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Indecisiveness, preference for flexibility, and a unique subjective state space

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Abstract

The objective of this study is to unify two major approaches for addressing uncertainty, namely, *indecisiveness* and *preference for flexibility*. Specifically, we assume preferences over alternatives and over menus as primitives, and axiomatize a joint representation of expected multi-utility (Dubra et al. 2004) and ordinal expected utility (Dekel et al. 2001), wherein the set of utility functions in the former is equivalent to the subjective state space in the latter. This result indicates that indecisiveness and preference for flexibility arise from the common underlying uncertainty about ex post tastes, that is, the subjective state space, albeit they may appear differently. Our key axiom is *dominance consistency*, which requires that the addition of an alternative to a menu strictly improves the menu evaluation if and only if the alternative is undominated by the menu. The main result can be extended to a specific class of ordinal expected utility, such as the additive representation. The relationship between the preference over alternatives and the commitment ranking, and the one-directional implications of dominance consistency, are also discussed.

Keywords: uncertainty, indecisiveness, preference for flexibility, subjective state space, dominance consistency

JEL Classification Numbers: D81

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1 Introduction

One of the assumptions behind Savage's (1954) subjective expected utility theory that has been criticized by many researchers is that the state space, that is, the set of all possible states of the world, is *exogenously* given. To address this issue, Kreps (1979, 1992) considered choice over menus (i.e., choice sets) of prizes and *endogenously* derived the subjective state space by assuming *preference for flexibility*, that is, larger menus are preferred to smaller ones. Moreover, Dekel et al. (2001) obtained the uniqueness of the subjective state space by considering the set of menus of lotteries, rather than menus of prizes, as the domain of choice.

The objective of this study is to extend this approach to the choice-over-lotteries framework by relating preference for flexibility to *indecisiveness*. To illustrate this point, consider a decision maker (DM) who chooses between two goods, namely, *sunscreen* and *umbrella*, the evaluation of which depends on the subjective state: the DM believes that the former alternative would be more helpful in sunny weather, whereas the latter would be more valuable in rainy weather.[1] Once the subjective state (weather, in this case) is fully realized, the DM can easily choose between the two alternatives. However, she may have difficulty making a decision if the state is uncertain because the "right" decision changes drastically according to the state.

In such a situation, the DM may exhibit preference for flexibility in the choice-over-menus framework; that is, she would choose menu $\{sunscreen, umbrella\}$ over singleton menus $\{sunscreen\}$ and $\{umbrella\}$ to better address both sunny and rainy weather. By imposing such an axiom, Dekel et al.'s (2001) ordinal expected utility (OEU) model evaluated menus using a strictly increasing aggregation function of indirect expected utility, which is generated by a unique subjective state space S, that is, a set of expected utility functions.[2] Moreover, their additive expected utility (AEU) model, a special case of OEU, also incorporated an additive aggregation function.

However, there is another possibility in the choice-over-alternatives framework: being indecisive between *sunscreen* and *umbrella* ex ante, the DM may defer the decision to an (often unmodeled) ex post stage wherein the subjective state is realized. This type of behavior can be characterized by incomplete preferences over alternatives, such as those studied by Bewley (1986) and Dubra et al. (2004). In particular, Dubra et al.'s expected multi-utility (EMU) theory proposed the unanimity ranking wherein one alternative is preferred to the other if and only if all the expected utility functions in some set $\mathcal{V}$ rank the former better than the latter.

[1] Despite using the expressions "sunny" and "rainy," which may be reminiscent of objectively observable states, we assume that these states are subjective; that is, they may be unobservable or unverifiable by a third party, and can be derived as part of the preference representations.

[2] Given each subjective state s in S, the state-dependent expected utility function in their model can be identified with a (state-independent) expected utility function.

To unify these two approaches, we first assume two preferences—preference $\succsim^*$ over *alternatives* and preference $\succsim$ over *menus*—as primitives, with the former interpreted as an incomplete and unobservable mental preference, and the latter as a complete revealed preference. Our key axiom, which is referred to as *dominance consistency*, relates the mental and revealed preferences in the following manner: for some alternative α and menu x, suppose that we can obtain an alternative β that is (weakly) preferred to α under the mental preference $\succsim^*$ by randomizing over alternatives of x; in this case, the DM would naturally regard α as $\succsim^*$-*dominated* by (the alternatives of) x.[3] Whenever the latter argument holds, we would assume that the revealed preference $\succsim$ attaches no value to adding α to x, that is, $x \cup \{\alpha\} \sim x$. However, if such β cannot be obtained, the DM would consider α as $\succsim^*$-*undominated*; accordingly, whenever this is the case, the revealed preference should assign a positive value to adding α to x, that is, $x \cup \{\alpha\} \succ x$. Dominance consistency formalizes this idea and also implies axioms discussed in the literature, such as monotonicity (Dekel et al., 2001; Kreps, 1979) and indifference to randomization (Dekel et al., 2001).

Our main theorems provide a joint axiomatization of EMU and OEU in the following two ways. First, we assume that both preferences $\succsim^*$ and $\succsim$ satisfy basic conditions such as continuity and non-triviality. Theorem 1 indicates that the mental preference $\succsim^*$ additionally satisfies the standard independence axiom and the preference pair $(\succsim^*, \succsim)$ jointly satisfies dominance consistency if and only if $\succsim^*$ and $\succsim$ admit a unique joint representation of EMU and OEU, wherein the set $\mathcal{V}$ of expected utility functions in the former is *equivalent* to the subjective state space S in the latter. Second, Theorem 2 indicates that the revealed preference $\succsim$ satisfies weak menu independence (Dekel et al., 2001) and $(\succsim^*, \succsim)$ jointly satisfies dominance consistency, in addition to the basic conditions, if and only if the preferences admit the joint representation stated above. These representation results imply that both indecisiveness and preference for flexibility can be generated by identical underlying uncertainties about ex post tastes, that is, the subjective state space, and one of the two ideas would rationalize the other, once we accept dominance consistency.

Moreover, we extend our analysis in the following three directions. First, by strengthening the independence axiom imposed on the menu preference $\succsim$, Corollary 3 indicates that our main result also applies for AEU, the additive form of OEU. Because AEU is more intuitive and more easily applicable to economic models, this corroborates the significance of our approach. Second, we compare the EMU mental preference $\succsim^*$ with the *commitment ranking*, that is, the OEU revealed preference $\succsim$ restricted to singleton menus (Gul and Pesendorfer, 2001). Proposition

[3]This includes the case in which β is a member of x, because β can be generated by a degenerated probability distribution that assigns probability one to itself.

4 concludes that the former ranking implies the latter, but the converse implication does not generally hold, because EMU offers the unanimity ranking whereas the commitment ranking is aggregated. In other words, the mental preference $\succsim^*$ conveys richer information than the commitment ranking, and the revealed preference $\succsim$ over the *entire* set of menus, rather than the commitment ranking, must be specified to derive the mental preference, even with the help of dominance consistency. Finally, we weaken the dominance consistency axiom so that one-directional implications are allowed, under the assumption of EMU and OEU representations. Proposition 5 indicates that the weakened axiom will replace the equivalence between the sets $\mathcal{V}$ and S of the expected utility functions, which is shown in Theorems 1 and 2, by set inclusion (i.e., $\mathcal{V} \subseteq S$ or $\mathcal{V} \supseteq S$). In other words, the weakened axiom may cause a discrepancy between the DM's perceptions of ex post tastes when alternatives and menus are evaluated.

The remainder of this paper is organized as follows. Section 2 proposes the basic framework and axioms that are the focus of this study. In Section 3, the main representation theorems are stated. We characterize the case of additive menu preference representation in Section 4, and compare the preference over alternatives with the commitment ranking in Section 5. One-directional implications of dominance consistency are explored in Section 6. We review the related literature in Section 7. Concluding remarks are made in Section 8.

2 Preliminaries and axioms

Let B be a finite set of prizes and $\Delta(B)$ denote the set of probability distributions, or *alternatives* (lotteries), on B, endowed with the weak convergence topology. A subset of $\Delta(B)$ is referred to as a *menu*. Let $\mathcal{K}(\Delta(B))$ be the set of menus, endowed with the Hausdorff topology.[4] For all x, $y \in \mathcal{K}(\Delta(B))$ and $\lambda \in [0, 1]$, we define $\lambda x + (1-\lambda)y \equiv \{\lambda\alpha + (1-\lambda)\beta : \alpha \in x, \beta \in y\}$. We assume two binary relations: the preference over alternatives, $\succsim^* \in \Delta(B) \times \Delta(B)$, and the preference over menus, $\succsim \in \mathcal{K}(\Delta(B)) \times \mathcal{K}(\Delta(B))$. The strict preferences $\succ^*$ and $\succ$, and the indifferences $\sim^*$ and $\sim$, are defined in the usual manner.

We interpret the preference $\succsim^*$ over alternatives as an *unobservable* mental preference that impacts the well-being of the DM, and the preference $\succsim$ over menus as a revealed (or behavioral) preference, which is observable to a third party.[5] This interpretation is relevant to many studies

[4] We elaborate on the Hausdorff topology in Appendix A1.

[5] There would be three alternative interpretations of the preferences $\succsim^*$ and $\succsim$. First, these two preferences may be the manifestation of an individual's responses in different evaluation modes, namely, choice over alternatives and choice over menus. Second, they may be the preferences of different individuals such as a layperson and a social planner; that is, $\succsim^*$ denotes the former individual's subjective preference over alternatives, whereas $\succsim$ describes the latter individual's choice over menus that is made on behalf of the former. Finally, the DM may in fact consist of various individuals and $\succsim^*$ and $\succsim$ describe their collective choice over alternatives and menus. In this interpretation, indecisiveness over alternatives may result from the disagreement of individual opinions,

that employ the (generally unobservable) underlying preference and the revealed preference in decision theory (e.g., Cerreia-Vioglio et al., 2016; Gilboa et al., 2010; Pejsachowicz and Toussaert, 2017) and in the context of freedom of choice (e.g., Arrow, 1995; Foster, 1993, 2011; Sen, 2002).[6]

In this study, we consider the following four axioms for preferences $\succsim^*$ over alternatives and $\succsim$ over menus, which are divided into three groups: the first axiom provides basic conditions that should be satisfied by both preferences, namely, preorder/weak order, continuity, and nontriviality; the next two axioms are independence, one of which is selectively imposed on the preference over alternatives or over menus; finally, we state our key axiom, *dominance consistency*, which relates the preferences $\succsim^*$ and $\succsim$.

We first explain the basic conditions assumed for each preference.[7]

Axiom 1 (Basic conditions) $\succsim^*$ and $\succsim$ satisfy Axioms 1.1–1.3.

Axiom 1.1 (Preorder/weak order)
(a) $\succsim^*$ is reflexive and transitive.
(b) $\succsim$ is complete and transitive.

Axiom 1.2 (Continuity)
(a) For all $\alpha \in \Delta(B)$, $\{\alpha' \in \Delta(B) : \alpha' \succsim \alpha\}$ and $\{\alpha' \in \Delta(B) : \alpha \succsim \alpha'\}$ are closed in the weak convergence topology.
(b) For all $x \in \mathcal{K}(\Delta(B))$, $\{x' \in \mathcal{K}(\Delta(B)) : x' \succsim^* x\}$ and $\{x' \in \mathcal{K}(\Delta(B)) : x \succsim^* x'\}$ are closed in the Hausdorff topology.

Axiom 1.3 (Nontriviality)
(a) There exist α, $\alpha' \in \Delta(B)$ such that $\alpha \succ^* \alpha'$.
(b) There exist x, $x' \in \mathcal{K}(\Delta(B))$ such that $x \succ x'$.

We explain the intuition behind Axioms 1.1–1.3. First, Axiom 1.1 requires the revealed preference $\succsim$ to be complete, whereas the mental preference $\succsim^*$ may be incomplete for some reasons, such as internal conflict and choice deferral; accordingly, we denote $\alpha \bowtie^* \beta$ for all α, $\beta \in \Delta(B)$ to imply that neither $\alpha \succsim^* \beta$ nor $\alpha \succsim^* \beta$, in which case the ranking between α and β

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whereas preference for flexibility can be interpreted as the expansion of a menu improving social welfare.

[6]The foundation for such a framework was also discussed by Mandler (2005) and Rubinstein and Salant (2008).

[7]These basic conditions are the counterparts of those in Gilboa et al. (2010), except that we consider preferences over alternatives and menus rather than two preferences over alternatives.

is referred to as *indecisive*. Second, Axiom 1.2 is the standard continuity axiom applied to the preferences over alternatives and menus.[8] Finally, Axiom 1.3 ensures that there exists a pair of alternatives or menus in which one choice is strictly preferred to the other.

Next, we state the independence axioms considered in this study. First, for the preference $\succsim^*$ over alternatives, we apply the following standard independence axiom.

Axiom 2 (Alternative independence) For all α, β, $\gamma \in \Delta(B)$ and $\lambda \in (0,1)$, $\alpha \succsim^* \beta$ if and only if $\lambda\alpha + (1-\lambda)\gamma \succsim^* \lambda\beta + (1-\lambda)\gamma$.

Second, we consider the following form of independence for the preference $\succsim$ over menus.

Axiom 3 (Weak menu independence) For all x, y, $z \in \mathcal{K}(\Delta(B))$ with $x \supsetneq y$ and $\lambda \in (0,1)$:

(a) $x \succ y$ if and only if $\lambda x + (1-\lambda)z \succ \lambda y + (1-\lambda)z$;

(b) $x \sim y$ if and only if $\lambda x + (1-\lambda)z \sim \lambda y + (1-\lambda)z$.

Condition (a) of this axiom was first proposed by Dekel et al. (2001). Later, Dekel et al. (2007) demonstrated that condition (b) is also needed to obtain an OEU representation, which we focus on in this study. This explains why both conditions (a) and (b) are assumed in this axiom.

Note that we only *selectively* impose either Axiom 2 or 3 in our main theorems; in other words, the main results of this study do *not* necessarily require that *both* preferences $\succsim^*$ and $\succsim$ satisfy the abovementioned independence axioms.

Finally, we state our key axiom, dominance consistency. To this end, we first define the following notion of dominance using the preference $\succsim^*$ over alternatives. We denote the convex hull of menu x by $\text{conv}(x)$.

Definition 1 ($\succsim^*$-dominance) For all $x \in \mathcal{K}(\Delta(B))$ and $\alpha \in \Delta(B)$:

(a) α is referred to as $\succsim^*$-*dominated by* x if there exists $\beta \in \text{conv}(x)$ such that $\beta \succsim^* \alpha$.

(b) α is referred to as $\succsim^*$-*undominated by* x if α is not $\succsim^*$-dominated by x.

Intuitively, this definition determines the mental ranking between α and the alternatives in menu x: suppose that the DM can apply mixed strategies to choose from x, that is, she can

[8] Although the continuity axiom imposed on the preference over alternatives by Axiom 1.2(a) is weaker than that assumed by Dubra et al. (2004), the former is equivalent to the latter in our setting because the set B of prizes is assumed to be finite.

randomize over alternatives in menu x, for which the generated alternative can naturally be identified with an element of $\mathrm{conv}(x)$. Definition 1 states that alternative α is $\succsim^*$-dominated by x if an alternative β that is *mentally* (weakly) preferred to α can be generated by such a randomization. However, α is $\succsim^*$-undominated by x if no such alternative β can be generated by a randomization over alternatives in x, which implies that, for all alternatives β generated by such a randomization, either α is mentally ranked as being strictly better than β (i.e., $\alpha \succ^* \beta$) or the mental ranking between α and β is indecisive (i.e., $\alpha \bowtie^* \beta$).

Now, using the abovementioned definition of dominance, the following axiom specifies exactly when adding an alternative to a menu is valuable.

Axiom 4 (Dominance consistency) For all $x \in \mathcal{K}(\Delta(B))$ and $\alpha \in \Delta(B)$, the following statements hold:

(a) $x \cup \{\alpha\} \succ x$ if and only if α is $\succsim^*$-undominated by x.

(b) $x \cup \{\alpha\} \sim x$ if and only if α is $\succsim^*$-dominated by x.

Statement (a) ensures that, if alternative α is $\succsim^*$- (or mentally) undominated by menu x, the DM would consider adding α to x as strictly valuable because she cannot reject the possibility that there exists a certain subjective state wherein α is better than all the alternatives β generated by a randomization over x; conversely, alternative α must be $\succsim^*$-undominated by menu x if the DM finds adding α to x strictly valuable. In contrast, statement (b) requires that adding α to x is of no value if alternative α is $\succsim^*$-dominated by menu x, because in this case, the DM is fully convinced that she can generate an alternative β that is better than α by randomizing over x; conversely, if the DM is indifferent to adding alternative α to menu x, α must be $\succsim^*$-dominated by x. Note that dominance consistency is relevant not only to the revealed preference $\succsim$, but also to the mental preference $\succsim^*$ because our definition of dominance is based on the latter preference.

Axiom 4 excludes the possibility of preference for commitment (i.e., $x \succ x \cup \{\alpha\}$ for some $\alpha \in \Delta(B)$), and thus it implies *monotonicity*; that is, for all x, $y \in \mathcal{K}(\Delta(B))$, $x \supseteq y$ obtains $x \succsim y$ (Dekel et al., 2001; Kreps, 1979). Moreover, as we will see later (Lemma 1 in the appendix), Axiom 4 also implies *indifference to randomization (IR)*, that is, $x \sim \mathrm{conv}(x)$ for all $x \in \mathcal{K}(\Delta(B))$ (Dekel et al., 2001), together with continuity, because it entails that $x \cup \{\alpha\} \sim x$ for all $\alpha \in \mathrm{conv}(x)$.

Finally, dominance consistency may be reminiscent of the *P-consistency* axiom introduced by Arlegi and Nieto (2001), who stated that adding an alternative α to a menu x is valuable if and only if there exists no alternative in x that is strictly preferred to α by some preorder P. However, they assumed the set of prizes (i.e., degenerated lotteries) as the domain of choice, and so their model is irrelevant to randomization over alternatives in the menu, which is crucial in our definition of dominance.[9]

3 Main theorems

This section states our main theorems, which axiomatize a joint representation of the preferences $\succsim^*$ over alternatives and $\succsim$ over menus. First, for the sake of simplicity, we identify an affine utility function $u : \Delta(B) \to \Re$ (i.e., $u(\lambda\alpha+(1-\lambda)\beta) = \lambda u(\alpha)+(1-\lambda)u(\beta)$ for all α, $\beta \in \Delta(B)$) with a vector $u = (u_{b_1}, \cdots, u_{b_{|B|}})$ in $\Re^B$ such that $u(\beta) \equiv \beta \cdot u = \sum_{b \in B} \beta(b)u_b$ for all $\beta \in \Delta(B)$. In the following analysis, we interchangeably interpret u as an affine function and as a vector. As the set of normalized utility functions, we use $\mathcal{U} = \{u \in \Re^B : \sum_{b \in B} u_b = 0, \sum_{b \in B} u_b^2 = 1\}$, which was employed by Ergin and Sarver (2010).[10] Moreover, we endow the set $\mathcal{U}$ with the standard Euclidean topology.[11]

We now focus on the following model of incomplete preference over alternatives.

Definition 2 (Expected multi-utility (EMU) representation, Dubra et al. 2004) We say that $\succsim^*$ admits an *expected multi-utility* representation with a closed convex $\mathcal{V} \subseteq \mathcal{U}$ if, for all α, $\beta \in \Delta(B)$,

$$\alpha \succsim \beta \Longleftrightarrow v(\alpha) \geq v(\beta) \text{ for } \text{ all } v \in \mathcal{V}.$$

This representation embodies the unanimity rule of a collection $\mathcal{V}$ of expected utility functions; that is, one alternative is preferred to the other if and only if the former is ranked better than the latter by *all* affine utility functions in $\mathcal{V}$. However, we obtain $\alpha \bowtie^* \beta$ if there exist v, $v' \in \mathcal{V}$ such that $v(\alpha) \geq v(\beta)$ and $v'(\alpha) < v'(\beta)$, which can be interpreted as indecisiveness caused by conflicting ex post tastes v and v'.

To explain the preference representation over menus considered in this study, let S be the set of subjective states. We assume that $S \subseteq \mathcal{U}$ because, given a state-dependent affine function $U : \Delta(B) \times S \to \Re$ and a state s in S, the function $U(\cdot, s)$ can be identified with $u \in \mathcal{U}$ such that $U(\alpha, s) = u(\alpha)$ for all $\alpha \in \Delta(B)$.[12] We refer to a state $s \in S$ as *relevant* if, for every neighborhood N of $u \in S$, there exist x, $x' \in \mathcal{K}(\Delta(B))$ such that $x \nsim x'$ and $\sup_{\beta \in x} u'(\beta) =$

[9] We will discuss the other implications of P-consistency in Section 7.

[10] It can readily be shown that for all $v \in \Re^B$, there exist $a > 0$, $b \in \Re$, and $u \in \mathcal{U}$ such that $v(\alpha) = au(\alpha) + b$ for all $\alpha \in \Delta(B)$.

[11] In our framework, where the set B of prizes is finite, this topology is equivalent to the topology over affine (expected utility) preferences considered by Dekel et al. (2001), which is discussed in Appendix A2.

[12] $U : \Delta(B) \times S \to \Re$ is a state-dependent affine function if $U(\cdot, s)$ is affine for all $s \in S$.

$\sup_{\beta \in x'} u'(\beta)$ for all $u' \in S \setminus N$. We also refer to $S \subseteq \mathcal{U}$ as *relevant* if all states $s \in S$ are relevant. Moreover, we define $\mathcal{U}^*(S) \equiv \left\{ \left(\sup_{\beta \in x} u(\beta) \right)_{u \in S} : x \in \mathcal{K}(\Delta(B)) \right\}$, that is, the set of vectors generated by indirect utility (i.e., the supremum utility levels).

In this study, we focus on the following representation of the preference over menus.

Definition 3 (Ordinal expected utility (OEU) representation, Dekel et al. 2001) For some relevant $S \subseteq \mathcal{U}$ and $g : \Re^S \to \Re$ that is continuous and strictly increasing on $\mathcal{U}^*(S)$, we say that $\succsim$ admits an *ordinal EU* representation (S, g) if $\succsim$ is represented by

$$W(x) = g\left(\left(\sup_{\beta \in x} u(\beta) \right)_{u \in S} \right).$$

In other words, OEU describes an aggregation rule for evaluating a given menu x: it aggregates the indirect utility, that is, the maximum utility that can be attained by alternatives of x, given each $u \in S$, and the aggregator g is continuous and strictly increasing on $\mathcal{U}^*$. Note that g being strictly increasing implies that preference for flexibility is exhibited, that is, an OEU menu preference $\succsim$ satisfies monotonicity.

Comparing EMU and OEU, we notice that they share some commonalities: they both are based on a set of expected utility functions ($\mathcal{V}$ and S, respectively) and can be interpreted as a model of addressing uncertainty about ex post tastes, as discussed in the Introduction. The following analysis indicates that these two preference representations can in fact be related by dominance consistency, along with other axioms.

3.1 Preference over alternatives to preference over menus

In this and the next subsections, we derive a joint representation of the preference pair $(\succsim^*, \succsim)$. As a starting point, in this subsection, we impose independence on the mental preference $\succsim^*$ over alternatives and basic conditions on the preference pair $(\succsim^*, \succsim)$. By additionally requiring dominance consistency, we obtain the joint representation of the preference pair, which can be summarized as the following theorem.

Theorem 1 *The following statements are equivalent:*

(i) $\succsim^*$ *satisfies basic conditions and alternative independence;* $\succsim$ *satisfies basic conditions;* $(\succsim^*, \succsim)$ *jointly satisfies dominance consistency.*
(ii) $\succsim^*$ *admits a unique EMU representation* $\mathcal{V}$ *and* $\succsim$ *admits a unique OEU representation* (S, g)*, with* $\mathcal{V} = S$*.*

A sketch of the proof is as follows. First, as Dubra et al. (2004) indicated, the basic conditions of $\succsim^*$ (specifically, preorder and continuity (a)) and alternative independence imply that $\succsim^*$ admits an EMU representation $\mathcal{V}$. The uniqueness of $\mathcal{V}$ follows from the construction of the set $\mathcal{U}$ of expected utility functions. Next, dominance consistency implies monotonicity and IR (Lemma 1). Thus, with the help of some basic conditions of $\succsim$ (specifically, weak order and continuity (b)), the argument by Dekel et al. (2001) implies that $\succsim$ admits a unique weak EU representation; that is, there exists a continuous (but not necessarily strictly increasing) aggregator g on $\mathcal{U}^*(S)$ that represents $\succsim$. In the key step of the proof, we further indicate that alternative independence and dominance consistency imply that g is strictly increasing on $\mathcal{U}^*(S)$ (Lemmas 2–4).

Finally, the equivalence of V and S can be obtained as follows. Given the OEU representation (S, g) of $\succsim$, and some $\alpha \in \Delta(B)$ and $x \in \mathcal{K}(\Delta(B))$, $x \cup \{\alpha\} \succ x$ implies that there exists $u \in S$ such that α is the unique maximizer of u in $\mathrm{conv}(x \cup \{\alpha\})$; that is, $u(\alpha) > u(\beta)$ for all $\beta \in \mathrm{conv}(x)$. In contrast, given the EMU representation $\mathcal{V}$ of $\succsim^*$, α being $\succsim^*$-undominated by x implies that there exists $v \in \mathcal{V}$ such that $v(\alpha) > v(\beta)$ for all $\beta \in \mathrm{conv}(x)$. Accordingly, dominance consistency implies that the first and the second conditions are equivalent, and so $\mathcal{V} = S$ (Lemma 5).

Some remarks follow. First, Theorem 1 indicates that the set $\mathcal{V}$ of expected utility functions in EMU can also be interpreted as the subjective state space. This underlies its counterpart in OEU, because we obtain the uniqueness of the joint representation and $\mathcal{V} = S$: as suggested in the Introduction, the EMU mental preference $\succsim^*$, which can be derived from basic conditions and alternative independence, may be indecisive between certain pairs of alternatives to better address uncertainty about ex post tastes generated by $\mathcal{V}$. However, the theorem states that the additional requirement of dominance consistency obtains the OEU revealed preference $\succsim$ with $S = \mathcal{V}$, which would exhibit preference for flexibility, to address exactly the same type of uncertainty. This result contrasts with existing approaches that discuss indecisiveness and preference for flexibility separately.[13]

Second, dominance consistency not only relates preferences $\succsim^*$ and $\succsim$, but also inherits the linear structure of the mental preference $\succsim^*$, which is implied by the alternative independence axiom, to the revealed preference $\succsim$: Axiom 4 implies that, for $\alpha \in \Delta(B)$ and $x \in \mathcal{K}(\Delta(B))$, $x \cup \{\alpha\} \succ x$ if and only if α is $\succsim^*$-undominated by x. Because the latter condition is relevant to the mental preference $\succsim^*$, alternative independence also establishes a linear structure for $\succsim$; that is, for all $\gamma \in \Delta(B)$ and $\lambda \in [0, 1]$, the addition of alternative $\lambda\alpha + (1-\lambda)\gamma$ to menu $\lambda x + (1-\lambda)\gamma$

[13]An exception is the *cautious deferral* axiom proposed by Pejsachowicz and Toussaert (2017), which we discuss in Section 6.

is strictly preferred by $\succsim$ whenever $\lambda\alpha + (1-\lambda)\gamma$ is $\succsim^*$-undominated by $\lambda x + (1-\lambda)\gamma$. This point is crucial for obtaining the OEU representation of $\succsim$ in this theorem *without* imposing weak menu independence.

Third, Theorem 1 *rationalizes* a preference $\succsim$ over menus by a preference $\succsim^*$ over alternatives, a point that has been discussed by many authors. As Kreps (1979) argued, a *single* underlying preference over alternatives that is represented by a utility function u trivially derives the indirect utility representation of preference over menus with the *single* subjective state u. In contrast, many authors assume *multiple* underlying preferences over alternatives to obtain a menu preference representation with *multiple* subjective states (Arrow, 1995; Foster, 1993, 2011; Sen, 2002). From this point of view, Theorem 1 unifies these two approaches: we derive the OEU preference $\succsim$ over menus with *multiple* subjective states from a *single* preference $\succsim^*$ over alternatives.[14] This is possible because we allow the preference over alternatives to be *incomplete* and consider the set of *lotteries*, rather than that of prizes, as the domain of choice; the obtained EMU preference $\succsim^*$ generates a *set* $\mathcal{V}$ of expected utility functions. As mentioned earlier, $\mathcal{V}$ is equivalent to the subjective state space in OEU.

Finally, this theorem focuses on OEU as a preference representation over menus, rather than more or less structured forms, because it includes various subclasses of menu preference representations that satisfy monotonicity, such as the additive EU model of Dekel et al. (2001) (which we discuss in the next section), the multiple prior model of Epstein et al. (2007), and the costly contemplation model of Ergin and Sarver (2010).[15] Accordingly, Theorem 1 associates these subclasses with the EMU preference over alternatives, by imposing dominance consistency.

To conclude this section, the following two polar cases are worth noting. First, if $\mathcal{V} = S = \{u\}$ for some $u \in \mathcal{U}$, the mental preference $\succsim^*$ is the standard (complete) expected utility, that is, $\succsim^*$ is decisive for all alternative pairs, whereas the revealed preference $\succsim$ admits the standard indirect utility representation, which evaluates menu x by the best alternative in x according to u. Second, if $\mathcal{V} = S = \mathcal{U}$, the mental preference $\succsim^*$ is *in*decisive for all alternative pairs and the revealed preference $\succsim$ strictly values the addition of *any* alternative α to menu x whenever it expands the convex hull of x, which can be interpreted as an extreme form of intrinsic preference for flexibility (Barberà et al., 2004; Foster, 2011); that is, the DM believes any such α to be the best (and thus, chosen) among all alternatives in the menu if a certain subjective state occurs.

[14] In this sense, the study closest to our approach would be Arlegi and Nieto (2001), who assumed a single (strict) preference over alternatives (prizes) to characterize a menu preference à la Kreps (1979). We will discuss the difference between our approach and theirs in Section 7.

[15] Of the two models considered by Epstein et al. (2007), their first model, called *short-run coarseness*, is a special case of OEU, whereas their second model is not.

3.2 Preference over menus to preference over alternatives

In the previous subsection, we assumed the independence axiom for the mental preference $\succsim^*$ over alternatives (along with the other axioms) to obtain a joint preference representation of $(\succsim, \succsim^*)$. In this subsection, we replace the independence axiom for the mental preference $\succsim^*$ with that for the revealed preference $\succsim$ over menus, and obtain the identical joint representation as before by assuming dominance consistency. The following theorem summarizes the result.

Theorem 2 *The following statements are equivalent:*

(i) $\succsim^*$ satisfies basic conditions; $\succsim$ satisfies basic conditions and weak menu independence; $(\succsim^*, \succsim)$ jointly satisfies dominance consistency.
(ii) $\succsim^*$ admits a unique EMU representation $\mathcal{V}$ and $\succsim$ admits a unique OEU representation (S, g), with $\mathcal{V} = S$.

The sketch of the proof is similar to that of Theorem 1: we first obtain the OEU representation (S, g) of the revealed preference $\succsim$ using basic conditions and weak menu independence. Dominance consistency then leads to the EMU representation $\mathcal{V}$ of the mental preference $\succsim^*$. Finally, the equivalence between S and $\mathcal{V}$ is derived in a manner similar to the proof of Theorem 1.

In contrast to Theorem 1, Theorem 2 provides a foundation for indecisiveness from the viewpoint of preference for flexibility: weak independence and basic conditions (b) obtain the OEU revealed preference $\succsim$ with the subjective state space S, which exhibits preference for flexibility to better address uncertainty about ex post tastes. However, Theorem 2 indicates that, under dominance consistency, S also describes the uncertainty that the EMU mental preference $\succsim^*$ assumes, and so $\succsim^*$ is indecisive whenever there is a disagreement among the evaluations of alternatives given by expected utility functions u in S.

Moreover, EMU can be obtained *without* assuming alternative independence in this theorem, because dominance consistency inherits the linear structure of the revealed preference $\succsim$, which is provided by weak menu independence, to the mental preference $\succsim^*$ in a manner similar to that of Theorem 1. The uniqueness result and the equivalence of $\mathcal{V}$ and S enable us to *pin down* the unobservable mental preference $\succsim^*$ by the revealed preference $\succsim$; that is, assuming dominance consistency and basic conditions, the implication of EMU mental preference $\succsim^*$ can be tested by simply checking whether the revealed preference $\succsim$ satisfies the OEU axioms. This also allows us to make a welfare judgment by focusing on the revealed preference over menus, although welfare is originally defined by the mental preference over alternatives in our framework.

4 Additive menu preference representation

Until this point, we have only obtained OEU as a preference representation over menus, which is more general but less structured than in many existing studies. This is because we imposed weak menu independence on the revealed preference $\succsim$. In contrast, this section derives an additive menu preference representation, which has been extensively considered in the literature, by considering the following axiom.

Axiom 3' (Menu independence) For all x, y, $z \in \mathcal{K}(\Delta(B))$ and $\lambda \in (0,1)$, $x \succ y$ if and only if $\lambda x + (1-\lambda)z \succ \lambda y + (1-\lambda)z$.

Clearly, this axiom is stronger than weak menu independence. Dekel et al. (2001) indicated that menu preference $\succsim$ satisfies menu independence, along with weak order, continuity (b), and monotonicity, if and only if $\succsim$ admits the following additive representation.

Definition 4 (Additive EU (AEU) representation, Dekel et al. 2001) For some relevant $S \subseteq \mathcal{U}$ and $\mu \in \Delta(S)$, we say that $\succsim$ admits an *additive EU* representation (S, μ) if $\succsim$ admits an OEU representation (S, g) and

$$W(x) = g\left(\left(\sup_{\beta \in x} u(\beta)\right)_{u \in S}\right) = \int_S \sup_{\beta \in x} u(\beta)\mu(du).$$

As the definition indicates, AEU is a special case of OEU. From Theorem 2, it is easy to show that the representation theorem also holds for this case.[16]

Corollary 3 *The following statements are equivalent:*

(i) $\succsim^*$ *satisfies basic conditions;* $\succsim$ *satisfies basic conditions and menu independence;* $(\succsim^*, \succsim)$ *jointly satisfies dominance consistency.*
(ii) $\succsim^*$ *admits a unique EMU representation* $\mathcal{V}$ *and* $\succsim$ *admits a unique AEU representation* (S, μ)*, with* $\mathcal{V} = S$*.*

A comparison of Corollary 3 and Theorem 2 indicates that both weak menu independence and menu independence are needed to derive AEU, because the former axiom only obtains

[16] Dekel et al. (2007) indicated that a continuity axiom called *L continuity*, in addition to the continuity axiom assumed in the current study, is required to obtain the additive representation of general (non-monotonic) menu preference. However, we do not need L continuity in Corollary 3 to derive AEU, because monotonicity is implied by dominance consistency.

a strictly increasing aggregation function g on $\mathcal{U}^*(S)$, whereas the latter further obtains the linearity of such an aggregation function g. In other words, the combination of alternative independence and dominance consistency, which is assumed in Theorem 1, is *not* strong enough to obtain the AEU menu preference, because it only implies weak independence.

5 Alternative preference vs. commitment ranking

In this section, we discuss the distinction between the preference $\succsim^*$ over alternatives, which is a primitive of this study, and the *commitment ranking*, or the preference $\succsim$ over menus restricted to singleton menus, which has often been considered in the literature (e.g., Gul and Pesendorfer, 2001). Because they both rank alternatives rather than menus, readers may suspect that these two rankings would coincide. However, the following result indicates that the alternative preference and the commitment ranking are not equivalent, albeit they are relevant.

Proposition 4 *Assume that $(\succsim^*, \succsim)$ admits EMU $\mathcal{V}$ and OEU (S, g) representations with $\mathcal{V}$ $= S$. Then, for all α, $\beta \in \Delta(B)$, the following statements hold.*

(a) $\alpha \succsim^* \beta$ implies $\{\alpha\} \succsim \{\beta\}$.
(b) $\{\alpha\} \succsim \{\beta\}$ implies $\alpha \succsim^* \beta$ or $\alpha \bowtie^* \beta$.

The proof is in the appendix. Given an EMU representation $\mathcal{V}$, the preference $\succsim^*$ over alternatives provides the *unanimity* ranking; that is, for some α, $\beta \in \Delta(B)$, $\alpha \succsim^* \beta$ implies that $v(\alpha) \geq v(\beta)$ for all expected utility functions $v \in \mathcal{V}$. It then follows from $\mathcal{V} = S$ that $u(\alpha)$ $\geq u(\beta)$ for all $u \in S$. Accordingly, we obtain $\{\alpha\} \succsim \{\beta\}$, given an OEU representation (S, g), which implies statement (a). However, the converse implication may not hold because the commitment ranking is an *aggregated* ranking: for some α, $\beta \in \Delta(B)$ and AEU representation (S, μ) with $S = \{u_1, u_2\}$ and $\mu(u_1) = \mu(u_2) = 1/2$, suppose that $u_1(\alpha) = 2$, $u_2(\beta) = 1$, and $u_1(\beta) = u_2(\alpha) = 0$. This clearly implies that $\{\alpha\} \succ \{\beta\}$. However, we have $\alpha \bowtie^* \beta$ because $u_1(\alpha) > u_1(\beta)$ while $u_2(\alpha) < u_2(\beta)$. In other words, Proposition 4 indicates that the preference $\succsim^*$ over alternatives conveys richer information than the commitment ranking. This is because of the different—unanimity and aggregated—calculations of ex post tastes when alternatives and menus are evaluated.[17]

Readers may suspect that, in experimental settings, it would be difficult to elicit information on the preference $\succsim^*$ over alternatives and the commitment ranking at the same time,

[17]One possible interpretation for this result is the *context effect* (Simonson and Tversky, 1993), which argues that identical objects can be evaluated differently in different contexts.

because the task of distinguishing between alternatives and singleton menus may be confusing for subjects. However, once dominance consistency and basic conditions are accepted, it is only necessary to check either the EMU mental preference $\succsim^*$ over alternatives or the OEU revealed preference $\succsim$ over the entire set of menus, *not* both, to test our model. As Theorem 2 indicates, the OEU revealed preference $\succsim$ can pin down both the EMU mental preference $\succsim^*$ and the commitment ranking; on the other hand, Theorem 1 and Proposition 4 imply that the EMU mental preference $\succsim^*$ uniquely determines the OEU revealed preference $\succsim$ as well as the commitment ranking. Because gathering information on only one of the preferences $\succsim^*$ and $\succsim$ is straightforward for subjects, our model can be tested more easily than it may appear.

6 One-directional implications of dynamic consistency

Thus far, we have presumed that the implication of dominance consistency is bilateral, that is, alternative α is $\succsim^*$-undominated by menu x *if and only if* the addition of α to x is strictly preferred, for example. In contrast, we now consider one-directional implications of the axiom.

For simplicity, we assume EMU and OEU representations of preference pair $(\succsim^*, \succsim)$ in this section; that is, the preference $\succsim$ over menus satisfies monotonicity. This is because our main focus is on the relationship between preference for flexibility and indecisiveness. Under this assumption, dominance consistency can be written in the following form.

Axiom 4' (Weak dominance consistency) For all $x \in \mathcal{K}(\Delta(B))$ and $\alpha \in \Delta(B)$, the following statements hold:

(a) α being $\succsim^*$-undominated by x implies $x \cup \{\alpha\} \succ x$.
(b) $x \cup \{\alpha\} \succ x$ implies α being $\succsim^*$-undominated by x.

Statement (a) ensures that alternative α being $\succsim^*$-undominated by menu x implies adding α to x is strictly preferred, whereas the converse does not necessarily hold; that is, it is possible that $x \cup \{\alpha\} \succ x$ and α is $\succsim^*$-*dominated* by x. Statement (b) establishes the converse, that is, adding α to x being strictly preferred implies that α is $\succsim^*$-undominated by x; however, it does not deny the possibility that α is $\succsim^*$-undominated by x and $x \cup \{\alpha\} \sim x$, which implies that the preference over menus can be *insensitive* to the addition of a certain $\succsim^*$-undominated alternative.

Axiom 4' is accompanied by the term "weak" because it does not necessarily exclude the possibility of preference for commitment, as the original dominance consistency does; that is, we may have $x \succ x \cup \{\alpha\}$ for some $\alpha \in \Delta(B)$ and $x \in \mathcal{K}(\Delta(B))$. However, under monotonicity, which is implied by the assumption of OEU preference $\succsim$, the combination of Axioms 4' (a) and (b) clearly implies dominance consistency.

The following proposition specifies the implications of Axioms 4' (a) and (b).

Proposition 5 *Assume that $(\succsim^*, \succsim)$ admits EMU $\mathcal{V}$ and OEU (S, g) representations. Then, the following statements hold:*

(i) Weak dominance consistency (a) is satisfied if and only if $\mathcal{V} \subseteq S$.

(ii) Weak dominance consistency (b) is satisfied if and only if $S \subseteq \mathcal{V}$.

The proof is in the appendix. Statement (i) implies that weak dominance consistency (a) holds, that is, the addition of $\succsim^*$-undominated alternative α to menu x is strictly preferred whenever $\mathcal{V}$ is included by S. This case can be interpreted as intrinsic preference for flexibility (Barberà et al., 2004; Foster, 2011); that is, because $\mathcal{V} \subseteq S$, there exists a $u \in S \setminus \mathcal{V}$ so that, for some $\alpha \in \Delta(B)$ and $x \in \mathcal{K}(\Delta(B))$, $x \cup \{\alpha\} \succ x$ even if α is $\succsim^*$-dominated by x. The latter argument implies that the addition of α to x would never improve welfare, which we assume to be defined by the mental preference $\succsim^*$. Statement (ii) entails that weak dominance consistency (b) holds, that is, a strict preference for the addition of alternative α to menu x obtains the $\succsim^*$-undominance of α by x whenever S is included by $\mathcal{V}$. This explains a context effect wherein the DM overlooks some subjective states when evaluating menus, as exhausting all the possible subjective states demands excessive cognitive resources, whereas pairwise comparisons of alternatives enhance the realization of subjective states by attracting the DM's attention to the states wherein the alternatives are evaluated differently.[18]

These set inclusion results may be reminiscent of Ahn and Sarver (2013), who characterized the implication of their consistency axioms between the ex ante preference over menus and the ex post random choice over alternatives. Requiring only consistency from the former to the latter corresponds to unforeseen contingencies, that is, the DM may ex ante overlook some subjective states that eventually realize and govern her ex post random choice of alternatives; in contrast, requiring only consistency from the latter to the former derives intrinsic preference for flexibility, that is, the DM may ex ante appreciate the addition of alternatives to the menu, even if they are not eventually chosen ex post. Unlike Ahn and Sarver (2013), however, we focus on consistency between the unobservable mental preference $\succsim^*$ and the revealed preference $\succsim$ in a *static* setting, rather than that between preferences/choices in different periods.

[18] The idea of restricting the number of alternatives presented to the DM to enhance her understanding of alternatives' values and improve decision making has been discussed in the marketing literature (e.g., Hsee et al., 1999; Jacoby, 1984).

7 Related literature

This section reviews the literature relevant to this study. First, Pejsachowicz and Toussaert (2017) considered psychological $\succsim_p$ and revealed $\succsim_r$ menu preferences, the former of which is unobservable, and related them by imposing an axiom called *cautious deferral*, which states that if the psychological ranking between menus x and y is indecisive (i.e., $x \bowtie_p^* y$), revealed preference for flexibility is exhibited (i.e., $x \cup y \succsim_r x$).

Although their motivation was relevant to our study, there are several major differences. First, their main focus is on relating psychological and revealed menu preferences in a general setting, whereas the present study relates more structured preferences over alternatives and over menus, namely, EMU and OEU. Second, their cautious deferral axiom only gives a one-directional implication, that is, indecisiveness between two menus implying preference for flexibility, but not the converse, whereas our dominance consistency axiom gives two-directional implications. This difference allows us to pin down the EMU mental preference from the OEU revealed preference and dominance consistency, as Theorem 2 indicates, whereas the psychological preference in their study is untestable.

Second, Arlegi and Nieto (2001) assumed a possibly incomplete strict preference P over alternatives and related it to the menu preference by imposing an axiom relevant to dominance consistency (which they referred to as *P-consistency*). Eventually, they obtained the indirect utility representation of the menu preference à la Kreps (1979). Although this result may also be reminiscent of our study, neither the uniqueness of subjective state space S nor a specific alternative preference representation, such as EMU, can be obtained in their approach, because they considered the set of prizes, rather than that of lotteries, as the domain of choice.[19] A lack of uniqueness also results in no equivalence result between the sets of utility functions in their model. This contrasts with the equivalence between $\mathcal{V}$ and S derived in the present study, which is crucial for unifying EMU and OEU.

Third, Danan et al. (2012) defined the dominance relation $\succsim_d$ over alternatives by the preference $\succsim_m$ over menus, so that $\alpha \succsim_d \beta$ if and only if $\alpha \sim_m \mathrm{conv}(\{\alpha, \beta\})$. As a result, they obtained a joint representation wherein the dominance relation admits an EMU representation and the menu preference is represented by the aggregation of the *infimum*, rather than supremum, of utility levels over a subjective state space. Although their model may appear similar to ours, there are some major differences. Aside from a difference in axiomatization, their menu preference representation depends only on the *subset* of the considered menu, because their

[19]This parallels the argument by Dekel et al. (2001) that it is crucial to consider the set of menus of lotteries than the menus of prizes (as in Kreps, 1979) as the domain of choice for obtaining a unique subjective state space.

indecisive aversion axiom requires the ranking between menus to be determined solely by the ranking among *undominated* alternatives (in their sense), rather than all alternatives, in the menus. Accordingly, the menu independence axiom does not hold in their model, whereas it is compatible with our model, as we indicated in Section 4.

Finally, Kraus and Sagi (2006) considered a dynamic menu choice model and obtained the ranking over menus that exhibits preference for flexibility (which is similar to OEU) by assuming an EMU ranking over alternatives and time consistency. Because their time consistency axiom links the rankings over alternatives and menus, it can be interpreted as the counterpart of the dominance consistency axiom in a dynamic setting.[20] Unlike time consistency, however, our dominance consistency directly relates EMU and OEU preferences; that is, it links the two static preferences, rather than connecting preferences in different periods of the dynamic model.

8 Concluding remarks

In this study, we have unified an incomplete preference over alternatives that exhibits indecisiveness and a menu preference that exhibits preference for flexibility, which are represented by EMU and OEU, respectively. The key axiom for this result is dominance consistency, which specifies exactly when the addition of an alternative to a menu is strictly valuable. A crucial property of Dekel et al.'s (2001) unique subjective state space model is IR, that is, indifference to the addition of an alternative to the menu if the alternative can be generated by a randomization over the alternatives in the menu. In contrast, we have shown that a key to characterizing the preference for flexibility and its relation to indecisiveness is our dominance consistency axiom, which derives indifference to the addition of alternative α to menu x whenever we can generate an alternative *weakly preferred* to α (according to $\succsim^*$) by randomizing over the alternatives in x; otherwise, the addition of α should be strictly valuable. Because the latter axiom implies the former, it is arguably a natural and compelling specification of Dekel et al.'s approach.

Moreover, the OEU representation of the preference over menus, which we have focused on, includes many subclasses considered in the literature, such as the linear and multiple prior models. This implies that our main results still hold for these models if stronger axioms are imposed, which also demonstrates the applicability of our approach. A possible direction for future research would be to extend our model to accommodate preference for commitment in addition to preference for flexibility. This requires a relaxation of the axioms assumed in this

[20]Precisely, an alternative in their model is recursively defined by a lottery over a consumption bundle in the current period and a menu to be faced in the next period. Accordingly, their time consistency axiom can be interpreted as linking the rankings over alternatives in the current period and over menus in the next period.

study, including dominance consistency.

Appendix

A. Topologies

A1. Hausdorff topology Let d be a distance on $\Delta(B)$ that generates the weak convergence topology. For all x, $y \in \mathcal{K}(\Delta(B))$, we refer to

$$d_h(x, y) = \max\{\sup_{\alpha \in x} \inf_{\beta \in y} d(\alpha, \beta), \sup_{\beta \in y} \inf_{\alpha \in x} d(\alpha, \beta)\}$$

as the *Hausdorff semimetric*. The Hausdorff topology is the topology generated by the Hausdorff semimetric.

A2. Topology on $\mathbb{P}(S, U)$ Let $\mathbb{P}^{EU}$ denote the set of all nontrivial expected utility preferences over alternatives (i.e., for all $\succsim^* \in \mathbb{P}^{EU}$, $\succsim^*$ is affine and $\alpha \succ^* \beta$ for some α, $\beta \in \Delta(B)$). We apply the following notion of a point convergence topology, which was considered by Dekel et al. (2001).

Definition A1 Given a sequence $\{\succsim_n\}_{n=1}^{\infty}$ of expected utility preferences over $\Delta(B)$, we say that $\succsim^*$ is a limit of the sequence if it is a nontrivial expected utility preference such that

$$\beta \succsim^* \beta' \text{ implies there exists } N \in \mathbb{N} \text{ such that } \beta \succsim_n \beta' \text{ for all } n \geq N.$$

As noted in Section 3, we can identify an expected utility preference as a point in $\mathcal{U}$, because B is finite. Under this interpretation, the topology above is equivalent to the usual (Euclidean) topology on this space.

B. Proofs

Proof of Theorem 1 We only prove the sufficiency part. First, it follows from Dubra et al. (2004) that preorder, continuity (a), and alternative independence imply that $\succsim^*$ admits an EMU representation $\mathcal{V}$. We normalize $\mathcal{V}$ so that each $v \in \mathcal{V}$ is a member of $\mathcal{U}$. Then, $\mathcal{V}$ is unique under this normalization.

The following lemma indicates that dominance consistency implies monotonicity and IR, which are the key axioms of Dekel et al. (2001).

Lemma 1 Under continuity (b), dominance consistency implies the following axioms:

(1) monotonicity: for all x, $y \in \mathcal{K}(\Delta(B))$, if $x \supseteq y$, $x \succsim y$.
(2) IR: for all $x \in \mathcal{K}(\Delta(B))$, $x \sim \text{conv}(x)$.

Proof First, dominance consistency clearly implies monotonicity. Next, dominance consistency implies that $x \cup \{\alpha\} \sim x$ for all $x \in \mathcal{K}(\Delta(B))$ and $\alpha \in \text{conv}(x)$ (note that $\alpha \in \text{conv}(x)$ itself is weakly preferred to α by ranking $\succsim^*$). It follows from the iterative application of this argument and continuity (b) that $x \sim \text{conv}(x)$. **Q.E.D.**

Accordingly, together with weak order, continuity (b), and nontriviality (b), Theorem 1 of Dekel et al. (2001) implies that $\succsim$ admits a weak EU representation (S, g); that is, there exist a relevant subjective state space $S \subseteq \mathcal{U}$ and a unique continuous aggregator $g : \Re^S \to \Re$ such that $\succsim$ is represented by

$$W(x) = g\left(\left(\sup_{\beta \in x} u(\beta)\right)_{u \in S}\right).$$

Because Dekel et al. also indicated that $x \sim \text{cl}(\text{conv}(x))$ for all $x \in \mathcal{K}(\Delta(B))$ in the weak EU representation, we first focus on the subset of all closed convex sets in $\mathcal{K}(\Delta(B))$, which is denoted by X, and extend the result to $\mathcal{K}(\Delta(B))$. For all $x \in X$ and $u = (u_1, \cdots, u_{|B|}) \in \mathcal{U}$, we define a support function $\sigma_x : \mathcal{U} \to \Re$ by $\sigma_x(u) \equiv \max_{\beta \in x} \beta \cdot u = \max_{\beta \in x} \sum_{i=1}^{|B|} \beta(b_i) u_i$, where $b_1, \cdots, b_{|B|}$ denote the members of B. Moreover, for all x, $x' \in X$, define $D(x, x') \equiv \{u \in \mathcal{U} : \sigma_x(u) \neq \sigma_{x'}(u)\}$; that is, $D(x, x')$ is the set of expected utility functions wherein menus x and x' generate different values of the support function, whereas $\mathcal{U} \setminus D(x, x')$ is the set of expected utility functions wherein x and x' generate identical values of the support function.

The next lemma indicates that the preference for the addition of an alternative to a menu is unaffected by mixing with a singleton menu, which is implied by alternative independence and dominance consistency.

Lemma 2 *For all $x \in X$, α, $\gamma \in \Delta(B)$, and $\lambda \in (0, 1)$, $x \cup \{\alpha\} \succ x$ if and only if $(\lambda x + (1 - \lambda)\{\gamma\}) \cup \{\lambda\alpha + (1 - \lambda)\gamma\} \succ \lambda x + (1 - \lambda)\{\gamma\}$.*

Proof Assume that $x \cup \{\alpha\} \succ x$. Then, dominance consistency implies that α is $\succsim^*$-undominated by x; that is, for all $\beta \in x$, $\alpha \succ^* \beta$ or $\alpha \bowtie^* \beta$. It follows from alternative independence that, for all $\beta \in x$, $\gamma \in \Delta(B)$, and $\lambda \in (0, 1)$, $\lambda\alpha + (1-\lambda)\gamma \succsim^* \lambda\beta + (1-\lambda)\gamma$ or $\lambda\alpha + (1-\lambda)\gamma \bowtie^* \lambda\beta + (1-\lambda)\gamma$. The latter argument implies that $\lambda\alpha + (1 - \lambda)\gamma$ is $\succsim^*$-undominated by $\lambda x + (1 - \lambda)\{\gamma\}$. It then follows from dominance consistency that $(\lambda x + (1 - \lambda)\{\gamma\}) \cup \{\lambda\alpha + (1 - \lambda)\gamma\} \succ \lambda x + (1 - \lambda)\{\gamma\}$. **Q.E.D.**

Lemma 2 implies that shifting the alternatives in menu x and alternative α in the direction of an arbitrary alternative γ does not alter the ranking between $x \cup \{\alpha\}$ and x. In particular, it implies that the preference for adding an alternative to a menu is unaffected by an enlargement or contraction; that is, this lemma also holds for all $x \in X$ and an interior point γ of x, which entails that $\lambda x + (1-\lambda)\{\gamma\} \subsetneq x$ and $\mathrm{conv}((\lambda x + (1-\lambda)\{\gamma\}) \cup \{\lambda\alpha + (1-\lambda)\gamma\}) \subsetneq \mathrm{conv}(x \cup \{\alpha\})$.

Next, for all alternatives α and closed convex menus x_1 and x_2, assume that x_1 and x_2 differ only by the alternatives whose optimality is unaffected by the addition of α, or equivalently, adding α to x_1 and x_2 only changes the maximizers of expected utility functions wherein x_1 and x_2 generate identical values of the support function, that is, $D(x_1 \cup \{\alpha\}, x_1) = D(x_2 \cup \{\alpha\}, x_2)$ $= \mathcal{U} \setminus D(x_1, x_2)$. The following lemma indicates that, for such x_1 and x_2, adding α to x_1 is strictly preferred whenever adding α to x_2 is also preferred.

Lemma 3 *Assume that there exist some x_1, $x_2 \in X$ and $\alpha \in \Delta(B)$ such that $D(x_1 \cup \{\alpha\}, x_1)$ $= D(x_2 \cup \{\alpha\}, x_2) = \mathcal{U} \setminus D(x_1, x_2)$. Then, $x_1 \cup \{\alpha\} \succ x_1$ if and only if $x_2 \cup \{\alpha\} \succ x_2$.*

Proof First, for some x_1, $x_2 \in X$ and $\alpha \in \Delta(B)$ that satisfy the assumption of the lemma, note that $D(x_i \cup \{\alpha\}, x_i) = \{u \in \mathcal{U} : \alpha \cdot u > \beta \cdot u$ for all $\beta \in x_i\}$ for $i = 1, 2$. Define $\bar{x}_i \equiv \cup_{u \in D(x_i \cup \{\alpha\}, x_i)} \mathrm{argmax}_{\beta \in x_i} \beta \cdot u$ for $i = 1, 2$. Then, $\bar{x}_1 = \bar{x}_2$ because x_1 and x_2 are convex, and $D(x_1 \cup \{\alpha\}, x_1) = D(x_2 \cup \{\alpha\}, x_2) = \mathcal{U} \setminus D(x_1, x_2)$. From this construction, it follows that, for all $u \in \mathcal{U}$ and $i = 1, 2$, $\alpha \cdot u > \beta \cdot u$ for all $\beta \in \bar{x}_i$ if and only if $\alpha \cdot u > \beta \cdot u$ for all $\beta \in x_i$.

Next, assume that $x_1 \cup \{\alpha\} \succ x_1$. It follows from dominance consistency that α is $\succsim^*$-undominated by x_1; that is, for all $\beta \in x_1$, $\alpha \succsim^* \beta$ or $\alpha \bowtie^* \beta$. Because $\succsim^*$ admits an EMU representation $\mathcal{V}$ and x_1 is convex, there exists $v_\alpha \in \mathcal{V}$ such that $\alpha \cdot v_\alpha > \beta \cdot v_\alpha$ for all $\beta \in x_1$, whereby the argument in the previous paragraph implies that $\alpha \cdot v_\alpha > \beta \cdot v_\alpha$ for all $\beta \in \bar{x}_1$. Accordingly, it follows from $\bar{x}_1 = \bar{x}_2$ that $\alpha \cdot v_\alpha > \beta \cdot v_\alpha$ for all $\beta \in \bar{x}_2$, which also implies that $\alpha \cdot v_\alpha > \beta \cdot v_\alpha$ for all $\beta \in x_2$. That is, α is $\succsim^*$-undominated by x_2, and so dominance consistency implies that $x_2 \cup \{\alpha\} \succ x_2$. **Q.E.D.**

Now, fix some $x^* \in X$ in the interior of X, which is denoted by $\mathrm{int}(X)$. Without loss of generality, we assume that x^* has a nonempty interior. We also fix some $u^* \in S$ and $\beta^* \in \partial x^*$ so that u^* is the normal vector of a tangent to x^* at β^*. For all $\lambda \in \Re$, let $\beta_\lambda \in \Delta(B)$ be such that $\beta_\lambda = \beta^* + \lambda u^*$ (note that there exists some $\lambda > 0$ such that $\beta_\lambda \in \Delta(B)$ because x^* is in the interior of X and both β^* and u^* can be identified with elements of $\Re^{|B|}$). From this construction, it then follows that $\beta_\lambda \notin x^*$ for all $\lambda > 0$.

The following lemma indicates that $x^* \cup \{\beta_\lambda\}$, which is an expansion of menu x^* in the direction of vector u^*, is strictly preferred to x^* for all $\lambda > 0$ because, otherwise, u^* must be irrelevant.

Lemma 4 *For all $\lambda > 0$, $x^* \cup \{\beta_\lambda\} \succ x^*$.*

Proof Lemma 1 implies that $\succsim$ satisfies monotonicity; thus, we have $x^* \cup \{\beta_\lambda\} \succsim x^*$ for all $\lambda > 0$. We assume that $x^* \cup \{\beta_{\hat{\lambda}}\} \sim x^*$ for some $\hat{\lambda} > 0$ and obtain a contradiction. It follows from dominance consistency that $\beta_{\hat{\lambda}}$ is $\succsim^*$-dominated by x^*; that is, $\hat{\beta} \succsim^* \beta_{\hat{\lambda}}$ for some $\hat{\beta} \in x^*$. Because $\succsim^*$ admits an EMU representation $\mathcal{V}$, the latter argument is equivalent to

$$\hat{\beta} \cdot v \geq \beta_{\hat{\lambda}} \cdot v \tag{1}$$

or

$$(\hat{\beta} - \beta^* - \hat{\lambda} u^*) \cdot v \geq 0 \tag{2}$$

for all $v \in \mathcal{V}$.

Now, for all $\lambda > 0$, define $C_1^\lambda \equiv \{\beta \in \Delta(B) : \beta \cdot u^* > \beta_\lambda \cdot u^*\}$ and $C_2^\lambda \equiv \{\beta \in \Delta(B) : \beta \cdot v \geq \beta_\lambda \cdot v$ for all $v \in \mathcal{V}\}$. By construction, we obtain $\beta_{\hat{\lambda}}, \hat{\beta} \in C_2^{\hat{\lambda}}$ and $C_1^\lambda \cap C_2^\lambda = \phi$ for all $\lambda > 0$. To clarify the implication of the lemma, examine the following two cases.

Case 1: $\beta^* \in C_2^{\hat{\lambda}}$. In this case, we can replace $\hat{\beta}$ in (1) by β^*, that is, $\beta^* \cdot v \geq \beta_{\hat{\lambda}} \cdot v = \beta^* \cdot v + \hat{\lambda} u^* \cdot v$ or $u^* \cdot v \leq 0$ for all $v \in \mathcal{V}$. This also implies that $\beta^* \cdot v \geq \beta_\lambda \cdot v$ for all $\lambda > 0$ and $v \in \mathcal{V}$, which gives $\beta^* \succsim^* \beta_\lambda$ for all $\lambda > 0$. Accordingly, dominance consistency implies that $x^* \cup \{\beta_\lambda\} \sim x^*$ for all $\lambda > 0$.

Case 2: $\beta^* \notin C_2^{\hat{\lambda}}$. For all $\xi \in [0, 1]$, define $\gamma_\xi \equiv (1-\xi)\beta^* + \xi\hat{\beta}$. This is included in x^* because x^* is convex and β^*, $\hat{\beta} \in x^*$. Now, fix some $\lambda \in (0, \hat{\lambda})$ and define $f(\xi, \lambda) \equiv \gamma_\xi \cdot v - \beta_\lambda \cdot v = (\xi(\hat{\beta} - \beta^*) - \lambda u^*) \cdot v$. By setting $\xi = \lambda/\hat{\lambda}$ or $\lambda = \xi\hat{\lambda}$, we obtain $f(\xi, \lambda) = (\xi(\hat{\beta} - \beta^*) - \xi\hat{\lambda} u^*) \cdot v$ $= \xi(\hat{\beta} - \beta^* - \hat{\lambda} u^*) \cdot v \geq 0$, where the latter inequality follows from (2). Thus, $\gamma_{\lambda/\hat{\lambda}} \cdot v \geq \beta_\lambda \cdot v$ for all $\lambda \in (0, \hat{\lambda})$ and $v \in \mathcal{V}$, which entails that $\gamma_{\lambda/\hat{\lambda}} \succsim^* \beta_\lambda$. Because $\gamma_{\lambda/\hat{\lambda}} \in x^*$ for all $\lambda \in (0, \hat{\lambda})$, dominance consistency implies that $x^* \cup \{\beta_\lambda\} \sim x^*$ for all $\lambda \in (0, \hat{\lambda})$.

Accordingly, both Cases 1 and 2 imply that $x^* \cup \{\beta_\lambda\} \sim x^*$ for all $\lambda \in (0, \hat{\lambda})$. Next, for a neighborhood N of u^*, we assume that there exists $\bar{x} \in \mathrm{int}(X)$ such that $\bar{x} \supsetneq x^*$, $\bar{x} \succ x^*$, and $\max_{\beta \in \bar{x}} \beta \cdot u = \max_{\beta \in x^*} \beta \cdot u$ for all $u \in S \setminus N$ (otherwise, u^* is not relevant). Because N can be arbitrarily small, we assume, without loss of generality, that $\mathrm{conv}(x^* \cup \{\beta_{\bar{\lambda}}\}) \supseteq \bar{x}$ for some $\bar{\lambda} \in (0, \hat{\lambda})$. It follows from monotonicity that $\mathrm{conv}(x^* \cup \{\beta_{\bar{\lambda}}\}) \succsim \bar{x} \succsim x^*$, which implies that $\mathrm{conv}(x^* \cup \{\beta_{\bar{\lambda}}\}) \sim \bar{x} \sim x^*$ because $x^* \cup \{\beta_{\bar{\lambda}}\} \sim x^*$ and IR holds. However, the latter argument contradicts the assumption that $\bar{x} \succ x^*$. Thus, given a sufficiently small neighborhood N of u^*, there exist no x, $x' \in X$ such that $x \succ x'$ and $\max_{\beta \in x} \beta \cdot u = \max_{\beta \in x'} \beta \cdot u$ for all $u \in S \setminus N$, which contradicts the assumption that u^* is relevant. Thus, we obtain $x^* \cup \{\beta_{\hat{\lambda}}\} \succ x^*$. Because

$\hat{\lambda} > 0$ can be chosen arbitrarily, this completes the proof. **Q.E.D.**

Now, for a neighborhood N of a given $u^* \in S$ and x_1, $x_2 \in X$, assume that $x_1 \supsetneq x_2$ and $\max_{\beta \in x_1} \beta \cdot u' = \max_{\beta \in x_2} \beta \cdot u'$ for all $u' \in S \backslash N$. Then, there exist $\lambda > 0$ and $\beta_\lambda = \beta^* + \lambda u^* \in \Delta(B)$ such that $\beta^* \in \partial x_2$, u^* is the normal vector of a tangent to x_2 at β^*, and $x_1 \supsetneq \operatorname{conv}(x_2 \cup \{\beta_\lambda\})$ $\supsetneq x_2$. For all such u^*, N, x_1, x_2, and λ, it follows from Lemmas 2, 3, and 4 that $x_1 \succsim x_2 \cup \{\beta_\lambda\}$ $\succ x_2$, and thus, $x_1 \succ x_2$. IR extends this result to all x_1, $x_2 \in \mathcal{K}(\Delta(B))$. This implies that g is strictly increasing on $\mathcal{U}^*(S)$, which entails that $\succsim$ admits an OEU representation.

Finally, the following lemma indicates the equivalence of $\mathcal{V}$ and S.

Lemma 5 $\mathcal{V} = S$.

Proof First, because $\succsim$ admits an OEU representation (S, g), for all $\alpha \in \Delta(B)$ and $x \in \mathcal{K}(\Delta(B))$, $x \cup \{\alpha\} \succ x$ if and only if $\alpha \in Y_x \equiv \bigcup_{u \in S} \{\alpha' \in \Delta(B) : \alpha' \cdot u > \beta \cdot u$ for all $\beta \in \operatorname{conv}(x)\}$. However, because $\succsim^*$ admits an EMU representation $\mathcal{V}$, for all $\alpha \in \Delta(B)$ and $x \in \mathcal{K}(\Delta(B))$, α $\in \Delta(B)$ is $\succsim^*$-undominated by x (i.e., $\alpha \succ^* \beta$ or $\alpha \bowtie^* \beta$ for all $\beta \in \operatorname{conv}(x)$) if and only if $\alpha \in$ $Z_x \equiv \bigcup_{v \in \mathcal{V}} \{\alpha' \in \Delta(B) : \alpha' \cdot v > \beta \cdot v$ for all $\beta \in \operatorname{conv}(x)\}$.

Accordingly, if we assume that $\mathcal{V} \setminus S$ is nonempty, $\alpha \in Z_x \setminus Y_x$ exists for some $x \in \mathcal{K}(\Delta(B))$. However, for such an α, we obtain $x \cup \{\alpha\} \sim x$ while α is $\succsim^*$-undominated by x, which contradicts dominance consistency. Thus, we have $\mathcal{V} \subseteq S$.

Conversely, assume that $S \setminus \mathcal{V}$ is nonempty, which implies that $\alpha \in Y_x \setminus Z_x$ exists for some $x \in \mathcal{K}(\Delta(B))$. It follows that $x \cup \{\alpha\} \succ x$, but α is $\succsim^*$-dominated by x. Again, this contradicts dominance consistency, which implies that $S \subseteq \mathcal{V}$. **Q.E.D.**

We have shown that $\succsim^*$ admits an EMU representation $\mathcal{V}$ and $\succsim$ admits an OEU representation (S, g) with $\mathcal{V} = S$, which concludes the proof. **Q.E.D.**

Proof of Theorem 2 We only prove the sufficiency part. First, Lemma 1 shows that dominance consistency implies monotonicity. Accordingly, weak order, continuity (b), and weak menu independence derive an OEU representation (S, g), as shown by Dekel et al. (2001, 2007). We normalize S so that each $u \in S$ (i.e., $U(\cdot, s)$ in Dekel et al.'s (2001) notation) is included in $\mathcal{U}$.

Next, let α, $\beta \in \Delta(B)$ and $x = \{\beta\}$ be such that $x \cup \{\alpha\} \sim x$. From dominance consistency, it follows that $\beta \succsim^* \alpha$. Because $x \cup \{\alpha\} \supsetneq x$, weak menu independence also implies that, for all $\lambda \in (0, 1)$ and $\gamma \in \Delta(B)$, $\lambda(x \cup \{\alpha\}) + (1 - \lambda)\{\gamma\} \sim \lambda x + (1 - \lambda)\{\gamma\}$. It then follows that $\{\lambda\alpha + (1 - \lambda)\gamma, \lambda\beta + (1 - \lambda)\gamma\} \sim \{\lambda\beta + (1 - \lambda)\gamma\}$. However, as a consequence of dominance consistency, $\lambda\beta+(1-\lambda)\gamma \succsim^* \lambda\alpha+(1-\lambda)\gamma$, which implies alternative independence for the weak preference $\succsim^*$. Continuity (a) also establishes alternative independence for the strict preference $\succ^*$ and the indecisive relation $\bowtie^*$.

As shown by Dubra et al. (2004), preorder, continuity (a), and alternative independence derive an EMU representation $\mathcal{V}$ of $\succsim^*$. The equivalence between $\mathcal{V}$ and S can be proved in a manner similar to that of Theorem 1. **Q.E.D.**

Proof of Corollary 3 We only prove the sufficiency part. First, Lemma 1 shows that dominance consistency implies monotonicity. Next, weak order, continuity (b), and menu independence derive an AEU representation (S, g), as shown by Dekel et al. (2001). We normalize S so that each $u \in S$ is included in $\mathcal{U}$.

Because AEU is a special case of OEU under monotonicity, which follows from dominance consistency, Theorem 2 implies that $\succsim^*$ admits an EMU representation $\mathcal{V}$ such that $\mathcal{V} = S$. **Q.E.D.**

Proof of Proposition 4 To prove statement (a), assume that $\alpha \succsim^* \beta$ for some given α, $\beta \in \Delta(B)$. As $\succsim^*$ admits an EMU representation $\mathcal{V}$, it follows that $\alpha \cdot v \geq \beta \cdot v$ for all $v \in \mathcal{V}$. However, because $\mathcal{V} = S$, this also implies that $(\alpha \cdot u)_{u\in S} \geq (\beta \cdot u)_{u\in S}$ (here, $(\alpha \cdot u)_{u\in S}$ and $(\beta \cdot u)_{u\in S}$ denote the vectors generated by $\alpha \cdot u$ and $\beta \cdot u$ for all $u \in S$). As $\succsim$ admits an OEU representation (S, g), it follows that $g((\alpha \cdot u)_{u\in S}) \geq g((\beta \cdot u)_{u\in S})$ because g is strictly increasing on $\mathcal{U}^*(S)$; in particular, $g((\alpha \cdot u)_{u\in S}) > g((\beta \cdot u)_{u\in S})$ unless $(\alpha \cdot u)_{u\in S} = (\beta \cdot u)_{u\in S}$. Accordingly, we have $\{\alpha\} \succsim \{\beta\}$.

To prove statement (b), assume that $\{\alpha\} \succsim \{\beta\}$ for some given α, $\beta \in \Delta(B)$. Because $\succsim$ admits an OEU representation (S, g), the latter condition implies that $g((\alpha \cdot u)_{u\in S}) \geq g((\beta \cdot u)_{u\in S})$. Now, suppose that $\beta \succ^* \alpha$. Then, the argument in the previous paragraph implies that $g((\beta \cdot u)_{u\in S}) > g((\alpha \cdot u)_{u\in S})$, which is a contradiction. Accordingly, we obtain $\alpha \succsim^* \beta$ or $\alpha \bowtie^* \beta$. **Q.E.D.**

Proof of Proposition 5 First, as α is $\succsim^*$-undominated by x for some $x \in \mathcal{K}(\Delta(B))$ and $\alpha \in \Delta(B)$, $\alpha \succ^* \beta$ or $\alpha \bowtie^* \beta$ for all $\beta \in \mathrm{conv}(x)$. Because $\succsim^*$ admits an EMU representation $\mathcal{V}$, the latter argument implies that there exists $v \in \mathcal{V}$ such that $\alpha \cdot v > \beta \cdot v$ for all $\beta \in x$. However, because $\succsim$ admits an OEU representation (S, g), $x \cup \{\alpha\} \succ x$ for some $\alpha \in \Delta(B)$ and $x \in \mathcal{K}(\Delta(B))$ implies that there exists $u \in S$ such that $\alpha \cdot u > \beta \cdot u$ for all $\beta \in x$.

Accordingly, weak dominance consistency (a) is satisfied if and only if $v \in \mathcal{V}$ implies that $v \in S$, from which it follows that $\mathcal{V} \subseteq S$. Similarly, weak dominance consistency (b) is satisfied if and only if $S \subseteq \mathcal{V}$. **Q.E.D.**

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