Problem 2

```
function [gammaeq,Teq,alphaeq,phieq,XdotSM,YdotSM,iflagterm,niter] = ...
            solvesteadystateaircraft01(Zeq,Veq,psieq,m,S,CLalpha,...
                                       CD0,oneoverpiARe,...
                                       nitermax)
%
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%
%  This function determines for the flight-path angle,
%  thrust, angle-of-attack, and roll angle that produce
%  steady, level, straight-line motion of a point-mass
%  3-dimensional aircraft that is operating over
%  a flat non-rotating Earth (i.e., without Coriolis effects
%  due to the Earth's rotation, without centrifugal
%  effects due to the Earth's rotation other than the
%  mean centrifugal effect at the origin of the coordinate
%  system, and with a constant gravitational acceleration).
%  While it uses a flat-Earth (i.e., constant) gravity field,
%  its constant gravity takes into account the Earth's J2
%  oblatness effect at the coordinate system center and
%  it subtracts off the centrifugal acceleration of the
%  coordinate system center as caused by the Earth's rotation.
%
%  This function works by reducing the problem to a
%  single equation in the single unknown value of
%  angle of attack, alphaeq.  This single nonlinear
%  equation is solved iteratively using Newton's
%  method.  Once the angle of attack has been
%  determined, the thrust Teq can be computed directly.
%  The flight-angle gammaeq and the roll angle
%  phieq are obvious.
%
%  This function also computes the steady-motion rates
%  of change of the northward displacement X and the
%  and eastward displacement Y, which are XdotSM
%  and YdotSM.
%
%
%
%  Inputs:
%
%    Zeq                    The steady-motion vertical
%                           displacement, in meters,
%                           relative to the origin of the
%                           local level coordinate system.
%                           A positive value is down.  This
%                           coordinate system is centered
%                           at the runway of the Blacksburg,
%                           VA, airport, which is at an
%                           altitude of 649.7 m above sea
%                           level.  Therefore, (-Zeq + 649.7)
%                           is the steady-motion aircraft
%                           altitude above sea level.
%
%    Veq                    The steady-motion aircraft
%                           velocity in meters/second.
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%
%    psieq                  The steady-motion aircraft
%                           heading angle in meters.
%                           psieq = 0 radians is due
%                           north (i.e., along the +X
%                           local-level axis) and
%                           psieq = +pi/2 radians is
%                           due east (i.e., along the
%                           +Y axis).
%
%    m                      The aircraft mass in kg.
%
%    S                      The wing area, in meters^2, which is
%                           the aerodynamic model's reference area.
%
%    CLalpha                The lift curve slope, dCL/dalpha, which
%                           is non-dimensional.
%
%    CD0                    The drag at zero lift, which is non-
%                           dimensional.
%
%    oneoverpiARe           = 1/(pi*AR*e), where AR is the non-
%                           dimensional aspect ratio of the wing
%                           and e is the Oswald efficiency factor.
%                           This composite input quantity is non-
%                           dimensional.  It is the coefficient
%                           of CL^2 in the drag coefficient model.
%
%    nitermax               The maximum number of Newton's
%                           method iterations that will be
%                           allowed before the algorithm
%                           quits with an error condition if
%                           it has not reached convergence
%                           prior to executing this many
%                           iterations.  A conservative value
%                           for this limit is 50.
%
%  Outputs:
%
%    gammaeq                The steady-motion flight-path
%                           angle in radians.  This will
%                           equal 0.
%
%    Teq                    The steady-motion thrust, in Newtons.
%
%    alphaeq                The steady-motion angle of attack,
%                           in radians.
%
%    phieq                  The steady-motion roll angle,
%                           in radians.  This will equal 0.
%
%    XdotSM                 The steady-motion northward component
%                           of velocity, in meters/second.
%
%    YdotSM                 The steady-motion eastward component
%                           of velocity, in meters/second.
%
%    iflagterm              A termination status flag that
%                           inticates whether the Newton's
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%                          method solution for alphaeq has
%                          converged.  Its possible values and
%                          their meanings are:
%
%                            0  Normal successful termination.
%
%                            1  Failure to converge in nitermax
%                               Newton iterations.  A warning
%                               message will be sent to the
%                               display in this case.
%
%    niter                 The number of Newton iterations that
%                          have been executed in the attempt
%                          to solve for alphaeq.
%


%
%  Set up the steady-motion flight-path angle and roll angle.
%
   gammaeq = 0;
   phieq = 0;
%
%  Compute the steady-motion northward and eastward velocities
%
   Veq_cosgammaeq = Veq*cos(gammaeq);
   XdotSM = Veq_cosgammaeq * cos(psieq);
   YdotSM = Veq_cosgammaeq * sin(psieq);
%
%  Compute the air density using a decaying exponential
%  model.  This model is good to about 1500 m altitude
%  (about 5000 ft).  This model recognizes that -Zeq + 649.7
%  is the aircraft altitude above sea level in meters.
%
   rho_sealevel = 1.225; % kg/m^3
   hscale = 10230.;      % meters
   rho = rho_sealevel*exp((Zeq - 649.7)/hscale); % kg/m^3
%
%  Set the flat-Earth gravitational acceleration at the
%  Blacksburg airport minus the effects of centrifugal
%  acceleration at the Blacksburg airport due to the
%  Earth's rotation vector.
%
   g = 9.79721; % meters/second^2
%
%  Determine the dynamic pressure.
%
   qbar = 0.5*rho*Veq*Veq;
%
%  Compute the product of qbar and S.
%
   qbar_S = qbar*S;
%
%  Compute the constant term in the equation that
%  will be solved using Newton's method in order to
%  determine alphaeq.  It takes the form:
%
%    0 = f(alphaeq) = tan(alphaeq)*CD(alphaeq) + CL(alphaeq) - C0
%
%  where C0 = 2*m*g/(rho*(Veq^2)*S) = m*g/(qbar*S);
```

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%
   C0 = m*g/qbar_S;
%
%  Initialize the guess of the steady-motion angle of attack
%  at zero.
%
   alphaeq = 0;
%
%  Initialize iflagterm at its nominal successful-case
%  value and initialize niter.
%
   iflagterm = 0;
   niter = 0;
%
%  This is the loop that performs one Newton's method
%  (also known as the Netwon-Raphson method for
%  a scalar equation in a single unknown) iteration
%  towards a better guess of alphaeq.  It also tests
%  whether the guess is close enough to stop
%  the iterations.
%
   testdone0 = 0;
   while testdone0 == 0
      niterp1 = niter + 1;
      if niterp1 > nitermax
         iflagterm = 1;
         disp('Warning in solvesteadystateaircraft01.m.: Newton''s')
         disp([' method did not converge in ',int2str(nitermax),...
               'iterations.'])
         break
      end
      niter = niterp1;
%
      CL = CLalpha*alphaeq;
      CD = CD0 + CL^2*(oneoverpiARe);
      tan_alphaeq = tan(alphaeq);
      f = tan_alphaeq*CD+CL-C0;
      dCL_dalpha = CLalpha;
      dCD_dalpha = 2*CLalpha*CLalpha*alphaeq*oneoverpiARe;
      dtan_alphaeq_dalpha = 1 + tan_alphaeq^2;
      df_dalpha = tan_alphaeq*dCD_dalpha+dtan_alphaeq_dalpha*CD+dCL_dalpha;
      deltaalphaeq = -f/df_dalpha;
      alphaeq = alphaeq + deltaalphaeq;
%
%  Test for convergence.
%
      if abs(deltaalphaeq) <= 1.e-10
         testdone0 = 1;
      end
   end
%
%  Compute the steady-motion thrust.
%
   CL = CLalpha*alphaeq;
   CD = CD0 + CL^2*(oneoverpiARe);
   D = qbar_S*CD;
   Teq = D/cos(alphaeq);
```

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Output:

gammaeq	0
Teq	4.829888745867086e+03
alphaeq	0.052510385639473
phieq	0
XdotSM	84.264888743087582
YdotSM	-70.706637065519317

Q1) The dynamic of the Rigid body Spacecraft Subject to gravity-gradient torque only.

① $\rightarrow \dot{q} = \frac{1}{2}\Omega(\Delta\vec{\omega}^b)q = \frac{1}{2}\Xi(q)\Delta\vec{\omega}^b$

where $\Delta\vec{\omega}^b = \vec{\omega}^b - R(q)\begin{bmatrix}0\\-\tilde{n}\\0\end{bmatrix}$

$\tilde{n} = \sqrt{\frac{\mu}{r_{orb}^3}}$ → mean motion

② $\rightarrow \dot{\vec{\omega}}^b = \left(I_{MOI}^b\right)^{-1}\left[-\vec{\omega}^b \times \left(I_{MOI}^b\vec{\omega}^b\right) + 3\tilde{n}^2 \ldots\right.$

$\left.\left(\left(R(q)\begin{bmatrix}0\\0\\-1\end{bmatrix}\right) \times \left\{I_{MOI}^b R(q)\begin{bmatrix}0\\0\\-1\end{bmatrix}\right\}\right)\right]$

for Equillibrium

$\dot{q} = \dot{\vec{\omega}}^b = 0 \Big|_{q_{eq},\, \vec{\omega}_{eq}^b}$

a) $q_{eq} = \begin{bmatrix}1/\sqrt{2}\\0\\0\\1/\sqrt{2}\end{bmatrix}$; $\vec{\omega}_{eq}^b = \begin{bmatrix}0\\0\\n\end{bmatrix}$

$\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$; ∴ its a 90° rotation about x-axis

$R(q_{eq}) = \begin{bmatrix}1 & 0 & 0\\0 & \cos 90^\circ & \sin 90\\0 & -\sin 90 & \cos 90\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & -1 & 0\end{bmatrix}$

$$\Delta\vec{\omega}^b_{eq} = \omega_{eq} - R(q_{eq})\begin{bmatrix}0\\-n\\0\end{bmatrix} = \begin{bmatrix}0\\0\\n\end{bmatrix} - \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & -1 & 0\end{bmatrix}\begin{bmatrix}0\\-n\\0\end{bmatrix}$$

$$= \begin{bmatrix}0\\0\\n\end{bmatrix} - \begin{bmatrix}0\\0\\n\end{bmatrix} = \vec{0}$$

$$\therefore \boxed{\frac{1}{2}\,\Xi(q_{eq})\,\Delta\vec{\omega}^b_{eq} = \vec{0} = \dot{\vec{q}}_{eq}}$$

$$R(q_{eq})\begin{bmatrix}0\\0\\-1\end{bmatrix} = \begin{bmatrix}0\\-1\\0\end{bmatrix}$$

$$I^b_{MOI}\begin{bmatrix}0\\-1\\0\end{bmatrix} = \begin{bmatrix}0\\-I^b_{22}\\0\end{bmatrix}$$ for a principal frame

$$R(q_{eq})\begin{bmatrix}0\\0\\-1\end{bmatrix} \times \left(I^b_{MOI}\,R(q_{eq})\begin{bmatrix}0\\0\\-1\end{bmatrix}\right) = \begin{bmatrix}0\\-1\\0\end{bmatrix} \times \begin{bmatrix}0\\-I^b_{22}\\0\end{bmatrix}$$ parallel vectors

$$= \vec{0}$$

Similarly

$$-\vec{\omega}^b_{eq} \times I^b_{MOI}\,\vec{\omega}^b_{eq} = \begin{bmatrix}0\\0\\-n\end{bmatrix} \times \begin{bmatrix}0\\0\\I^b_{33}n\end{bmatrix}$$ parallel vectors

$$= \vec{0}$$

$$\therefore \boxed{\dot{\vec{\omega}}^b_{eq} = \left(I^b_{MOI}\right)^{-1}\left\{\vec{0} + 3\tilde{n}^2\vec{0}\right\} = \vec{0}}$$

b) $q_{eq} = \begin{bmatrix} 0 \\ 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$ or $\begin{bmatrix} 0 \\ 0 \\ \sin\frac{90}{2} \\ \cos\frac{90}{2} \end{bmatrix}$; $\vec{\omega}^{b}_{eq} = \begin{bmatrix} -n \\ 0 \\ 0 \end{bmatrix}$

$q_{eq} = \begin{bmatrix} 0 \\ 0 \\ \sin\frac{90}{2} \\ \cos\frac{90}{2} \end{bmatrix}$ A Rotation about z-axis

$$R(q_{eq}) = \begin{bmatrix} \cos 90 & \sin 90 & 0 \\ -\sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Delta\vec{\omega}^{b}_{eq} = \underbrace{\begin{bmatrix} -n \\ 0 \\ 0 \end{bmatrix}}_{\vec{\omega}^{b}_{eq}} - \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R(q_{eq})} \begin{bmatrix} 0 \\ -n \\ 0 \end{bmatrix} = \begin{bmatrix} -n \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -n \\ 0 \\ 0 \end{bmatrix} = \vec{0}$$

$$\boxed{\therefore \dot{\vec{q}}_{eq} = \tfrac{1}{2} \Xi(q_{eq}) \Delta\vec{\omega}^{b}_{eq} = \vec{0}}$$

$$R(q_{eq}) \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\therefore R(q_{eq}) \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \times I^{b}_{MOI} R(q_{eq}) \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -I^{b}_{33} \end{bmatrix} = \vec{0}$$

$$-\vec{\omega}^{b}_{eq} \times I^{b}_{MOI} \vec{\omega}^{b}_{eq} = \begin{bmatrix} n \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} -I_{33} n \\ 0 \\ 0 \end{bmatrix} = \vec{0}$$

$$\boxed{\dot{\vec{\omega}}^{b}_{eq} = \left(I^{b}_{MOI}\right)^{-1} \left(\vec{0} + 3n^2 \vec{0}\right) = \vec{0}}$$

c) $q_{eq} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \sin\frac{180}{2} \\ 0 \\ \cos\frac{180}{2} \end{bmatrix}$ → A 180° degree Rotation about Y-Axis

$$R(q_{eq}) = \begin{bmatrix} \cos 180 & 0 & -\sin 180 \\ 0 & 1 & 0 \\ \sin 180 & 0 & \cos 180 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\vec{\omega}^b_{eq} = \begin{bmatrix} 0 \\ -n \\ 0 \end{bmatrix} \quad \therefore \Delta\vec{\omega}^b_{eq} = \begin{bmatrix} 0 \\ -n \\ 0 \end{bmatrix} - \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ -n \\ 0 \end{bmatrix}} = \vec{0}$$

$$\boxed{\therefore \dot{q}_{eq} = \frac{1}{2} \begin{pmatrix} - \end{pmatrix} (q_{eq}) \Delta\vec{\omega}^b_{eq} = \vec{0}} \begin{bmatrix} 0 \\ -n \\ 0 \end{bmatrix}$$

$$R(q_{eq}) \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R(q_{eq}) \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \times I^b_{MOI} R(q_{eq}) \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ I^b_{33} \end{bmatrix} = \vec{0}$$

$$-\vec{\omega}^b_{eq} \times I^b_{MOI} \vec{\omega}^b_{eq} = \begin{bmatrix} 0 \\ +n \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ -nI_{22} \\ 0 \end{bmatrix} = \vec{0}$$

$$\boxed{\dot{\vec{\omega}}^b_{eq} = \left(I^b_{MOI}\right)^{-1} \left(\vec{0} + 3n^2\vec{0}\right) = \vec{0}}$$

Q3) $A = \underset{(n\times n)}{\dfrac{\partial \underline{F}}{\partial \underline{x}}}$; $B = \underset{n\times m}{\dfrac{\partial \underline{F}}{\partial \underline{u}}}$

$$\frac{\partial \underline{F}}{\partial \underline{x}} = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_n}{\partial x_1} & \cdots & \frac{\partial F_n}{\partial x_n} \end{bmatrix}_{n\times n} ; \quad \frac{\partial \underline{F}}{\partial \underline{u}} = \begin{bmatrix} \frac{\partial F_1}{\partial u_1} & \cdots & \frac{\partial F_1}{\partial u_m} \\ \vdots & & \\ \frac{\partial F_n}{\partial u_1} & \cdots & \frac{\partial F_n}{\partial u_m} \end{bmatrix}_{n\times m}$$

$$\boxed{A_{13} = \left.\frac{\partial F_1}{\partial x_3}\right|_{\underline{x}_{eq},\, u_{eq}}}$$

$$x_{eq} = [\overset{x_1}{0}\ \overset{x_2}{0}\ \overset{x_3}{0}\ \overset{x_4}{1}\ ;\ \overset{x_5}{0},\ \overset{x_6}{-n},\ \overset{x_7}{0}]^T \quad ; \quad u_{eq} = [\overset{u_1}{0}\ \overset{u_2}{0}\ \overset{u_3}{0}]^T$$

$$\underbrace{0\ 0\ 0\ 1}_{q_{eq}} \quad \underbrace{0,\ -n,\ 0}_{u_{eq}}$$

$$F_1 = \tfrac{1}{2}\Big(x_4[x_5 + 2n(x_1x_2 + x_3x_4)] - x_3[x_6 + n(-x_1^2 + x_2^2 - x_3^2 + x_4^2)] + x_2[x_7 + 2n(x_2x_3 - x_1x_4)]\Big)$$

$$F_1 = \left(\frac{1}{2}\right)\Big\{x_4x_5 + 2nx_1x_2x_4 + 2nx_3x_4^2 - x_3x_6 + nx_1^2x_3 - nx_2^2x_3 + nx_3^3 - nx_3x_4^2 + x_2x_7 + 2nx_2^2x_3 - 2nx_1x_2x_4\Big\}$$

$$\left.\frac{\partial F_1}{\partial x_3}\right|_{x_{eq}} = \frac{1}{2}\Big[2n\underset{1}{\cancel{x_4^2}} - \underset{-n}{\cancel{x_6}} + n\underset{0}{\cancel{x_1^2}} - n\underset{0}{\cancel{x_2^2}} + 3n\underset{0}{\cancel{x_3^2}} - n\underset{1}{\cancel{x_4^2}} + 2n\underset{0}{\cancel{x_2^2}}\Big]$$

at x_{eq}

$$= \frac{1}{2}\left[2n + \cancel{n} - \cancel{n}\right] = \frac{1}{2}\left[2n\right]$$

$$\boxed{A_{13} = n}$$

$$\boxed{A_{26} = \left.\frac{\partial F_2}{\partial x_6}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}}$$

$$F_2 = \left(\frac{1}{2}\right) x_4 x_6$$ (Ignoring all the terms not containing x_6)

$$\left.\frac{\partial F_2}{\partial x_6}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}} = \left.\frac{\cancel{x_4}^{\,1}}{2}\right|_{\text{at } x_{eq},\, u_{eq}} = 1/2$$

$$\boxed{A_{26} = 1/2}$$

$$\boxed{A_{34} = \left.\frac{\partial F_3}{\partial x_4}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}}$$

$$F_3 = \left(\frac{1}{2}\right)\left[\cancel{-2n x_2 x_3 x_4} + \cancel{n x_1 x_4^2} + x_4 x_7 + \cancel{2n x_2 x_3 x_4} - 2n x_1 x_4^2\right.$$

$$F_3 = \left(\frac{1}{2}\right)\left(x_4 x_7 - n x_1 x_4^2\right)$$ Ignoring terms without x_4

$$\left.\frac{\partial F_3}{\partial x_4}\right|_{\text{at } \underline{x}_{eq}, \underline{u}_{eq}} = \left(\frac{1}{2}\right) \left.\cancel{x_7}^{\to 0} - 2n\cancel{x_1}^{\to 0} x_4\right|_{\text{at } x_{eq}\, u_{eq}} = 0$$

$$\therefore \boxed{A_{34} = 0}$$

$$\boxed{A_{45} = \left.\frac{\partial F_4}{\partial x_5}\right|_{\text{at } \underline{x}_{eq}, \underline{u}_{eq}}}$$

$$F_4 = \frac{-x_1 x_5}{2}$$ (Ignoring term without x_5)

$$\left.\frac{\partial F_4}{\partial x_5}\right|_{\text{at } \underline{x}_{eq}, \underline{u}_{eq}} = \frac{-\cancel{x_1}^{\to 0}}{2} = 0$$

$$\therefore \boxed{A_{45} = 0}$$

$$\boxed{A_{57} = \left.\frac{\partial F_5}{\partial x_7}\right|_{\text{at } \underline{x}_{eq}, \underline{u}_{eq}}}$$

$$F_5 = \left(\frac{I_{22}^b - I_{33}^b}{I_{11}^b}\right) x_6 x_7$$ (Ignoring terms not containing x_7)

$$\left.\frac{\partial F_5}{\partial x_7}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}} = \frac{-n\left(I_{22}^b - I_{33}^b\right)}{I_{11}^b} = \frac{n\left(I_{33}^b - I_{22}^b\right)}{I_{11}^b}$$

$$\therefore \boxed{A_{57} = n\left(\frac{I_{33}^b - I_{22}^b}{I_{11}^b}\right)}$$

$$\boxed{A_{62} = \left.\frac{\partial F_6}{\partial x_2}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}}$$

$$F_6 = \left(\frac{1}{I_{22}^b}\right)\left[\begin{array}{l} -6n^2\left(I_{11}^b - I_{33}^b\right)x_1 x_2^2 x_3 \ldots \\ -6n^2\left(I_{11}^b - I_{33}^b\right)\left(-x_1^2 x_2 x_4 - x_2^3 x_4 + x_2 x_4 x_3^2 + x_2 x_4^3\right) \end{array}\right]$$

(dropping terms not containing x_2)

$$\left.\frac{\partial F_6}{\partial x_2}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}} = \frac{1}{I_{22}^b}\left(\begin{array}{l} -12n^2\left(I_{11}^b - I_{33}^b\right)\cancel{x_1 x_2 x_3}^{\to 0} \\ -6n^2\left(I_{11}^b - I_{33}^b\right)\left(-\cancel{x_1^2}^{\to 0} x_4 - 3\cancel{x_2^2}^{\to 0} x_4 + x_4 \cancel{x_3^2}^{\to 0} + \cancel{x_4^3}_{\to 1}\right) \end{array}\right)$$

$$= \frac{1}{I_{22}^b}\left(-6n^2\left(I_{11}^b - I_{33}^b\right)\right)$$

$$\therefore \boxed{A_{62} = 6n^2\left(\frac{I_{33}^b - I_{11}^b}{I_{22}^b}\right)}$$

$$\boxed{A_{76} = \left.\frac{\partial F_7}{\partial x_6}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}}$$

$$F_7 = \left(\frac{I_{11}^b - I_{22}^b}{I_{33}^b}\right) x_5 x_6$$ (dropping terms not containing x_6)

$$\left.\frac{\partial F_7}{\partial x_6}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}} = \left.\left(\frac{I_{11}^b - I_{22}^b}{I_{33}^b}\right) \cancel{x_5}^{\,0}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}$$

$$\therefore \boxed{A_{76} = 0}$$

$$\boxed{B_{62} = \left.\frac{\partial F_6}{\partial u_2}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}}$$

$$F_6 = \frac{u_2}{I_{22}^b}$$ (dropping terms not containing u_2)

$$\left.\frac{\partial F_6}{\partial u_2} = \frac{1}{I_{22}^b}\right\}$$ already linear in u_2

$$\therefore \boxed{B_{62} = \frac{1}{I_{22}^b}}$$

Q4)

$$\boxed{A_{14} = \frac{\partial F_1}{\partial x_4}\Big|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}}$$

$$F_1 = x_4 \cos(x_5)\cos(x_6)$$

$$\frac{\partial F_1}{\partial x_4}\Big|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}} = \cos(x_{5eq})\cos(x_{6eq})\Big|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}$$

from problem 2, $x_{5eq} = 0 \Rightarrow \cos(x_{5eq}) = 1$

$$\therefore \boxed{A_{14} = \cos(x_{6eq})}$$

$$\boxed{A_{25} = \frac{\partial F_2}{\partial x_5}\Big|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}}$$

$$F_2 = x_4 \cos(x_5)\sin(x_6)$$

$$\frac{\partial F_2}{\partial x_5}\Big|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}} = -x_{4eq}\cancel{\sin(x_{5eq})}^{\,0}\sin(x_{6eq})\Big|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}$$

as $x_{5eq} = 0 \Rightarrow \sin(x_{5eq}) = 0$

$$\therefore \boxed{A_{25} = 0}$$

$$\boxed{A_{35} = \left.\frac{\partial F_3}{\partial x_5}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}}$$

$$F_3 = -x_4 \sin(x_5)$$ ~~(not a function~~

$$\left.\frac{\partial F_3}{\partial x_5}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}} = -x_{4_{eq}} \cos(x_{5_{eq}}) \Big|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}$$

$$x_{5_{eq}} = 0 \Rightarrow \cos(x_{5_{eq}}) = 1$$

$$\therefore \boxed{A_{35} = -x_{4_{eq}}}$$

$$\boxed{A_{43} = \left.\frac{\partial F_4}{\partial x_3}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}}$$

$$F_4 = -\frac{1}{2m}\left(\rho(x_3)\, x_4^2\, S C_D(u_2)\right)$$ (Ignoring terms not containing x_3)

$$\left.\frac{\partial F_4}{\partial x_3}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}} = -\frac{1}{2m}\left(x_{4_{eq}}^2\, S C_D(u_{2_{eq}})\right) \rho'(x_{3_{eq}})$$

where $\rho'(x_3) = \dfrac{d\rho(x_3)}{dx_3}$

$$\therefore \boxed{A_{43} = -\left(x_{4_{eq}}^2\, S C_D(u_{2_{eq}})\right)\left(\rho'(x_{3_{eq}})\right) / 2m}$$

$$\boxed{A_{54} = \left.\frac{\partial F_5}{\partial x_4}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}}$$

$$x_{5eq} = 0 \quad \& \quad \dot{x}_{5eq} = 0 \Rightarrow f_{5_{eq}} = 0$$

$$F_{5eq} = F_5(\underline{x}_{eq}, \underline{u}_{eq})$$

$$F_5 = \underbrace{\left(\frac{1}{x_4}\right)}_{\substack{\text{call it} \\ f_5^1}} \underbrace{\left[\frac{1}{m}\left(u_1 \sin(u_2) + \frac{1}{2}\rho(x_3)\, x_4^2\, S\, C_L(u_2)\right)\cos(u_3) \ldots - g\cos(x_5)\right]}_{\substack{\text{Call it} \\ f_5^2}}$$

$$F_5 = f_5^1 f_5^2 \rightarrow \text{not a power}$$

$$\left.\frac{\partial F_5}{\partial x_4}\right|_{\substack{\text{at} \\ \underline{x}_{eq},\, \underline{u}_{eq}}} = \frac{\partial f_5^1}{\partial x_4} f_5^2 \rightarrow \text{not a power} + f_5^1 \left.\frac{\partial f_5^2}{\partial x_4}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}$$

$$f_5^2 = x_4 F_5$$

$$f_{5_{eq}}^2 = x_{4eq} \cancel{F_{5eq}}^{\,0} \Rightarrow \boxed{f_{5eq}^2 = 0}$$

$$\left.\frac{\partial F_5}{\partial x_4}\right|_{\text{at } x_{eq}, u_{eq}} = -\frac{1}{x_{4eq}^2}\,0 + \frac{1}{x_{4eq}} \times \left.\frac{\partial f_5^2}{\partial x_4}\right|_{\text{at } x_{eq}, u_{eq}}$$

$$\left.\frac{\partial f_5^2}{\partial x_4}\right|_{\text{at } x_{eq}\, u_{eq}} = \left(\cancel{\tfrac{1}{2m}}\,\rho(x_{3eq})\,\cancel{2}\,x_{4eq}\,S C_L(u_{2eq})\right)\cos(u_{3eq})$$

$$\cos(u_{3eq}=0) = 1$$

$$\left.\frac{\partial f_5^2}{\partial x_4}\right|_{\text{at } x_{eq}\, u_{eq}} = \frac{\rho(x_{3eq})\,x_{4eq}\,S C_L(u_{2eq})}{m}$$

$$\left.\frac{\partial F_5}{\partial x_4}\right|_{\text{at } x_{eq}\, u_{eq}} = \frac{1}{\cancel{x_{4eq}}}\,\frac{\rho(x_{3eq})\,\cancel{x_{4eq}}\,S C_L(u_{2eq})}{m}$$

$$\therefore \boxed{A_{54} = \frac{\rho(x_{3eq})\,S C_L(u_{2eq})}{m}}$$

$$A_{64} = \left.\frac{\partial F_6}{\partial x_4}\right|_{\text{at } x_{eq},\, u_{eq}}$$

$$F_6 = \sin(u_3)\underbrace{\left(\frac{1}{m x_4 \cos(x_5)}\left(u_1 \sin(u_2) + \tfrac{1}{2}\ell(x_3) x_4^2 S_4(u_2)\right)\right)}_{\text{dependent on } x_4}$$

call it $\bar{F}_6$

$$F_6 = \sin(u_3)\, \bar{F}_6(\underline{x}, \underline{u})$$

$$\left.\frac{\partial F_6}{\partial x_4}\right|_{\text{at } \underline{x}_{eq}\, \underline{u}_{eq}} = \sin(u_{3eq}) \left.\frac{\partial \bar{F}_6}{\partial x_4}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}$$

as $u_{3eq} = 0 \Rightarrow \sin(u_{3eq}) = 0$

$$\therefore \boxed{A_{64} = 0}$$

$$\boxed{B_{63} = \left.\frac{\partial F_6}{\partial u_3}\right|_{\text{at } \underline{x}_{eq},\, \underline{u}_{eq}}}$$

$$\frac{\left(F_5 + \frac{g\cos(x_5)}{x_4}\right)}{\cos(u_3)} = \frac{1}{x_4 m}\left(u_1 \sin(u_2) + \tfrac{1}{2}\ell(x_3) x_4^2 S_4(u_2)\right)$$

$$F_6 = \frac{1}{\cos(x_5)} \times \sin(u_3) \times \left[\frac{1}{x_4 m}\left(u_1 \sin(u_2) + \frac{1}{2}\rho(x_3) x_4^2 S C_L(u_2)\right)\right]$$

$$F_6 = \frac{\sin(u_3)}{\cos(x_5)}\left(\frac{F_5 + \frac{g\cos(x_5)}{x_4}}{\cos(u_3)}\right) = \underbrace{\tan(u_3)}_{F_6^1}\underbrace{\left(\frac{F_5}{\cos x_5} + \frac{g}{x_4}\right)}_{F_6^2}$$

$$\left.\frac{\partial F_6}{\partial u_3}\right|_{\text{at } x_{eq}, u_{eq}} = \frac{1}{\cos^2(u_3)}\left(\frac{F_5}{\cos x_5} + \frac{g}{x_4}\right) + \frac{\tan(u_3)}{\cos x_5}\left.\frac{\partial F_5}{\partial u_3}\right|_{\text{at } x_{eq}, u_{eq}}$$

$$u_{3eq} = 0 \Rightarrow \cos^2(u_{3eq}) = 1 \; ; \Rightarrow \tan(u_{3eq}) = 0$$

$$x_{5eq} = 0 \Rightarrow \cos(x_{5eq}) = 1$$

$$\left.\frac{\partial F_6}{\partial u_3}\right|_{\text{at } x_{eq}, u_{eq}} = \left(\cancel{F_{5eq}}^{\to 0} + \frac{g}{x_{4eq}}\right) + \frac{\cancel{\tan(u_{3eq})}^{\to 0}}{\cancel{\cos(x_{5eq})}^{\to 1}}\left.\frac{\partial F_5}{\partial u_3}\right|_{\text{at } x_{eq}, u_{eq}}$$

$$\frac{\partial F_6}{\partial u} = \frac{g}{x_{4eq}}$$

$$\boxed{B_{63} = \frac{g}{x_{4eq}}}$$