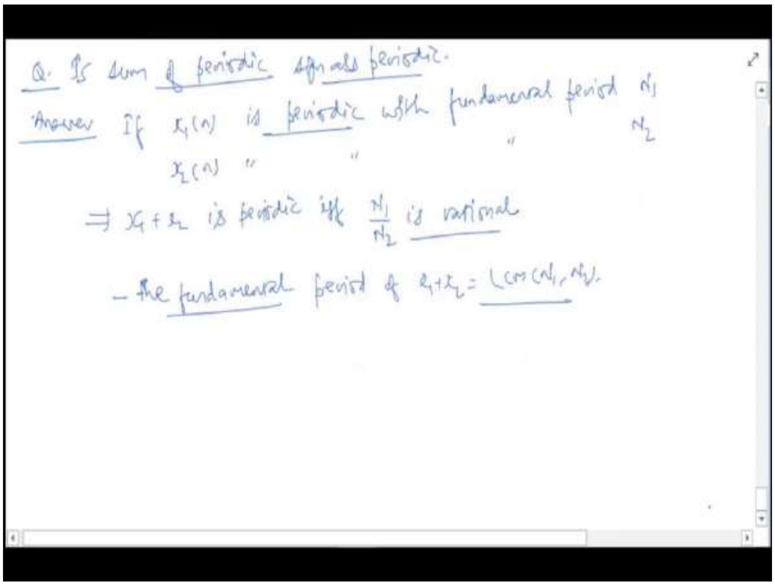

Next question, as in continuous time case we can ask if we have two periodic signals will their sum always be periodic? The answer is, so is sum of periodic signals, periodic. Well, the answer is similar to the continuous-time case, if $x_1(n)$ is periodic with fundamental period N_1 and $x_2(n)$ is periodic with fundamental period N_2, then $x_1 + x_2$ is periodic if and only if $\frac{N_1}{N_2}$ is rational and if $x_1 + x_2$ is periodic fundamental period of $x_1 + x_2$ is LCM of N_1 and N_2. But we did not exactly, we did not define the fundamental period of discrete-time periodic signals.

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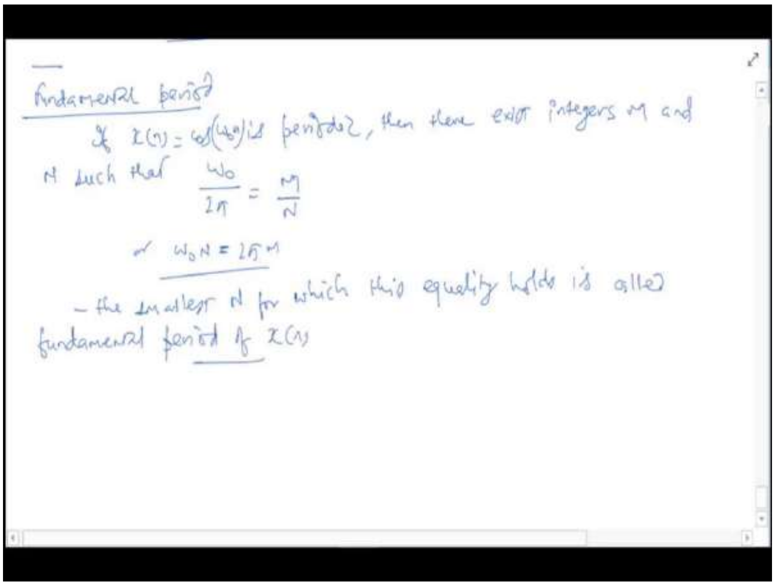

So, for that let us see the following fundamental period. If x(n) is periodic then we saw that, so let us see if $x(n) = \cos(\omega_0 n)$ is periodic then we saw that $\frac{\omega_0}{\pi}$ is rational. So, then there exists integers M and N. Such that $\frac{\omega_0}{2\pi} = \frac{M}{N}$ or $\omega_0 N = 2\pi M$. The smallest M, sorry the smallest N for which equality holds is called fundamental period of x. So, for which this equality holds is called fundamental period of x(n). Let us see an example.

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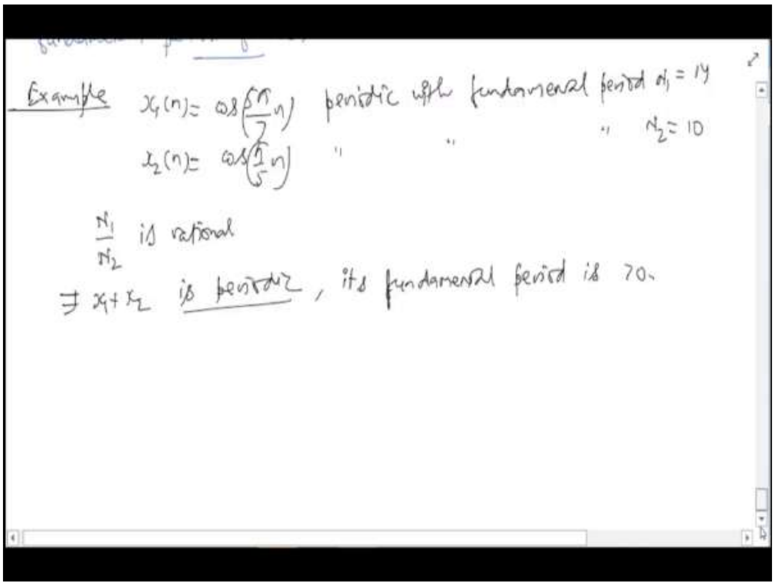

Consider $x_1(n) = \cos(\frac{5\pi}{7}n)$. Following above definition, it can be seen that $x_1(n)$ is periodic with fundamental period $N_1 = 14$. We have already seen that $x_2(n) = \cos(\frac{\pi}{5}n)$ periodic with fundamental period. Then notice that $\frac{N_1}{N_2}$ is rational, so $x_1 + x_2$ is periodic and its period is, its fundamental period rather is LCM of 14 and 10 which is 70. So, I have not define periodic signals. Let us now move to definition of discrete-time Fourier series.

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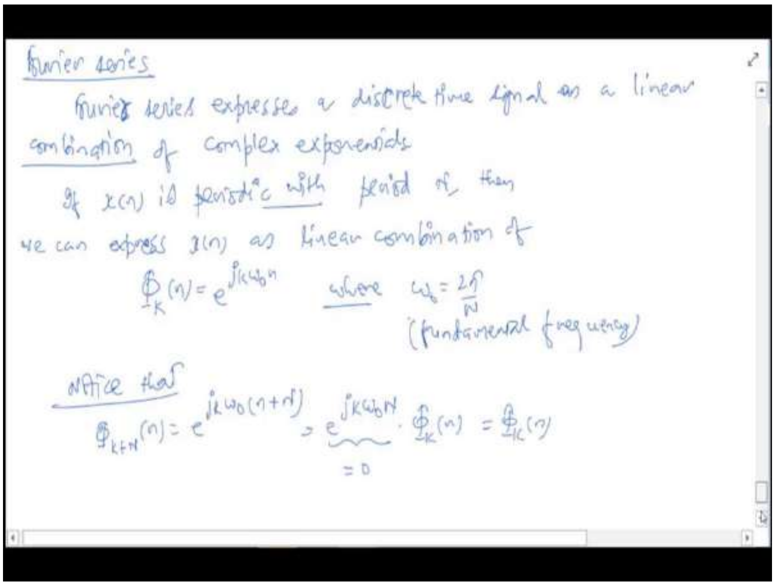

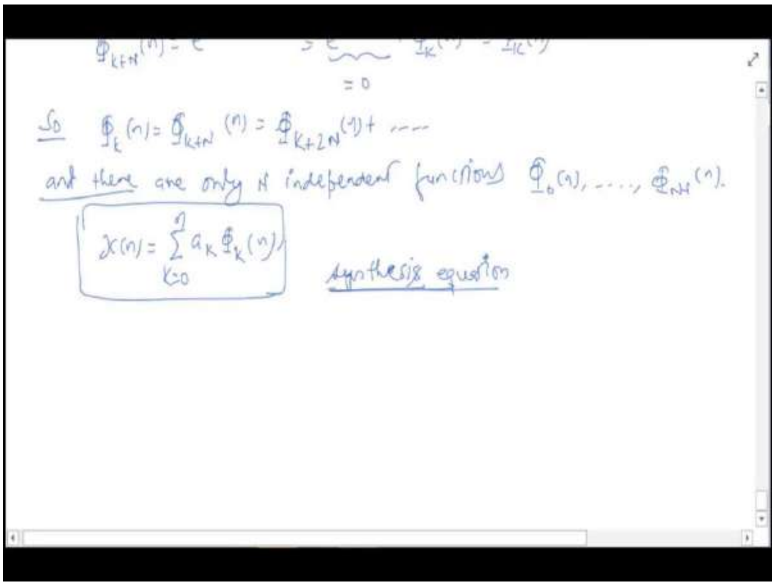

As before, Fourier series expresses a sequence a discrete-time signal, with period n as linear combination of complex exponentials, so Fourier series expresses a discrete-time signal as linear combination or weighted sum combination of complex exponentials.

For instance, if x(n) is periodic with fundamental period n, then we can express x(n) as summation of or okay as rather I would say, x(n) as, we can express x(n) as linear combination of complex exponentials

φ_k(n) = $e^{jk\omega_0}$, $\omega_0 = \frac{2\pi}{N}$.

And as in continuous time case, it is called fundamental frequency. Notice that

φ_{k+N}(n)= $e^{jk\omega_0(N+n)} = e^{jk\omega_0 N}\varphi_k(n)$ and this term is 1. Why? Because $\omega_0 N = 2\pi$, so this is φ_k(n). So, we see that there are only finitely many independent functions φ_k(n)'s.

So, we see that φ_k(n) = φ_{k+N}(n) plus which is, which in turn is equal to φ_{k+2N}(n), and so on and there are only capital N independent functions. Namely, φ_1(n), let me start with 0 φ_0(n)…φ_{N-1}(n). So, as a consequence, we can represent x(n) as a finite sum.

So, unlike continuous-time case where we had an infinite series in discrete time, a periodic signal x(n) is represented as a finite sum of complex exponentials, more precisely I can write x_k(n) as this. As in continuous time case, this equation is called synthesis equation. Let us see an example.

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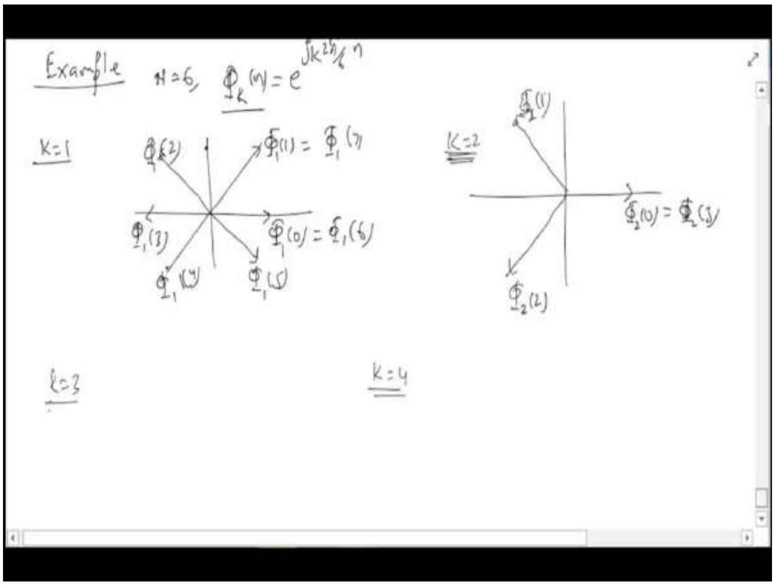

Let us just fix capital N =6. In this case,

φ_k(n) = $e^{jk\frac{2\pi}{6}n}$

And we have already seen that for any value of small n there are only capital N independent or rather, yeah there are only capital N independent functions φ_0(n) to φ(n) or φ_1(n) to φ 2 6 n. Let us see how this, how these values will look like.

So, K =1, in this case, we will have φ's complex exponential as follows. So, here is φ_1(0), φ_1(1), this will be φ_1(2), φ_1, sorry so this is φ_1(2), φ_1(3), φ_1(4), and this is φ_1(5). From what we discussed earlier φ_1(7) will be $=$ φ_1(1), φ_1(0) will be $=$ φ_1(6), and so on.

Similarly, if I choose K =2, I get only three distinct complex exponentials which are as follows. Here, I have φ_2(0), then this is φ_1(1), so φ_2(1), and this is φ_2(2), φ_2(3) because your periodicity will be same as φ_2(0). Similarly, I can write compound, complex exponentials for K =3 and K =4 as well, I leave those as an exercise.

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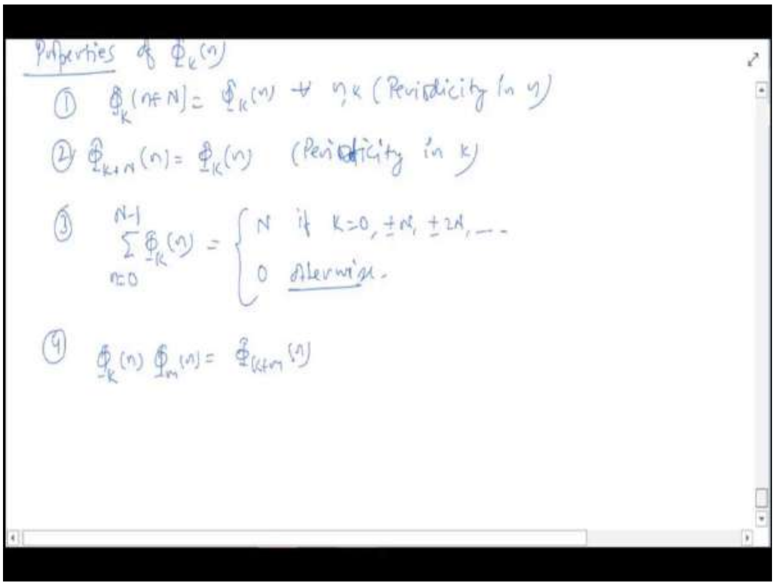

Now, let us see a few properties of complex exponentials, properties of φ_2(n). So, first thing is periodicity in n, that is φ_k(n+N) = φ_k(n) ∀n, and this is called a periodicity. Similarly, we have φ_{k+N}(n) = φ_k(n) and this is called periodicity in K. This is the property that we just saw, periodicity in K.

The next property is if we add these complex exponentials and it equals to let us say 0 to N-1, the summation turns out to be N, if K = 0, ±N, ±2N, etcetera and it is 0 otherwise. This is a property that can be easily verified by carrying out the summation. Finally, the fourth property says that

φ_k(n)φ_m(n) = φ_{k+m}(n).

So, next question is given a periodic signal x(n), how do we find this component, these coefficients, Fourier series coefficients.

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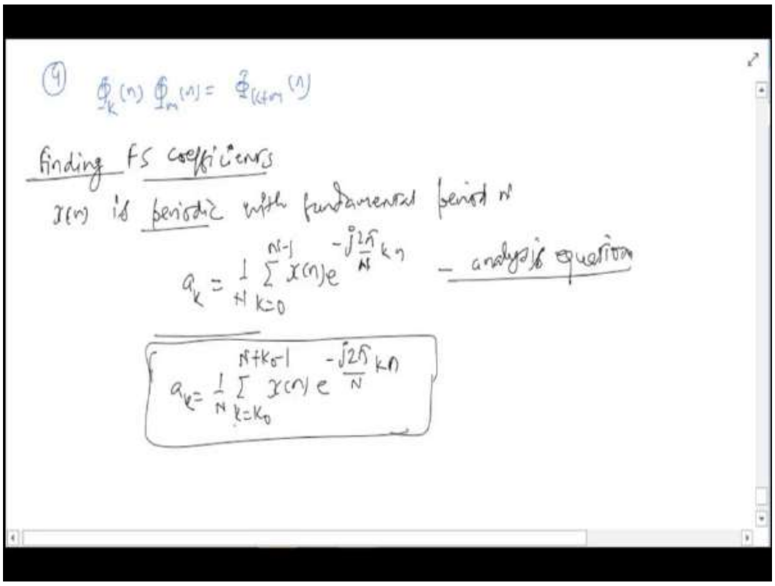

Just as we pose this question in continuous-time case. So, findings Fourier series coefficients, it turns out that if x(n) is periodic with fundamental period capital N, then a_k Fourier series coefficients are given by

$a_k = \frac{1}{N} \sum_{k=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$

Notice that $\frac{2\pi}{N}$ is what we call fundamental frequency. Also as in the continuous-time case, we could change the limits of summation to any value as long as the terms in the summation are capital N. So, we can sum from K $=K_0$ to $N+K_0-1$. And we would get that same Fourier coefficients, Fourier series coefficients. So, this is a formula that is equally valid. This expectedly is called analysis equations. Let us see an example.

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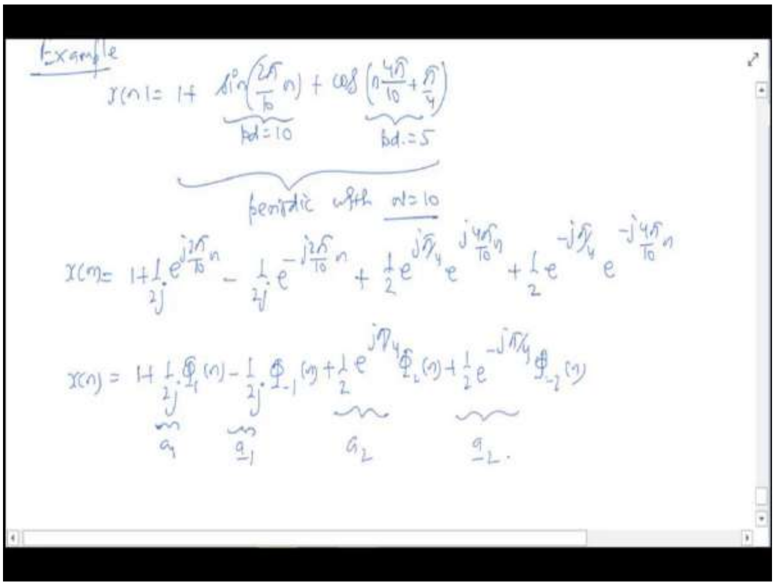

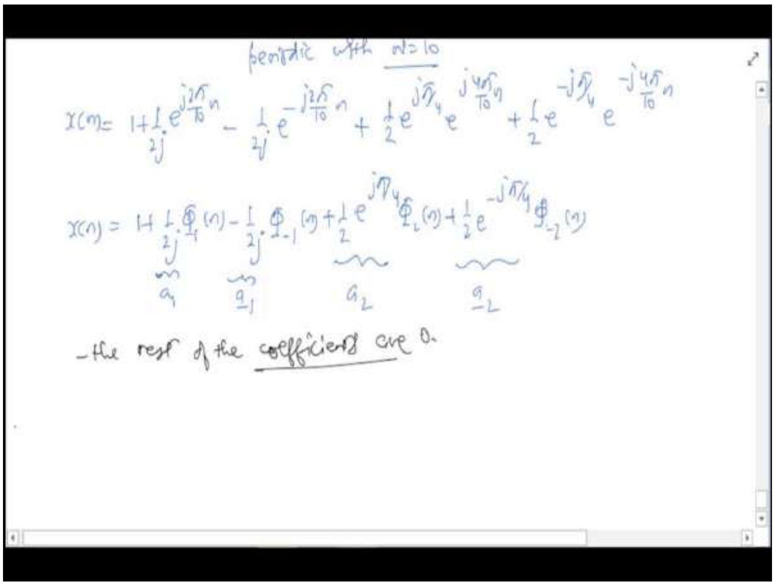

Let us consider

$x(n) = 1 + \sin\left(\frac{2\pi}{10}n\right) + cos(\frac{4\pi}{10}n + \frac{\pi}{4})$

Notice that this signal is periodic with period 10, whereas this is periodic with period 5. So, their summation will be periodic with period which is equal to LCM of 10 and 5 that is 10. So, this whole thing is periodic with capital N =10.

Now, we can see that

$$x(n) = 1 + \frac{1}{2j}e^{j\frac{2\pi}{10}n} - \frac{1}{2j}e^{-j\frac{2\pi}{10}n}$$

This is by just writing this sinusoid terms of first sinusoid, in terms of complex exponentials. Similarly, the second term can be written as summation of the following two terms

$$\frac{1}{2}e^{j\frac{\pi}{4}}e^{j\frac{4\pi}{10}n} + \frac{1}{2}e^{-j\frac{\pi}{4}}e^{-j\frac{4\pi}{10}n}$$

So, in this special case, we see that

$$x(n) = 1\ + \frac{1}{2j}\varphi_1(\mathrm{n}) - \frac{1}{2j}\varphi_{-1}(\mathrm{n}) + \frac{1}{2}e^{j\frac{\pi}{4}}\varphi_2(\mathrm{n}) + \frac{1}{2}e^{-j\frac{\pi}{4}}\varphi_{-2}(\mathrm{n})$$

If we do term by term comparison, we find that this is a_1, this is a_{-1}, this is a_2, and this is a_{-2}. The rest of the Fourier series coefficients are 0 in this special case, the rest of the coefficients are 0. Now, before we go ahead let us see the following property of Fourier series for discrete-time signals. This is analogous to similar property for continuous case.

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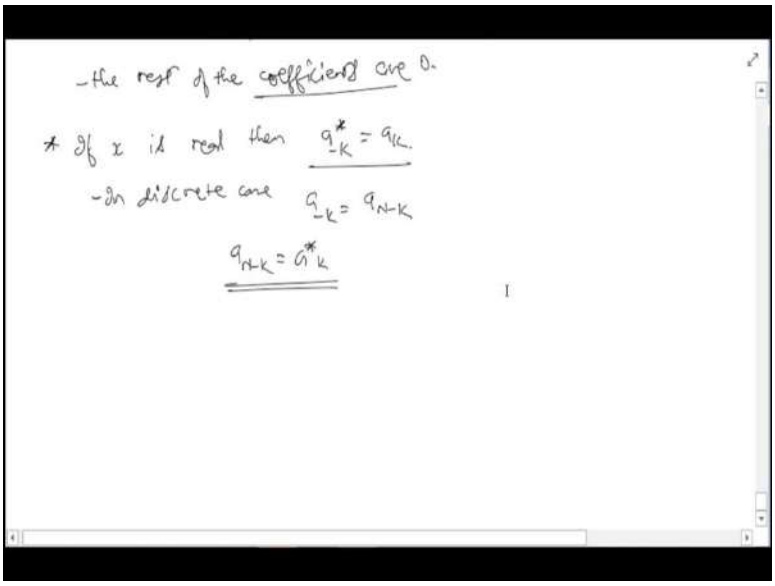

Properties says that if x is real then $a^*_{-k}=a_k$. Notice that in discrete case the Fourier series coefficients are also periodic, they repeat. So, $a_{-k} = a_{n-k}$. Combining the qualities, we obtain that $a_{-k} = a^*_k$. So, this is an important property of Fourier series coefficients in discrete-time case. This is all that we have to see about Fourier series, we end this lecture by summarizing the properties of Fourier series in continuous and discrete-time.

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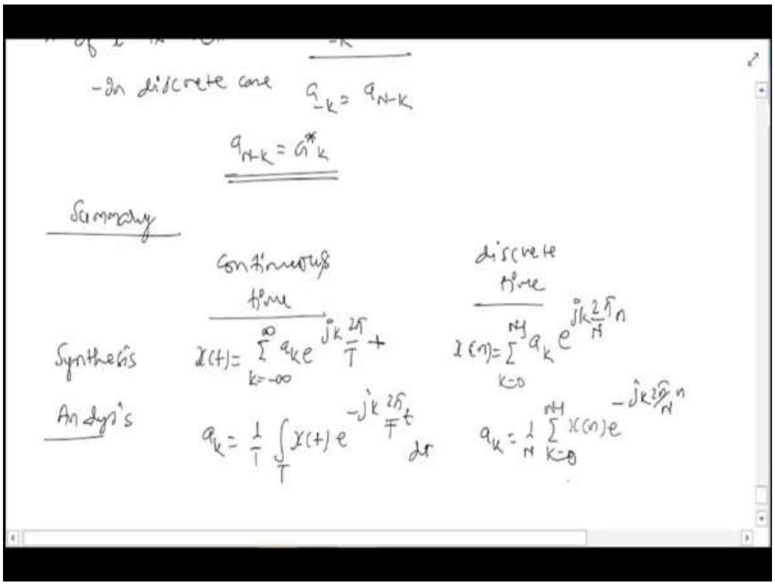

Here is the summary, we have continuous-time and the discrete-time. We saw synthesis equations and analysis equations; very quickly what these equations are

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

In case of discrete time,

$$x(n) = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{N}n}$$

As far as analysis equations goes,

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\frac{2\pi}{T}t} dt$$

And in discrete case,

$$a_k = \frac{1}{N} \sum_{k=0}^{N-1} x(n) e^{-jk\frac{2\pi}{N}n}$$

We see a symmetry in synthesis and analysis equations in both cases namely, the signs of the exponents in complex exponential they reverse. In case of synthesis equations, we have positive exponents. Whereas in case of analysis equations, they become negative exponents. This brings us to the end of this chapter. Thank you.