Q1) The definition of I^b_{MoI} is as follows, in $\mathcal{F}^b$

$$I^b_{MoI} = \sum_{i=1}^{N} m_i \left((\Delta \vec{r}_i^{\,b})^T (\Delta \vec{r}_i^{\,b}) I_{3\times3} - (\Delta \vec{r}_i^{\,b})(\Delta \vec{r}_i^{\,b})^T \right)$$

where, $\Delta \vec{r}_i^{\,b} = \vec{r}_i - \vec{r}_{CM}$

then let $\mathcal{F}^c$ be the principle axis frame.

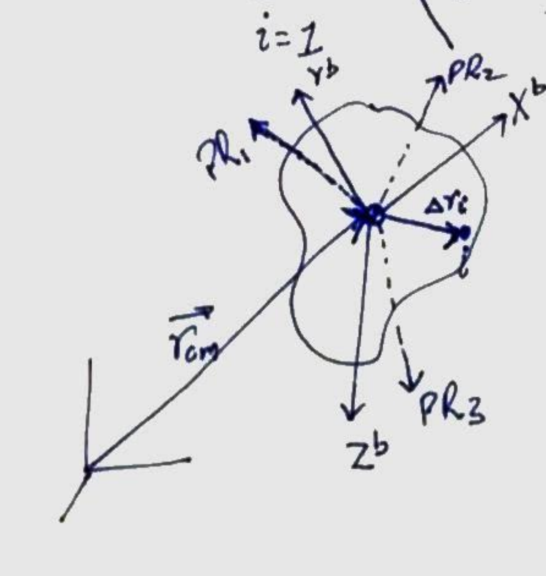
$$I^c_{MoI} = R^T_{pr} I^b_{MoI} R_{pr}$$

where, R^T_{pr} is a rotation matrix from $\mathcal{F}^b$ to $\mathcal{F}^c$.

So, we can write the principle moments of Inertia as follows.

$$I^c_{MoI} = R^T_{pr} \left[\sum_{i=1}^{N} m_i \left((\Delta \vec{r}_i^{\,b})^T (\Delta \vec{r}_i^{\,b}) I_{3\times3} - (\Delta \vec{r}_i^{\,b})(\Delta \vec{r}_i^{\,b})^T \right) \right] R_{pr}$$

$$= \sum_{i=1}^{N} m_i \left((\Delta \vec{r}_i^{\,b})^T (\Delta \vec{r}_i^{\,b}) \underbrace{R^T_{pr} I_{3\times3} R_{pr}}_{I_{3\times3}} - \overbrace{\left(R^T_{pr} \Delta \vec{r}_i^{\,b}\right)}^{\Delta \vec{r}_i^{\,c}} \underbrace{\left(\Delta \vec{r}_i^{\,bT} R_{pr}\right)}_{\Delta \vec{r}_i^{\,cT}} \right)$$



because $\mathcal{F}^c$ and $\mathcal{F}^b$ are only rigid Rotations, the magnitude of, $|\Delta \vec{r}_i^{\,c}| = |\Delta \vec{r}_i^{\,b}|$

$$\therefore (\Delta \vec{r}_i^{\,b})^T (\Delta \vec{r}_i^{\,b}) = (\Delta \vec{r}_i^{\,c})^T (\Delta \vec{r}_i^{\,c})$$

This fact can also be proved as below

$$\Delta\vec{r}_i^{\,b} = R_{pr}^T \Delta\vec{r}_i^{\,c} \; ; \quad \left(\Delta\vec{r}_i^{\,b}\right)^T = \Delta\vec{r}_i^{\,c\,T} R_{pr}$$

$$\left(\Delta\vec{r}_i^{\,b}\right)^T\left(\Delta\vec{r}_i^{\,b}\right) = \left(\Delta r_i^{c}\right)^T \underbrace{R_{pr} R_{pr}^T}_{I_{3\times3}} \left(\Delta\vec{r}_i^{\,c}\right)$$

$$= \left(\Delta r_i^{c}\right)^T\left(\Delta\vec{r}_i^{\,c}\right)$$

So rewriting I_{MoI}^{C} in its final form in terms $\Delta\vec{r}_i^{\,c}$

$$I_{MoI}^{C} = \sum_{i=1}^{N} m_i\left[\left(\Delta\vec{r}_i^{\,c}\right)^T\left(\Delta\vec{r}_i^{\,c}\right) I_{3\times3} - \left(\Delta r_i^{c}\right)\left(\Delta\vec{r}_i^{\,c}\right)^T\right]$$

All the off-diagonal term are zero because of the property of the Principle axes. So the diagonal term are as follows.

$$I_{pr1} = \sum_{i=1}^{N} m_i\left(\left(\Delta Y_i^c\right)^2 + \left(\Delta Z_i^c\right)^2\right); \quad I_{pr2} = \sum_{i=1}^{N} m_i\left(\left(\Delta X_i^c\right)^2 + \left(\Delta Z_i^c\right)^2\right)$$

$$I_{pr3} = \sum_{i=1}^{N} m_i\left(\left(\Delta X_i^c\right)^2 + \left(\Delta Y_i^c\right)^2\right)$$

$$I_{PR1} + I_{PR2} = \sum_{i=1}^{N} m_i\left(\underbrace{\left(\Delta X_i^c\right)^2 + \left(\Delta Y_i^c\right)^2}_{I_{PR3}} + 2\left(\Delta Z_i^c\right)^2\right)$$

from the last two relations it can be infrered that

$$I_{PR1} + I_{PR2} \geq I_{PR3}$$

Problem 2

```
function [Mtot,rcmtot,IMoItot] =
 momentofinertia01(mvec,rcmmat,IMoIarray)
%
%  Copyright (c) 2019 Mark L. Psiaki.  All rights reserved.
%
%  This function computes the total mass, the center of mass,
%  and the total moment of inertia of a collection of
%  rigid bodies that, taken together, form a larger rigid body.
%
%  All position vectors, those of the individual rigid bodies'
%  centers of mass, rcmi = rcmmat(:,i) for i = 1:N, and that of
%  the system center of mass, rcmtot, are given in a common
%  coordinate system as are the individual moment-of-inertia
%  matrices, IMoIi = IMoIarray(:,:,i), and the final total
%  system moment-of-inertia matrix, IMoItot.
%
%
%  Inputs:
%
%    mvec                      The 1-by-N vector that contains the
%                              masses of the individual rigid-body
%                              components, in kg units.  mi = mvec(1,i)
%                              is the mass of the ith rigid body.
%
%    rcmmat                    The 3-by-N matrix that contains the
%                              positions of the centers of mass
%                              of the individual rigid bodies,
%                              given in meters units and along the
%                              common axes that are used
%                              throughout these calculations.
%                              rcmi = rcmmat(:,i) is the center-
%                              of-mass position of the ith
%                              rigid body.
%
%    IMoIarray                 The 3-by-3-by-N array that contains
%                              the moment-of-inertia matrices of the
%                              individual rigid bodies about their
%                              respective centers of mass, in
%                              kg-m^2 units and along the common
%                              axes that are used throughout these
%                              calculations. IMoIi =  IMoIarray(:,:,i)
%                              is the moment-of-inertia matrix of
%                              the ith rigid body about its own
%                              center of mass.
%
%  Outputs:
%
%    Mtot                      The total mass of the composite
%                              rigid body, in kg.
%
```

```
%    rcmtot                    The 3-by-1 vector that gives the
%                              center of mass of the composite rigid
%                              body, in meters and along the common
%                              axes that are used throughout these
%                              calculations.
%
%    IMoItot                   The 3-by-3 moment-of-inertia matrix
%                              of the composite rigid body about its
%                              center of mass, in kg-m^2 and along
%                              the common axes that are used
%                              throughout these calculations.
%

%
%  Compute the total mass.
%
   Mtot = sum(mvec);
%
%  Compute the composite rigid body's center of mass.
%
   N = size(mvec,2);
   Mtot_rcmtot = zeros(3,1);
   for i = 1:N
      mi = mvec(1,i);
      rcmi = rcmmat(:,i);
      Mtot_rcmtot = Mtot_rcmtot + mi*rcmi;
   end
   rcmtot = Mtot_rcmtot/Mtot;
%
%  Compute the composite rigid body's moment-of-inertia
%  matrix about its center of mass.
%
   IMoItot = zeros(3,3);
   for i = 1:N
      mi = mvec(1,i);
      rcmi = rcmmat(:,i);
      deltarcmi = rcmi - rcmtot;
      IMoIi =  IMoIarray(:,:,i);
      deltaIMoIi = mi*((deltarcmi'*deltarcmi)*eye(3)-
(deltarcmi*deltarcmi'));
      IMoItot = IMoItot + IMoIi + deltaIMoIi;
   end
```

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Problem 3

```
clc; clear;
format long

load moicalcs01_data

[my_Mtot,my_rcmtot,my_IMoItot] =
 momentofinertia01(mvec,rcmmat,IMoIarray);

Mtot = 2.058306536059932e+02;

rcmtot = [0.409101895949622;0.526651850364819;0.058154823743388];

IMoItot = 1.0e+04 *[...
1.047260719697208 0.028550325166975 0.040407207761532;...
0.028550325166975 1.063729452526229 0.010880405279456;...
0.040407207761532 0.010880405279456 1.146269471862332];

disp('Checking with data set moicalcs01_data')
disp('Error in Mtot')
disp(Mtot-my_Mtot);
disp('Error in rcmtot')
disp(rcmtot-my_rcmtot);
disp('Error in IMoItot')
disp(IMoItot-my_IMoItot);


clear;

load moicalcs02_data
disp('Results with data set moicalcs02_data')
[Mtot,rcmtot,IMoItot] = momentofinertia01(mvec,rcmmat,IMoIarray)
```

```
Checking with data set moicalcs01_data
Error in Mtot
     2.842170943040401e-14

Error in rcmtot
   1.0e-15 *

   0.111022302462516
  -0.333066907387547
  -0.485722573273506

Error in IMoItot
   1.0e-11 *

   0.545696821063757   0.471800376544706  -0.170530256582424
   0.471800376544706                   0   0.406430444854777
  -0.170530256582424   0.406430444854777  -0.181898940354586
```

```
Results with data set moicalcs02_data

Mtot =

     2.797748586051755e+03


rcmtot =

   1.166967057877594
   0.342883847595047
   1.288814940207648


IMoItot =

   1.0e+06 *

   0.839391302803123  -0.006045159569299  -0.415001439219734
  -0.006045159569299   1.095252203770579  -0.018055436804117
  -0.415001439219734  -0.018055436804117   0.395817419209069
```

Problem 4

```
clear;
a = 0.4;
b = 0.2;
c = 0.8;

l = 2.1;
w = 0.6;

M = 20;
m = 0.6;
theta = 0.523599;

rcm_panel_left = [0;-b/2-l/2;c/2];
rcm_box = [0;0;0];
rcm_panel_right = [0;b/2+l/2;c/2];

rcmmat = [rcm_panel_left,rcm_box,rcm_panel_right];
mvec = [m,M,m];

I_polar_panel = (m/12)*(l^2+w^2);
I_alongl_panel = (m/12)*(w^2);
I_alongw_panel = (m/12)*(l^2);

R2theta_pr = [cos(-theta) 0 -sin(-theta);...
              0           1            0;...
              sin(-theta) 0  cos(-theta)];
```

```
Ipanel_left_principle = diag([I_alongw_panel I_alongl_panel
 I_polar_panel]);
Ipanel_right_principle = diag([I_alongw_panel I_alongl_panel
 I_polar_panel]);

Ipanel_left_b = R2theta_pr*Ipanel_left_principle*R2theta_pr';
Ipanel_right_b = R2theta_pr*Ipanel_right_principle*R2theta_pr';

Ibox_i_b = (M/12)*(b^2+c^2);
Ibox_j_b = (M/12)*(c^2+a^2);
Ibox_k_b = (M/12)*(a^2+b^2);

Ibox_b = diag([Ibox_i_b Ibox_j_b Ibox_k_b]);

IMoIarray(:,:,1) = Ipanel_left_b;
IMoIarray(:,:,2) = Ibox_b;
IMoIarray(:,:,3) = Ipanel_right_b;

[my_Mtot,my_rcmtot,my_IMoItot] =
 momentofinertia01(mvec,rcmmat,IMoIarray);



Mtot = 21.200000000000003;
rcmtot = [0 ; 0;0.022641509433962];

IMoItot =[...
3.351465415801186 0 0.015588461307349;...
0 1.550465408805032 0;...
0.015588461307349 0 2.388333326337180];

disp('Checking with test data')
disp('Error in Mtot')
disp(Mtot-my_Mtot);
disp('Error in rcmtot')
disp(rcmtot-my_rcmtot);
disp('Error in IMoItot')
disp(IMoItot-my_IMoItot);

clear;
a = 0.3;
b = 0.4;
c = 0.6;

l = 1.1;
w = 0.5;

M = 15;
m = 0.8;
theta = 0.34906585;

rcm_panel_left = [0;-b/2-l/2;c/2];
rcm_box = [0;0;0];
rcm_panel_right = [0;b/2+l/2;c/2];
```

```
rcmmat = [rcm_panel_left,rcm_box,rcm_panel_right];
mvec = [m,M,m];

I_polar_panel = (m/12)*(l^2+w^2);
I_alongl_panel = (m/12)*(w^2);
I_alongw_panel = (m/12)*(l^2);

R2theta_pr = [cos(-theta) 0 -sin(-theta);...
              0           1            0;...
              sin(-theta) 0  cos(-theta)];

Ipanel_left_principle = diag([I_alongw_panel I_alongl_panel
 I_polar_panel]);
Ipanel_right_principle = diag([I_alongw_panel I_alongl_panel
 I_polar_panel]);

Ipanel_left_b = R2theta_pr*Ipanel_left_principle*R2theta_pr';
Ipanel_right_b = R2theta_pr*Ipanel_right_principle*R2theta_pr';

Ibox_i_b = (M/12)*(b^2+c^2);
Ibox_j_b = (M/12)*(c^2+a^2);
Ibox_k_b = (M/12)*(a^2+b^2);

Ibox_b = diag([Ibox_i_b Ibox_j_b Ibox_k_b]);

IMoIarray(:,:,1) = Ipanel_left_b;
IMoIarray(:,:,2) = Ibox_b;
IMoIarray(:,:,3) = Ipanel_right_b;

[my_Mtot,my_rcmtot,my_IMoItot] =
 momentofinertia01(mvec,rcmmat,IMoIarray);



Mtot = 21.200000000000003;
rcmtot = [0 ; 0;0.022641509433962];

IMoItot =[...
3.351465415801186 0 0.015588461307349;...
0 1.550465408805032 0;...
0.015588461307349 0 2.388333326337180];

disp('Results:')
disp('Mtot')
disp(my_Mtot);
disp('rcmtot')
disp(my_rcmtot);
disp('IMoItot')
disp(my_IMoItot);
```

Checking with test data
Error in Mtot
0

```
Error in rcmtot
   1.0e-15 *

                   0
                   0
  -0.260208521396521

Error in IMoItot
   1.0e-15 *

   0.444089209850063                   0   0.085001450322864
                   0   0.222044604925031                   0
   0.067654215563095                   0                   0

Results:
Mtot
  16.600000000000001

rcmtot
                   0
                   0
   0.028915662650602

IMoItot
   1.845353074533848                   0   0.010713126817924
                   0   0.725953815261044                   0
   0.010713126817924                   0   1.403267407393863
```

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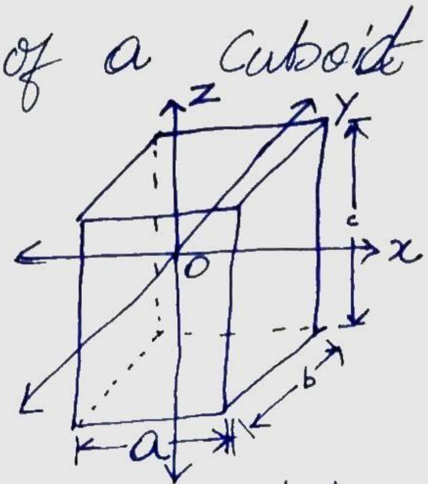
Theory for problem - 4:

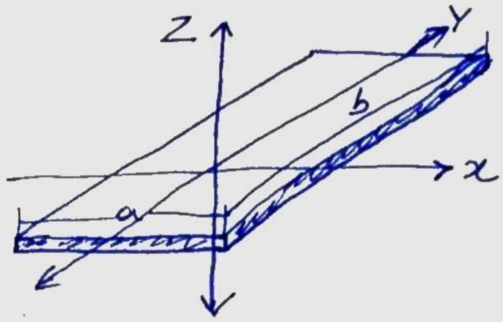
The moment of Inertia of a Cuboid

$$I_{Cuboid} = \frac{M}{12}\begin{bmatrix} b^2+c^2 & 0 & 0 \\ 0 & c^2+a^2 & 0 \\ 0 & 0 & a^2+b^2 \end{bmatrix}$$



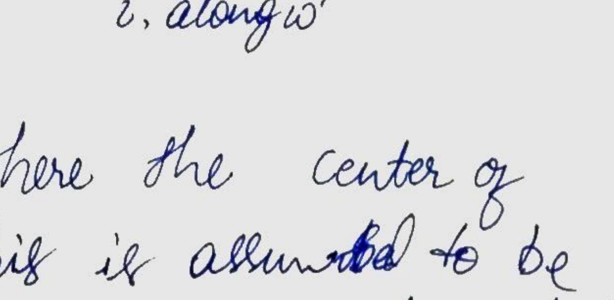
If $c << a$ & $c << b$, a thin plate,
The c^2 is insignificant Compared to a^2 & b^2
So the moment of Inertia is as follows

$$I_{plate} = \frac{M}{12}\begin{bmatrix} b^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2+b^2 \end{bmatrix}, \quad c^2 \simeq 0$$



In the Context of the Satellite,

$$I_{panel} = \frac{m}{12}\begin{bmatrix} l^2 & 0 & 0 \\ 0 & w^2 & 0 \\ 0 & 0 & l^2+w^2 \end{bmatrix}$$



$$I_{box} = \frac{M}{12}\begin{bmatrix} b^2+c^2 & 0 & 0 \\ 0 & c^2+a^2 & 0 \\ 0 & 0 & a^2+b^2 \end{bmatrix}$$

where the center of this is assumed to be the Origin of the whole Satellite.

In order to Express I_{panel} along the main Coord-Syst along the box, we can write this Relation

$$I_{panel}^{in\ box} = R_2(-\theta)\, I_{panel}\, R_2(-\theta)^T$$

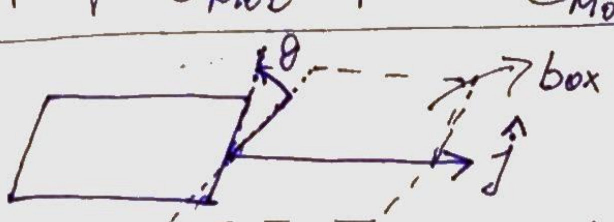
Proof:

I_{panel} is the Principle MoI Matrix & we know the

$$\underbrace{I^{c}_{MoI}}_{principle} = R_{pr}^{T}\, I^{b}_{MoI}\, R_{pr}$$

$$R_{pr}\, I^{c}_{MoI} = \underbrace{R_{pr} R_{pr}^{T}}_{I_{3\times3}}\, I^{b}_{MoI}\, R_{pr}$$ premultiplyin with R_{pr}

$$\boxed{R_{pr}\, I^{c}_{MoI}\, R_{pr}^{T} = I^{b}_{MoI}}$$ (post multiplying with R_{pr}^{T}



because the panel is rotated by $+\theta$, a rotation of $-\theta$ of the principle axis will change to the box (θ) main Co-ord System

$$\therefore \boxed{R_{pr} = R_2(-\theta)}, \text{ where } R_2(\theta) = \begin{bmatrix} \cos(-\theta) & 0 & -\sin(-\theta) \\ 0 & 1 & 0 \\ \sin(-\theta) & 0 & \cos(-\theta) \end{bmatrix}$$

So, the moment of Inertias of the three Compo-
-nents are

I_{box}, $I^{in\ box}_{panel}$, $I^{in\ box}_{panel}$ } all Expressed in the main Co-ord System

$\downarrow$ box, $\downarrow$ Right, $\downarrow$ left

the $\vec{r}_{cm}$ for the Compoments are

$$\vec{r}_{cm,\ box} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \vec{r}_{cm,\ left\ panel} = \begin{bmatrix} 0 \\ -\frac{b}{2}-\frac{l}{2} \\ \frac{c}{2} \end{bmatrix} ; \vec{r}_{cm,\ Right\ panel} = \begin{bmatrix} 0 \\ \frac{b}{2}+\frac{l}{2} \\ \frac{c}{2} \end{bmatrix}$$

The Following Eq (also Coded in the function)

$$M_{tot} = M_{box} + 2\, M_{panel} = M + 2m$$

$$\vec{r}_{cm,tot} = \frac{1}{M_{tot}} \left[\vec{r}_{cm,box} M_{box} + \vec{r}_{cm,\ left\ panel} M_{panel} + \ldots \vec{r}_{cm,\ right\ panel} M_{panel} \right]$$

$$I^{in\ box}_{MOI,\ tot} = \sum_{i=1}^{3} \left(I^{in\ box}_{MOI,\ i} + M_i \left\{ \left(\vec{r}_{cm,i} - \vec{r}_{cm,tot}\right)^T \left(\vec{r}_{cm,i} - \vec{r}_{cm,tot}\right) I_{3\times3} \ldots - \left(\vec{r}_{cm,i} - \vec{r}_{cm,tot}\right)\left(r_{cm,i} - r_{cm,tot}\right)^T \right\} \right)$$

$i = 1, 2, 3$ are the three compoments box, left panel & Right panel