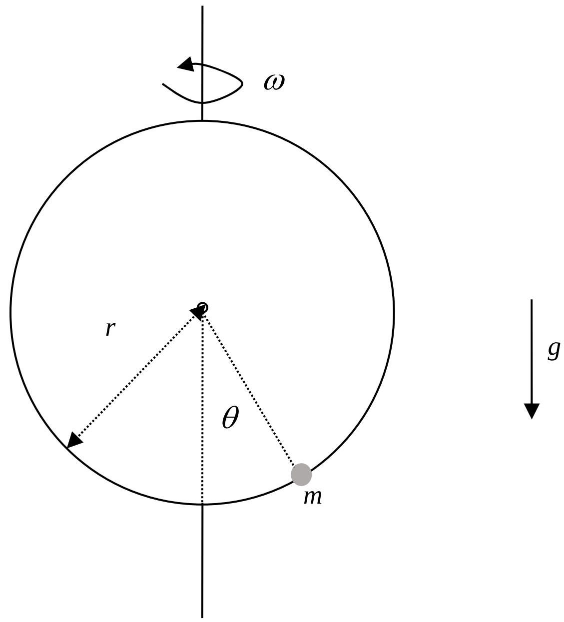
Posting Date: Monday Nov. 4th.
Due Date: Monday Nov. 11th.

1. Consider the following mechanical system:



It consists of a bead of mass m that is constrained to move under the influence of gravity on a circular wire of radius r. The plane of the circular wire is vertical, and it rotates about the vertical axis that goes through the center of the circle. It rotates counterclockwise, as viewed from above, at the constant angular velocity ω. The angle of the mass on the wire measured from the lowest point of the circle is θ. Use Lagrange's Equations to derive the equation of motion for this system's generalized coordinate θ.

2. Consider the following model of the vertical motion of a person on a pogo stick. It is pictured below. The model consists of two masses, one above the other. The bottom mass is connected to the ground via a spring that represents the pogo stick's spring. The top mass is connected to the bottom mass via a spring and a pneumatically driven piston. These two components act in parallel The bottom mass represents the mass of the moving part of the pogo stick and of the parts of the person that move with the pogo stick, i.e., the person's feet, hands, lower legs, and lower arms. The upper mass represents the remainder of the person's mass. The spring and piston between the two masses approximate the effects of the person's arms and legs. This part of the system provides passive support through the spring, but it also has the capability to input or extract energy via the piston. The two generalized coordinates of this system are the deflection of the bottom mass relative to the ground, y_1, and the deflection of the upper mass relative to the lower mass, y_2. An increase in y_1 indicates an increase in the height of mass m_1. An increase in y_2 indicates an increase in the distance between masses m_1 and m_2. At $y_1 = 0$ the lower spring is neither stretched nor

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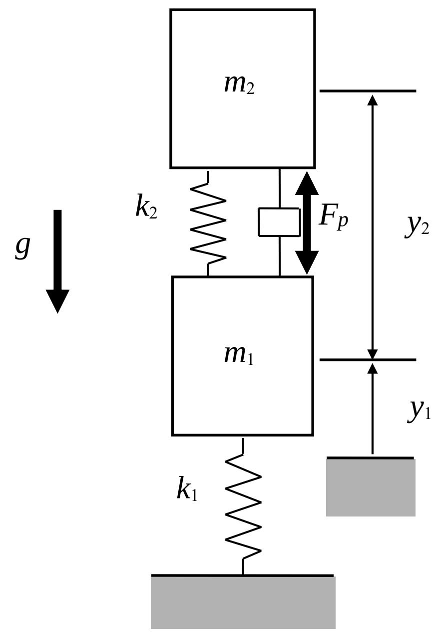
compressed. Similarly, at $y_2 = 0$ the upper spring is neither stretched nor compressed. The force exerted by the position on the two masses is F_p. It is commanded by the jumper, as though it had a pneumatic control input. It is not a conservative force. F_p is positive-valued when it is pushing the two masses further apart. Use Lagrange's Equations to derive the equations of motion for this system's generalized coordinates y_1 and y_2. The pneumatic piston force time history $F_p(t)$ is assumed to be a known function of time.



Note: a real pogo stick would not be connected to the ground. Therefore, spring k_1 would only be able to supply compressive forces. The equations you develop will only be valid when $y_1 \leq 0$. You need not consider this fact in order to derive the correct equations of motion for this situation.

3. Consider the model of a tricycle that is shown below. It moves in a horizontal plane. The Cartesian coordinates of the mid-point between its rear wheels are X and Y. Its heading angle is θ, with $\theta = 0$ orienting it in the $+X$ direction (i.e., due east) and with $\theta = \pi/2$ orienting it in the $+Y$ direction (i.e., due north). The steer angle of the front wheel relative to the heading is γ, with a positive γ corresponding to a leftward turn and a negative γ corresponding to a rightward turn. The horizontal distance from the mid-point between the two rear wheels to the ground contact point of the front wheel is b_w. The horizontal distance from the mid-point between the two rear wheels to the total system center of mass (i.e., of the tricycle plus rider) is b_{cm}. The total mass of the tricycle plus rider is m. The total moment of inertial of the tricycle plus rider about the system center of mass is I_{33}. This moment of inertia is for rotation about the axis that points up, i.e., perpendicular to the horizontal plane of motion.

The tricycle and rider can be treated as a single rigid body. The moments of inertia of three wheels about their axles and the corresponding kinetic energy and angular momentum can be ignored because these moments of inertia are negligible. Similarly, the moment of inertial of the front-wheel/handlebar system about the vertical steering column and the corresponding kinetic energy and angular momentum can be ignored because this moment of inertial is negligible.

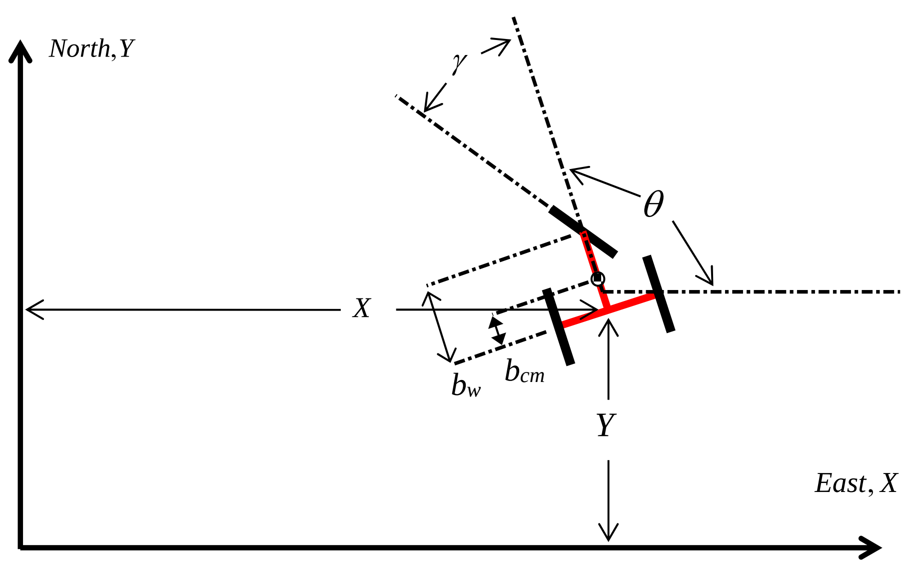
The three wheels all roll without slipping. For purposes of modeling the motion of this vehicle, the important constraints implied by the no-slip condition are that none of the wheels can slip sideways. The rear wheels' no-side-slip condition can be written in the form

$$[-\sin\theta]\,\dot{X} + [\cos\theta]\,\dot{Y} = 0$$

The front wheel's no-side-slip condition can be written in the form:

$$[-\sin(\theta+\gamma)]\dot{X} + [\cos(\theta+\gamma)]\dot{Y} + [b_w\cos\gamma]\dot{\theta} = 0$$

Use Lagrange's Equations to derive the equations of motion for this system's generalized coordinates X, Y and θ. Deal properly with this system's two non-holonomic constraints in your equations. Note that the steer angle $\gamma(t)$ is assumed to be a known input time history that is applied by the rider. There is no need to derive an equation of motion for this angle because it is a system input rather than a generalized coordinate.



Hint: Be careful to correctly derive the kinetic energy of this system. The correct formula should include the length b_{cm} and the moment of inertia I_{33}.