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Online Retailer's Contingent Free-Shipping Decisions under Large-Scale Promotions Considering Delayed Delivery

Xiaxia Ma [1], Wenliang Bian [1,*], Xiqing Yang [1], Shengnan Niu [2], Yongming Cai [3], Jie Guan [4] and Wenbin Wang [4]

1 School of Economics and Management, Beijing Jiaotong University, Beijing 100044, China
2 Beijing Sankuai Online Technology Co., Ltd., Beijing 100083, China
3 School of Business, University of Jinan, Jinan 250002, China
4 School of Economics and Management, China University of Mining and Technology, Xuzhou 221116, China
* Correspondence: wlbian@bjtu.edu.cn

Abstract: Large-scale promotions lead to a huge number of orders, and the quantity of deliveries grows sharply, which puts considerable strain on cities' logistics and imposes high related shipping costs. To alleviate these consequences, in this paper we provide a new contingent free shipping policy with delayed delivery (DD-CFS) for an online retailer during large-scale promotions and study its CFS threshold decisions, considering consumers' different sensitivities to delivery time delays.We start by analyzing a consumer's purchasing decision based on consumer utility theory. Next, we establish a mathematical model to help the online retailer find the optimal CFS threshold to maximize its expected profit. Finally, we analyze the benefit of delayed delivery to the online retailer and conduct a sensitivity analysis to examine the impacts of important parameters on the online retailer's CFS threshold decisions, profit, and the value of the delayed delivery. We find that the DD-CFS policy can lead to more profits during the large-scale promotions period compared with the traditional CFS policy. As the delayed delivery time and the consumer's negative attitude towards delayed delivery time increase, the online retailer should reduce the low CFS threshold value. On the other hand, as the shipping fee and the consumer's negative attitude towards the shipping fee increase, the online retailer should raise the high and low CFS threshold values.

Keywords: contingent free shipping; delay delivery; large-scale promotions; consumer utility

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1. Introduction

In recent years, online shopping has become increasingly popular with the development of e-commerce, and the competition in online markets is becoming increasingly fierce. In particular, due to the impact of uncertain factors such as the outbreak of COVID-19 and partial supply interruptions, more and more offline retailing demand has been transferred to online shops. To successfully compete, online retailers frequently implement promotions to increase sales and expand their markets. In reality, many e-commerce platforms regularly carry out large-scale promotions every year, such as the US's Cyber Monday and China's Double 11 Day, resulting in a short-term surge in order quantities. According to a CNN Business report, sales on the US's Cyber Monday reached 10.7 billion dollars in 2021 [1], whereas the sales of China's e-platform Tmall.com reached 84.5 billion dollars during China's Double 11 Day in 2021, representing an increase of 8.5% over the same period last year [2]. Therefore, despite ongoing global sourcing and supply chain disruptions caused by the COVID-19 pandemic, consumers' appetites for shopping remain strong, and online shopping demands still surge in a short term under large-scale promotions.

Large-scale promotions lead to a huge number of orders, and the quantity of deliveries grows sharply. According to the statistics of the State Post Bureau of China, during China's Double 11 Day in 2021 (1 November to 16 November), the express delivery business volume reached 6.8 billion, and the average daily delivery volume reached 400 million. Although the sales of online retailers are relatively stable on normal days, their logistics capabilities cannot match the surge in express delivery business volume, thus causing many problems such as the loss of goods, delivery errors, and delivery delays. To alleviate these problems, online retailers usually temporarily expand their production capacity in terms of logistics and distribution and increase their investment in manpower and equipment, which is bound to increase the related shipping costs significantly. On the other hand, online retailers often use demand postponement strategies to shift consumer demand later, so that the modified demand can be better matched by logistics capabilities. During large-scale online promotions, many consumers stockpile goods such as household items and staple foods when sales promotions take place to offset possible price increases in the future [3]. Thus, consumers have different sensitivities to the delivery times of orders. Taking another Chinese e-commerce platform, JD.com, as an example, they offer online coupons for consumers to induce them to choose delayed delivery during the large-scale promotions period.

In practice, many e-commerce platforms adopt the contingent free shipping (CFS) policy. Under the CFS policy, the online retailer pays the shipping fees only for those orders equal to or larger than a predefined threshold. For example, JD.com offers consumers free shipping for every order over CNY99. For each order under CNY 99, consumers are charged CNY 6 for the shipping fee. The CFS policy has been the dominant practice for online retailers, designed to maximize profit by increasing order size to compensate for shipping costs [4]. The CFS threshold is an important factor that influences consumers' online purchase decisions. If the CFS threshold is high, it deters consumers from placing online orders. Otherwise, a low CFS threshold imposes large operational and shipping costs on the online retailer. Thus, when an online retailer adopts the CFS policy, it should determine the optimal CFS threshold to maximize the expected profit. Most researchers have studied CFS threshold decisions based on consumers' different sensitivities to the shipping fee. However, few studies have considered consumers' differing sensitivities to delivery time delays in order to determine the optimal CFS threshold value. Thus, a critical question for the CFS policy remains: what CFS threshold should be set by an online retailer when demand surges in a short time during large-scale promotions?

In this paper, we present a new CFS policy with delayed delivery (DD-CFS) for online retailers during large-scale promotions. Two CFS threshold values are set in the DD-CFS policy, a high and a low CFS threshold value. Online retailers use a low CFS threshold to induce some consumers to choose delayed delivery to earn more time for themselves to process orders and relieve delivery pressures. We divide consumers into two types based on this study, namely, bargain hunters at the expense of late delivery and consumers who require on-time delivery. The former care more about the shipping fee instead of the delivery time, and we refer to the former as shipping-fee-sensitive consumers. Consumers with stricter time requirements will not opt for delayed delivery. For these two types of consumers, if their orders are equal to or larger than the high CFS threshold value, the online retailer pays the shipping fee, and their orders will be delivered normally. Otherwise, when consumers' orders are less than the high CFS threshold value, the consumers pay the shipping fee, and their orders also will be delivered normally. For the former type of consumer, although their orders may be less than the high CFS threshold value, they can choose the delayed delivery to get the free shipping service if their orders are equal to or larger than the low CFS threshold value.

In this study, we examined the online retailer's CFS threshold decisions under large-scale promotions considering delayed delivery. More specifically, we set out to present a basic model without considering delayed delivery as a benchmark. Next, we established a developed model with delayed delivery. We aimed to find the optimal high and low CFS threshold values that would maximize an online retailer's expected profit during a large-scale promotion period. Finally, we analyzed the benefit of delayed delivery to the online retailer and performed sensitivity analysis to assess the impact of important parameters on the value of delayed delivery through numerical experiments. Furthermore, when building the models, we first analyzed the consumer's optimal purchasing decisions.

Then, we established the online retailer's expected profit function using our analytical results relating to consumers' purchase decisions.

The rest of the paper is organized as follows. In Section 2, we review the primary relevant literature. In Section 3, we establish the basic model without considering delayed delivery and the developed model with delayed delivery, respectively. Numerical studies are conducted in Section 4 to examine our developed model's performance. Finally, in Section 5 we conclude this study and propose directions for future research.

2. Literature Review

Our research was based on three main streams of literature: the CFS policy, the demand postponement strategy, and the firm's operational decisions during price promotions.

2.1. The CFS Policy

The existing literature on the CFS policy can be divided into two perspectives: empirical analyses of the impact of CFS policies on consumers' purchase behavior and mathematical models of CFS policy. In terms of empirical analysis, Lewis et al. [5] demonstrated that the CFS policy is the most effective type of free shipping policy for generating larger order amounts. Lewis [6] showed that the CFS policy was the most effective policy in increasing the revenues of online retailers. Koukova et al. [7] investigated consumers' responses to a threshold-based free shipping policy and a flat rate shipping policy. Huang et al. [8] examined the responses of consumers to two economically equivalent but different CFS policies based on quantity (free shipping for orders with four or more items) and value (free shipping for orders over one hundred dollars). The retailer's decisions regarding the CFS threshold relative to the retailer's self-interest were analyzed. Huang et al. [9] examined the effect of the CFS policy on online shoppers' willingness to pay for shipping.

Although it is important for online retailers to implement the CFS policy, its effect largely depends on the design of the CFS threshold. In terms of mathematical models of CFS policies, Jiang et al. [10] optimized an online retailer's shipping-fee charges for single- and multiple-product transactions. Zhou et al. [11] examined the impact of free shipping quantity on the optimal policy of a retailer using a stochastic inventory system. Kwon et al. [12] extended the base model presented by Zhou et al. [11] with a free shipping policy to further study the optimal ordering strategy, and they analyzed the impacts of the minimum free shipping quantity and the shipping fee on the performance of the extended model. In addition, some studies have focused on the optimal CFS threshold value. Many studies have focused on the use of the CFS policy in B2B transactions. For example, Leng and Parlar [13] presented a game-theoretic analysis of the free shipping problem between a seller and a buyer in a B2B transaction, in which the seller decides the free shipping threshold level and the buyer decides the purchase amount. Similarly, Hua et al. [14] studied the retailer's replenishment problem with a free shipping policy in the B2B context, in which the retailer decides the optimal order lot size and the optimal price given the threshold level of the free shipping quantity. Many studies have focused on the use of the CFS policy in a B2C transaction. Leng and Becerril-Arreola [15] examined the impact of an online retailer's joint pricing and CFS free-shipping decisions on the purchase behavior of consumers and computed the optimal price and CFS cutoff level for the retailer. Boone and Ganeshan [16] investigated how to structure CFS strategies from the online retailer's perspective, and their model presented the optimal CFS threshold value and order quantity that would maximize profits. Becerril-Arreola et al. [17] considered a two-stage decision problem in which the online retailer first makes optimal decisions on the profit margin and the CFS threshold, and then determines the optimal inventory level that maximizes the expected profit for a promotion period. Song et al. [18] explored optimizing decisions about the CFS threshold by coordinating the delivery operator and the online retailer.

The above papers investigated the effects of CFS policy on consumers' purchase behavior and focused on determining the CFS threshold level, with relatively few studies considering consumers' different sensitivities to delayed delivery times in their models.

Besides, the CFS threshold is set to inspire consumers to buy more products to qualify for free shipping.In our study, however, we consider two types of consumers—so-called on-time delivery required consumers and shipping-fee-sensitive consumers—with different sensitivities to delayed delivery times.Moreover, we develop a new CFS policy with delayed delivery for online retailers. We set high and low CFS threshold values as part of this new CFS policy. The low CFS threshold is set to induce some consumers to choose a delayed delivery to qualify for free shipping.

2.2. Demand Postponement Strategy

Our work is also related to the literature on demand postponement strategies. Demand postponement was first proposed by Iyer and Wu [19]. They analyzed demand postponement as a strategy to handle potential demand surges. Customers whose demands are postponed can obtain a reimbursement per unit. They considered a two-stage capacity planning problem under demand postponement conditions. However, this problem was modeled in B2B transactions in their paper, in which customers behaved passively and were forced to accept demand postponement decisions made by firms. Real customers are heterogeneous in their sensitivity to price and order fulfillment time. Thus, in contrast with this approach, several studies have considered more realistic cases in which customers play an active role, i.e., they can select whether to participate in demand postponement or not through their choice of whether to accept a price discount. Wu and Wu [20] considered an opaque selling strategy by a firm that used a price discount to induce demand postponement. In the case of demand postponement, the firm offers a price discount to advance consumers in exchange for the option to fulfill their orders after the spot demand has been satisfied. The price discount enables the firm to create a capacity buffer for the urgent spot demand. Alim and Beullens [21] developed a model to study the value of flexible delivery in a continuous review inventory and distribution system. They only offered the postponement when the inventory level reached a critical value and offered a discount to induce customers to accept flexible delivery. Shin et al. [22] proposed a robust multiperiod inventory model with a new type of buy-one-get-one promotion. Consumers who purchased the promotional product can receive a giveaway and pick it up another day.

Previous studies focused on demand postponement strategies in the retail industry, and consumers in these papers were more sensitive to price and time. Most of them used a price discount to convince consumers to shift demand to a later date. Different from the previous literature,in this study, we considered consumers who were more sensitive to the shipping fee and time and used free shipping to induce consumers to choose demand postponement.

2.3. Firms' Operational Decisions during Price Promotions

Since price promotions have proven to be an effective marketing strategy, firms' operational decisions during price promotions have been an important research topic, which is also related to our study. Papers on this topic have mainly studied discount pricing and joint inventory and promotional pricing decisions. In terms of research on discount pricing, many researchers exploring discount pricing have focused on how to design the optimal discount policy to maximize the firm's profits [23–27], mainly focusing on finding an optimal discount pricing strategy to attract consumers for revenue increases. Price promotions influence inventory decisions by influencing market demand, so many researchers have studied the joint optimization of promotional pricing and inventory management [28–31]. For example, Zhang et al. [29] studied a single-item, finite-horizon, periodic review model in which the demand was influenced by price and promotions, and the objective was to obtain maximum profit through joint optimization decisions related to pricing, promotion, and inventory control. Chen [30] investigated a new production-inventory optimization problem for perishable products with the consideration of pricing and promotions for a single-vendor multi-buyer system. The goal was to maximize the profits of the manufacturer and retailers by obtaining the optimal sales price, promotion effort, and inventory replenishment frequency for the retailers, and the production lot size and raw material purchasing frequency for the manufacturer. Moreover, there have been several publications on price discounts in two-stage supply chains [32–34]. For example, Kurata and Liu [34] studied how a retailer can reasonably decide both the depth and frequency of a price discount promotion under a two-stage supply chain framework, when the retailer wants to maximize the expected revenue and the supplier tries to minimize the expected inventory cost.

The above studies investigated the firm's operational decisions in the context of regular price promotions. In this context, although demand does increase during regular price promotions, it does not surge in a short time. On the other hand, large-scale promotions can lead to tremendous amounts of sales, and demand is characterized by a large volume, resulting in a mismatch between supply and demand. Hence, it is crucial to study firms' operational decisions during large-scale promotions. Shu et al. [3] studied the pure contract procurement strategy of a risk-averse retailer in the presence of both strategic customer behavior and unreliable supply during large-scale promotions. Extending the work of Shu et al. [3], Wu et al. [35] studied the optimal portfolio procurement strategy that incorporated a long-term contract and a spot market for an e-retailer in the context of a large-scale promotion. In addition, consumers change their purchasing behaviors significantly during large-scale promotions. Zeng et al. [36], Xu et al. [37], and Xu et al. [38] explored the characteristics of consumers' purchasing behaviors, the factors that affected customer behaviors, and the determinants of consumers' behaviors related to shopping platform usage during the online shopping carnival in China, respectively.

Most of the above studies have investigated firms' operational decisions during regular price promotions, and the literature on firms' operational decisions in the context of large-scale promotions is limited. Moreover, the existing literature has mainly solved the problem of supply and demand mismatches during large-scale promotions from the supply side. In contrast with the previous literature, in this paper, we consider solving this problem from the demand side and use free shipping to induce consumers to shift demand to a later date, so as to better match the demand and supply during large-scale promotion periods.

3. Model

3.1. Notations and Assumptions

The following notations and assumptions are used throughout this paper.

Notations:

For convenience, a list of notations used in this paper is provided in Table 1.

Assumptions:

(1) We consider a single type of goods that have the same information. Thus, we assume that a single type of goods is discussed in this paper.

(2) Each order has a similar shipping cost, and the shipping fee is related to the quantity of the delivered orders. Moreover, the number of orders delivered equals the number of consumer orders placed on the retailer's website. Order splitting and merging are not considered.

(3) The demand function is assumed to be linear with the CFS threshold value and shipping fee, according to the work of Song et al. [18]. The demand function is $Q_1 = a - \delta V_h - \theta C$ in the basic model. Under the DD-CFS policy, there are high and low CFS threshold values, and the demand is affected by these two CFS threshold values. The low CFS threshold can attract some consumers to buy online, thereby increasing market demand. We use the average of the high and low CFS threshold values to calculate the market demand, and similar assumptions have been made in the literature, such as the work of Yu et al. [39]. Therefore, we assume that the demand function is $Q_2 = a - \dfrac{\delta(V_h + V_l)}{2} - \theta C$ in the developed model .

(4) To make online retailers profitable, we assume that the average expected benefit from the unit consumer is larger than the shipping fee of the unit order, that is, $\dfrac{mb}{2} > C$.

Table 1. List of notations.

Notations	
m	The online retailer's profit margin
C	The shipping fee of the unit order
T	The length of the delivery delay time
a	Potential market size
θ	Consumer sensitivity to the shipping fee
δ	Consumer sensitivity to CFS threshold
Q_1	The demand for consumer orders in the basic model
Q_2	The demand for consumer orders in the developed model
x_0	The consumer's planned purchase amount (in dollars), which follows the uniform distribution $U[0, b]$
v	Consumer's marginal utility when the real purchase amount is less than the planned purchase amount, $v > 1$
d	Consumer's marginal utility when the real purchase amount is not less than the planned purchase amount, $0 < d < 1$
β	Indicator of consumers' negative attitude toward the shipping fee, $\beta > 0$
η	Indicator of consumers' negative attitude toward the delayed delivery time, which follows the uniform distribution $U[0, \omega]$
k	The discount on the shipping fee for the delayed delivery
R	The total shipping fees
E	The total expected revenues
π	The online retailer's total expected profit
Subscript	
n	The shipping-fee-sensitive consumer
s	The on-time delivery required consumer
Decision variables	
x	Consumer's real purchase amount (in dollars)
V_h	The high CFS threshold value
V_l	The low CFS threshold value

3.2. The Basic Model

In this section, the consumer's delivery time sensitivities are not considered. The online retailer offers a traditional CFS policy to the consumers by setting a high CFS threshold value. If the consumer's real purchase amount is equal to or larger than the high CFS threshold, the online retailer bears the shipping fee. Otherwise, the consumer must pay the shipping fee. Moreover, delayed delivery is not considered, and the orders will be delivered normally.

3.2.1. Consumer's Purchasing Decision

The CFS policy affects consumers' purchasing behavior. In this paper, a consumer has a planned purchase amount (in dollars) before purchasing, which is called the planned purchase amount, x_0. The consumer's utility function is assumed as follows (Khouja et al. [40]):

$$U(x) = \begin{cases} vx & 0 \le x \le x_0 \\ vx_0 + d(x - x_0) & x > x_0 \end{cases} \tag{1}$$

where $v > 1$ and $0 < d < 1$.

The disutility of the consumer buying goods comes from the consumer's payment for the goods and the shipping fee for delivery. Therefore, the consumer's net utility function when ignoring the shipping fee is $G(x) = U(x) - x$, that is,

$$G(x) = \begin{cases} (v-1)x & 0 \le x \le x_0 \\ (v-1)x_0 - (1-d)(x - x_0) & x > x_0 \end{cases} \tag{2}$$

Note that $v-1$ is the marginal net utility in the case that the real purchase amount is less than or equal to the planned purchase amount, and $-(1-d)$ is the marginal net utility in the case that the real purchase amount is larger than the planned purchase amount. Since $v>1$ and $0<d<1$, there exists $v-1>0$ and $-(1-d)<0$. This implies that the consumer benefits from continuous purchasing if the real purchase amount is less than the planned purchase amount. Otherwise, the consumer's net utility will decrease if the real purchase amount exceeds the planned purchase amount. This explains why most consumers are reluctant to continuously purchase beyond their planned purchase amount in reality. For convenience of calculation, we assume that $v-1=1-d$ in this paper.

When considering the shipping fee, consumers' net utility will change because they must pay a shipping fee. This change may lead to changes in consumers' purchasing decisions. We assume that the disutility of paying the shipping fee is $-\beta C$. The larger the value of β, the greater the negative utility for the consumer paying the shipping fee, and consumers are therefore more reluctant to pay the shipping fee.

The consumer's purchasing decision is analyzed in the following two cases, as shown in Figure 1.

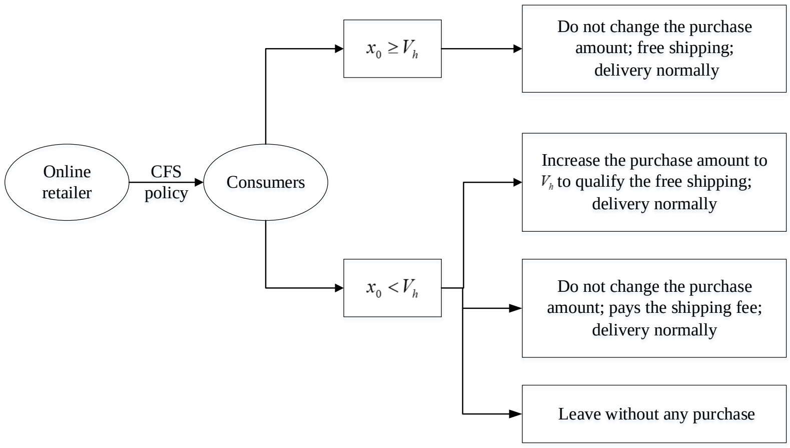

Figure 1. The consumer's purchasing decision under the traditional CFS policy.

Case 1: $x_0 \geq V_h$

When $x_0 \geq V_h$, free shipping occurs, the order will be delivered normally, and the consumer does not need to change the optimal purchase amount, $x^* = x_0$.

Case 2: $x_0 < V_h$

When $x_0 < V_h$, the consumer has three purchasing options. First, the consumer must decide to increase the purchase amount to V_h to qualify for free shipping, $x^* = V_h$. The consumer's net utility is G_1=$(v-1)x_0-(1-d)(V_h-x_0)$. Second, the consumer keeps the purchase amount and pays the shipping fee, $x^* = x_0$. When ignoring the shipping fee, the consumer's net utility is $(v-1)x_0$. In practice, consumers' net utility is affected by the disutility of paying the shipping fee, and the disutility of paying the shipping fee is $-\beta C$; thus, the consumer's net utility is G_2=$(v-1)x_0-\beta C$. Third, the consumer can leave without making any purchase, $x^*=0$. The consumer's net utility is zero.

The consumer determines the real purchase amount by comparing G_1 to G_2. Figure 2 presents the consumer's net utility when $x_0 < V_h$. As we can see, when $0 \leq x_0 \leq \frac{\beta C}{v-1}$, the consumer's net utility is zero and the optimal purchase amount is $x^*=0$. If $G_2 > G_1$ and $G_2 > 0$, that is, $(v-1)x_0-\beta C > (v-1)x_0-(1-d)(V_h-x_0)$ and $(v-1)x_0-\beta C > 0$, then $\frac{\beta C}{v-1} < x_0 < V_h - \frac{\beta C}{1-d}$, the consumer's optimal purchase amount is the original decision to purchase the dollar value of the goodsand the optimal decision is $x^* = x_0$. If $G_1 \geq G_2$ and $G_1 > 0$, that is, $(v-1)x_0-(1-d)(V_h-x_0) \geq (v-1)x_0-\beta C$ and $(v-1)x_0-(1-d)(V_h-x_0) > 0$, then $V_h - \frac{\beta C}{1-d} \leq x_0 < V_h$, the consumer would increase the purchase amount to V_h, and the optimal decision is $x^* = V_h$.

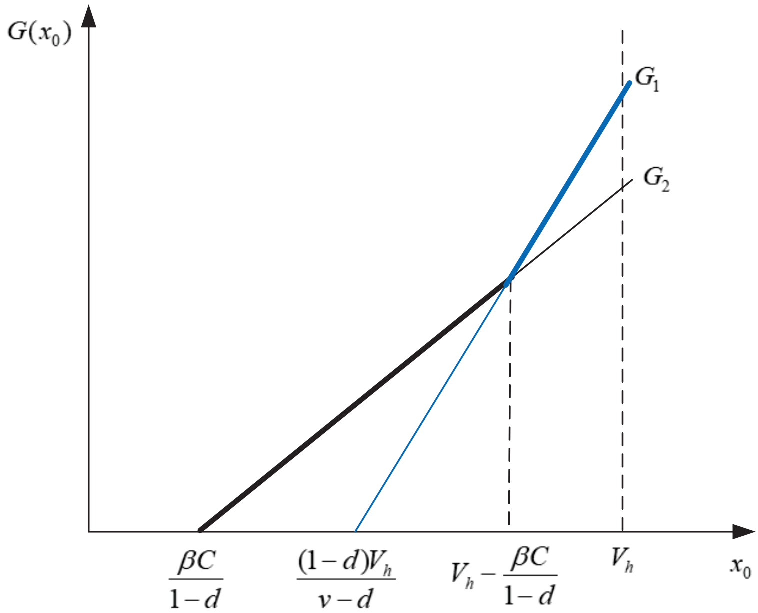

Figure 2. The consumer's net utility when $x_0 < V_h$.

Based on the above analysis, we can obtain Theorem 1.

Theorem 1. *In the single CFS threshold situation, the optimal purchasing decision (x^*) of the consumer with a particular x_0 is*

$$x^* = \begin{cases} 0 & 0 \le x_0 \le \frac{\beta C}{v-1} \\ x_0 & \frac{\beta C}{v-1} < x_0 < V_h - \frac{\beta C}{1-d} \\ V_h & V_h - \frac{\beta C}{1-d} \le x_0 < V_h \\ x_0 & V_h \le x_0 \le b \end{cases}$$

.

Theorem 1 implies that a consumer's purchasing decision depends on the high CFS threshold value and the planned purchase amount. Given a specific planned purchase amount x_0, if the high CFS threshold value is too large, the negative utility brought about by increasing the purchase amount is larger than the positive utility brought about by the free shipping, and the consumer keeps the planned purchase amount x_0. If the high CFS threshold value is moderate, x_0 is close to V_h, and the consumer will benefit more by increasing the purchase amount to qualify for free shipping. If the high CFS threshold value is small, the planned purchase amount x_0 qualifies for free shipping, and the consumer would not consider changing the planned purchase amount to receive the free shipping service.

3.2.2. Online Retailer's Decision

According to the consumer's purchasing decision, the online retailer determines the optimal high CFS threshold value to maximize its expected profit. The expected profit is comprised of the following two components:

(1) The total sales revenues (E_1):

$$E_1 = Q_1 m \left(\int_{\frac{\beta C}{v-1}}^{V_h - \frac{\beta C}{1-d}} x_0 f(x_0) dx_0 + \int_{V_h - \frac{\beta C}{1-d}}^{V_h} V_h f(x_0) dx_0 + \int_{V_h}^{b} x_0 f(x_0) dx_0 \right) = \frac{(a - \delta V_h - \theta C) m b}{2} \tag{3}$$

(2) The total shipping fees (R_1):

$$R_1 = Q_1 C \int_{V_h - \beta C/(1-d)}^{b} f(x_0) dx_0 = \frac{(a - \delta V_h - \theta C) C}{b} \left(b - V_h + \frac{\beta C}{1-d} \right) \tag{4}$$

Therefore, the expected profit (π_1) is given by

$$\pi_1 = E_1 - R_1 = \frac{(a - \delta V_h - \theta C)mb}{2} - \frac{(a - \delta V_h - \theta C)C}{b}\left(b - V_h + \frac{\beta C}{1-d}\right) \tag{5}$$

Taking the first-order derivative and the second-order derivative of π_1 with respect to V_h, that is,

$$\frac{d\pi_1}{dV_h} = -\frac{\delta mb}{2} - \frac{2\delta C V_h}{b} + \delta C + \frac{\delta \beta C^2}{b(1-d)} + \frac{aC}{b} + \frac{\theta C^2}{b} \tag{6}$$

$$\frac{d^2\pi_1}{d{V_h}^2} = -\frac{2\delta C}{b} \tag{7}$$

Noting $\frac{d^2\pi_1}{d{V_h}^2} < 0$, there may exist V_h^* to maximize the expected profit. The optimal high CFS threshold value (V_h^*) may be solved through $\frac{d\pi_1}{dV_h} = 0$. Based on the above analysis, we can obtain Theorem 2.

Theorem 2. *The optimal high CFS threshold value of the online retailer is*

$$V_h^* = \frac{1}{2}\left(b + \frac{\beta C}{1-d} + \frac{a - \theta C}{\delta} - \frac{mb^2}{2C}\right) \tag{8}$$

From Theorem 2, we have the following corollary.

Corollary 1. *The optimal high CFS threshold value V_h^* increases with the increase of β.*

Since $\frac{dV_h^*}{d\beta} = \frac{C}{2(1-d)} > 0$, as the value of β increases, V_h^* will increase. This reveals that when the consumer's negative attitude towards shipping fees is significant, the online retailer should increase the high CFS threshold value.

3.3. The Developed Model with Delayed Delivery

In this section, the online retailer offers consumers the DD-CFS policy. We set two CFS threshold values for the DD-CFS policy, a high and a low CFS threshold value. Since the traditional CFS policy with a single high threshold has been the dominant practice for online retailers during the ordinary sales period, the single high CFS threshold is a long-term and stable decision that does not change frequently. However, the setting of a low CFS threshold is a short-term decision for online retailers during a large-scale promotion period. When the large-scale promotion ends, the online retailer still adopts the traditional CFS policy with a single high threshold. Therefore, in this section we establish a developed model to determine the optimal low CFS threshold value based on the optimal high CFS threshold value in Theorem 1.

3.3.1. Consumer's Purchasing Decision

We consider two types of consumers, on-time delivery required consumer and the shipping-fee-sensitive consumer, with different delay delivery time sensitivities. Assume that the disutility of paying for the shipping fee is $-\beta C$, and the disutility of the delayed delivery is $-\eta T$. If $\eta T < \beta C$, the consumer who is more shipping-fee-sensitive but less delivery-time-sensitive is willing to delay their delivery; this type of consumer is defined as the shipping-fee-sensitive consumer ($\eta < \frac{\beta C}{T}$). Otherwise, if $\eta T > \beta C$, the consumer who is more delivery-time-sensitive but less shipping-fee-sensitive will not choose the delayed delivery; this type of consumer is defined as the on-time delivery required consumer ($\eta > \frac{\beta C}{T}$).

The on-time delivery required consumers have a stricter time requirement and will not opt for delayed delivery. Thus, their purchasing behaviors are not affected by the low

CFS threshold. The on-time delivery required consumer's optimal purchase decision is the same as the consumer's optimal purchase decision in the basic model.

Shipping-fee-sensitive consumers are more shipping-fee-sensitive but less delivery-time-sensitive. Their purchasing behaviors are affected by the high and low CFS threshold values. The shipping-fee-sensitive consumer's purchasing decision is analyzed in the following three cases, as shown in Figure 3.

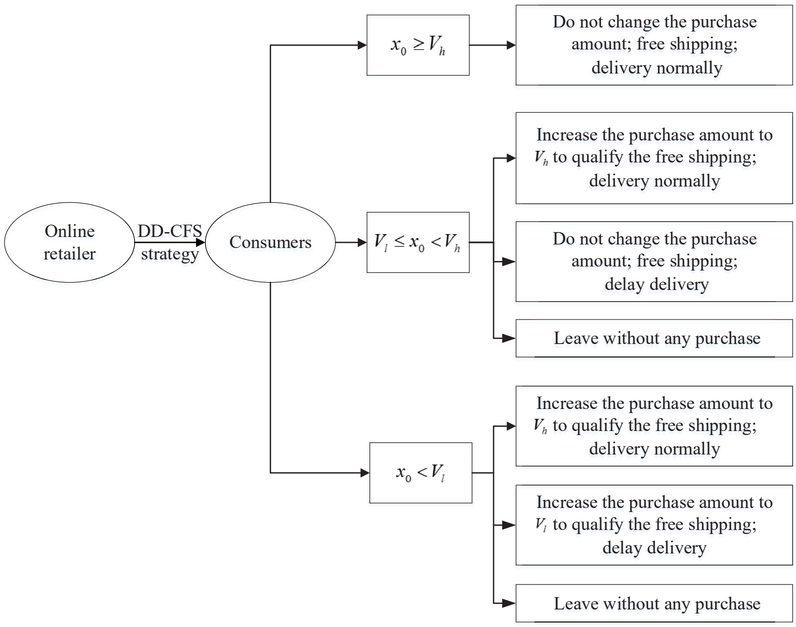

Figure 3. The shipping-fee-sensitive consumer's purchasing decision under the DD-CFS policy.

Case 1: $x_0 \geq V_h$

When $x_0 \geq V_h$, free shipping occurs, the order will be delivered normally, and the optimal purchasing decision is $x_n^* = x_0$.

Case 2: $V_l \leq x_0 < V_h$

When $V_l \leq x_0 < V_h$, the shipping-fee-sensitive consumer has three purchasing options. First, the shipping-fee-sensitive consumer must decide to increase the purchase amount to V_h to benefit from the free shipping, $x_n^* = V_h$. The order will be delivered normally, and the shipping-fee-sensitive consumer's net utility is G_{n1}=$(v-1)x_0 - (1-d)(V_h - x_0)$. Second, the shipping-fee-sensitive consumer keeps the purchase amount and chooses the delayed delivery to qualify for free shipping, $x_n^* = x_0$. The shipping-fee-sensitive consumer's net utility is G_{n2} = $(v-1)x_0 - \eta T$. Third, the shipping-fee-sensitive consumer leaves without any purchase, $x_n^* = 0$. The shipping-fee-sensitive consumer's net utility is zero.

The shipping-fee-sensitive consumer determines the real purchase amount by comparing G_{n1} to G_{n2}. Figure 4 shows the shipping-fee-sensitive consumer's net utility when $V_l \leq x_0 < V_h$. As shown in Figure 4, to ensure that some shipping-fee-sensitive consumers will choose the delayed delivery, the low CFS threshold should be less than $V_h - \frac{\eta T}{1-d}$, that is, $V_l < V_h - \frac{\eta T}{1-d}$. At this point, it is necessary to compare V_l and $\frac{\eta T}{v-1}$ to analyze the shipping-fee-sensitive consumer's purchasing decision.

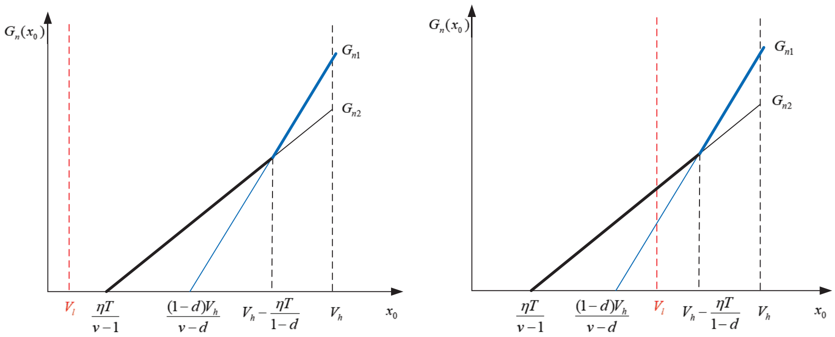

Figure 4. The shipping-fee-sensitive consumer's net utility when $V_l \leq x_0 < V_h$.

Case 3: $x_0 < V_l$

When $x_0 < V_l$, the shipping-fee-sensitive consumer also has three purchasing options. First, the shipping-fee-sensitive consumer must decide to increase the purchase amount to V_h to benefit from the free shipping, $x_n^* = V_h$. The order will be delivered normally. The shipping-fee-sensitive consumer's net utility is $G_{n3} = (v-1)x_0 - (1-d)(V_h - x_0)$. Second, the shipping-fee-sensitive consumer must decide to increase the purchase amount to V_l and choose the delayed delivery to qualify for free shipping, $x_n^* = V_l$. The shipping-fee-sensitive consumer's net utility is $G_{n4} = (v-1)x_0 - (1-d)(V_l - x_0) - \eta T$. Thirdly, the shipping-fee-sensitive consumer will leave without any purchase, $x_n^* = 0$. The shipping-fee-sensitive consumer's net utility is zero.

The shipping-fee-sensitive consumer determines the real purchase amount by comparing G_{n3} to G_{n4}. Figure 5 shows the shipping-fee-sensitive consumer's net utility when $x_0 < V_l$. As shown in Figure 5, G_{n4} is always larger than G_{n3}. To ensure that some shipping-fee-sensitive consumers will choose the delayed delivery, the online retailer should set $V_l > \frac{(1-d)V_l+\eta T}{v-d}$. Hence, $V_l > \frac{\eta T}{v-1}$.

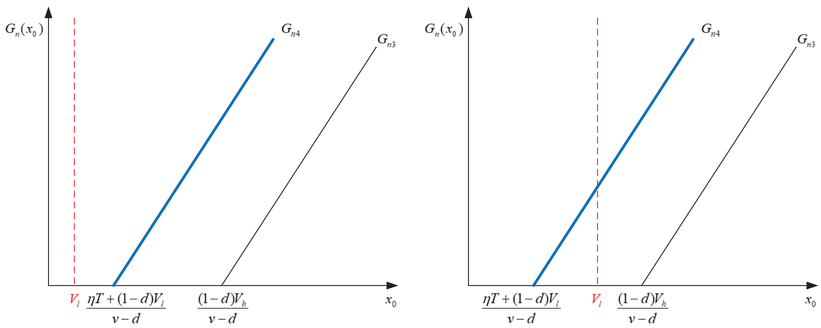

Figure 5. The shipping-fee-sensitive consumer's net utility when $x_0 < V_l$.

Based on the above analysis, we can obtain the shipping-fee-sensitive consumer's optimal purchase decision for Cases 2 and 3 as follows:

For Case 2 ($V_l \leq x_0 < V_h$), the consumer determines the real purchase amount by comparing G_{n1} to G_{n2}. When $V_l \leq x_0 < V_h$, the optimal purchase decision is shown in Figure 4. If $G_{n2} > G_{n1}$ and $G_{n2} > 0$, that is, $(v-1)x_0 - \eta T > (v-1)x_0 - (1-d)(V_h - x_0)$ and $(v-1)x_0 - \eta T > 0$, then $\frac{\eta T}{v-1} < x_0 < V_h - \frac{\eta T}{1-d}$, since $V_l > \frac{\eta T}{v-1}$, so when $x_0 \in \left[V_l, V_h - \frac{\eta T}{1-d}\right)$, the shipping-fee-sensitive consumer's optimal purchase decision is $x_n^* = x_0$, and the delivery of the order will be delayed. If $G_{n1} \geq G_{n2}$ and $G_{n1} > 0$, that is, $(v-1)x_0 - (1-d)(V_h - x_0) \geq (v-1)x_0 - \eta T$ and $(v-1)x_0 - (1-d)(V_h - x_0) > 0$, then $V_h - \frac{\eta T}{1-d} \leq x_0 < V_h$. Thus, when $x_0 \in \left[V_h - \frac{\eta T}{1-d}, V_h\right)$, the shipping-fee-sensitive consumer's optimal purchase decision is $x_n^* = V_h$, and the order will be delivered normally.

For Case 3, when $x_0 < V_l$, the optimal purchase decision is shown in Figure 5. If $x_0 \in \left[0, \frac{\eta T+(1-d)V_l}{v-d}\right]$, the shipping-fee-sensitive consumer gives up purchasing, $x_n^* = 0$. If

$x_0 \in \left(\frac{\eta T+(1-d)V_l}{v-d}, V_l\right)$, the shipping-fee-sensitive consumer's optimal purchase decision is $x_n^* = V_l$, and the delivery of the order will be delayed.

In summary, the consumer's optimal purchasing decisions are shown in Figure 6, and Theorem 3 can be obtained.

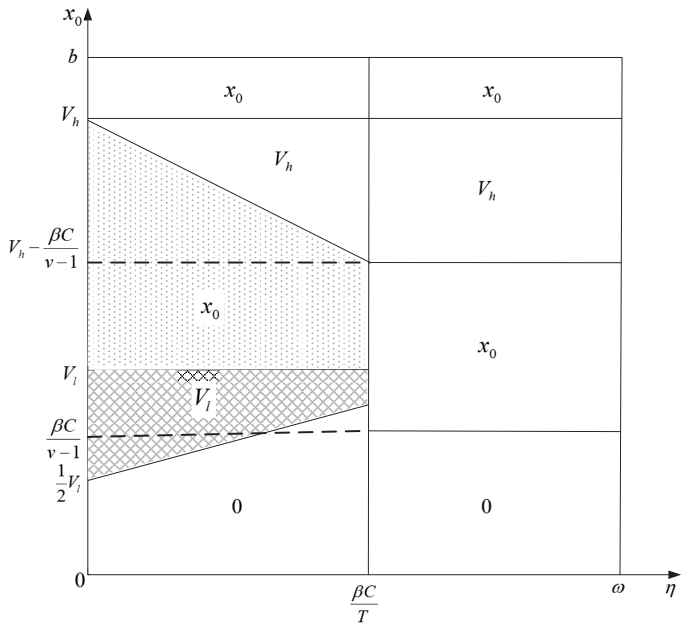

Figure 6. The consumer's optimal purchase decisions.

The shaded part of Figure 6 shows that the low CFS threshold value can successfully induce some shipping-fee-sensitive consumers to choose the delayed delivery to receive the free shipping service. Specifically, when consumers' planned purchase amount is less than V_h and larger than $V_h - \frac{\beta C}{v-1}$, all consumers who require on-time delivery would increase their purchase amount to V_h to qualify for free shipping, whereas the shipping-fee-sensitive consumers can make choices according to their willingness. Some shipping-fee-sensitive consumers would be willing to maintain their purchase amount and choose the delayed delivery to qualify for free shipping, and the rest of the shipping-fee-sensitive consumers would increase their purchase amount to V_h to qualify for free shipping. Although the total real purchase amount of the shipping-fee-sensitive consumers is less than that of the on-time delivery required consumers, the satisfaction of shipping-fee-sensitive consumers will be higher than that of customers who require on-time delivery. When consumers' planned purchase amount is less than $V_h - \frac{\beta C}{v-1}$ and larger than V_l, all shipping-fee-sensitive consumers are also willing to keep their purchase amount and choose the delayed delivery to qualify for free shipping, but all on-time delivery required consumers would pay for the shipping fee. When the consumers' planned purchase amount is less than V_l and larger than $\frac{\beta C}{v-1}$, most shipping-fee-sensitive consumers are likely to increase their purchase amount to V_l and choose the delayed delivery to qualify for free shipping, but all on-time delivery required consumers would maintain their purchase amount and pay the shipping fee. Hence, the total real purchase amount of the shipping-fee-sensitive consumers is larger than that of the consumers who require on-time delivery. When consumers' planned purchase amount is less than $\frac{\beta C}{v-1}$, all on-time delivery required consumers would leave without any purchase, whereas a small fraction of shipping-fee-sensitive consumers are still likely to increase their purchase amount to V_l to receive the free shipping service.

Theorem 3. *Under the DD-CFS policy,*

(i) the on-time delivery required consumer's optimal purchasing decision (x_s^*) is

$$x_s^* = \begin{cases} 0 & \eta \in \left[\frac{\beta C}{T}, \omega\right], x_0 \in \left[0, \frac{\beta C}{v-1}\right] \\ x_0 & \eta \in \left[\frac{\beta C}{T}, \omega\right], x_0 \in \left(\frac{\beta C}{v-1}, V_h - \frac{\beta C}{1-d}\right) \\ V_h & \eta \in \left[\frac{\beta C}{T}, \omega\right], x_0 \in \left[V_h - \frac{\beta C}{1-d}, V_h\right) \\ x_0 & \eta \in \left[\frac{\beta C}{T}, \omega\right], x_0 \in [V_h, b] \end{cases}$$

(ii) the shipping-fee-sensitive consumer's optimal purchasing decision (x_n^*) is

$$x_n^* = \begin{cases} 0 & \eta \in \left[0, \frac{\beta C}{T}\right], x_0 \in \left[0, \frac{\eta T + (1-d)V_l}{v-d}\right] \\ V_l & \eta \in \left[0, \frac{\beta C}{T}\right], x_0 \in \left(\frac{\eta T + (1-d)V_l}{v-d}, V_l\right) \\ x_0 & \eta \in \left[0, \frac{\beta C}{T}\right], x_0 \in \left[V_l, V_h - \frac{\eta T}{1-d}\right) \\ V_h & \eta \in \left[0, \frac{\beta C}{T}\right], x_0 \in \left[V_h - \frac{\eta T}{1-d}, V_h\right) \\ x_0 & \eta \in \left[0, \frac{\beta C}{T}\right], x_0 \in [V_h, b] \end{cases}$$

Theorem 3 suggests that a shipping-fee-sensitive consumer's purchasing decision depends on the low CFS threshold value, the planned purchase amount, and the consumer's negative attitude towards the delayed delivery time. As the low CFS threshold value and the consumer's negative attitude towards the delayed delivery time increase, the shipping-fee-sensitive consumers with small planned purchase amounts are likely to leave without any purchase because they are neither willing to pay the shipping fee nor increase the purchase amount. When the shipping-fee-sensitive consumer's planned purchase amount is close to V_h, if the consumer's negative attitude towards the delayed delivery time is large, he or she is likely to increase their purchase amount to receive the free shipping service. Otherwise, if the consumer's negative attitude towards the delayed delivery time is small, shipping-fee-sensitive consumers are likely to maintain the purchase amount and choose delayed delivery to qualify for free shipping, despite the fact that their planned purchase amounts are close to V_h. When the shipping-fee-sensitive consumer's planned purchase amount is moderate ($V_l \le x_0 < V_h - \frac{\beta C}{1-d}$), he or she would no doubt maintain the purchase amount and choose delayed delivery to qualify for free shipping, regardless of the range of the value of η.

3.3.2. Online Retailer's Decision

Based on the optimal high CFS threshold value in Theorem 1, the online retailer determines the optimal low CFS threshold value to maximize the expected profit. The expected profit is also composed of the following two components:

(1) The total sales revenues (E_2):

$$\begin{aligned} E_2 = Q_2 m \Bigg[& \int_{\frac{\beta C}{T}}^{\omega} \int_{\frac{\beta C}{v-1}}^{V_h^* - \frac{\beta C}{1-d}} x_0 f(x_0) g(\eta) dx_0 d\eta + \int_{\frac{\beta C}{T}}^{\omega} \int_{V_h^* - \frac{\beta C}{1-d}}^{V_h^*} V_h^* f(x_0) g(\eta) dx_0 d\eta + \int_{\frac{\beta C}{T}}^{\omega} \int_{V_h^*}^{b} x_0 f(x_0) g(\eta) dx_0 d\eta + \\ & \int_0^{\frac{\beta C}{T}} \int_{\frac{\eta T + (1-d)V_l}{v-d}}^{V_l} V_l f(x_0) g(\eta) dx_0 d\eta + \int_0^{\frac{\beta C}{T}} \int_{V_l}^{V_h^* - \frac{\eta T}{1-d}} x_0 f(x_0) g(\eta) dx_0 d\eta + \int_0^{\frac{\beta C}{T}} \int_{V_h^*}^{b} x_0 f(x_0) g(\eta) dx_0 d\eta \\ & + \int_0^{\frac{\beta C}{T}} \int_{V_h^* - \frac{\eta T}{1-d}}^{V_h^*} V_h^* f(x_0) g(\eta) dx_0 d\eta \Bigg] \\ = & \left[a - \frac{\delta(V_h^* + V_l)}{2} - \theta C\right] \left\{ \frac{mb}{2}\left(1 - \frac{\beta C}{T\omega}\right) + \frac{m\beta C}{2b\omega T}\left[b^2 + \frac{\beta^2 C^2}{3(1-d)^2} - \frac{\beta C V_l}{2(1-d)}\right] \right\} \end{aligned} \quad (9)$$

(2) The total shipping fees (R_2):

$$R_2 = Q_2C\left[\int_{\frac{\beta C}{T}}^{\omega}\int_{V_h^*-\frac{\beta C}{1-d}}^{b} f(x_0)g(\eta)dx_0d\eta + \int_0^{\frac{\beta C}{T}}\int_{V_h^*-\frac{\eta T}{1-d}}^{b} f(x_0)g(\eta)dx_0d\eta + k\int_0^{\frac{\beta C}{T}}\int_{\frac{\eta T+(1-d)V_l}{v-d}}^{V_h^*-\frac{\eta T}{1-d}} f(x_0)g(\eta)dx_0d\eta\right]$$
$$= \frac{\beta C^2}{b\omega T}\left[a - \frac{\delta(V_h^*+V_l)}{2} - \theta C\right]\left\{b - V_h^* + \frac{\beta C}{2(1-d)} + k\left[V_h^* - \frac{1}{2}V_l - \frac{3\beta C}{4(1-d)}\right]\right\} \tag{10}$$
$$+ \frac{C}{b}\left[a - \frac{\delta(V_h^*+V_l)}{2} - \theta C\right]\left(1 - \frac{\beta C}{T\omega}\right)\left(b - V_h^* + \frac{\beta C}{1-d}\right)$$

Therefore, the expected profit (π_2) is given by

$$\pi_2 = E_2 - R_2$$
$$= \left[a - \frac{\delta(V_h^*+V_l)}{2} - \theta C\right]\left\{\frac{mb}{2}\left(1 - \frac{\beta C}{T\omega}\right) + \frac{m\beta C}{2b\omega T}\left[b^2 + \frac{\beta^2C^2}{3(1-d)^2} - \frac{\beta C V_l}{2(1-d)}\right]\right\}$$
$$- \frac{\beta C^2}{b\omega T}\left\{b - V_h^* + \frac{\beta C}{2(1-d)} + k\left[V_h^* - \frac{1}{2}V_l - \frac{3\beta C}{4(1-d)}\right]\right\}\left[a - \frac{\delta(V_h^*+V_l)}{2} - \theta C\right] \tag{11}$$
$$- \frac{C}{b}\left(1 - \frac{\beta C}{T\omega}\right)\left(b - V_h^* + \frac{\beta C}{1-d}\right)\left[a - \frac{\delta(V_h^*+V_l)}{2} - \theta C\right]$$

Taking the first-order derivative and the second-order derivative of π_2 with respect to V_l, that is,

$$\frac{d\pi_2}{dV_l} = \left[\frac{\delta C}{2b}\left(b - V_h^* + \frac{\beta C}{1-d}\right) - \frac{\delta mb}{4}\right]\left(1 - \frac{\beta C}{T\omega}\right) - \frac{\delta m\beta C}{4b\omega T}\left[b^2 + \frac{\beta^2C^2}{3(1-d)^2}\right]$$
$$+ \frac{\beta C}{2b\omega T}\left(a - \frac{\delta V_h^*}{2} - \theta C\right)\left[kC - \frac{m\beta C}{2(1-d)}\right] - \frac{\delta\beta C V_l}{2b\omega T}\left[kC - \frac{m\beta C}{2(1-d)}\right], \tag{12}$$
$$+ \frac{\delta\beta C^2}{2b\omega T}\left\{b - V_h^* + \frac{\beta C}{2(1-d)} + k\left[V_h^* - \frac{3\beta C}{4(1-d)}\right]\right\}$$

$$\frac{d^2\pi_2}{dV_l^2} = -\frac{\delta\beta C}{2b\omega T}\left[kC - \frac{m\beta C}{2(1-d)}\right]. \tag{13}$$

If $kC - \frac{m\beta C}{2(1-d)} > 0$, $\frac{d^2\pi_2}{dV_l^2} < 0$. Hence, there exists V_l^* to maximize the expected profit. The optimal low CFS threshold value (V_l^*) may be solved through $\frac{d\pi_2}{dV_l} = 0$. Based on the above analysis, Theorem 4 can be obtained.

Theorem 4. *When the discount of the shipping fee for the delayed delivery is large ($k > \frac{m\beta}{2(1-d)}$), the optimal low CFS threshold value of the online retailer is*

$$V_l^* = \frac{\left[2C\left(b - V_h^* + \lambda\right) - mb^2\right](T\omega - \beta C)}{2\beta C\Delta} - \frac{m}{2\Delta}\left(b^2 + \frac{\lambda^2}{3}\right) + \frac{a - \theta C}{\delta} - \frac{V_h^*}{2} + \frac{C}{\Delta}\left[b - V_h^* + \frac{\lambda}{2} + k\left(V_h^* - \frac{3\lambda}{4}\right)\right], \tag{14}$$

where $\lambda = \frac{\beta C}{1-d}$, $\Delta = kC - \frac{m\lambda}{2}$.

From Theorem 4, we have the following corollary.

Corollary 2. *The optimal low CFS threshold value V_l^* decreases with an increase in ω and T.*

From Figure 2, we can observe that $V_h - \lambda > \lambda$, then $V_h > 2\lambda$. Since $\frac{mb}{2} > C$, the first-order derivatives of V_l^* with respect to ω and T are

$$\frac{dV_l^*}{d\omega} = \frac{\left[b(2C - mb) - 2C\left(V_h^* - \lambda\right)\right]T}{2\beta C\Delta} < 0, \tag{15}$$

$$\frac{dV_l^*}{dT} = \frac{\left[b(2C - mb) - 2C\left(V_h^* - \lambda\right)\right]\omega}{2\beta C\Delta} < 0. \tag{16}$$

Therefore, V_l^* will decrease as ω and T increase. This reveals that when the delayed delivery time and the consumer's negative attitude towards the delayed delivery time are large, the online retailer should reduce the low CFS threshold value.

4. Numerical Example

In this part, we first present a numerical example to illustrate the benefit of delayed delivery. Then, we conduct numerical experiments to explore the impacts of four important parameters (T, C, ω, and β) on the online retailer's optimal CFS threshold decisions, profit, and the value of the delayed delivery.

4.1. Numerical Results

The numerical example is based on the following parameter values: $m = 6\%$, $a = 6{,}000{,}000$ orders, C = ¥8/order, $T = 5$ days, $\delta = 47{,}150$, $\theta = 10{,}000$, $v = 1.6$, $d = 0.4$, $k = 0.7$, $\beta = 2$, b = ¥160, $\omega = 10$. The computational results are shown in Table 2.

Table 2. The online retailer's optimal CFS threshold values and performance.

	CFS Threshold	Demand	The Expected Profit
The basic model	$V_h^* = 108.11$	8.2253×10^5	7.1746×10^5
The developed model	$V_l^* = 59.73$	1.9630×10^6	7.8459×10^5

As can be observed from Table 2, compared to the basic model, the market demand for the developed model with the delayed delivery increases by 138.65%, and the online retailer's profit increases by 9.36%. Thus, it is profitable for the online retailer to implement the DD-CFS policy. Figure 7 shows the impact of the potential market size a on the online retailer's CFS threshold values and profit. As we can see, as potential market size a increases, the high CFS threshold will increase, the low CFS threshold will decrease, and the total profits (π_1^* and π_2^*) will increase. This suggests that when the potential market size is very large, the online retailer should set a larger high CFS threshold and a smaller low CFS threshold to earn more profits. Setting a larger high CFS threshold aims to induce consumers to increase their purchase amounts to qualify for free shipping. This result can be observed in an online bookstore in China. The bookstore increased the high CFS threshold value from CNY 39 in 2011 to CNY 49 in 2016 (Song et al. [18]). At the same time, setting a smaller low CFS threshold aims to attract more consumers to choose the delayed delivery, which can reduce order processing expenses. In addition, we can also see from Figure 7 that when the potential market size is not very large, the total profit of the basic model is larger than the total profit of the developed model with the delayed delivery ($\pi_1^* > \pi_2^*$). When the potential market size is enormous, the total profit of the developed model with the delayed delivery is larger than the total profit of the basic model ($\pi_2^* > \pi_1^*$). Moreover, as the potential market size a increases, the increased profit ($\pi_2^* - \pi_1^*$) increases. This illustrates that the online retailer should apply the traditional CFS policy during the ordinary sales period. During the large-scale promotions period, the online retailer should apply the DD-CFS policy with two threshold values.

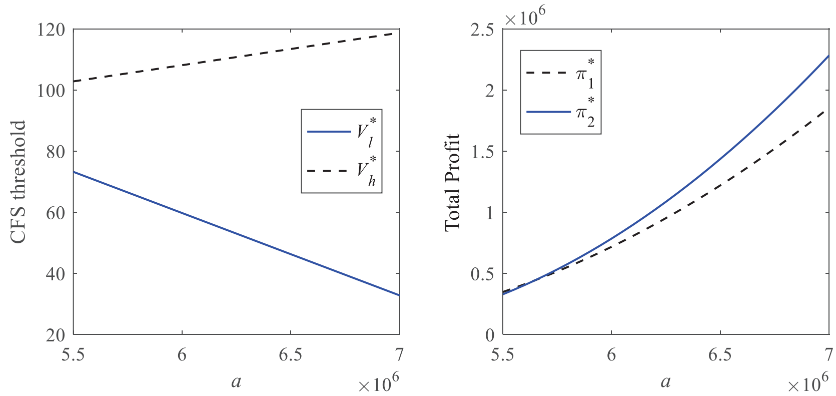

Figure 7. The impact of the potential market size a on the online retailer's CFS threshold values and profit.

During the large-scale promotions period, the order volume surges in a short time, followed by a sharp increase in delivery volume. However, the logistics and distribution capabilities are limited, and it is necessary to temporarily expand the logistics and distribution capacities and increase investment in human resources and equipment, which is bound to increase the related shipping costs greatly. Under the DD-CFS policy with two threshold values, some shipping-fee-sensitive consumers will choose delayed delivery to qualify for free shipping. Thus, some orders can be delayed, and the online retailer can ship orders in off-peak periods, thereby effectively reducing the related shipping costs. Under the traditional CFS policy, however, all orders must be delivered immediately, so the shipping costs will increase significantly. In addition, since the demand is negatively correlated with the CFS threshold value, the DD-CFS policy with two threshold values can take advantage of the low CFS threshold to increase market demand. The market demand under the DD-CFS policy is larger than that under the traditional CFS policy. Therefore, the total profit under the DD-CFS policy is larger than that under the traditional CFS policy.

4.2. Sensitivity Analysis

The developed model with delayed delivery yields a higher profit than the basic model. We refer to the percentage profit increase between the two models as the value of the delayed delivery, and the percentage increase in profits is calculated as

$$\sigma = \frac{(\pi_2^* - \pi_1^*)}{\pi_1^*} \cdot 100\% \tag{17}$$

The higher value of σ, the higher value the online retailer can obtain from the delayed delivery. In this section, we study the impact of important parameters T, C, ω, and β on the online retailer's optimal CFS threshold values, profit, and the value of the delayed delivery, and some notable results can be drawn.

4.2.1. The Impact of T

Result 1. *As the length of the delayed delivery time increases, the online retailer should reduce the low CFS threshold value, and the value of the delayed delivery will increase.*

We begin by examining the effect of the length of the delayed delivery time T on the online retailer's optimal CFS threshold, profit, and the value of the delayed delivery. In this sensitivity analysis, we increase the value of T from 4 to 6.5 in increments of 0.5 and compute the optimal CFS threshold values, profit, and the percentage profit increase for each value of T. We find from Figure 8 that as the length of the delayed delivery time T increases, the low CFS threshold value (V_l^*) will decrease, whereas the profit of the developed model (π_2^*) and the percentage of the increase in profit (σ) will increase. As T

increases, the negative utility brought about by the delayed delivery increases, consumers are more reluctant to choose delayed delivery, and some consumers may abandon their shopping carts. Thus, the online retailer should accordingly reduce the low CFS threshold value to induce more consumers to choose delayed delivery. In addition, since the market demand is negatively correlated with the CFS threshold value, the market demand will increase with the decrease in the low CFS threshold. Hence, profits in the developed model will increase with increases in T. The longer the delayed delivery time, the more value the online retailer can obtain from the delayed delivery.

As shown in Figure 8, we found that when the length of the delayed delivery time T is short, the profit of the developed model is less than the profit of the basic model ($\pi_1^* > \pi_2^*$). When the length of the delayed delivery time T is long, the profit of the developed model is larger than the profit of the basic model ($\pi_2^* > \pi_1^*$). This implies that when the online retailer implements the DD-CFS policy, the length of the delayed delivery time should not be set too short. This is because a delayed delivery time that is too short cannot gain much an increase in processing time for the online retailer. Only an adequately large length of the delayed delivery time can obtain more order processing time, relieving the pressure on logistics and reducing the related shipping fees for the online retailer.

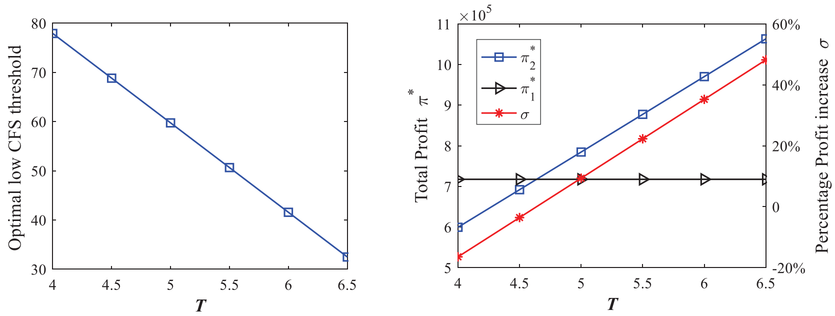

Figure 8. The impact of T on the online retailer's optimal CFS threshold, profit, and the value of the delayed delivery.

4.2.2. The Impact of C

Result 2. *Increasing the shipping fee forces the online retailer to raise the high and low CFS threshold values, and the larger the shipping fee, the greater the value of the delayed delivery.*

We investigated how changing the shipping fee C impacts the online retailer's optimal CFS threshold, profit, and the value of the delayed delivery. For the sensitivity analysis, the value of C was increased from 7.5 to 8.5 in increments of 0.25. As can be seen from Figure 9, as C increases, the high CFS threshold (V_h^*) and low CFS threshold (V_l^*) will increase. As the shipping fee increases, the online retailer should accordingly raise the CFS threshold values to reduce its shipping-related expenses, which may prevent some consumers from buying online. This result can also be observed in Yihaodian, an online supermarket in China. The shipping cost of fresh products is higher than that of ordinary products. China's Yihaodian offered consumers free shipping for every order of ordinary products over CNY 68 and every order of fresh products over CNY 99 in 2016. In addition, with the increase in C, the profit of the basic model (π_1^*) and the profit of the developed model (π_2^*) will decrease. This implies that increasing the shipping fee always inevitably harms the performance of an online retailer which cannot change its decisions to eliminate negative impacts. This result is supported by Leng and Becerril-Arreola's study [15].

As shown in Figure 9, we also found that as C increases, the profit of the developed model decreases more rapidly than that of the basic model. Thus, the percentage of the profit increase (σ) will decrease with an increase in C. However, when the shipping fee is very high, the profit of the basic model is larger than the profit of the developed model ($\pi_1^* > \pi_2^*$). This can be explained by the following reason. The large shipping fee enables the online retailer to have very high shipping expenses, and the total shipping expenses of the developed model are larger than that of the basic model due to the delayed delivery. On the other hand, the order processing costs of the developed model are reduced compared to the basic model. When the shipping fee is very large, the reduced order processing costs are not sufficient to make up for the high shipping expenses, so the profit of the developed model is lower than the profit of the basic model. Therefore, the online retailer should consider the shipping fee when deciding on the CFS strategy. When the shipping fee is very high, the DD-CFS policy is not beneficial for the online retailer. At this point, the online retailer should choose the traditional CFS policy, and set a high CFS threshold to reduce its shipping-related expenses.

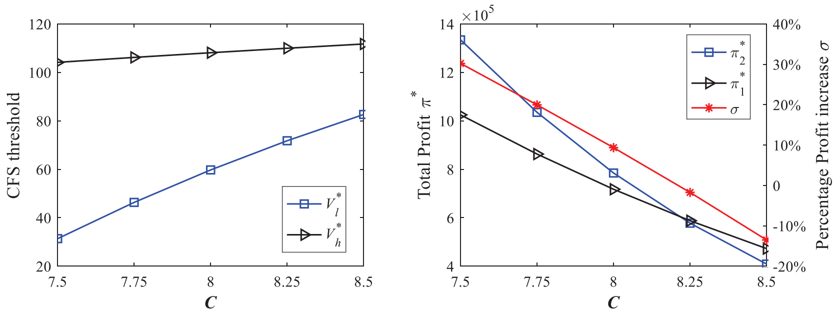

Figure 9. The impact of C on the online retailer's optimal CFS threshold, profit, and the value of the delayed delivery.

4.2.3. The Impact of ω

Result 3. *As the consumer's negative attitude towards the delayed delivery time increases, the online retailer should decrease the low CFS threshold value, and the value of the delayed delivery will increase.*

For this sensitivity analysis, the value of ω was increased from 8 to 12 in increments of 1. As shown in Figure 10, the numerical results suggest that the heterogeneity of consumers attitudes to delayed delivery time affects the online retailer's contingent free-shipping decision. More specifically, as the value of ω increases, the online retailer should decrease its low CFS threshold value. This means that when the consumer's negative attitude towards the delayed delivery time is high, the online retailer should set a smaller low CFS threshold to induce more consumers to choose the delayed delivery.

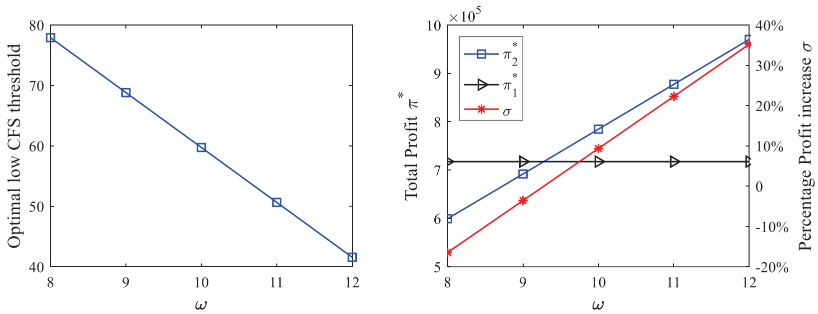

Figure 10. The impact of ω on the online retailer's optimal CFS threshold, profit, and the value of the delayed delivery.

In addition, as Figure 10 shows, as the value of ω increases, the profit of the developed model (π_2^*) and the percentage of the profit increase (σ) will increase. This implies that the larger the value of ω, the more value the online retailer can gain from the delayed delivery. However, when the value of ω is small, the profit of the basic model is larger than that of the developed model ($\pi_1^* > \pi_2^*$). This happens because when the consumer's negative attitude towards the delayed delivery time is low, more consumers will choose the delayed delivery to qualify for the free shipping. At this point, compared with the traditional CFS policy, the online retailer needs to pay more shipping fees under the DD-CFS policy, which results in less profit. Thus, when the consumer's negative attitude towards the delayed delivery time is small, the online retailer should choose the traditional CFS policy. Otherwise, the online retailer should choose the DD-CFS policy. Moreover, the larger the consumer's negative attitude towards the delayed delivery time, the greater value the online retailer can obtain from the delayed delivery.

4.2.4. The Impact of β

Result 4. *As the consumer's negative attitude towards the shipping fee increases, the online retailer should increase the high and low CFS threshold values, and the value of the delayed delivery will decrease.*

For this sensitivity analysis, we increased the value of β from 1.6 to 2.4 in steps of 0.2. As shown in Figure 11, we found that increasing the value of β forces the online retailer to increase its optimal high and low CFS thresholds. This occurs because of the following reason. As the value of β increases, consumers are more reluctant to pay the shipping fee. Under the traditional CFS policy, more consumers will choose to increase their purchase amount to qualify for free shipping, which gives the online retailer very high shipping expenses. Under the DD-CFS policy, with the increase in β, the proportion of shipping-fee-sensitive consumers will increase, and the proportion of shipping-fee-sensitive consumers who choose to increase the purchase amount and opt for the delayed delivery to qualify for free shipping will increase. This also means that the online retailer has to pay more shipping fees. Thus, the online retailer should increase the high and low CFS threshold values to stimulate consumers to increase their purchase amounts to compensate for high shipping expenses.

As shown in Figure 11 we also found that with the increase in β, the profit of the basic model (π_1^*) and the profit of the developed model (π_2^*) will decrease, and the profit of the developed model decreases more rapidly than that of the basic model. Thus, the percentage of profit increase (σ) will decrease as β increases. This implies that the smaller the consumer's negative attitude towards the shipping fee, the more value the retail store can gain from the delayed delivery. However, when the value of β is large, the profit of the developed model is lower than the profit of the basic model ($\pi_2^* < \pi_1^*$). This is because when the value of β is large, the fraction of shipping-fee-sensitive consumers is very large. This means that more consumers will choose the delayed delivery or increase the purchase amount to receive the free shipping service. Thus, the shipping expenses of the supply chain under the DD-CFS policy are larger than that under the traditional CFS policy, which leads to less profit for the DD-CFS policy. Hence, the online retailer should implement the traditional CFS policy when the consumer's negative attitude towards shipping fee is very high.

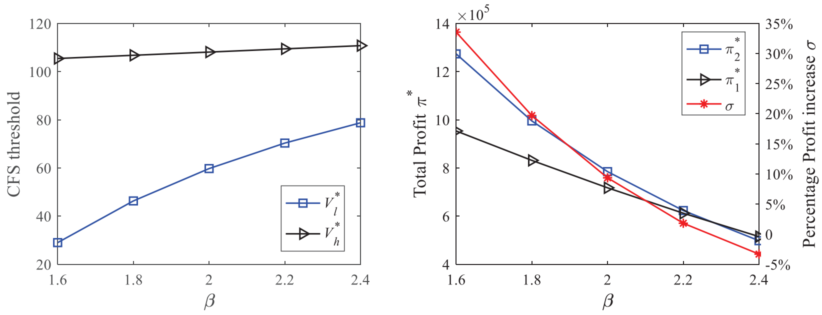

Figure 11. The impact of β on the online retailer's optimal CFS threshold, profit, and the value of the delayed delivery.

5. Conclusions

In this paper, we have provided a new CFS policy with delayed delivery (DD-CFS) for online retailers during large-scale promotions and studied retailers' CFS threshold decisions. We investigated this problem by considering two types of consumers, on-time delivery required consumer and the shipping-fee-sensitive consumer, with different delay delivery time sensitivities. After analyzing a consumer's purchase decision given an online retailer's CFS threshold, we then established a mathematical model to find the optimal CFS threshold that maximized the online retailer's expected profit. More specifically, we first presented a basic model without considering the delayed delivery, and the optimal high CFS threshold was obtained. Then, we established a developed model with delayed delivery to find the optimal low CFS threshold. Finally, we performed a sensitivity analysis to examine the impacts of important parameters on the online retailer's CFS threshold decisions, profit, and the value of the delayed delivery.

The key findings of this study are as follows. (1) Compared with the traditional CFS policy, the DD-CFS policy can bring about more profits during a large-scale promotion period. Moreover, as potential market size increases, the amount of increased profit will rise. (2) As the length of the delayed delivery time and the consumer's negative attitude towards the delayed delivery time increase, the online retailer should reduce the low CFS threshold value, and the value of the delayed delivery will increase. However, if the length of the delayed delivery time and the consumer's negative attitude towards delayed delivery time are small, the DD-CFS policy is not beneficial for the online retailer. (3) As the shipping fee and the consumer's negative attitude towards shipping fee increase, the online retailer should raise the high and low CFS threshold values, and the value of the delayed delivery will decrease. When the shipping fee and the consumer's negative attitude towards shipping fee are very high, the online retailer should implement the traditional CFS policy.

This study also has several limitations. In the future, we may extend this work as follows. First, we assumed that the demand was a function of the CFS threshold value and shipping fee. However, demand in real life is also affected by the length of the delayed delivery time. Future research can extend the demand to be dependent on the length of the delayed delivery time. Second, for the convenience of calculation, we assumed that the consumer's planned purchase amount obeyed a uniform distribution. Further studies may also be conducted to analyze other distribution functions of consumers' planned order values, such as a normal distribution. Third, we may consider the competition between the online retailer and brick-and-mortar stores because the presence of physical stores significantly influences customers' purchasing decisions. In reality, a consumer may choose between picking up the products from a local store and increasing the online order size to qualify for free shipping. These extensions may provide some consequential managerial insights for online retailers.

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