HW8 – Floyd-Hoare Logic

CS 476, Fall 2018
Due Nov. 28 at 2 PM

1 Instructions

This assignment is to be completed by hand (or in LaTeX if you know how to use it). Submit your answers as a PDF file via Gradescope. If you don't have easy access to a scanner, you can use the one in SEO 1120, the main CS office – the staff will be happy to help you. As always, please don't hesitate to ask for help on Piazza (`https://piazza.com/class/jkh8q52qrh06v`).

2 Floyd-Hoare Logic

Here are the inference rules of Floyd-Hoare Logic for a simple imperative programming language.

$$\frac{}{\{P\}\ \text{skip}\ \{P\}} \qquad \frac{\{P\}\ c_1\ \{Q\} \qquad \{Q\}\ c_2\ \{R\}}{\{P\}\ c_1; c_2\ \{R\}} \qquad \frac{}{\{[x \mapsto e]P\}\ x := e\ \{P\}}$$

$$\frac{P_1 \Rightarrow P_2 \qquad \{P_2\}\ c\ \{Q_2\} \qquad Q_2 \Rightarrow Q_1}{\{P_1\}\ c\ \{Q_1\}}$$

$$\frac{\{P \wedge (e = \text{true})\}\ c_1\ \{Q\} \qquad \{P \wedge (e = \text{false})\}\ c_2\ \{Q\}}{\{P\}\ \text{if}\ e\ \text{then}\ c_1\ \text{else}\ c_2\ \{Q\}} \qquad \frac{\{P \wedge (e = \text{true})\}\ c\ \{P\}}{\{P\}\ \text{while}\ e\ \text{do}\ c\ \{P \wedge (e = \text{false})\}}$$

3 Problems

There are five problems in all. Use as much space as you need for each problem. You can add extra pages if you need to.

1. (3 points) Using the rules above, construct a proof tree for the Hoare triple $\{x = 3\}$ x := 4 $\{x = 4\}$. You will need to use both the assignment rule and the rule of consequence.

2. (6 points) Construct a proof tree for the Hoare triple

$$\{true\} \text{ if } x = y \text{ then } z := y - x \text{ else } z := y - y \ \{z = 0\}$$

Make sure to check all the necessary implications!

3. (4 points) Construct a proof tree for the Hoare triple

$$\{x = a \wedge y = b\} \; z := x; \; x := y; \; y := z \; \{y = a \wedge x = b\}$$

Assume that sequencing is right-associative, so that $z := x; \; x := y; \; y := z$ is the same as $z := x; \, (x := y; \; y := z)$.

4. (2 points) Write an informative precondition and postcondition for the following program.

```
x := 1;

y := b;

while y > 0 (

  x := x * a;

  y := y - 1

)
```

5. (10 points) Annotate the program from the previous problem with conditions showing the outline of a Hoare logic correctness proof. For full credit, also show any logical implications that need to hold for the proof to be correct.