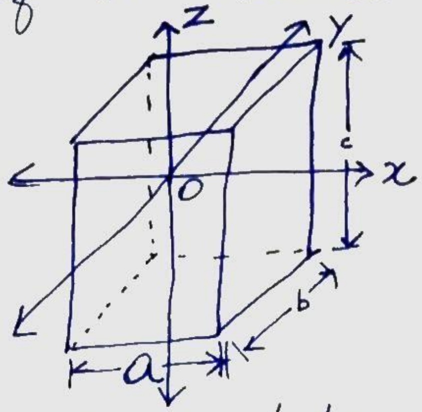
Theory for problem - 4:

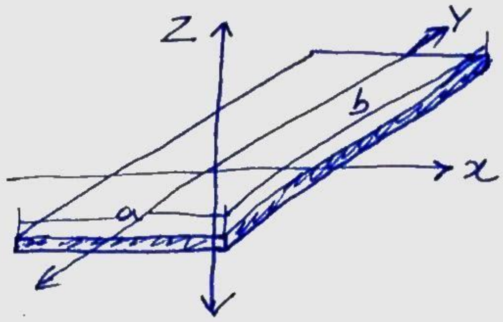
The moment of Inertia of a Cuboid

$$I_{cuboid} = \frac{M}{12}\begin{bmatrix} b^2+c^2 & 0 & 0 \\ 0 & c^2+a^2 & 0 \\ 0 & 0 & a^2+b^2 \end{bmatrix}$$



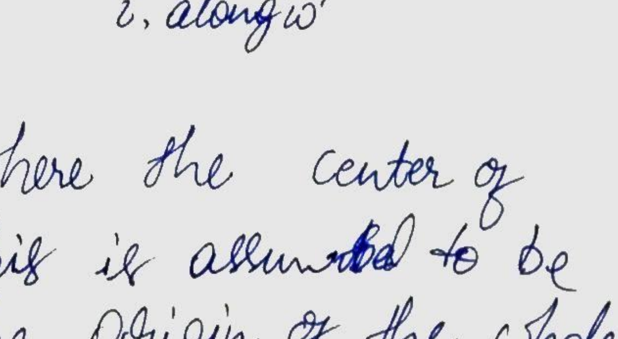
If $c << a$ & $c << b$, a thin plate,
the c^2 is insignificant compared to a^2 & b^2
So the moment of Inertia is as follows

$$I_{plate} = \frac{M}{12}\begin{bmatrix} b^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2+b^2 \end{bmatrix}, \quad c^2 \simeq 0$$



In the context of the satellite,

$$I_{panel} = \frac{m}{12}\begin{bmatrix} l^2 & 0 & 0 \\ 0 & w^2 & 0 \\ 0 & 0 & l^2+w^2 \end{bmatrix}$$



$$I_{box} = \frac{M}{12}\begin{bmatrix} b^2+c^2 & 0 & 0 \\ 0 & c^2+a^2 & 0 \\ 0 & 0 & a^2+b^2 \end{bmatrix}$$

where the center of this is assumed to be the Origin of the whole Satellite.

In order to Express I_{panel} along the main Coord-Syst along the box, we can write this Relation

$$I_{panel}^{in\ box} = R_2(-\theta)\, I_{panel}\, R_2(-\theta)^T$$

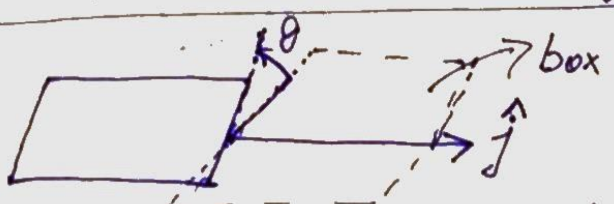
Proof:

I_{panel} is the Principle MoI Matrix & we know the

$$\underbrace{I_{MOI}^{c}}_{principle} = R_{pr}^T\, I_{MOI}^{b}\, R_{pr}$$

$$R_{pr} I_{MOI}^{c} = \underbrace{R_{pr} R_{pr}^T}_{I_{3\times3}} I_{MOI}^{b} R_{pr}$$ premultplying with R_{pr}

$$\boxed{R_{pr} I_{MOI}^{c} R_{pr}^T = I_{MOI}^{b}}$$ (post multiplying with R_{pr}^T



because the panel is rotated by $+\theta$, a rotation of $-\theta$ of the principle axis will change to the box (θ) main Co-ord System

$$\therefore \boxed{R_{pr} = R_2(-\theta)}, \text{ where } R_2(-\theta) = \begin{bmatrix} \cos(-\theta) & 0 & -\sin(-\theta) \\ 0 & 1 & 0 \\ \sin(-\theta) & 0 & \cos(-\theta) \end{bmatrix}$$

So, the moment of Inertias of the three Compo-nents are

I_{box}, $I_{panel}^{in\ box}$, $I_{panel}^{in\ box}$ } all Expressed in the main Co-ord System

$\downarrow$ box, $\downarrow$ Right, $\downarrow$ left

The $\vec{r}_{cm}$ for the Compoments are

$$\vec{r}_{cm,\ box} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \vec{r}_{cm,\ left\ panel} = \begin{bmatrix} 0 \\ -\frac{b}{2}-\frac{l}{2} \\ \frac{c}{2} \end{bmatrix} ; \vec{r}_{cm,\ Right\ panel} = \begin{bmatrix} 0 \\ \frac{b}{2}+\frac{l}{2} \\ \frac{c}{2} \end{bmatrix}$$

The Following Eq (also Coded in the function)

$$M_{tot} = M_{box} + 2 M_{panel} = M + 2m$$

$$\vec{r}_{cm,tot} = \frac{1}{M_{tot}} \left[\vec{r}_{cm,box} M_{box} + \vec{r}_{cm,\ left\ panel} M_{panel} + \ldots \vec{r}_{cm,\ right\ panel} M_{panel} \right]$$

$$I_{MOI,\ tot}^{in\ box} = \sum_{i=1}^{3} \left(I_{MOI,\ i}^{in\ box} + M_i \left\{ \left(\vec{r}_{cm,i} - \vec{r}_{cm,tot}\right)^T \left(\vec{r}_{cm,i} - \vec{r}_{cm,tot}\right) I_{3\times3} \ldots - \left(\vec{r}_{cm,i} - \vec{r}_{cm,tot}\right)\left(r_{cm,i} - r_{cm,tot}\right)^T \right\} \right)$$

$i = 1, 2, 3$ are the three compoments box, left panel & Right panel