

# CS 476 Midterm

Sandeep Joshi

TOTAL POINTS

**100 / 100**

## QUESTION 1

### Problem 1 20 / 20

#### 1.1 1a 10 / 10

+ 0 pts graded

- ✓ + 1 pts O production 1 takes string
- ✓ + 2 pts O production 2 takes int \* string
- ✓ + 1 pts I production 1 takes ast\_O \* ast\_O
- ✓ + 1 pts I production 2 takes ast\_O \* ast\_O
- ✓ + 2 pts I production 3 takes ast\_I \* int
- ✓ + 2 pts no extra productions
- ✓ + 1 pts mostly correct syntax

#### 1.2 1b 10 / 10

+ 0 pts graded

- ✓ + 1 pts root: do
- ✓ + 2 pts L: take
- ✓ + 1 pts R: 5
- ✓ + 2 pts LL: pieces
- ✓ + 1 pts LR: box
- ✓ + 1 pts LLL: 3
- ✓ + 1 pts LLR: paper
- ✓ + 1 pts correct types/tree structure

## QUESTION 2

### 2 Problem 2 25 / 25

+ 0 pts graded

- ✓ + 5 pts assignment rule
- ✓ + 3 pts expression has type int
- ✓ + 5 pts if-then-else rule
- ✓ + 3 pts bool rule
- ✓ + 3 pts num rule
- ✓ + 3 pts var rule
- ✓ + 3 pts proof tree structure
  - 1 pts Extra rules
  - 1 pts ill-formed rule applications
  - 2 pts missing one top line

- 3 pts missing multiple top lines

1 No rule needed here

## QUESTION 3

### 3 Problem 3 15 / 15

+ 0 pts graded

- ✓ + 3 pts typecheck e
- ✓ + 3 pts typecheck e1
- ✓ + 3 pts typecheck e2
- ✓ + 3 pts e must be Tbool
- ✓ + 3 pts e1 and e2 must be t
- + 2 pts e1 and e2 same type, but not t
- 3 pts only typechecks one side

## QUESTION 4

### 4 Problem 4 15 / 15

+ 0 pts graded

- ✓ + 5 pts if-then-else-command rule
- ✓ + 5 pts correct next step
- ✓ + 5 pts evaluate true
  - 2 pts missing top line
  - 2 pts Extra rules
  - + 3 pts (if others 0) Correct tree structure
  - + 2.5 pts false above the line, true below
  - + 2 pts turned true into false

## QUESTION 5

### 5 Problem 5 25 / 25

✓ + 0 pts graded

- ✓ + 5 pts gave small-step or hybrid rules
- ✓ + 7 pts handles c0 and c1 cases
- ✓ + 5 pts handles default case
- ✓ + 5 pts evaluates e correctly
- ✓ + 3 pts default case only applies when e is not 0 or 1

- + **3 pts** evaluates  $e$  only in non-default cases
- **2 pts** Didn't use switch in bottom rule
- **2 pts** extra conditions
- + **3 pts** has the right idea intuitively

# CS 476 Fall 2018 Midterm

Name:	SANDEEP JOSHI
NetID:	Sjoshi37

- You have 50 minutes to complete this exam.
- This is a closed-book exam.
- Do not share anything with other students. Do not talk to other students. Do not look other students' exams. Do not expose your exam to easy viewing by other students. Violation of any of these rules will count as cheating.
- If you believe there is an error or an ambiguous question, you may seek clarification from the instructor. Please speak quietly or write your question out.
- Including this cover sheet and rules at the end, there are 8 pages to the exam, including one blank page for workspace. Please verify that you have all 8 pages.
- Please write your name and NetID in the spaces above, and also in the provided space at the top of every sheet.
- Show your work. Partial credit will be given for incomplete answers.
- If you finish with time remaining, check your work!

Question	Points	Score
1	20	
2	25	
3	15	
4	15	
5	25	
Total:	100	

**Problem 1.** (20 points)

Consider the following BNF grammar:

 $O ::= \langle \text{ident} \rangle \mid \langle \# \rangle \text{ pieces of } \langle \text{ident} \rangle$  $I ::= \text{take } O \text{ from } O \mid \text{put } O \text{ in } O \mid \text{do } I \langle \# \rangle \text{ times}$ 

- (a) (10 points) Write OCaml datatypes `ast_O` and `ast_I` that encode the abstract syntax trees of  $O$  and  $I$  respectively. You may represent  $\langle \text{ident} \rangle$  with the `string` type and  $\langle \# \rangle$  with the `int` type.

`ast_O = Item of string | Pieces of int * string`

`ast_I = Takefrom of ast_O * ast_O | Putin of ast_O * ast_O |  
Do of ast_I * int`

- (b) (10 points) Write the instance of the `ast_I` type corresponding to the AST for the term

do (take 3 pieces of paper from box) 5 times

If you prefer, you can instead draw the AST for the term.

`Do (Takefrom (Pieces (3, "paper"), Item("box")), 5)`

**Problem 2.** (25 points)

The typing rules for a simple imperative language are given in Appendix A. Write a proof tree for the judgment

$$\Gamma \vdash x := \text{if false then } 5 \text{ else } x : \text{ok}$$

given that  $\Gamma(x) = \text{int}$ .

$$\begin{array}{c}
 \begin{array}{c} \Gamma(x) = \text{int} \\ \hline \Gamma \vdash x : \text{int} \end{array} \quad \begin{array}{c} \text{false is a boolean} \quad 5 \text{ is a number} \\ \hline \Gamma \vdash \text{false} : \text{bool} \quad \Gamma \vdash 5 : \text{int} \end{array} \quad \begin{array}{c} \Gamma(x) = \text{int} \\ \hline \Gamma \vdash x : \text{int} \end{array} \\
 \hline
 \Gamma \vdash x := \text{if false then } 5 \text{ else } x : \text{ok}
 \end{array}$$

**Problem 3.** (15 points)

Suppose you were writing a type-checking function `typecheck_exp : context -> exp -> typ -> bool` that takes a type context `gamma`, an expression `e`, and a type `t` that is either `Tint` or `Tbool`, and returns `true` if  $\gamma \vdash e : t$  and `false` otherwise. Fill in the skeleton below by translating the typing rule for the if-then-else expression into OCaml code.

```
let rec typecheck_exp (gamma : context) (e : exp) (t : typ) : bool =
```

```
  match e with
```

```
  | If (e, e1, e2) -> (match typecheck_exp gamma e Tbool with
```

```
    | true -> typecheck_exp gamma e1 t && typecheck_exp
```

```
    | true -> typecheck_exp gamma e1 t && typecheck_exp gamma e2 t)
```

```
    | false ->
```

```
    | _ -> false )
```

**Problem 4.** (15 points)

The operational semantics rules for a simple imperative language are shown in Section B. Write a proof tree for the next step that

(if true then  $y := 5$  else skip,  $\{y = 3\}$ )

takes.

$$\frac{\frac{\text{true is a boolean}}{(true, \{y=3\}) \Downarrow true}}{(if \text{ true then } y := 5 \text{ else skip, } \{y=3\}) \rightarrow (y := 5, \{y=3\})}$$

**Problem 5.** (25 points)

Write small-step operational semantics rules for the construct

$$\text{switch } e \{ \text{case } 0: c_0; \text{case } 1: c_1; \text{default: } c_2 \}$$
executes  $c_0$  if the value of  $e$  is 0,  $c_1$  if the value of  $e$  is 1, and  $c_2$  if the value of  $e$  is anything else.

Hint: Think about which existing construct is most similar to switch, and look at its rules.

~~$v$  is a number~~  
 $(e, \sigma) \Downarrow 0$

---


$$(\text{switch } e \{ \text{case } 0: c_0; \text{case } 1: c_1; \text{default: } c_2 \}, \sigma) \rightarrow (c_0, \sigma)$$

~~$v$  is a number~~  
 $(e, \sigma) \Downarrow 1$

---


$$(\text{switch } e \{ \text{case } 0: c_0; \text{case } 1: c_1; \text{default: } c_2 \}, \sigma) \rightarrow (c_1, \sigma)$$

~~$v$  is a number~~  
 $(e, \sigma) \Downarrow v$

~~$v$  is a number~~  $((v=0), \sigma) \Downarrow \text{false}$   $((v=1), \sigma) \Downarrow \text{false}$

---


$$(\text{switch } e \{ \text{case } 0: c_0; \text{case } 1: c_1; \text{default: } c_2 \}, \sigma) \rightarrow (c_2, \sigma)$$



## A Typing Rules for a Simple Imperative Language

$$\begin{array}{c}
\frac{(n \text{ is a number})}{\Gamma \vdash n : \text{int}} \quad \frac{(b \text{ is a boolean})}{\Gamma \vdash b : \text{bool}} \quad \frac{(\Gamma(x) = \tau)}{\Gamma \vdash x : \tau} \\
\\
\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 \oplus e_2 : \text{int}} \quad \text{where } \oplus \text{ is an arithmetic operator} \\
\\
\frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \text{bool}}{\Gamma \vdash e_1 \otimes e_2 : \text{bool}} \quad \text{where } \otimes \text{ is a boolean operator} \quad \frac{\Gamma \vdash e_1 : \tau \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1 = e_2 : \text{bool}} \\
\\
\frac{\Gamma \vdash e : \text{bool} \quad \Gamma \vdash e_1 : \tau \quad \Gamma \vdash e_2 : \tau}{\text{if } e \text{ then } e_1 \text{ else } e_2 : \tau} \\
\\
\frac{(\Gamma(x) = \tau) \quad \Gamma \vdash e : \tau}{\Gamma \vdash x := e : \text{ok}} \quad \frac{\Gamma \vdash c_1 : \text{ok} \quad \Gamma \vdash c_2 : \text{ok}}{\Gamma \vdash c_1; c_2 : \text{ok}} \\
\\
\frac{\Gamma \vdash e : \text{bool} \quad \Gamma \vdash c_1 : \text{ok} \quad \Gamma \vdash c_2 : \text{ok}}{\Gamma \vdash \text{if } e \text{ then } c_1 \text{ else } c_2 : \text{ok}} \quad \frac{\Gamma \vdash e : \text{bool} \quad \Gamma \vdash c : \text{ok}}{\Gamma \vdash \text{while } e \text{ do } c : \text{ok}}
\end{array}$$

## B Operational Semantics for a Simple Imperative Language

$$\begin{array}{c}
\frac{(n \text{ is a number})}{(n, \sigma) \Downarrow n} \quad \frac{(b \text{ is a boolean})}{(b, \sigma) \Downarrow b} \quad \frac{(\sigma(x) = v)}{(x, \sigma) \Downarrow v} \\
\\
\frac{(e_1, \sigma) \Downarrow v_1 \quad (e_2, \sigma) \Downarrow v_2 \quad (v_1 \oplus v_2 = v)}{(e_1 \oplus e_2, \sigma) \Downarrow v} \quad \text{where } \oplus \text{ is an arithmetic or boolean operator} \\
\\
\frac{(e, \sigma) \Downarrow \text{true} \quad (e_1, \sigma) \Downarrow v}{(\text{if } e \text{ then } e_1 \text{ else } e_2, \sigma) \Downarrow v} \quad \frac{(e, \sigma) \Downarrow \text{false} \quad (e_2, \sigma) \Downarrow v}{(\text{if } e \text{ then } e_1 \text{ else } e_2, \sigma) \Downarrow v} \\
\\
\frac{(e, \sigma) \Downarrow v}{(x := e, \sigma) \rightarrow (\text{skip}, \sigma[x \mapsto v])} \\
\\
\frac{(c_1, \sigma) \rightarrow (c'_1, \sigma')}{(c_1; c_2, \sigma) \rightarrow (c'_1; c_2, \sigma')} \quad \frac{}{(\text{skip}; c_2, \sigma) \rightarrow (c_2, \sigma)} \\
\\
\frac{(e, \sigma) \Downarrow \text{true}}{(\text{if } e \text{ then } c_1 \text{ else } c_2, \sigma) \rightarrow (c_1, \sigma)} \quad \frac{(e, \sigma) \Downarrow \text{false}}{(\text{if } e \text{ then } c_1 \text{ else } c_2, \sigma) \rightarrow (c_2, \sigma)} \\
\\
\frac{}{(\text{while } e \text{ do } c, \sigma) \rightarrow (\text{if } e \text{ then } (c; \text{while } e \text{ do } c) \text{ else skip}, \sigma)}
\end{array}$$

## C Scratch Space