HW6 – Lambda Calculus

CS 476, Fall 2018
Due Oct. 31 at 2 PM

1 Instructions

This assignment is to be completed by hand (or in LaTeX if you know how to use it). Submit your answers as a PDF file via Gradescope. If you don't have easy access to a scanner, you can use the one in SEO 1120, the main CS office – the staff will be happy to help you. As always, please don't hesitate to ask for help on Piazza (`https://piazza.com/class/jkh8q52qrh06v`).

2 Lambda Calculus

1. (5 points) Write the lambda calculus terms corresponding to the following functions:

   - The function that takes an argument and applies that argument to itself.

   - The function that takes two arguments, and returns the first argument. (Hint: "the function that takes two arguments and does X" is the same as "the function that takes an argument, and returns a function that takes an argument and does X".)

   - The function that takes two arguments, and applies the second argument to the first argument.

2. (7 points) For each of the following terms, draw a line from each variable occurrence to the place where it is bound.

   - $(\lambda x.\ x\ x)\ (\lambda x.\ x\ x)$

   - $(\lambda x.\ (\lambda x.\ x)\ x)$

   - $(\lambda x.\ \lambda y.\ \lambda x.\ x\ y\ x)(\lambda z.\ z)$

   - $(\lambda x.\ (\lambda x.\ x)\ x)$

3. Fully evaluate each of the following terms according to call-by-value semantics. You do not need to write proof trees (though you can), but you do need to show each step of the evaluation if there is more than one step. Be sure to rename variables when necessary to avoid variable capture. (Hint: your final result for each term should be a λ-expression.)

   - $(\lambda x.\ x\ x)\ (\lambda z.\ z)$

   - $(\lambda x.\ \lambda y.\ x)\ (\lambda y.\ y)$

   - $(\lambda x.\ \lambda y.\ \lambda x.\ x)\ (\lambda z.\ z)$

   - $((\lambda x.\ \lambda y.\ y\ x)\ (\lambda x.\ x))\ (\lambda y.\ y)$

4. Write the type of each of the following terms in the simply-typed lambda calculus. Again, you do not need to write proof trees, but it might help. Make sure to write parentheses when needed: the $\to$ is right-associative by default, so that $\text{int} \to \text{int} \to \text{int}$ means $\text{int} \to (\text{int} \to \text{int})$ and not $(\text{int} \to \text{int}) \to \text{int}$.

   - $\lambda x : \text{int}.\ \lambda y : \text{int}.\ x + y$

   - $\lambda y : \text{int}.\ \lambda f : \text{int} \to \text{int}.\ f\ y$

   - $(\lambda y : \text{int}.\ \lambda f : \text{int} \to \text{int}.\ f\ y)\ (4)$

   - $(\lambda f : \text{int} \to \text{int}.\ \lambda y : \text{int}.\ f\ (f\ y))\ (\lambda x : \text{int}.\ x + 1)$