We can construct an arbitrary length hash function  $H^s(x)$  from a fixed length hash function  $h^s(x)$  with inputs of length 2n and output of length n. If (Gen, h) is a fixed length collision resistant hash function, then (Gen, H) is also a collision resistant hash function.

Let (Gen, h) be a fixed length collision resistant hash function for input of length 2l(n) and output of length l(n). We can set l(n) = l. On input a key s and a string x of any length L such that  $L < 2^{l(n)}$ , a variable length hash function (Gen, H) can be constructed as follows using Merkle-Demgard transform:

- Set the length L = length of the string x. Let there be B =  $\lceil L/1 \rceil$  blocks. Pad x with zeros so that its length is a multiple of I. Parse the result as the sequence of I-bit blocks x1...xB. Set  $x_{B+1} = L$  where L is encoded using exactly I bits.
- Set  $z_0 = 0^1$  as initialising vector
- For i=1 to B+1, compute  $z_i = h^s(z_{i-1} || x_i)$
- Output z<sub>B+1</sub>

