Baryon Acoustic Oscillation

Punyam Pandey, Simran Agrawal, Arshia Anjum, Sandeepan Sahoo

December 2023

Abstract

Baryon Acoustic Oscillations (BAO) are fluctuations in the density of the primordial baryon-photon plasma in the pre-recombination era which were responsible for much of the structure formation in the universe. In this paper we discuss BAO in the context of the thermal history of the universe and introduce the Sound Horizon as a standard ruler. We further explore some observational results for BAO obtained in the Sloan Digital Sky Survey.

1 Introduction

The origin of BAO can be traced back to the early universe, when it was filled with a hot and dense plasma of electrons and baryons (normal matter). Tiny fluctuations in the density of the plasma created sound waves that propagated through it. When the universe cooled enough for the plasma to form neutral atoms, the sound waves stopped and left a characteristic pattern of density variations. These variations later influenced the formation of galaxies and clusters, creating a preferred distance between them that corresponds to the size of the sound horizon at the time of recombination. By measuring this distance at different cosmic epochs, astronomers can learn about the history and fate of the universe.

Baryon Acoustic Oscillation can be measured by using large-scale surveys of galaxies and quasars that cover a wide range of redshifts. Cosmologists can use spectroscopic observations to obtain the radial and angular positions of these objects, and then analyze the statistical properties of their spatial distribution. By comparing the observed clustering pattern with the theoretical prediction based on the standard ruler, they can infer the angular diameter distance and the Hubble parameter at different cosmic epochs, and thus constrain the dark energy model.

2 Primordial Plasma Expansion

2.1 Baryon Photon Fluid

In the early universe, between the end of inflation and the start of the recombination epoch, the universe consisted of a hot and dense form of matter (and an almost equal amount of anti-matter) coupled with radiation. Neutrinos had decoupled earlier at MeV scale energies and dark matter still earlier at GeV, radiation coupled to matter consisted of a baryon-photon fluid which we call the primordial plasma.
The primordial plasma was spread-out throughout the universe but there were fluctuations in the homogeneity of the plasma all the time. Due to these fluctuations in the mass distribution, certain regions became overdense while others were overdense. Gravitationally these overdensities would attract more and more matter and become heavier. However due to the photons consistently interacting with matter (annihilation, Thomson scattering etc) the heat generated from these reactions would exert an outward pressure the plasma.
The forces of gravity and thermal pressure acting in opposite directions will create oscillations in the baryon photon plasma. Since the waves generated in these oscillations are of a mechanical nature, i.e. propagating due to the disturbances in the pressure levels, they are acoustic and as they propagate in baryonic matter, these oscillations are known as Baryon Acoustic Oscillation.

2.2 Expansion

The speed at which these waves propagated is the speed of sound c_s n the medium. These acoustic waves move with c_s outwards from the origin of the overdensity. Cold Dark matter (CDM), which is no longer interacts with photons and hence is not affected by the outward pressure remains concentred around the center of the overdensity due to gravity. Consequently the mass profile of the baryon-photon fluid only expands spherically outwards.
The shifting mass profile with comoving distance over time can be depicted in Fig.1. [1]

2.3 Recombination and Sound Horizon

After recombination, the baryons have cooled down enough to decouple from the photons. As photons are no longer forced to move in small mean free paths due to Thomson scattering, they are free to move and therefore stream out of the plasma at the speed of light. The baryons on the other hand stop expanding as the CDM in concentrated around the centers of these spheres are gravitational potential well which attract the baryons. The distance or radius travelled by the baryon photon fluid until recombination is known as the Sound Horizon.

After recombination, the mass profile of baryonic matter is affected by gravitational interaction with CDM. CDM pulls baryonic matter towards the center,

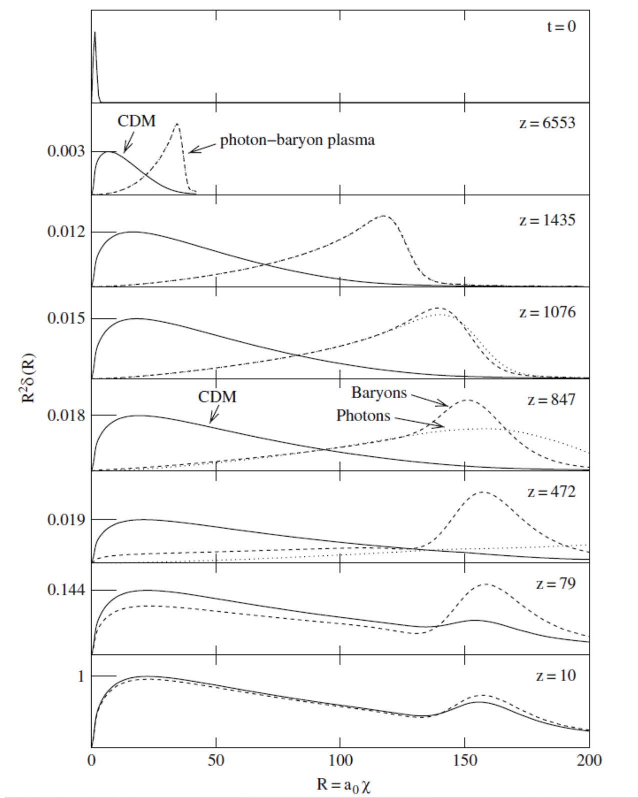

Figure 1: The time evolution overdensity in the universe. Initial baryons, CDM, photons and neutrinos are concentrated together. With increasing time, baryons and photons move away from the original.

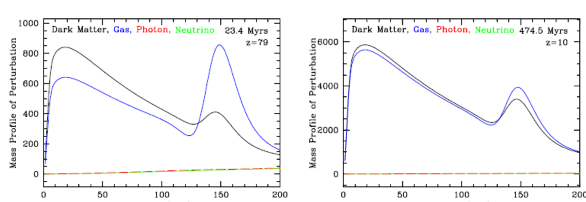

Figure 2: Change in Mass Profile of matter after recombination caused due to interaction between Baryonic shells and CDM.

hence the peak near the origin, however as most of the baryonic matter initially froze at the sound horizon forming shells, it also attracts CDM towards itself causing another peak at sound horizon as shown Fig 2, [2]. To calculate the sound horizon $a_0\chi_s$, we use c_s. Considering the baryon photon plasma to be a perfect fluid (characterized only by its energy density ρ and pressure p), the sound velocity can be expressed as [1]

$$c_s = c\sqrt{\frac{dp}{d\rho}} \tag{1}$$

where c is the speed of light. The expansion of the fluid is considered to be adiabatic since there is no change in the overall heat content of a volume element. Therefore the following relation holds

$$d(\rho V) = -pdV \tag{2}$$

Then,

$$d\rho = -(p+\rho)\frac{dV}{V} \tag{3}$$

Since the outward expanding pressure is called solely due to the photons, we have

$$p = \frac{\rho_\gamma}{3} = \frac{\rho - \rho_b}{3} \tag{4}$$

where ρ_γ and ρ_B are the energy density of photons and baryons respectively. Using eqs (2),(3), (4) in eq (1), we get the speed of sound as

$$c_s = c\sqrt{\frac{d\rho_\gamma}{d\rho_b + d\rho_\gamma}} = c\sqrt{(3\frac{d\rho_b + d\rho_\gamma}{d\rho_\gamma})^{-1}} = \frac{c}{\sqrt{3}}(1 + \frac{d\rho_b}{d\rho_\gamma})^{-1/2} \tag{5}$$

To solve the last step, one uses the dependence of energy densities on the scale factor a. Since we know that

$$\rho_b = \frac{\Omega_b}{a^3} \tag{6}$$

$$\rho_\gamma = \frac{\Omega_b}{a^4} \tag{7}$$

We have $\frac{d\rho_b}{d\rho_\gamma} = \frac{3a^{-4}\Omega_b}{4a^5\Omega_\gamma} = \frac{3\rho_b}{4\rho_\gamma}$ so we have

$$c_s = \frac{c}{\sqrt{3(1+\frac{3\rho_b}{4\rho_\gamma})}} \tag{8}$$

Then we can find out the sound horizon using $\frac{d\chi}{dt} = \frac{c_s}{a(t)}$ and the Friedman equation

$$(\frac{\dot{a}}{a})^2 = H_0^2(\Omega_M a^{-3} + \Omega_R a^{-4} + \Omega_\lambda + \Omega_K a^{-2}) \tag{9}$$

we can get

$$a_0\chi_s = \int_0^{a_{rec}} \frac{c_s dt}{a(t)} = \int_0^{a_{rec}} \frac{c_s da}{\dot{a}} = \int \frac{c_s da}{a^2(\Omega_M a^{-3} + \Omega_R a^{-4} + \Omega_\lambda + \Omega_K a^{-2})^{1/2}} \tag{10}$$

For the recombination era one can consider Ω_λ, i.e. the specific vacuum energy density to be zero and approximate specific total energy density $\Omega_T \approx 1$ so that $\Omega_k = 1 - \Omega_T = 0$. As the recombination happens around a redshift of z = 1100 and a = 1/(1+z) we can using $a_{rec} \approx 1/1100$. Inserting the appropriate values of the other parameters obtained from observation, we get the value of the sound horizon

$$a_0\chi_s = 153 Mpc.$$

3 Galactic Correlators and Standard Ruler

4 Galactic correlation function

Consider the distribution of galaxies. The correlation function as function of distance describes the probability of finding two galaxies separated by that distance (excess over the probability if the galaxies were uniformly distributed. Since the baryon shells formed around CDM perturbations are responsible for eventual formation of galactic structures and they expand till the sound horizon, it would appear that at time t pairs of galaxies are more likely to be separated by distance a(t)χ_s. Indeed the galactic correlation function, when plotted against the comoving distance, shows a peak at the sound horizon as demonstrated in Fig. 3.

The sound horizon at recombination also appears to be the preferred distance between hot and cold regions of the universe as observed in the cosmic microwave background. This makes sense as it is the distance travelled by matter when the universe was hot till the recombination, i.e. when it became cool.

5 Standard Ruler

When things are far away they appear small and this is how we jugde how far away an object is in daily life. On cosmological scales however, the curvature of space-time is akin to looking at an object through a lens of unknown curvature, which means the size of an object is not an indicator of its exact distance from the observer.

Therefore one requires a standard ruler, i.e. an object of known size at a given redshift z (or scale factor a) [2] or a collection of objects whose size is a well defined function of the redshift. The Baryon Acoustic Oscillation are such a set of objects among others. The Sound Horizon being at anytime t being equal to

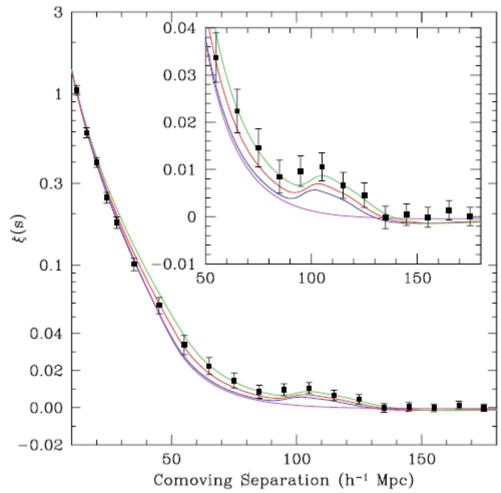

Figure 3: The galactic correlation function as function of comoving distance

$a(t)\chi_s =$ and a = 1/(1+z) can therefore be used as a standard ruler.
In fact BAO are an ideal candidate for a statistical standard ruler where in one utilizes the concept that the distribution of galaxies have a preferred scale, which in our case happens to be the sound horizon and observing it at different redshifts one can constrain the angular diameter distance.
At a redshift z, the likely distance between two galaxies would be $a_0\chi_s/(1+z)$ which would give us a likely angular separation.

$$\theta = \frac{a_0\chi_s}{(1+z)d_A} \tag{11}$$

where d_A is the angular diameter distance. And a preferred redshift

$$\delta z = \frac{a_0\chi_s}{H(z)^{-1}} \tag{12}$$

where H(z) is the expansion rate at time t.
The matter power spectrum is defined as the difference between the local density and average density, i.e. the matter overdensity, as function of the distance scale. It is the Fourier transform of the galactic correlation function.
As a characteristic distance scale, such as the sound horizon, would show a peak in the galactic correlation function due to plenty of clustering of galaxies at said distance, so too the power spectrum would show characteristic qualities associated with the distance scale. Let, $\delta(r)$ be the matter overdensity

$$\delta(r) = \frac{\rho(r) - \bar{\rho}}{\rho} \tag{13}$$

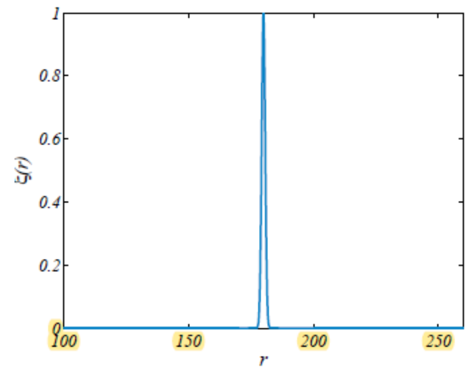

Figure 4: Galactic correlator $\xi(r)$ v r showing a sharp peak at some characteristic distance

where $\bar{\rho}$ is the average density of galaxies over all space; $\xi(r)$ be the galactic correlator as function of distance r

$$\xi(r) = \langle \delta(x)\delta(x') \rangle \propto \int d^3x \delta(x)\delta(x-r) \tag{14}$$

where r = x - x'; and P(k) be its one dimensional power spectrum. Then they are related by

$$P(k) = \int_{-\infty}^{\infty} \frac{d^3r}{(2\pi)^3} \xi(r) exp(-ikr) \tag{15}$$

Let's understand how a characteristic distance might affect the power spectrum given prior knowledge of correlation function's behavior. For a simple example say the galactic correlator shows a peak in the form of a Dirac delta function at distance r_s, then power spectrum P(r) will, according to eq (15) will go as e$^{-ikr_s}$ and therefore show oscillatory behaviour. These are indeed the acoustic oscillations we have been discussing so far, albeit a bit oversimplified.

6 Non Linearity

Till now we had been considering the sound horizon as a reliable standard ruler due to us taking the clustering the galaxies to be linear and not considering any scale dependent bias. But the fact remains that the distribution of galaxies deviates from regular or linear arrangement and this puts a limit on the use of sound horizon as a standard ruler.

The error arising due to non-linearity manifests as a shift in the peak of the correlation function. Firstly is the smooth correlation function without the peak (broadband correlator) shifts then so does the peak at sound horizon. Next, consider the fact the galactic correlation, which we have been consider as joint

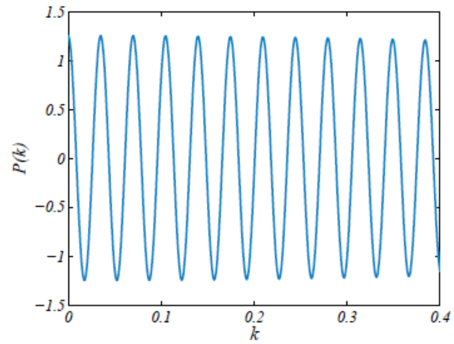

Figure 5: Power spectrum in Fourier space. Oscillations in P(k) corresponding to the peak in Fig. 4.

probability dependent on positions of both galaxies, can be considered as a conditional probability: given a galaxy at the origin what is the probability of finding another galaxy at some distance r. Think of the non-linear evolution of a spherical shell (as an acoustic wave) with a galaxy at the center. After recombination, it would undergo some kind of collapse due to its own self-gravity and due to the gravity of the galaxy at the center thereby reducing the radius of the shell and in consequence the standard ruler [2] by a small portion.
Non-linearity can also smoothen out and broaden the acoustic peak of the correlation function. Say a galaxy forms on the spherical shell subject to simultaneous forces of over and under densities. The net force might pull it outwards or inwards depending on the density configuration, thus affecting the radius if the shell. The effect varies from galaxy to galaxy and to calculate the correlation function, when we average over all galaxies, we get a broadening of the peak.
Earlier for the sharp peak, the power spectrum was a sinusoid. This broadening can be thought of as damping of the oscillations, i.e. adding a damping constant to the index of the exponential e^{-ikr} This makes the standard ruler less accurate. But this damping is only significant at large values of k (small scales).
For example consider the galactic correlation function having a Gaussian form centered at r_s. Now it has broader peak than the sharp Dirac delta.

$$\xi(r) = exp(-\alpha(r - r_s)^2) \tag{16}$$

Here α is a constant. The Power spectrum in one dimension is then

$$P(k) = \int_{-\infty}^{\infty} exp(-ikr)exp(-\alpha(r - r_s)^2)dr = \frac{\pi}{2}exp(-ikr_s)exp(-\frac{k^2}{2\alpha}) \tag{17}$$

Here the term $exp(-\frac{k^2}{2\alpha})$ acts as a dampener of the oscillations. The width of the peak is accounted for by $\frac{1}{\sqrt{2\alpha}}$. With increased broadening, the oscillations become harder to detect.

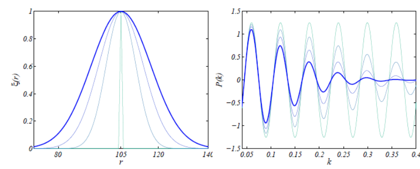

Figure 6: Demonstration of the effect of broadening of $\xi(r)$ in turn washing out the oscillations of the power spectrum.

7 Sloan Digital Sky Survey

8 Summary

References

[1] James Rich, Fundamentals of Cosmology.

[2] Bruce A. Basset, Renee Hlozek. Baryon Acoustic Oscillations