Distributed Dynamic Pricing for Car-sharing Systems with Stochastic Demand Shift

Kazunori Sakurama and Takanori Aoki

***Abstract*— This study investigated one-way car-sharing systems under dynamic pricing. Despite their convenience, one-way car-sharing systems have the limitation that vehicles can be unevenly parked according to the demand of customers. To distribute car parking, we consider introducing dynamic pricing in which customers can shift their demand (i.e., change origins and destinations by walking) according to usage prices. A model of this system is developed with consideration of stochastic processes representing human behavior in the demand shift. Furthermore, we develop a common, distributed dynamic pricing policy to minimize the unevenness of the vehicles according to the network topology representing the layout of stations. Numerical examples using a realistic traffic simulator demonstrate the effectiveness of the developed method.**

I. Introduction

Car-sharing services have become popular as a new transportation form due to several advantages, including low cost and traffic reduction. In particular, in a station-based one-way car-sharing service, customers can return the vehicle to any available station. This service is promising in urban areas because normal parking slots can be utilized as stations. This service has rapidly spread in Europe and the U.S. In Japan, the round-trip model was initially employed due to legal restrictions, but one-way service has been legally allowed since 2014. Therefore, one-way car sharing is currently being developed, and many field trials are underway [1], including Ha:mo Ride Toyota [2].

One-way car-sharing services have been eagerly investigated, as reviewed in [3]. Despite the convenience, this service has the critical limitation that vehicles become unevenly distributed among stations due to customer demand, which causes the problem that no vehicles are parked in some stations, while no parking slots are available in other stations. To solve this problem, vehicles must be redeployed to maintain an even distribution. This redeployment is generally performed by the service staff driving the vehicles one at a time, which involves significant labor costs. Hence, an inexpensive redeployment strategy is indispensable to the practical implementation of the one-way car-sharing service. Previous studies have considered the problem of reducing the cost of redeployment. In [4], the model predictive control method was employed to minimize the cost. In [5], a model was developed in which the behavior of staff for redeployment was considered. In [6], a simulation-based study was conducted to investigate a car-sharing system in a resort area in Southern California to minimize the number of redeployments.

Another solution is to introduce dynamic pricing, which can naturally encourage customers to evenly distribute vehicles by adjusting prices appropriately at every time interval. In [7], a model was developed in which the price was determined to maximize both the customers' and operating company's profits. The model developed in [8] aimed at maximizing profits with consideration of the allocation of redeployment staff. In [9], optimal pricing and charge scheduling of an electric vehicle sharing system were investigated. Additionally, [10] and [11] investigated pricing policies for the one-way car sharing service. In most models presented in these papers, only the increases/decreases in demand were considered. In contrast, the authors considered a model with demand shift [13], which represents the customer behavior of shifting links (i.e., changing origins and destinations by walking) to cheaper ones. This scenario was first adopted for bike sharing systems [12], while it is also feasible in car-sharing systems in urban areas because numerous stations are located within walking distance of each other.

In this paper, a one-way car sharing system with dynamic pricing is theoretically analyzed with consideration of demand shift. First, this system is modeled with four components: parking-slot, reservation, demand, and demand-shift models. In particular, a demand-shift model, which represents the voluntary behavior of shifting to other links according to prices, is introduced with stochastic processes to describe the uncertain behavior of customers. Subsequently, based on the models, the best common, distributed dynamic pricing policy is designed to minimize the unevenness of occupied parking slots. Finally, the effectiveness of the developed method is illustrated through numerical examples using the realistic traffic simulator, Simulation On Urban road Network with Dynamic route choice (SOUND) [14].

The differences from the previous papers [12], [13] considering demand shift are as follows. First, this paper theoretically analyzes the system and designs the best distributed dynamic pricing policy, while [12], [13] did not provide any theoretical results. In particular, by simplifying the models in this paper, the essence of the dynamics of the target system is revealed such that the dynamics are equivalent to a consensus control system with external disturbance, which has been well-studied in the field of control engineering [15]. Second, we design a common, distributed dynamic pricing policy, which is advantageous for large-scale systems

This work was partially supported by the joint project of Kyoto University and Toyota Motor Corporation, titled "Advanced Mathematical Science for Mobility Society".

Kazunori Sakurama and Takanori Aoki are with Graduate School of Informatics, Kyoto University, Yoshida-honmachi, Kyoto, Japan sakurama@i.kyoto-u.ac.jp, aoki.takanori.35z@st.kyoto-u.ac.jp.

because the price of each link can be easily computed with the information on neighboring stations. This contrasts with other centralized optimization-based methods, including [12]. Third, this paper verifies the validity of the models through a realistic traffic simulator, while [12], [13] did not conduct such simulations.

The remainder of this paper is organized as follows. Section 2 describes models in the target system and provides a control objective. In Section 3, as a main result, the best distributed dynamic pricing policy is developed. Section 4 presents simulation results. Finally, Section 5 concludes the paper.

II. Problem formulation

A. Notation

Let $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{Z}$, and $\mathbb{Z}_+$ be the sets of real numbers, non-negative real numbers, integers, and non-negative integers, respectively. For $\kappa \in \mathbb{R}$, define $\kappa\mathbb{Z} = \{\ldots, -2\kappa, -\kappa, 0, \kappa, 2\kappa, \ldots\}$. The floor and ceiling functions with respect κ are defined for $x \in \mathbb{R}$ as follows:

$$\lfloor x \rfloor_\kappa = \max\{y \in \kappa\mathbb{Z} : y \leq x\},$$
$$\lceil x \rceil_\kappa = \min\{y \in \kappa\mathbb{Z} : y \geq x\}.$$

The identity matrix is denoted by $I \in \mathbb{R}^{n\times n}$, the unit vector with the ith entry 1 is denoted by $e_i \in \mathbb{R}^n$, and $\mathbf{1} = [1 \ \cdots \ 1]^\top$. For $a_1, \ldots, a_n \in \mathbb{R}$, $\mathrm{diag}(a_1, \ldots, a_n)$ represents the diagonal matrix whose ith diagonal entry is a_i. For a vector $a = [a_1 \ \cdots \ a_n]^\top \in \mathbb{R}^n$, $\mathrm{diag}(a) = \mathrm{diag}(a_1, \ldots, a_n)$.

Let $x : \mathbb{Z}_+ \to \mathbb{Z}_+$ be a stochastic variable of time $t \in \mathbb{Z}_+$ with a domain of non-negative integers. The probability that $x(t)$ takes the value $x \in \mathbb{Z}_+$ is represented as $\Pr(x(t) = x) \in [0, 1]$. For a function $f : \mathbb{Z}_+ \to \mathbb{R}$ of $x(t)$, the expectation of $f(x(t))$ is defined as

$$\mathrm{E}[f(x(t))] = \sum_{x=0}^{\infty} f(x)\Pr(x(t) = x).$$

For random variables $x, y : \mathbb{Z}_+ \to \mathbb{Z}_+$ of time t, $\Pr(x(t) = x, y(t) = y) \in [0, 1]$ represents the joint probability that $x(t), y(t)$ take the values of $x, y \in \mathbb{Z}_+$, respectively. Let $\Pr(x(t) = x | y(t) = y)$ represent the conditional probability that $x(t)$ takes the value of $x \in \mathbb{Z}_+$ under the condition that $y(t)$ takes the value of $y \in \mathbb{Z}_+$, which is given as follows:

$$\Pr(x(t) = x | y(t) = y) = \frac{\Pr(x(t) = x, y(t) = y)}{\Pr(y(t) = y)}.$$

For a function $f : \mathbb{Z}_+ \to \mathbb{R}$ of $x(t)$, the conditional expectation of $f(x(t))$ under the condition that $y(t)$ takes the value of $y \in \mathbb{Z}_+$ is defined as

$$\mathrm{E}[f(x(t))|y(t) = y] = \sum_{x=0}^{\infty} f(x)\Pr(x(t) = x|y(t) = y).$$

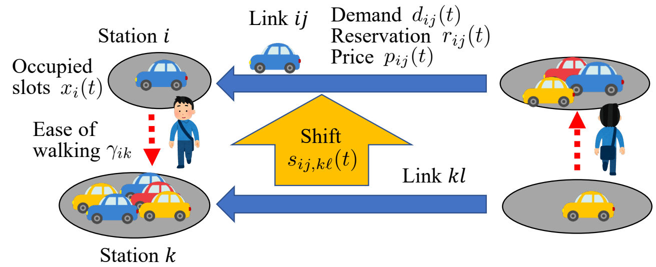

Fig. 1: Sketch of the target car-sharing system.

TABLE I: Variables and constants in the models.

$x_i(t)$	# of occupied and reserved parking slots at station i
$x_i^{\max}$	capacity of the parking slots at station i
$x_i^+(t)$	expected next number of $x_i(t)$ for demands $d_{ij}(t)$
$r_{ij}(t)$	# of reservations of vehicles for link ij
$d_{ij}(t)$	# of demands for link ij
$\hat{d}_{ij}(t)$	# of original demands for link ij
$p_{ij}(t)$	price of link ij
$\hat{p}_{ij}$	standard price of link ij
$u_{ij}(t)$	# of unfulfilled demands for link ij
δ_{ij}	expectation of the original demand number $\hat{d}_{ij}(t)$
γ_{ik}	ease of changing stations from k to i by walking
$s_{ij,k\ell}(t)$	# of demand shifts from link $k\ell$ to ij
$c_{ij,k\ell}(t)$	credible # of demand shifts from link $k\ell$ to ij
n	# of stations
m	total # of vehicles
$m^{\max}$	sum of all capacities
κ	unit of price in the car-sharing service
$\phi(\cdot)$	sensitivity of customers to prices

B. System models

The car-sharing system considered in this paper consists of $n \in \mathbb{Z}_+$ stations. Let $\mathcal{N} = \{1, 2, \ldots, n\}$ be the index set of stations. A specific station is denoted as $i \in \mathcal{N}$, and the travel from station j to i is called link ij, that is, the origin and destination of link ij are stations j and i, respectively. The sampling time of the system is determined from the time interval of changing prices, corresponding to the discrete time $t \in \mathbb{Z}_+$.

This system consists of four models: parking-slot, reservation, demand, and demand-shift models. Figure 1 shows a sketch of the system. The variables and constants in the models are summarized in Table I.

1) Parking-slot model: First, we model the number of occupied and reserved parking slots at station $i \in \mathcal{N}$, denoted by $x_i(t) \in \mathbb{Z}_+$.

For capacity $x_i^{\max} \in \mathbb{Z}_+$ of the parking slots at station i, $x_i(t)$ must satisfy

$$0 \leq x_i(t) \leq x_i^{\max} \ \forall t \in \mathbb{Z}_+. \tag{1}$$

The sum of the numbers $x_i(t)$ is preserved, i.e.,

$$\sum_{i\in\mathcal{N}} x_i(t) = m \tag{2}$$

always holds, where $m \in \mathbb{Z}_+$ is the total number of vehicles. For the sum of capacities of the stations

$$m^{\max} = \sum_{i\in\mathcal{N}} x_i^{\max}, \tag{3}$$

m is assumed to be considerably smaller than $m^{\max}$.

Let $r_{ij}(t) \in \mathbb{Z}_+$ be the number of reservations of the vehicles for link ij. Parking slots are reserved according to $r_{ij}(t)$, and $x_i(t)$ varies as

$$x_i(t+1) = x_i(t) + \sum_{j \in \mathcal{N}} (r_{ij}(t) - r_{ji}(t)), \tag{4}$$

where $r_{ij}(t)$ ($r_{ji}(t)$) represents the number of reservations of vehicles entering station i (j) from station j (i).

2) Reservation model: Next, we consider the number of reservations $r_{ij}(t)$, which is determined according to three rules: (i) the reservations must be less than the demands, i.e., $r_{ij}(t) \leq d_{ij}(t)$, where $d_{ij}(t) \in \mathbb{Z}_+$ is the number of demands for link ij, (ii) $x_i(t+1)$ in (4) must satisfy the capacity restriction of (1), (iii) the unfulfilled demand $u_{ij}(t) = d_{ij}(t) - r_{ij}(t) \in \mathbb{Z}_+$ must be minimized.

The service is usually conducted according to the first-come-first-served rule, so (iii) cannot be strictly satisfied. However, $u_{ij}(t)$ can be zero, i.e., $r_{ij}(t) = d_{ij}(t)$, as long as the capacities of the stations are satisfied with the demands, i.e.,

$$0 \leq x_i^+(t) \leq x_i^{\max},\ 0 \leq x_j^+(t) \leq x_j^{\max} \tag{5}$$

hold for

$$x_i^+(t) = x_i(t) + \sum_{j \in \mathcal{N}} (d_{ij}(t) - d_{ji}(t)). \tag{6}$$

Under this condition, we can assign $r_{ij}(t) = d_{ij}(t)$, and $x_i(t+1)$ in (4), equal to $x_i^+(t)$, satisfies (1).

From this viewpoint, to fulfill (iii), we must simply control $x_i^+(t)$ to satisfy (5) for all $i, j \in \mathcal{N}$.

3) Demand model: Next, we model $d_{ij}(t)$, the number of demands for vehicles for link ij, which is changed according to the prices of links.

Let $\hat{d}_{ij}(t) \in \mathbb{Z}_+$ be the number of original demands for link ij with a standard price. We assume that the original demand number $\hat{d}_{ij}(t)$ follows a random variance with expectation $\delta_{ij} > 0$, i.e.,

$$\mathrm{E}[\hat{d}_{ij}(t)] = \delta_{ij}. \tag{7}$$

A typical example of such a random variance is the Poisson distribution. We assume that $\hat{d}_{ij}(t)$ are independent for any $i, j \in \mathcal{N}$ and $t \in \mathbb{Z}_+$.

Customers shift the origins and destinations of links by walking according to the prices of links if changing the origins and destinations is easy. Let $s_{ij,k\ell}(t) \in \mathbb{Z}_+$ be the number of customers who shift links from $k\ell$ to ij, that is, the origin changes from ℓ to j and the destination changes from k to i. Thus, the demand number $d_{ij}(t)$ is changed from the original one $\hat{d}_{ij}(t)$ by the demand shifts $s_{ij,k\ell}(t)$ as follows:

$$d_{ij}(t) = \hat{d}_{ij}(t) + \sum_{k\ell \in \mathcal{N}^2} (s_{ij,k\ell}(t) - s_{k\ell,ij}(t)), \tag{8}$$

where $s_{ij,k\ell}(t)$ ($s_{k\ell,ij}(t)$) represents the number of customers who shift into link ij from other links (who shift away from link ij to other links).

4) Demand-shift model: Next, we model the demand shift number $s_{ij,k\ell}(t)$, which is determined from the prices of links and ease of changing links.

Let $p_{ij}(t) \in \kappa\mathbb{Z}$ be the price of link ij, where a positive integer $\kappa \in \mathbb{Z}_+$ ($\kappa > 0$) represents the unit of price in the car-sharing service (e.g., one dollar or ten yen). The ease of walking between stations i and k is described by $\gamma_{ik} (= \gamma_{ki}) \geq 0$, which can be evaluated according to the distance between stations. In general, the farther apart stations i and k are, the smaller the ease γ_{ik} is. Customers are informed of two pieces of information by the operator: (i) the difference $p_{k\ell}(t) - p_{ij}(t)$ of prices of links (the benefit received from shifting links) and (ii) the ease $\gamma_{ik}, \gamma_{j\ell}$ of changing stations by walking (the cost of shifting links). The shift is assumed to randomly occur with expectation as

$$\mathrm{E}[s_{ij,k\ell}(t) | P(t) = P] = \gamma_{ik}\gamma_{j\ell}\phi(p_{ij} - p_{k\ell}), \tag{9}$$

where $P(t) \in \mathbb{Z}^{n \times n}$ represents a matrix whose (i,j)-entry is $p_{ij}(t)$, and similarly $P \in \mathbb{Z}^{n \times n}$ has (i,j)-entry p_{ij}. Here, the function $\phi : \mathbb{Z} \to \mathbb{R}_+$ is a monotonically non-increasing function satisfying $\phi(p) = 0$ for $p \geq 0$, which implies that customers tend to shift to cheaper links and do not shift to more expensive ones.

C. Control objective

The control objective is to distribute car parking to reduce the unfulfilled demands. As discussed at the end of Section II-B.2, this is achieved by controlling $x_i^+(t)$ to satisfy the inequality condition of (5) for all $i \in \mathcal{N}$. Here, we employ a soft constraint to solve (5) by considering the discrepancy $\bar{x}_i^+(t) = x_i^+(t) - x_i^{\max}/2$ of the occupied and reserved parking slots from the desired number (half the capacity) as a penalty. Our goal is to achieve an even distribution of penalties across all stations, which can be evaluated with the sample variance of the penalties, defined as

$$V_{\mathrm{x}}(x(t)) = \frac{1}{n} \sum_{i \in \mathcal{N}} \mathrm{E}\left[\bar{x}_i^+(t) - \frac{1}{n} \sum_{i \in \mathcal{N}} \bar{x}_i^+(t)\right]^2, \tag{10}$$

where $x(t) = [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^\top$.

To maintain the even distribution, we employ a common, distributed dynamic pricing policy, which is advantageous for large-scale systems because the price of each link can be easily computed with the information on neighboring stations. Let $\hat{p}_{ij} = \hat{p}_{ji} \in \kappa\mathbb{Z}$ be the standard price between stations i and j in the car-sharing service, and the price $p_{ij}(t)$ is adjusted on the common, distributed dynamic pricing policy

$$p_{ij}(t) = \hat{p}_{ij} + \pi(\bar{x}_i(t), \bar{x}_j(t)) \tag{11}$$

with a function $\pi : \mathbb{Z}_+^2 \to \kappa\mathbb{Z}$, where $\bar{x}_i(t) = x_i(t) - x_i^{\max}/2$. Then, π is common among links and is distributed. The discrepancy of the price $p_{ij}(t)$ from the standard price $\hat{p}_{ij}$ is preferably small for customer convenience. From (11), the squared discrepancy is evaluated according to

$$V_{\mathrm{p}}(x(t)) = \frac{1}{n^2} \sum_{i,j \in \mathcal{N}} \mathrm{E}[\pi(\bar{x}_i(t), \bar{x}_j(t))]^2, \tag{12}$$

which should be minimized. For simplicity, $\pi : \mathbb{Z}_+^2 \to \kappa\mathbb{Z}$ is assumed to be an affine function given as

$$\pi(\bar{x}_i, \bar{x}_j) = \hat{\pi}_{\rm a}\bar{x}_i + \hat{\pi}_{\rm b}\bar{x}_j + \hat{\pi}_{\rm c},\ \ \hat{\pi}_{\rm a}, \hat{\pi}_{\rm b}, \hat{\pi}_{\rm c} \in \kappa\mathbb{Z}, \tag{13}$$

where $\hat{\pi}_{\rm a}, \hat{\pi}_{\rm b}, \hat{\pi}_{\rm c} \in \kappa\mathbb{Z}$ are design parameters.

We expect to design a common, distributed dynamic pricing policy to minimize $V_{\rm x}(x(t))$ and $V_{\rm p}(x(t))$. Here, we evaluate the terminal value of the sum of these values, i.e.,

$$\begin{aligned} V(\hat{\pi}_{\rm a}, \hat{\pi}_{\rm b}, \hat{\pi}_{\rm c}) = \max_{m\in\mathbb{Z}} \lim_{t\to\infty} (V_{\rm x}(\bar{x}(t)) + \mu V_{\rm p}(x(t))) \\ + \nu(\hat{\pi}_{\rm a}^2 + \hat{\pi}_{\rm b}^2 + \hat{\pi}_{\rm c}^2) \end{aligned} \tag{14}$$

with constants $\mu, \nu > 0$, where the last term is a regularization term. The maximum with respect to $m \in \mathbb{Z}$ is used in (14) to design a scalable dynamic pricing policy π, which is valid regardless of the number of vehicles m.

The target problem in this paper is summarized as follows.

Problem 1: Design a common, distributed dynamic pricing policy of (11) with a function $\pi : \mathbb{Z}_+^2 \to \kappa\mathbb{Z}$ of the form (13) that is the solution to the optimization problem

$$\min_{\hat{\pi}_{\rm a}, \hat{\pi}_{\rm b}, \hat{\pi}_{\rm c} \in \kappa\mathbb{Z}} V(\hat{\pi}_{\rm a}, \hat{\pi}_{\rm b}, \hat{\pi}_{\rm c}) \tag{15}$$

for the system consisting of the models of (1)–(9).

III. Main result

To obtain an analytic solution to Problem 1, we consider the linearized system valid in the neighborhood of the equilibrium point, and derive the following theorem.

Theorem 1: Let $x_i(t) \in \mathbb{Z}_+$, $i \in \mathcal{N}$ be the solution of the linearized system of (1)–(9), where $\mathrm{d}\phi/\mathrm{d}p(p_*) = -\hat{\phi}$ for some constant $\hat{\phi} > 0$ at the equilibrium point $p_* < 0$. Let $\Delta, \Gamma \in \mathbb{R}^{n\times n}$ be the matrices whose (i,j)-entries are δ_{ij}, γ_{ij}, respectively. For $L = \mathrm{diag}(\Gamma\mathbf{1}) - \Gamma \in \mathbb{R}^{n\times n}$, let $\lambda_2, \ldots, \lambda_n (\lambda_2 \le \cdots \le \lambda_n)$ be the positive eigenvalues of L with corresponding eigenvectors given by $v_2, \ldots, v_n \in \mathbb{R}^n$. Assume that the graph with adjacency matrix Γ is connected. Subsequently, the solution to the optimization problem of (15) is obtained for $\pi : \mathbb{Z}_+^2 \to \kappa\mathbb{Z}$ in (11) of the form (13) only if $\hat{\pi}_{\rm a}, \hat{\pi}_{\rm b}, \hat{\pi}_{\rm c} \in \kappa\mathbb{Z}$ satisfy

$$\hat{\pi}_{\rm a} \in \left\{ \left\lfloor \sqrt{\frac{\|h\|}{2\sqrt{2n\nu}}} \right\rfloor_\kappa, \left\lceil \sqrt{\frac{\|h\|}{2\sqrt{2n\nu}}} \right\rceil_\kappa, \left\lfloor \frac{1}{\lambda_n \sum_{i,j\in\mathcal{N}} \gamma_{ij}} \right\rfloor_\kappa \right\}, \tag{16}$$

$$\hat{\pi}_{\rm a} \le \left\lfloor \frac{1}{\lambda_n \sum_{i,j\in\mathcal{N}} \gamma_{ij}} \right\rfloor_\kappa, \tag{17}$$

$$\hat{\pi}_{\rm b} = -\hat{\pi}_{\rm a}, \tag{18}$$

$$\hat{\pi}_{\rm c} = 0, \tag{19}$$

where

$$h = \frac{\sum_{i\in\{2,\ldots,n\}} \lambda_i^{-1} v_i v_i^\top (\Delta - \Delta^\top)\mathbf{1}}{\hat{\phi} \sum_{i,j\in\mathcal{N}} \gamma_{ij}}. \tag{20}$$

The minimum in (15) is derived as follows:

$$\min_{\hat{\pi}_{\rm a}, \hat{\pi}_{\rm b}, \hat{\pi}_{\rm c} \in \kappa\mathbb{Z}} V(\hat{\pi}_{\rm a}, \hat{\pi}_{\rm b}, \hat{\pi}_{\rm c}) = \frac{1}{n}\frac{\|h\|^2}{4\hat{\pi}_{\rm a}^2} + 2\nu\hat{\pi}_{\rm a}^2. \tag{21}$$

Three candidates for solution $\hat{\pi}_{\rm a}$ are given in (16). If one of the first two terms is less than the last one, it is the solution. If both the first two terms are less than the last one, we can verify which term is the solution by computing the value of (21). Otherwise, the last term in (16) is the solution. Equations (18) and (19) show that the dynamic pricing policy π in (13) should be skew-symmetrical, i.e., $\pi(\bar{x}_i, \bar{x}_j) = -\pi(\bar{x}_j, \bar{x}_i)$, for effective pricing.

Proof: Due to the limitation of space, a brief proof is given. From (4), (7), (8), (11), and (13), the expectation of the discrepancy $\bar{x}_i(t) = x_i(t) - x_i^{\max}/2$ is governed around the equilibrium point by the following equation:

$$\begin{aligned} &\mathrm{E}[\bar{x}(t+1)] \\ &\quad = (I - (\hat{\pi}_{\rm a} - \hat{\pi}_{\rm b})\hat{\phi}\mathbf{1}^\top\Gamma\mathbf{1}L)\mathrm{E}[\bar{x}(t)] + (\Delta - \Delta^\top)\mathbf{1}, \end{aligned} \tag{22}$$

where $\bar{x}(t) = [\bar{x}_1(t) \ \cdots \ \bar{x}_n(t)]^\top \in \mathbb{R}^n$. From the eigenvalue analysis, the stability condition of (22) is derived as

$$(\hat{\pi}_{\rm a} - \hat{\pi}_{\rm b})\lambda_n \sum_{i,j\in\mathcal{N}} \gamma_{ij} < 2. \tag{23}$$

Under this condition, The limit $\lim_{t\to\infty} \mathrm{E}[\bar{x}(t)]$ exists and

$$\lim_{t\to\infty} \mathrm{E}[\bar{x}(t)] = \frac{\sum_{i\in\{2,\ldots,n\}} \lambda_i^{-1} v_i v_i^\top (\Delta - \Delta^\top)\mathbf{1}}{(\hat{\pi}_{\rm a} - \hat{\pi}_{\rm b})\hat{\phi} \sum_{i,j\in\mathcal{N}} \gamma_{ij}} + \frac{\bar{m}\mathbf{1}}{n} \tag{24}$$

holds, where $\bar{m} = m - m^{\max}/2$.

We assume that the solution to the optimization problem of (15) is achieved for $\pi : \mathbb{Z}_+^2 \to \kappa\mathbb{Z}$ in (11) of the form (13) with some $\hat{\pi}_{\rm a}, \hat{\pi}_{\rm b}, \hat{\pi}_{\rm c} \in \kappa\mathbb{Z}$. Therefore, $\lim_{t\to\infty} \mathrm{E}[\bar{x}(t)]$ exists, and thus (23) holds. Furthermore, the limit is given as (24), which can be rewritten with h in (20) as

$$\lim_{t\to\infty} \mathrm{E}[\bar{x}(t)] = \frac{h}{\hat{\pi}_{\rm a} - \hat{\pi}_{\rm b}} + \frac{\bar{m}\mathbf{1}}{n}. \tag{25}$$

From (13), (12), and (25),

$$\begin{aligned} &\lim_{t\to\infty} V_{\rm p}(x(t)) \\ &\quad = \frac{1}{n^2} \sum_{i,j\in\mathcal{N}} \left(\frac{(\hat{\pi}_{\rm a} e_i + \hat{\pi}_{\rm b} e_j)^\top h}{\hat{\pi}_{\rm a} - \hat{\pi}_{\rm b}} + \frac{(\hat{\pi}_{\rm a} + \hat{\pi}_{\rm b})\bar{m}}{n} + \hat{\pi}_{\rm c} \right)^2 \end{aligned} \tag{26}$$

is derived. Because $\max_{m\in\mathbb{Z}_+} \lim_{t\to\infty} V_{\rm p}(x(t))$ is bounded, $\hat{\pi}_{\rm a} + \hat{\pi}_{\rm b} = 0$ holds from (26) and $\bar{m} = m - m^{\max}/2$. This yields (18), and

$$\lim_{t\to\infty} V_{\rm p}(x(t)) = \frac{1}{n^2} \sum_{i,j\in\mathcal{N}} \left(\frac{(e_i - e_j)^\top h}{2} + \hat{\pi}_{\rm c} \right)^2 \tag{27}$$

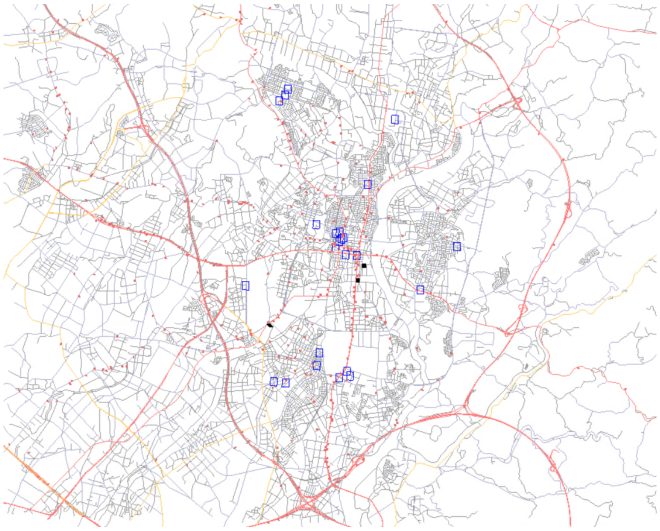

Fig. 2: Scene of simulation using SOUND: $n = 25$ stations (blue squares), sharing cars (solid black squares), and regular cars (solid red squares).

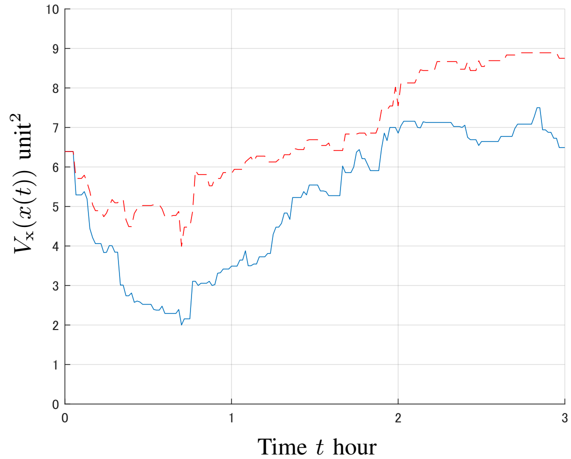

Fig. 3: Sample variance $V_{\mathrm{x}}(x(t))$ of penalties of parking slots with DP (solid line) and without DP (dashed line).

is obtained from (26). Note that $x_i^+(t) = x_i(t+1)$ holds from (4) and (6) because $r_{ij}(t) = d_{ij}(t)$ holds around the equilibrium point. Subsequently, from (10) and (25),

$$\lim_{t\to\infty} V_{\mathrm{x}}(x(t)) = \lim_{t\to\infty} \frac{1}{n}\|\mathrm{E}[C\bar{x}^+(t)]\|^2 = \frac{1}{n}\frac{\|h\|^2}{4\hat{\pi}_{\mathrm{a}}^2} \tag{28}$$

is derived, where $C = I - \mathbf{1}\mathbf{1}^\top/n$ and the facts that $Ch = h$ and $C\mathbf{1} = 0$ are used. From (14), (27), and (28),

$$V(\hat{\pi}_{\mathrm{a}}, \hat{\pi}_{\mathrm{b}}, \hat{\pi}_{\mathrm{c}}) = \frac{1}{n}\frac{\|h\|^2}{4\hat{\pi}_{\mathrm{a}}^2} + \frac{\mu}{n^2}\sum_{i,j\in\mathcal{N}}\left(\frac{(e_i - e_j)^\top h}{2} + \hat{\pi}_{\mathrm{c}}\right)^2 + \nu(2\hat{\pi}_{\mathrm{a}}^2 + \hat{\pi}_{\mathrm{c}}^2) \tag{29}$$

is derived. Equation (29) can be minimized separately using $\hat{\pi}_{\mathrm{a}}$ and $\hat{\pi}_{\mathrm{c}}$. For $\hat{\pi}_{\mathrm{c}} \in \kappa\mathbb{Z}$,

$$\hat{\pi}_{\mathrm{c}} = \frac{1}{n^2}\sum_{i,j\in\mathcal{N}}\frac{(e_i - e_j)^\top h}{2} = 0$$

is obtained, which yields (19). For $\hat{\pi}_{\mathrm{a}}$, from (18) and (23), (17) must be satisfied. If $\hat{\pi}_{\mathrm{a}}$ is a real value and

$$\hat{\pi}_{\mathrm{a}} = \sqrt{\frac{\|h\|}{2\sqrt{2n\nu}}} \tag{30}$$

satisfies (17), (30) is the solution to (29). Because (29) is convex according to $\hat{\pi}_{\mathrm{a}}$, the optimizer $\hat{\pi}_{\mathrm{a}} \in \kappa\mathbb{Z}$ is given by rounding (30) into $\kappa\mathbb{Z}$, which corresponds to the first two terms in (16). If the rounded values do not satisfy (17), the boundary of the inequality provides the solution, which corresponds to the last term in (16). From (29), (21) is derived. ■

IV. Simulation

To verify the effectiveness of the developed method, simulations were conducted using the realistic traffic simulator SOUND [14]. Figure 2 shows a scene of the simulations, in which $n = 25$ stations were scattered over an area of approximately 6×8 km with sharing and general cars. The capacities of the parking slots were all set to 10, i.e., $x_i^{\max} = 10$. $m = 164$ was the number of sharing cars. We considered the system consisting of the models of (1)–(9) with the common, distributed dynamic pricing policy of (11) and (13). The time interval of changing prices was 5 min, which determined the sampling time.

The system parameters were as follows: price elasticity of demand $\hat{\phi} = 2.5 \times 10^{-5}$; expectation of demand $\delta_{ij} = 0.005 \times 60^{-1}$ to 3×60^{-1}; ease of shifting stations $\gamma_{ik} = e^{-\eta\|\rho_i - \rho_k\|}$, where $\rho_i \in \mathbb{R}^2$ is the position of station i in Figure 2 and $\eta = 4.5 \times 10^{-4}$. According to Theorem 1, the pricing policy π in (13) was designed with $\mu = \nu = 0.01$ and $\kappa = 1$. Subsequently, the parameters $\hat{\pi}_{\mathrm{a}} = -\hat{\pi}_{\mathrm{b}} = 1$ and $\hat{\pi}_{\mathrm{c}} = 0$ were designed according to (16), (18), and (19) by solving the optimization problem of (15). Simulations were executed under two conditions: with dynamic pricing (DP) and without DP (fixed price).

Figures 3 and 4 show the sample variance of the penalties of the parking slots $V_{\mathrm{x}}(x(t))$ in (10) and the discrepancy of the prices $V_{\mathrm{p}}(x(t))$ in (12), respectively, with DP (solid lines) and without DP (dashed lines). The value in Figure 3 is restrained with DP (solid line), compared with that without DP increases over time (dashed line). This result demonstrated that the unevenness of the vehicle distribution decreases when the developed pricing policy was used.

Figures 5 and 6 show the incremental values of the unfulfilled demands $\sum_{ij} u_{ij}(t)$ and total income $\sum_{ij} r_{ij}(t)p_{ij}(t)$, respectively, with DP (solid lines) and without DP (dashed lines). These figures indicate that the unfulfilled demand with DP (solid line) was lower than that without DP (dashed line) and the total income with DP (solid line) was consequently larger. This result suggested that introducing DP can increase incomes of car-sharing services owing to the decreases in unfulfilled demand.

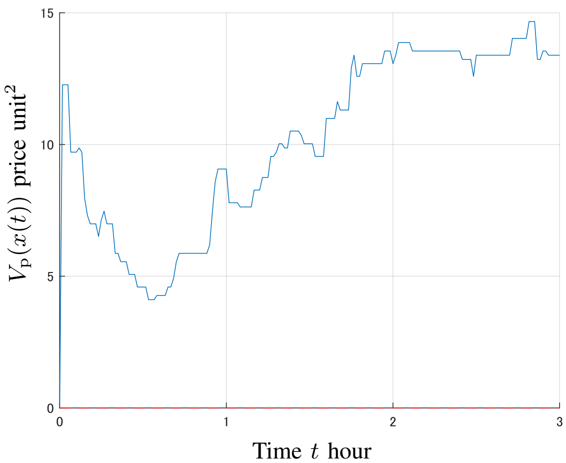

Fig. 4: Discrepancy $V_{\mathrm{p}}(x(t))$ of adjusted prices with DP (solid line) and without DP (dashed line).

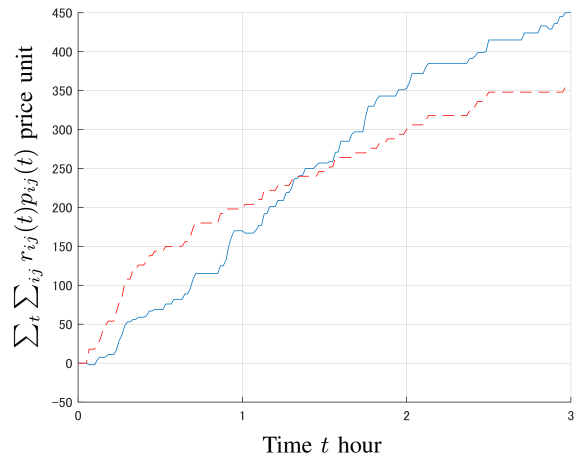

Fig. 6: Incremental value of total income $\sum_{ij} r_{ij}(t)p_{ij}(t)$ with DP (solid line) and without DP (dashed line).

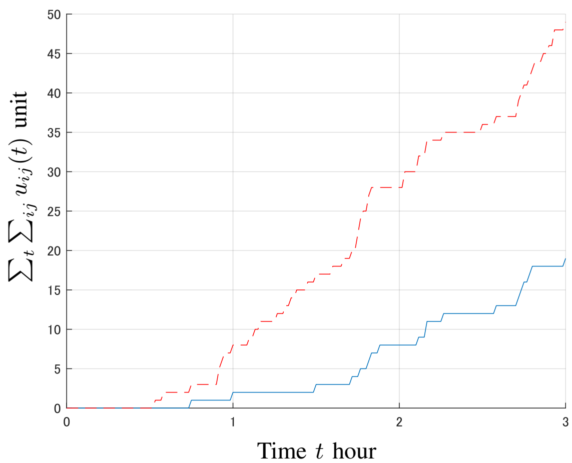

Fig. 5: Incremental value of the numbers of unfulfilled demands $\sum_{ij} u_{ij}(t)$ with DP (solid line) and without DP (dashed line).

V. Conclusions

This study investigated the potential of dynamic pricing to solve the problem of uneven distribution of vehicles in a one-way car-sharing system. To design a dynamic pricing policy, this system was modeled with parking-slot, reservation, demand, and demand-shift models. In particular, the voluntary demand shift of customers was newly modeled with a stochastic process. Based on the simplified models, the unevenness of penalties of the parking slots was estimated based on the results of consensus control systems. Subsequently, the common, distributed dynamic pricing policy was designed with the aim of minimizing the unevenness of penalties and price discrepancies. The effectiveness of the developed method was illustrated through simulations using the realistic traffic simulator by comparing the results with and without DP.

References

[1] Shoshi Mizokami, Takuya Maruyama, Junya Hashimoto, Toshikatsu Mori, Daisuke Sunaga: Research and development on the possibility of introducing a one-way car sharing system and new ways to utilize road space; *New Road Technology Conference* (2019)

[2] Toyota Motor Corp.: Toyota to Start Verification Tests in Japan Using "Ha:mo Ride" Driving Data; https://global.toyota/en/newsroom/corporate/28866957.html (last visited Feb. 25, 2022)

[3] Stefan Illgen, Michael Hock: Literature review of the vehicle relocation problem in one-way car sharing networks; *Transportation Research Part B*, Vol. 120, pp. 193–204 (2019)

[4] Giuseppe Carlo Calafiore, Christian Bongiorno, Alessandro Rizzo: A control-oriented model for mobility on demand systems; *IEEE Conference on Decision and Control (CDC)* (2018)

[5] Stephen L. Smith, Marco Pavone, Mac Schwager, Emilio Frazzoli, Daniela Rus: Rebalancing the Rebalancers: Optimally routing vehicles and drivers in mobility on-demand systems; *American Control Conference (ACC)* (2013)

[6] Matthew Barth, Michael Todd: Simulation model performance analysis of a multiple station shared vehicle system; *Transportation Research Part C: Emerging Technologies*, Vol. 7, No. 4, pp. 237–258 (1999)

[7] Diana Jorge, Goran Molnar, Goncalo H. A. Correia: Trip pricing of one-way station based carsharing networks with zone and time of day price variations; *Transportation Research Part B: Methodological*, Vol. 8, No. 2, pp. 461–482 (2015)

[8] Min Xu, Qiang Meng, Zhiyuan Liu: Electric vehicle fleet size and trip pricing for one-way carsharing services considering vehicle relocation and personnel assignment; *Transportation Research Part B: Methodological*, Vol. 111, pp. 60–82 (2018)

[9] Rui Xie, Wei Wei, Qiuwei Wu, Tao Ding, Shengwei Mei: Optimal service pricing and charging scheduling of an electric vehicle sharing system; *IEEE Transactions on Vehicular Technology*, Vol. 69, No. 1, pp. 78-89 (2020)

[10] Lei Wang, Wanjing Ma: Pricing approach to balance demands for one-way car-sharing systems; *Proc. of the 22nd IEEE Intelligent Transportation Systems Conference (ITSC)* (2019)

[11] Toya Kamatani, Yusuke Nakata, Sachiyo Arai: Dynamic pricing method to maximize utilization of one-way car sharing service; *2019 IEEE International Conference on Agents (ICA)* (2019)

[12] Zulqarnain Haidera, Alexander Nikolaevb, Jee Eun Kangb, and Changhyun Kwona: Inventory rebalancing through pricing in public bike sharing systems *European Journal of Operational Research*, Vol. 270, pp. 103-117 (2018)

[13] Kazuki Sumida, Kazunori Sakurama, Takanori Aoki: Demand Shift Model of a One-Way Car-Sharing System with Real-Time Pricing; *Proc. of the 24th IEEE International Intelligent Transportation Systems Conference (ITSC)* (2021)

[14] i-Transport Lab. Co., Ltd. SOUND Ver.5, a traffic simulator for wide area road networks; https://www.i-transportlab.jp/en/index/products/sound/ (last visited Feb. 25, 2022)

[15] Mehran Mesbahi and Magnus Egerstedt, Graph Theoretic Methods in Multiagent Networks, *Princeton University Press* (2010)