Simultaneous Localization and Mapping Survey Based on Filtering Techniques

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***Abstract*— Simultaneous Localization and Mapping (SLAM) problem has been an active area of research in robotics for more than two decades. This paper reviews SLAM based on different filtering techniques used to do the state estimation of the mobile robot. The filtering techniques included in this study are kalman filter, particle filter, H infinity filter. It can be concluded that each filtering technique has its own advantages and disadvantages as it is very dependent on the situations. Kalman filter is much suitable for dealing with Gaussian distribution. Particle filter is selected for large-scale environment as its computation complexity is logarithmic compared to Kalman filter which has quadratic complexity. H infinity filter is used to improve the convergence of SLAM system.**

Keywords—SLAM, Simultaneous Localization and Mapping, Mobile Robot, Kalman Filter, Particle Filter, H infinity filter.

I. INTRODUCTION

Autonomous mobile robot which is an intelligent agent must be able to explores and navigates through any kind of environment either it is known or unknown without any human interference in order to achieve its desired goals. Leonard and Durrant-Whyte [1] summarized the problem of navigation into answering these three questions: “where am I?”, “where am I going?” and “how should I get there?” The first question is about localization problem, which intend to obtain robot’s pose estimations based on data obtained from robot’s sensors and previously obtained information about the environment. The second question is specifying a goal and the third question is being able to do path planning to achieve the specified goal.

Thrun [2] stated that in order to build truly autonomous mobile robots, one of the most important problems is the mapping problem. During localization problem, the mobile robot need refer to some reference system and a map is required to be constructed for its navigation purpose. By solving these two problems together, robot’s pose and the map of the environment can be estimated and this solution is known as simultaneous localization and mapping. It is a problem that if a mobile robot is placed at an unknown location in a priori unknown environment, the mobile robot is able to build a map of the environment using local information perceived by its sensor while estimating its position within the map simultaneously [3, 4].

There are some uncertainties and limitations need to be considered and overcame when performing the SLAM solutions. First, data association error [5] as mobile robot is unable to correctly identify a landmark perceived from different mobile robot’s poses as the same landmark that it has perceived previously. If that landmark is wrongly associated, mapping will be corrupted. Besides, divergence between the mobile robot’s assumed motion and its actual motion also contribute to data association uncertainty. Second, measurement or sensor’s noise from imperfect sensors used especially incremental sensors as errors in navigation control accumulate gradually and condition the way in which succeeding measurements are analyzed. Third, computational complexity grows exponentially when the environment is of large scale as amount of landmarks are huge. Finally, the dynamic environments [6] are usually being simplified to controlled environments to avoid dealing with those troublesome dynamics.

Many reviews on SLAM have been published over a decade. Thrun [2] reviewed different type of probabilistic techniques that can be applied on robotic mapping. Freese [7] discussed in details the structure and properties of SLAM problem. Durrant-Whyte and Bailey [3, 4] surveyed the development of SLAM algorithm in state-space and particle-filter form, computational complexity, data association and environment representation. Lu et al. [8] classified SLAM by process, sensors and uncertainty calculation to study their performance. Dissanayake et al. [9] reviewed the fundamental

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properties of feature based SLAM, such as observability, convergence, consistency and computational efficiency. Zamora and Yu [10] summarize most of the popular SLAMs solution in term of their main contributions, insights and limitation. Boal et al. [11] discussed the topological SLAM in terms of feature detection, map matching and map fusion which is used by most famous techniques.

II. SLAM PROBLEM

According to Durrant-Whyte and Bailey [3], the following probabilistic distribution needs to be computed for all time T to solve SLAM problem. This probabilistic distribution described the joint posterior distribution of robot's pose and landmark location at time T given the control inputs and observations up to and including time T.

$$p(x_{0:T}, m | z_{1:T}, u_{1:T}) \tag{1}$$

where $x_{0:T} = \{x_0, x_1, \dots, x_T\}$ is all the robot's pose, $m = \{m_1, m_2, \dots, m_T\}$ is the set of all landmarks, $z_{1:T} = \{z_1, z_2, \dots, z_T\}$ is the set of all landmark observations and $u_{1:T} = \{u_1, u_2, \dots, u_T\}$ is all the control inputs.

Equation (1) is known as Full SLAM which estimates the entire path as in Fig. 1. There is another SLAM solution known as online SLAM which seeks to recover only the most recent robot's pose as in Fig. 2. Its probabilistic distribution is as follows.

$$p(x_t, m | z_{1:t}, u_{1:t}) \tag{2}$$

where x_t is the robot's current pose, $m, z_{1:t}, u_{1:t}$ is same as Full SLAM. Online SLAM is marginalizing out all the previous robot's poses by recursive integration once at a time as shown in the following.

$$p(x_t, m | z_{1:t}, u_{1:t})$$
$$= \int_{x_0} \dots \int_{x_{t-1}} p(x_{0:T}, m | z_{1:T}, u_{1:T}) dx_{t-1} \dots dx_0 \tag{3}$$

In order to solve these problems, two more mathematical models are introduced which are motion model and observation model. The motion model is probability distribution of the robot's estimated pose, x_t using the robot's previous pose, x_{t-1} and a control input, u_t.

$$p(x_t | x_{t-1}, u_t) \tag{4}$$

The observation model is probability distribution of the estimated measurement, z_t using the estimated robot's pose, x_t and the landmarks position within the map, m.

$$p(z_t | x_t, m) \tag{5}$$

Fig. 1 and Fig. 2 represent the graphical model of Full SLAM and Online SLAM respectively. Both Fig. 1 and Fig. 2 illustrated the relationship of the variables involved in SLAM. These graphical models show the sequences of robot's poses, sensor's measurements, control inputs, map of the environment and their relationship.

III. KALMAN FILTER

The Kalman filter is an exact filter derived from recursive Bayesian rule under the assumptions that the system is linear and the noise being Gaussian distributed. The derivation of it can be found in [12]. Thus, state space model is used to represents the robot and environment in Kalman filter SLAM with the Gaussian noise. Kalman filter SLAM solved the localization problem and mapping problem by estimating the robot's pose and landmarks' location concurrently.

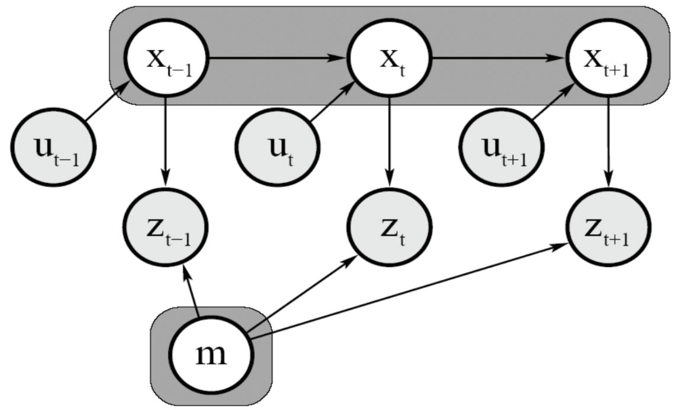

Fig. 1. Full SLAM Graphical Model

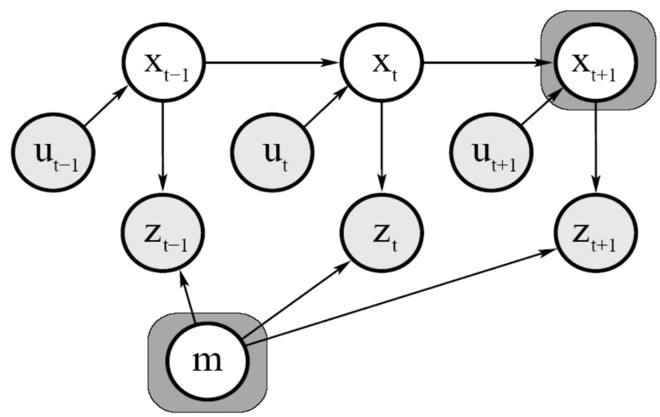

Fig. 2. Online SLAM Graphical Model

A. *Extended Kalman Filter*

In mobile robot domain, linear models assumed by Kalman filter are generally non-linear, thus Gaussian noise assumption cannot be applied. In order to overcome it, a nonlinear extension of Kalman filter known as Extended Kalman filter (EKF) is introduced which linearizes about an estimate of the current mean and covariance.

Extended Kalman filter has its own disadvantages in terms of convergence and consistency. For convergence, if the process is modelled wrongly, the filter diverges very quickly due to its linearization. For consistency, without the addition of "stabilizing noise", the true covariance matrix is tends to be underestimated by the estimated covariance matrix and result in becoming inconsistent. Thus, EKF-SLAM requires additional techniques to eliminate those disadvantages. Huang and Dissanayake [13, 14] investigate the consistency and convergence of the EKF-SLAM algorithm and concluded that robot orientation error play an important role in EKF-SLAM convergence and consistency. Rodriguez-Lasada et al. [15]

reviewed various techniques such as shape constraints, robocentric mapping, local maps fusion and using a tree transformation to overcome the inconsistency problem of EKF-SLAM.

Another major problem of EKF-SLAM is that its computational complexity of the order of $O(N^2)$ where N is the states used to represent robot's pose and landmarks. In order to reduce the computational complexity of EKF-SLAM, the following solutions are proposed:

i. Postponement method by extending the standard Kalman filter to model the information gained in local are with a cost ~ $O(N_a^2)$ where N_a is the number of landmarks in the local area, and then transferred to the overall map by only one iteration at full SLAM computational cost [16].
ii. Map Management Scheme by limiting the number of features that are included in the map by selecting landmarks for removal from the map without making the map building process inconsistent [17].
iii. Introducing the Global Map Postponement method, approximation necessary for ensuring linear computational complexity of EKF-based SLAM are delayed multiple time steps [18].
iv. Intersecting covariance by fusion rule for combining two estimates when the correlation between them are unknown [19].
v. Mapping only lines that are orthogonal which is parallel or perpendicular to each other to represent the main structure of most indoor environment [20].

Besides EKF-SLAM, there are unscented Kalman filter (UKF) and cubature Kalman Filter (CKF) which were proposed to minimize the localization error and the estimated error of the landmarks and improve the accuracy and robustness of the algorithms.

B. *Unscented Kalman Filter*

The motivation for unscented Kalman filter (UKF) is that linear models are needed in normal Kalman filter and Taylor expansion is used to do linearization of non-linear function in EKF. UKF used the Unscented Transform (UT) to estimate the result of a probability distribution by computing a finite set of weighted sigma points and transform each of those sigma points through a nonlinear function [21].

Martinez-Cantin and Castellanos [22] are the first to introduce UKF to solve SLAM problem. Their main objective is to avoid the analytical linearization based on Taylor-series expansion of motion model and measurement model by using the unscented filter instead of classical extended Kalman filter approach.

The results obtained by using UKF on linear models are the same as EKF. Despite the results' differences are small, UKF is still a better approximation for nonlinear models compared to EKF. UKF has same computational complexity with EKF; although UKF's computational time is slower than EKF and no Jacobians matrix are needed in UKF.

Rudolph and Eric proposed square-root unscented Kalman filter (SR-UKF) algorithm to estimate sate and parameter [23]. This algorithm is introduced into SLAM problem by Li and Ni [24] to overcome the sigma points calculation by square root of the state covariance matrix at each update step in UKF-based SLAM. SR-UKF propagates the square root of the state covariance matrix directly in SLAM and resulting in estimation of robot poses and landmark location became more accurate than UKF-SLAM. Computational Complexity of SR-UKF is reduced to an order of magnitude by re-arrangement of the state and re-setting of the QR decomposition algorithm and inverse depth features initialization to deal with the rank deficiency [25].

C. *Extended Information Filter*

The extended information filter (EIF) was introduced and developed to reduce the computational complexity of EKF-SLAM in large scale environments. The most beneficial aspect of EIF is that its sparseness property of the information matrix reduced the time complexity [26]. Inversion of information matrix is done to obtain the relationship between landmark locations and the variables associated with the landmarks so that localization of robot, mapping of environment and data association can be carried out. This process can be done when the number of landmarks is small, but the computational cost of the inversion will be unacceptable with huge number of landmarks.

Thurn et al. [27] developed sparse extended information filter (SEIF) algorithm to represent maps by graphical networks of features that are locally interconnected, where constraints represent relative information between pairs of nearby features and robot's pose relative to the maps. The size of the maps do not affect the computational complexity of the measurement and motion updates in SEIF as they are always executed in constant time. An amortized constant-time coordinate descent algorithm was proposed to recover state estimates of nonlinear function from the information matrix.

The main limitation of SEIF is that it may easily become overconfident and causing the map building process becomes inconsistent. This overconfidence mainly causes by the approximation in the sparsification step. Eustice et al. [28] discussed the inconsistency due to sparseness approximation in details. Another limitation is SEIF inherits the use of Taylor expansion to linearize the motion and measurement model which can cause the map to diverge.

Walter, Eustice and Leonard [29] proposed exactly sparse extended information filter (ESEIF) while maintaining the global and local consistency relative to the EKF-SLAM to overcome the overconfidence issue of the SEIF. The sparseness of the information matrix is controlled by formation of constraints between the number of active landmarks and robot's poses. The robot's poses are deliberately marginalized out to preserve sparsity, thus robot need to re-localize itself within the map with only those active landmarks observed.

D. *Cubature Kalman Filter*

The cubature Kalman filter (CKF) is proposed by Arasaratnam and Haykin [30] by deriving a third-degree spherical-radial cubature rule that provides a set of cubature points scaling linearly with the state-vector dimension using a set of cubature points to capture the mean and covariance in each step update. The usage of CKF provides systematic solution for high-dimension nonlinear filtering problems with more accurate filtering than existing Gaussian based filter.

The cubature rule is derivative-free meaning there is no need to compute the Jacobian and Hessians matrix even in complicated nonlinearities. Chandra, Gu and Postlethwaite [31] proposed the use of CKF for the estimation of the SLAM augmented state vector. The augmented state vector of robot's pose and landmark locations are estimated using CKF.

Yu et al. [32] introduced Sage-Husa noise statistic estimator to estimates the statistical parameters of the unknown system noise and combined it with CKF-SLAM. This combination formed a new adaptive cubature Kalman filter SLAM (ACKF-SLAM) algorithm that can reduce the state estimated uncertainty significantly and improves the navigation accuracy of the SLAM system effectively.

IV. Particles Filter

Particles filter or known as Sequential Monte Carlo (SMC) [33] method is a recursive Bayesian filter that approximates the exact probability distribution through a set of state samples. The distribution is model by using samples compared to Kalman filter which used Gaussian distributed model. Thus, particle filter can be used to represent any multi-modal distribution. Computational cost of particle filter in high-dimensional space is costly and becoming not appropriate for real time applications [34]. Thus, particle filter are normally used to do localization of the robot and not the mapping of the environment.

The FastSLAM proposed by Montemerlo, Thrun and Siciliano [35] is based on the assumption that if the robot pose is known, the landmarks are conditionally independent of each other. Rao–Blackwellized Particles Filter (RBPF) can be applied to estimate the robot path and to estimate the landmark position by several low-dimensional EKFs by using this assumption. FastSLAM models only the robot's path by sampling and each particle makes its own local data association. The SLAM problem is break down by the algorithm into a collection of landmark estimation problems and a robot localization problem that are conditioned on the robot pose estimate. The FastSLAM's computational complexity is O (M log N), where M is the number of landmarks which can be constant and N is the number of particles.

Below are some advantages of using FastSLAM:

i. Data association is more robust to association error since each particle views the landmarks differently causing multiple data association.
ii. Robot's motion model and measurement model do not need to be linearized.
iii. Nonlinear and non-Gaussians system can be handled better.

There are some disadvantages of FastSLAM as follow:

i. The computational cost is expensive since each particle need to perform data association independently.
ii. It is sensitive to divergence in sparse and noisy SLAM.
iii. Its consistency is always lost for long time execution of the algorithm.

Active SLAM problem and exploration with Rao–Blackwellized Particles Filter is investigated by Carlone, Du, Ng, Bona and Indri [36]. They proposed an application of Kullback-Leiber divergence to evaluate the particle-based SLAM posterior estimation. This metric is then employed in the definition of expected information from a policy, which allows the robot to self-decide the best motion strategy to simultaneously explore the environment and reduce the uncertainty in SLAM posterior. The probabilistic interpretation of the proposed information gain is then validated, by comparing it with state-of-art metrics of information gain. The proposed technique enhances robot awareness in detecting loop closing occasions compared to other approaches.

Mullane, Ba-Ngu, Adams and Ba-Tuong [37] used the Gaussian mixture probability hypothesis density (PHD) filter to represents the map of the environment as a random finite set (RFS) of landmarks. The main idea to this approach is to adopt the natural finite-set representation of the map and to use the tools of finite-set statistic for error estimation using sets, instead of conventional state vector. Particles are used to represent robot's trajectories and Rao–Blackwellized PHD filter for each particle's map, robot's location and the number of features that have passed through the field of view (FOV) of robot's sensor are estimated [38].

Particle filter SLAM algorithm is being improved to be implemented in environments that look very similar to each other. The main problem that needs to be solved with similar environments is data association in order to perform SLAM. A large amount of particles are needed to reduce the precision error in particle filtering SLAM in environments which look identical. Zhang, Yao, Tang and Xu proposed an improved particle filter SLAM algorithm based on particle swarm optimization in similar environments [39]. The information from the odometry and the laser scanning are fused to form a multimode proposal distribution with particles which are concentrated to the region of each posterior probability distribution maximum value by particle swarm optimization. Thus, it improved the performance of the conventional particle filter SLAM.

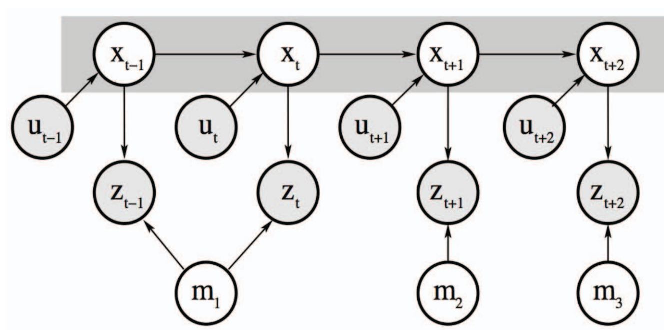

Fig. 3. Particle Filter SLAM Graphical Model

Fig. 3 shows the graphical model of particle filter SLAM which illustrate that landmarks are independence of each other given that the robot's pose is known. Thus, landmark's location can be estimated using several low-dimensional EKFs.

V. H_∞ Filter

H_∞ method is normally used to integrate controller to achieve promising performance by expressing the control problem as a mathematical optimization problem. H_∞ filter does not require any prior knowledge of the noise distribution as compared to conventional Kalman filter which requires an accurate system model as well as the knowledge of noise distribution.

The H_∞ filter approach to solve SLAM problem has been proposed by Chandra, Gu and Postlethwait [40]. H_∞ overcome the limitation of conventional Kalman filter which suffers from the assumption of statistical noises and estimation accuracy is improved under non-Gaussian noise distribution [41]. H_∞ filter is applicable for system that is either linear or nonlinear and is good alternative for an environment with unknown noise distribution. Another important aspect that H_∞ filter can improve is the robot's orientation estimation is much more accurate.

Unscented H_∞ filter based SLAM which is capable for a nonlinear system with non-Gaussian noise is introduced by Ni and Li [42]. This proposed filter is derived from H_∞ filter fused with a UKF. The robustness of Unscented H_∞ SLAM is more robust compared to EKF based methods when the measurement noise is non-Gaussian because Unscented H_∞ SLAM can provide a guarantee to limit the estimation error in the worst case. Besides, Unscented H_∞ SLAM also gained more accurate estimate than EKF-SLAM and UKF-SLAM.

The advantages of Unscented H_∞ SLAM:

i. It can deal with systems with either Gaussian or non-Gaussian noises.
ii. It does not need to compute Jacobians for nonlinear state estimation.
iii. It is a more robust filter compared to conventional filter like EKF-SLAM and UKF-SLAM.

Although studies showed that H_∞ filter may be one of the best methods to mitigate navigation issues for SLAM for environment with unknown noise characteristic. However, H_∞ filter also has some limitations such as the robot need to keep observing any landmarks of the environment to avoid the state covariance of H_∞ filter being diverged. H_∞ filter has to fulfill sufficient conditions [43, 44] i.e. state covariance initialization, measurement noise and process noise for estimation in order to avoid wrong estimation which will cause finite escape time [45] that is undesirable situation in SLAM.

VI. Conclusion

In this paper, we have briefing survey the most used filtering techniques in SLAM over decades. These filtering strategies are Kalman filter with its variants (EKF, UKF, EKF and CKF), particle filter and H_∞ filter. Even though SLAM problems are considered solved, there are still some insights need to be overcome i.e. computational complexity and dynamic environments since most of the studies are conducted in controlled environment. Other approaches such as Graph-based SLAM [46] which used optimization techniques and topological SLAM [47] are being studied in recent years.

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