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"Advanced Engineering Mathematics"

"Unit – I"

Optimization without constraints.

(i) multivariable $\rightarrow$ Hessian Matrix method.

$f(x,y)$ $\qquad\qquad$ $f(x,y,z)$

Neccessary condition

$\frac{\partial f}{\partial x} = 0$ $\qquad\qquad$ $\frac{\partial f}{\partial x} = 0$

$\frac{\partial f}{\partial y} = 0$ $\qquad\qquad$ $\frac{\partial f}{\partial y} = 0$

(2-variable) $\qquad\qquad$ $\frac{\partial f}{\partial z} = 0$ (3-variable)

$\Rightarrow$ Solving to get the values of x, y, z.

"Sufficient Conditions":-

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\ \frac{\partial^2 f}{\partial z \partial x} & \frac{\partial^2 f}{\partial z \partial y} & \frac{\partial^2 f}{\partial z^2} \end{bmatrix} \Leftrightarrow \begin{bmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{bmatrix}$$

H_1 = determinent value of f_{xx}.

H_2 = determinent value of $\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$

H_3 = determinent value of $\begin{vmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{vmatrix}$

	Case I	Case II	
$H_1 =$	$-$	$+$	Otherwise neither maxima or nor minima (saddle point)
$H_2 =$	$+$	$+$	
$H_3 =$	$-$	$+$	
	maxima	minima	

Que. Find maxima and minima of the function.

$f(x,y,z) = x^2+y^2+z^2+4x+6y+8z+56.$

Necessary condition: $\frac{\partial f}{\partial x} = 2x+4 = 0 \Rightarrow 0 \Rightarrow x = -2$

$\frac{\partial f}{\partial y} = 2y+6 \Rightarrow 0 \Rightarrow y = -3$

$\frac{\partial f}{\partial z} = 0 = 2z+8 = 0 \Rightarrow z = -4$

Sufficient condition. $H = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$H_1 = 2 \; +ve$
$H_2 = 4 \; +ve$
$H_3 = 8 \; +ve$

minima

$F(x,y,z)$ is minima.

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Ques: Find maxima or minima

$f(x,y) = x^3 + y^3 + 2x^2 + 4y^2 + 6$

Necess. Condition: $\Rightarrow \frac{\partial f}{\partial x} = 3x^2 + 4x$ —— (1) $\quad x(3x+4) = 0$

$\Rightarrow x = 0, \frac{-4}{3}$

$\frac{\partial f}{\partial y} = 3y^2 + 8y$ —— (2) $\quad y(3y+8) = 0 \Rightarrow$

$y = 0, \frac{-8}{3}$

$A(0,0)$; $B\left(0, \frac{-8}{3}\right)$; $C\left(\frac{-4}{3}, 0\right)$; $D\left(\frac{-4}{3}, \frac{-8}{3}\right)$

Sufficient condition $\quad H_2 = \begin{bmatrix} 6x+4 & 0 \\ 0 & 6y+8 \end{bmatrix}$

$H_1 = 6x+4$

$H_2 = (6x+4)(6y+8) \Rightarrow 36xy + 48x + 24y + 32$

At, $A(0,0)$; $\left.\begin{array}{l} H_1 = 4 \text{ +ve} \\ H_2 = 32 \text{ +ve} \end{array}\right\}$ - minima

At, $B\left(0, \frac{-8}{3}\right)$; $\left.\begin{array}{l} H_1 = 4 \text{ +ve} \\ H_2 = -8 \text{ -ve} \end{array}\right\}$ saddle point

At; $C\left(\frac{-4}{3}, 0\right)$; $\left.\begin{array}{l} H_1 = -4 \\ H_2 = -32 \end{array}\right\}$ -ve saddle point

At; $D\left(\frac{-4}{3}, \frac{-8}{3}\right)$; $\left.\begin{array}{l} H_1 = -4 \\ H_2 = \text{+ve} \end{array}\right\}$ - maxima

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Ques. Find maxima or minima

$f(x,y,z) = x^2 + 4y^2 + 4z^2 + 4xy + 4xz + 36yz$

$\frac{\partial f}{\partial x} = 2x + 4y + 4z \Rightarrow 2x + 4y + 4z = 0$

$\frac{\partial f}{\partial y} = \cancel{2}\, 8y + \cancel{8}\, 4x + 36z \Rightarrow \cancel{8y} \cancel{+} 4x + 8y + 36z = 0$

$\frac{\partial f}{\partial z} = 8z + 4x + 36y \Rightarrow 4x + \cancel{8}\, 36y + 8z = 0$

①, ② & ③ are the Homognous eq^n which has o trival set i.e. $x = y = z = 0$

$$H = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 8 & 36 \\ 4 & 36 & 8 \end{bmatrix} \Rightarrow 2(64 - 36^2) - 4(32 - 144) + 4(144 - 32)$$

$$\left.\begin{aligned} H_1 &= 2 \quad + \\ H_2 &= 0 \quad + \\ H_3 &= -1568 \quad - \end{aligned}\right\} \text{Saddle Point}$$

II. Optimization with Constraints :

Equality Constraints → Langrange's multiplier method.

Inequality Constraints → K-T Conditions

Langrange's Multiplier Method

(i) Case I : 3 variables & 1 constraints.

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Optimize the $f(x, y, z)$ — ①
s.t. $g(x, y, z) = 0$ — ②

Langrangian Function:-
$L(x, y, z; \lambda) = f + \lambda g$ — ③

Necessary Condition, $\frac{\partial L}{\partial x} = 0$ — ④, $\frac{\partial L}{\partial y} = 0$ — ⑤

$\frac{\partial L}{\partial z} = 0$ — ⑥, $\frac{\partial L}{\partial \lambda} = 0$ — ⑦

solving ④, ⑤, ⑥ & ⑦ to get value of x, y, z & λ.

Sufficient Condition

$$\begin{vmatrix} L_{xx}-K & L_{xy} & L_{xz} & \partial g/\partial x \\ L_{yx} & L_{yy}-K & L_{yz} & \partial g/\partial y \\ L_{zx} & L_{zy} & L_{zz}-K & \partial g/\partial z \\ \frac{\partial g}{\partial x} & \partial g/\partial y & \partial g/\partial z & 0 \end{vmatrix} = 0$$

Conclusion:-

If all values of K are negative (-ve) Function is "Maxima"

If all values of K are positive (+ve) Function is "minima".

otherwise saddle point

K = (-) ve All ⇒ maxima

(+) ve All ⇒ minima

Case II: 3 Variables / 2 Constraints

$f(x,y,z)$ — (1); $g(x,y,z)$ — (2); $h(x,y,z)$ — (3)

Langrangian function

$$L(x,y,z;\lambda_1,\lambda_2) = f + \lambda_1 g + \lambda_2 h \quad \text{— (4)}$$

$$\begin{vmatrix} L_{xx}-K & L_{xy} & L_{xz} & \partial g/\partial x & \partial h/\partial x \\ L_{yx} & L_{yy}-K & L_{yz} & \partial g/\partial y & \partial g/\partial y \\ L_{zx} & L_{zy} & L_{zz}-K & \partial g/\partial z & \partial h/\partial z \\ \partial g/\partial x & \partial g/\partial y & \partial g/\partial z & 0 & 0 \\ \partial h/\partial x & \partial h/\partial y & \partial h/\partial z & 0 & 0 \end{vmatrix} = 0$$

Conclusion: All, $K = -ve \Rightarrow$ Maxima
$K = +ve \Rightarrow$ Minima
otherwise Saddle Point

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Ques 1. Solve by Lagrange's Multiplier Method.

max. $F(x,y,z) = xyz$ —(1)

S.t. $g(x,y,z) = x+y+z-1 = 0$ —(2)

$L(x,y,z,\lambda) = F+\lambda g = xyz + \lambda(x+y+z-1)$ —(3)

Nec. condition.

$\frac{\partial L}{\partial x} = 0 \Rightarrow yz+\lambda = 0$ —(4) $\lambda = -yz$

$\frac{\partial L}{\partial y} = 0 \Rightarrow xz+\lambda = 0$ —(5) $\lambda = -xz$

$\frac{\partial L}{\partial z} = 0 \Rightarrow xyz+\lambda = 0$ —(6) $\lambda = -xy$

$\overbrace{\cancel{-}yz = \cancel{-}xz}^{x=y} = \cancel{-}xy$, $\underbrace{xz = xy}_{z=y}$

$\frac{\partial L}{\partial \lambda} = x+y+z-1 = 0$ —(7) $x=y=z$

(7) $\Rightarrow x+x+x-1 = 0 \Rightarrow \boxed{x = \frac{1}{3}}$ $\therefore y = z = \frac{1}{3}$

$\therefore \boxed{\lambda = -\frac{1}{9}}$

Sufficient Condition

$$\begin{vmatrix} 0-k & z & y & 1 \\ z & 0-k & x & 1 \\ y & x & 0-k & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = 0$$

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Multiply the matrix by 3

$$\begin{vmatrix} -K & \frac{1}{3} & \frac{1}{3} & 1 \\ \frac{1}{3} & -K & \frac{1}{3} & 1 \\ \frac{1}{3} & \frac{1}{3} & -K & 1 \\ 1 & 1 & 1 & -K \end{vmatrix} = 0$$

$$\begin{vmatrix} -3K & 1 & 1 & 3 \\ 1 & -3K & 1 & 3 \\ 1 & 1 & -3K & 3 \\ 3 & 3 & 3 & -3K \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{vmatrix} -3K & 1 & 1 & 3 \\ 1+3K & -3K-1 & 0 & 0 \\ 1+3K & 0 & -3K-1 & 0 \\ 3 & 3 & 3 & 0 \end{vmatrix} = 0$$

$$-3\begin{vmatrix} (1+3K) & -(3K+1) & 0 \\ 1+3K & 0 & -(3K+1) \\ 3 & 3 & 3 \end{vmatrix} = 0$$

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$$3\left[1+3k(9k+3) + 1+3k(\cancel{3}+\cancel{9k}-\cancel{3}-\cancel{9k})\right] = 0$$

$$3/3\,(1+3k)(1+3k)) = 0$$

$$(1+3k)^2 = 0$$

$$K = \frac{-1}{3}, \frac{-1}{3}$$

All K (-ve) $\Rightarrow$ "<u>Maxima</u>"

Ques. Solve the question by Lagrange's method,

max. $F(x,y,z) = x^2+y^2+z^2$ — (1)

$g(x,y,z) = ax+by+cz-p = 0$ — (2)

$L(x,y,z,\lambda) = f+\lambda g = x^2+y^2+z^2+\lambda(ax+by+cz-p)$ — (3)

$\frac{\partial L}{\partial x} = 0 \Rightarrow 2x+\lambda a = 0$ — (4) $x = \frac{-\lambda a}{2}$

$\frac{\partial L}{\partial y} = 0 \Rightarrow 2y+\lambda b = 0$ — (5) $y = \frac{-\lambda b}{2}$

$\frac{\partial L}{\partial z} = 0 \Rightarrow 2z+\lambda c = 0$ — (6) $z = -\frac{\lambda c}{2}$

$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow ax+by+cz-p = 0$ — (7)

$$a\left(-\frac{a\lambda}{2}\right) + b\left(-\frac{b\lambda}{2}\right) + c\left(\frac{-c\lambda}{2}\right) - p = 0$$

$$\lambda(a^2+b^2+c^2) = 2p$$

$$\boxed{\lambda = \frac{-2p}{a^2+b^2+c^2}}$$

putting (4), (5), (6)

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$x = \frac{ap}{a^2+b^2+c^2}$, $y = \frac{bp}{a^2+b^2+c^2}$, $z = \frac{cp}{a^2+b^2+c^2}$

Sufficient condition

$$\begin{vmatrix} 2-K & 0 & 0 & a \\ 0 & 2-K & 0 & b \\ 0 & 0 & 2-K & c \\ a & b & c & 0 \end{vmatrix} = 0$$

$$2-K \begin{vmatrix} 2-K & 0 & b \\ 0 & 2-K & c \\ b & c & 0 \end{vmatrix} - a \begin{vmatrix} 0 & 2-K & 0 \\ 0 & 0 & 2-K \\ a & b & c \end{vmatrix} = 0$$

$$2-K\left[2-K(-c^2) - 0 + b\{-2b+bK\}\right] - a\left[-2+K(-2a+2K)\right]$$

$(-b(2-K))$ $\quad -(2-K)-a(-2-K)$

$$(2-K)^2\left[-c^2-b^2\right] - a(2-K)^2 = 0$$

$$(2-K)^2(-a^2-b^2-c^2) = 0$$

$$(2-K)^2 = 0$$

$$K = 2, 2$$

All (+) K , so $\Rightarrow$ minima

(3.) Solve Find the point in the plane $x+2y+3z=1$ which is nearest to the point $(-1,0,1)$.

- Let us, assume $P(x,y,z)$ be the any point on the plane $x+2y+3z=1$, which is nearest to $A(-1,0,1)$.

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$$PA = \sqrt{(x^{2}+1)^2 + y^2 + (z-1)^2}$$

$$PA^2 = f(x,y,z) = (x+1)^2 + y^2 + (z-1)^2 \quad \text{--- (1)}$$

$$g(x,y,z) = x + 2y + 3z - 1 = 0 \quad \text{--- (2)}$$

$$L(x,y,z,\lambda) = f + \lambda g = (x+1)^2 + y^2 + (z-1)^2 + \lambda(x+2y+3z-1) \quad \text{--- (3)}$$

$$\frac{\partial L}{\partial x} = 2x + 2 + \lambda = 2(x+1) + \lambda = 0 \quad \text{--- (4)} \qquad x = \frac{-\lambda}{2} - 1$$

$$\frac{\partial L}{\partial y} = 2y + 2\lambda = 0 \quad \text{--- (5)} \qquad y = -\lambda$$

$$\frac{\partial L}{\partial z} = 2(z-1) + 3\lambda = 0 \quad \text{--- (6)} \qquad z = \frac{-3\lambda}{2} + 1$$

$$\frac{\partial L}{\partial \lambda} = x + 2y + 3z - 1 = 0 \quad \text{--- (7)}$$

$$(7) \Rightarrow \lambda = \frac{1}{7}$$

$$(4), (5), (6) \Rightarrow x = \frac{-15}{14}, \quad y = \frac{-1}{7}, \quad z = \frac{11}{14}$$

Sufficient Condition.

$$\begin{vmatrix} 2-K & 0 & 0 & 1 \\ 0 & 2-K & 0 & 2 \\ 0 & 0 & 2-K & 3 \\ 1 & 2 & 3 & 0 \end{vmatrix} = 0$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\begin{vmatrix} 2-K & 0 & 0 & 1 \\ -2(2-K) & 2-K & 0 & 0 \\ -3(2-K) & 0 & 2-K & 0 \\ 1 & 2 & 3 & 0 \end{vmatrix} = 0$$

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$$-1\begin{bmatrix} -2(2-K) & 2-K & 0 \\ -3(2-K) & 0 & 2-K \\ 1 & 2 & 3 \end{bmatrix} = 0$$

$$-1\left[-2(2-K)(-2(2-K)) - (2-K)(-9(2-K)-(2-K))\right] = 0$$

$$-1\left[-4(2-K)^2 - (2-K)^2(-10)\right] = 0$$

$$-(2-K)^2(6) = 0$$

$$(2-K)^2 = 0$$

$$K = 2, 2$$

All K (+)ve ⇒ minima & distance will be minima

Ques 5. Solve by Lagrange's method.

min. $f(x,y,z) = \frac{1}{2}(x^2+y^2+z^2)$ —— (1)

s.t. $g(x,y,z) = x-y = 0$ —— (2)

$h(x,y,z) = x+y+z-1 = 0$ —— (3)

$L(x,y,z;\lambda_1,\lambda_2) = f+\lambda_1 g+\lambda_2 h = \frac{1}{2}(x^2+y^2+z^2)+\lambda_1(x-y)$
$+\lambda_2(x+y+z-1)$ —— (4)

$\frac{\partial L}{\partial x} = x+\lambda_1+\lambda_2 = 0 \Rightarrow x = -(\lambda_1+\lambda_2)$ —— (5)

$\frac{\partial L}{\partial y} = y-\lambda_1+\lambda_2 = 0 \Rightarrow y = \lambda_1-\lambda_2$ —— (6)

$\frac{\partial L}{\partial z} = z+\lambda_2 = 0 \Rightarrow z = -\lambda_2$ —— (7)

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$\frac{\partial L}{\partial \lambda_1} = x - y = 0$ — (8) $\Rightarrow x = y$ — (9)

$\frac{\partial L}{\partial \lambda_2} = x + y + z - 1 = 0 \Rightarrow x + y + z = 1$

from (5), (6), (8) $-\lambda_1 - \cancel{\lambda_2} - \lambda_1 + \cancel{\lambda_2} = 0$

$-2\lambda_1 = 0$

$\therefore \boxed{\lambda_1 = 0}$ //

$-\cancel{\lambda_1} - \lambda_2 + \cancel{\lambda_1} - \lambda_2 - \lambda_2 = 1$

$-3\lambda_2 = 1$

$\therefore \boxed{\lambda_2 = \frac{-1}{3}}$ //

$x = -\left(\frac{-1}{3}\right) = \frac{1}{3}$

$y = -\left(\frac{-1}{3}\right) = \frac{1}{3}$

$z = -\left(\frac{-1}{3}\right) = \frac{1}{3}$

Sufficient Condition.

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#. Multi-Variable Optimization :-

K-T Conditions (For Inequality Constraints).

Case I :- Suppose $F = F(x, y, z)$ — ①
(Optimize)

s.t. $g_j(x, y, z) \leq 0$ — ②

Inequality Converted into Equality

$$G_j = g_j + y_j^2$$

$L(x, y, z; \lambda_j) = F + \lambda_j G_j$ — ③

Neccessary Conditions : $\frac{\partial L}{\partial x} = 0, \frac{\partial L}{\partial y} = 0, \frac{\partial L}{\partial z} = 0,$

& $\lambda_j g_j = 0$

Case II : Optimize $F = F(x, y, z)$ — ①

s.t. $g_j(x, y, z) \geq 0$ — ②

$G_j = g_j - y_j^2$

$L(x, y, z; \lambda_j) = F + \lambda_j G_j$ — ③

Neccessary Condition: $\frac{\partial L}{\partial x} = 0, \frac{\partial L}{\partial y} = 0, \frac{\partial L}{\partial z} = 0,$

& $\lambda_j \cdot g_j = 0$

: Sufficient Conditions
(Common for Both Cases).

Constraints	Maxima	Minima
≤ 0	$\lambda_j \leq 0$	$\lambda_j \geq 0$
≥ 0	$\lambda_j \geq 0$	$\lambda_j \leq 0$

Imp.

Ques① Solve by K-T Conditions.

min. $F(x,y,z) = x^2+y^2+z^2+20x+10y$ — ①

s.t. $g_1(x,y,z) = x-40 \geq 0$ — ②

$g_2(x,y,z) = x+y-80 \geq 0$ — ③

$g_3(x,y,z) = x+y+z-120 \geq 0$ — ④

Case II

$G_1 = g_1 - y_1^2, \quad G_2 = g_2 - y_2^2, \quad G_3 = g_3 - y_3^2$

$L(x,y,z;\lambda_1,\lambda_2,\lambda_3) = f + \lambda_1 G_1 + \lambda_2 G_2 + \lambda_3 G_3$

$= F + \lambda_1(g_1 - y_1^2) + \lambda_2(g_2 - y_2^2) + \lambda_3(g_3 - y_3^2)$

$= x^2+y^2+z^2+20x+10y+\lambda_1(x-40-y_1^2)+\lambda_2(x+y-80-y_2^2)$
$+\lambda_3(x+y+z-120-y_3^2)$ — ⑤

$\frac{\partial L}{\partial x} = 2x+20+\lambda_1+\lambda_2+\lambda_3 = 0$ — ⑥

$\frac{\partial L}{\partial y} = 2y+10+\cancel{\lambda_1}+\lambda_2+\lambda_3 = 0$ — ⑦

$\frac{\partial L}{\partial z} = 2z+\lambda_3 = 0$ — ⑧

$\lambda_j g_j = 0 \Rightarrow \lambda_1 g_1 = 0, \Rightarrow \lambda_1(x-40) = 0$ — ⑨

$\lambda_2 g_2 = 0, \Rightarrow \lambda_2(x+y-80) = 0$ — ⑩

$\lambda_3 g_3 = 0, \Rightarrow \lambda_3(x+y+z-120) = 0$ ⑪

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* Let, $\lambda_1 \neq 0$, $\lambda_2 \neq 0$, $\lambda_3 \neq 0$

$\therefore$ (9) $x - 40 = 0 \Rightarrow x = 40$

$\therefore$ (10) $x + y - 80 = 0 \Rightarrow y = 80 - 40 = 40$

$\therefore$ (11) $x + y + z - 120 = 0 \Rightarrow z = 120 - 40 - 40 = 40$

Using values of x, y, z in (6), (7) & (8)

(8) $2 \times 40 + \lambda_3 = 0$

$\lambda_3 = -80$ (-ve)

(7) $2 \times 40 + 10 + \lambda_2 - 80 = 0$

$\lambda_2 = \cancel{80} - \cancel{80} - 10$

$\lambda_2 = -10$ (-ve)

(6) $\cancel{2 \times 40} + 20 + \lambda_1 - 10 - \cancel{80} = 0$

$\lambda_1 + 10 = 0$

$\lambda_1 = -10$ (-ve)

$\because$ $\boxed{\text{All } \lambda_j \leq 0 \text{, "Function gives minima"}}$

V.V. Imp.

Ques(2) Solve the following problem:

min. $f(x, y, z) = x^2 + y^2 + z^2$ —— (1)

$$\left.\begin{aligned} \text{s.t.} \quad g_1(x, y, z) &= 2x + y - 5 \leq 0 \\ g_2(x, y, z) &= x + y - 2 \leq 0 \\ g_3(x, y, z) &= 1 - x \leq 0 \\ g_4(x, y, z) &= 2 - y \leq 0 \\ g_5(x, y, z) &= -z \leq 0 \end{aligned}\right] \text{—— (2)}$$

Case I. $G_1 = g_1 + y_1^2$, $G_2 = g_2 + y_2^2$; $G_3 = g_3 + y_3^2$

$G_4 = g_4 + y_4^2$ & $G_5 = g_5 + y_5^2$

$L(x, y, z : \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5) = F + \lambda_1 G_1 + \lambda_2 G_2 + \lambda_3 G_3 + \lambda_4 G_4 + \lambda_5 G_5$

$= F + \lambda_1(g_1 + y_1^2) + \lambda_2(g_2 + y_2^2) + \lambda_3(g_3 + y_3^2) + \lambda_4(g_4 + y_4^2) + \lambda_5(g_5 + y_5^2)$

$= x^2 + y^2 + z^2 + \lambda_1(2x + y - 5 + y_1^2) + \lambda_2(x + y - 2 + y_2^2)$
$+ \lambda_3(1 - x + y_3^2) + \lambda_4(2 - y + y_4^2) + \lambda_5(-z + y_5^2)$ — ③

$\frac{\partial L}{\partial x} = 2x + 2\lambda_1 + \lambda_2 - \lambda_3 = 0$ — ④

$\frac{\partial L}{\partial y} = 2y + \lambda_1 + \lambda_2 - \lambda_4 = 0$ — ⑤

$\frac{\partial L}{\partial z} = 2z - \lambda_5 = 0$ — ⑥

$\lambda_j g_j \Rightarrow \lambda_1 g_1 = \lambda_1(2x + y - 5) = 0$ — ⑦
$\lambda_2 g_2 = \lambda_2(x + y - 2) = 0$ — ⑧
$\lambda_3 g_3 = \lambda_3(1 - x) = 0$ — ⑨
$\lambda_4 g_4 = \lambda_4(2 - y) = 0$ — ⑩
$\lambda_5 g_5 = \lambda_5(-z) = 0$ — ⑪

(A)

- Let, $\lambda_1 \neq 0$, $\lambda_2 \neq 0$, $\lambda_3 \neq 0$, $\lambda_4 \neq 0$, $\lambda_5 \neq 0$

$\therefore$ ⑪ $\Rightarrow z = 0$
⑩ $\Rightarrow 2 - y = 0 \Rightarrow y = 2$
⑨ $\Rightarrow 1 - x = 0 \Rightarrow x = 1$

At the values of x, y, z Equation-⑦ and ⑧ are not satisfied, therefore,
$\lambda_1 \neq 0$, $\lambda_2 \neq 0$ is the "Contradiction"
therefore,
$\lambda_1 = 0$, $\lambda_2 = 0$

from ⑥ $2(0) - \lambda_5 = 0$

$\lambda_5 = 0$

⑤ $2\times 2 + 0 + 0 - \lambda_4 = 0$

$\lambda_4 = 4$

④ $2\times 1 + 2(0) + \cancel{2}\, 0 - \lambda_3 = 0$

$\lambda_3 = 2$

" $\boxed{\text{All } \lambda_j \geq 0 \text{, " Function gives Minima"}}$ "

Ques3. Solve the following Optimization Problem.

Max. $F = -x - y$

s.t $x^2 + y \geq 2$

$\cancel{x+y\geq}\ x + 3y \geq 4$

$x + y^4 \leq 30$

Show that K-T satisfied at design vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

max $F(x,y) = -x - y$ —①

s.t $\left.\begin{aligned} g_1(x,y) &= x^2 + y^2 - 2 \geq 0 \\ g_2(x,y) &= x + 3y - 4 \geq 0 \\ g_3(x,y) &= -x - y^4 + 30 \geq 0 \end{aligned}\right\}$ —②

Case II

$G_1 = g_1 - y_1^2$

$L(x,y,\lambda_1,\lambda_2,\lambda_3) = F + \lambda_1 G_1 + \lambda_2 G_2 + \lambda_3 G_3$

$= -x - y + \lambda_1(x^2 + y - 2 - y_1^2) + \lambda_2(x + 3y - 4 - y_2^2) + \lambda_3(-x - y^4 + 30 - y_3^2)$ —③

$\frac{\partial L}{\partial x} = -1 + 2x\lambda_1 + \lambda_2 - \lambda_3 = 0$ —④

$\frac{\partial L}{\partial y} = -1 + \lambda_1 + 3\lambda_2 - 4y^3\lambda_3 = 0$ —⑤

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Eq. $\lambda_j g_j = 0 \Rightarrow \lambda_1 g_1 = 0 \Rightarrow \lambda_1(x^2+y-2) = 0$ — (6)

$\lambda_2 g_2 = 0 \Rightarrow \lambda_2(x+3y-4) = 0$ — (7)

$\lambda_3 g_3 = 0 \Rightarrow \lambda_3(-x-y^4+30) = 0$ — (8)

(*) Let, $\lambda_1 \neq 0$, $\lambda_2 \neq 0$, "$\lambda_3 \neq 0$"

given, $x=1$, $y=1$

putting $x=1$ and $y=1$ in Eq (6), (7), (8)

we get Eq (6) and Eq (7) are satisfied but Eq (8) is not satisfied which is a "Contradiction" to our assumption.

i.e. $\lambda_3 \neq 0$ therefore $\boxed{\lambda_3 = 0}$

by x, y, λ_3 using in (4) & (5)

(4) $-1 + 2(1)\lambda_1 + \lambda_2 - 0 = 0$

$2\lambda_1 + \lambda_2 = 1$

(5) $-1 + \lambda_1 + 3\lambda_2 - 4y^3(0) = 0$

$\lambda_1 + 3\lambda_2 = 1$

$\cancel{\lambda_1} + 3\lambda_2 = 1 \times 2$

$\cancel{2}\lambda_1 + \cancel{3}\lambda_2 = 1$

$5\lambda_2 = 1$

$\boxed{\lambda_2 = \frac{1}{5}}$

$\lambda_1 + \frac{3}{5} = 1$

$\lambda_1 = 1 - \frac{3}{5}$

$\boxed{\lambda_1 = \frac{2}{5}}$

$\boxed{\text{All } \lambda_j \geq 0, \text{ "function gives Maxima"}}$