CS 476 HW8 -- Floyd-Hoare Logic

Sandeep Joshi

TOTAL POINTS

15 / 25

QUESTION 1

1 1 **3 / 3**

- ✓ **+ 1 pts assignment rule and rule of consequence**
- ✓ **+ 1 pts correct precondition**
- ✓ **+ 1 pts overall structure**

1 I think this is a typo.

QUESTION 2

2 2 **1 / 6**

- ✓ **+ 1 pts true and false case**
- **+ 1 pts** preconditions for each case
- **+ 2 pts** rule of consequence and assignment rule
- **+ 1 pts** correct implications
- **+ 1 pts** assignment rule correctly applied
- **- 1 pts** extra steps
- **+ 1 pts** only rule of consequence (no assignment rule)
- **+ 1 pts** only assignment rule (no rule of consequence)

QUESTION 3

3 3 **1 / 4**

- ✓ **+ 1 pts first application of sequence rule**
- **+ 1 pts** second application of sequence rule
- ✓ **+ 1 pts intermediate assertion 1**
- **+ 1 pts** intermediate assertion 2
- **- 0.5 pts** Extra steps
- ✓ **- 1 pts Included "x = a" in preconditions**
- **- 0.5 pts** missing top line for assignment rule

2 This should only be "{z = a ^ y = b}"

3 The two right-most sequence statements need to be grouped as one for the first sequence rule application.

QUESTION 4

4 4 **2 / 2**

- ✓ **+ 0.5 pts right general form**
- ✓ **+ 0.5 pts precondition is reasonable**
- ✓ **+ 0.5 pts postcondition is correct**
- ✓ **+ 0.5 pts postcondition is informative**

4 The program sets the values of x and y, so they don't need to be set in the precondition.

QUESTION 5

5 5 **8 / 10**

- ✓ **+ 2 pts first two annotations**
- ✓ **+ 3 pts loop invariant**
- ✓ **+ 3 pts loop body annotations**
- **+ 1 pts** implications for loop invariant
- **+ 1 pts** implications where necessary for assignments
- **+ 0 pts** graded
- **+ 2.5 pts** loop invariant almost right
- **- 0.5 pts** one assignment rule misapplied
- **+ 1.5 pts** applied sequence rule on loop body
- **+ 1.5 pts** first two annotations mostly correct
- **+ 2 pts** invariant and body annotations exist, but are incorrect
- **+ 1.5 pts** loop invariant half right
- **+ 1 pts** one annotation for initial assignments
- **+ 1 pts** body annotations exist, but are incorrect

5 These annotations are correct, but you also need to show how to use the rule of consequence to put the preconditions in the right form for the assignment rule.

6 You need to show that this follows from the precondition before the loop...

7 and that this implies the final postcondition.

1. (3 points) Using the rules above, construct a proof tree for the Hoare triple {x = 3} x := 4 {x = 4}. You will need to use both the assignment rule and the rule of consequence.

$$\frac{\{\mathrm{x{=}3} \Rightarrow \mathrm{4{=}4}\} \qquad \dfrac{}{\{4 = 4\}x := 4\{x = 4\}}}{\{x = 3\}x := 4\{x = 4\}}$$

2. (6 points) Construct a proof tree for the Hoare triple

$$\{\mathrm{true}\}\ \mathrm{if\ x = y\ then\ z := y - x\ else\ z := y - y}\ \{\mathrm{z = 0}\}$$

Make sure to check all the necessary implications!

$$\frac{\{x = y \wedge y - x = 0\}x := y - x\{z = 0\} \quad \{x \neq y\}z : y - y\{z = 0\}}{\{\mathrm{true}\}\ \mathrm{if\ x = y\ then\ z := y - x\ else\ z := y - y}\ \{\mathrm{z = 0}\}}$$

1 1 3 / 3

- ✓ + 1 pts assignment rule and rule of consequence
- ✓ + 1 pts correct precondition
- ✓ + 1 pts overall structure

1 I think this is a typo.

1. (3 points) Using the rules above, construct a proof tree for the Hoare triple {x = 3} x := 4 {x = 4}. You will need to use both the assignment rule and the rule of consequence.

$$\dfrac{\{\mathrm{x{=}3} \Rightarrow \mathrm{4{=}4}\} \qquad \dfrac{}{\{4 = 4\}x := 4\{x = 4\}}}{\{x = 3\}x := 4\{x = 4\}}$$

2. (6 points) Construct a proof tree for the Hoare triple

$$\{\text{true}\} \text{ if x = y then z := y} - \text{x else z := y} - \text{y } \{\text{z = 0}\}$$

Make sure to check all the necessary implications!

$$\dfrac{\{x = y \wedge y - x = 0\}x := y - x\{z = 0\} \quad \{x \neq y\}z : y - y\{z = 0\}}{\{\text{true}\} \text{ if x = y then z := y} - \text{x else z := y} - \text{y } \{\text{z = 0}\}}$$

3. (4 points) Construct a proof tree for the Hoare triple

$$\{\mathrm{x} = a \land \mathrm{y} = b\}\ \mathrm{z} := \mathrm{x};\ \mathrm{x} := \mathrm{y};\ \mathrm{y} := \mathrm{z}\ \{\mathrm{y} = a \land \mathrm{x} = b\}$$

Assume that sequencing is right-associative, so that z := x; x := y; y := z is the same as z := x; (x := y; y := z).

$$\frac{\{x = a \land y = b\}z := x\{x = a \land y = b \land z = a\}x := y\{x = b \land y = b \land z = a\}y := z\{x = b \land y = a\}}{\{\mathrm{x} = a \land \mathrm{y} = b\}\ \mathrm{z} := \mathrm{x};\ \mathrm{x} := \mathrm{y};\ \mathrm{y} := \mathrm{z}\ \{\mathrm{y} = a \land \mathrm{x} = b\}}$$

2 2 1 / 6

- ✓ + **1 pts** true and false case
- + **1 pts** preconditions for each case
- + **2 pts** rule of consequence and assignment rule
- + **1 pts** correct implications
- + **1 pts** assignment rule correctly applied
- - **1 pts** extra steps
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$$\{\mathrm{x} = a \land \mathrm{y} = b\}\ \mathrm{z} := \mathrm{x};\ \mathrm{x} := \mathrm{y};\ \mathrm{y} := \mathrm{z}\ \{\mathrm{y} = a \land \mathrm{x} = b\}$$

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3 3 **1 / 4**

- ✓ **+ 1 pts** **first application of sequence rule**
- **+ 1 pts** second application of sequence rule
- ✓ **+ 1 pts** **intermediate assertion 1**
- **+ 1 pts** intermediate assertion 2
- **- 0.5 pts** Extra steps
- ✓ **- 1 pts** **Included "x = a" in preconditions**
- **- 0.5 pts** missing top line for assignment rule

2 This should only be "{z = a ^ y = b}"

3 The two right-most sequence statements need to be grouped as one for the first sequence rule application.

4. (2 points) Write an informative precondition and postcondition for the following program.

```
Pre-Condition : {x=1 /\ y=b /\ a=a}

x := 1;

y := b;

while y > 0 (

  x := x * a;

  y := y - 1

)

Post-Condition : {x = a^b /\ y=0}
```

4 4 2 / 2

✓ + 0.5 pts right general form

✓ + 0.5 pts precondition is reasonable

✓ + 0.5 pts postcondition is correct

✓ + 0.5 pts postcondition is informative

4 The program sets the values of x and y, so they don't need to be set in the precondition.

5. (10 points) Annotate the program from the previous problem with conditions showing the outline of a Hoare logic correctness proof. For full credit, also show any logical implications that need to hold for the proof to be correct.

```
{a = a}
x := 1;
{a = a /\ x = 1}
y := b;
{a = a /\ x = 1 /\ y = b}
while y > 0 (   {x * a^y = a^b}
  {x * a^y = a^b /\ y > 0}
  x := x * a;
  {x * a^(y-1) = a^b }
  y := y - 1
  {x * a^y = a^b}

)             {x * a^y = a^b /\ y <= 0}

{x = a^b /\ y = 0}
```

5 5 8 / 10

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