Posting Date: Monday Oct. 21st.
Due Date: Monday Oct. 28th.

1. Suppose that I_{pr1}, I_{pr2}, and I_{pr3} are the principle moments of inertia of a rigid body. Suppose that they are ordered so that $I_{pr1} \geq I_{pr2} \geq I_{pr3}$. Prove that $(I_{pr2} + I_{pr3}) \geq I_{pr1}$.

   Hint: Suppose that the moment-of-inertia matrix has been calculated in some body-fixed frame $\mathcal{F}_b$ and suppose that the direction-cosines transformation matrix from frame $\mathcal{F}_b$ axes to principle axis frame $\mathcal{F}_c$ is R_{pr}^{T} so that the transformed moment of inertial matrix is

$$\begin{bmatrix} I_{pr1} & 0 & 0 \\ 0 & I_{pr2} & 0 \\ 0 & 0 & I_{pr3} \end{bmatrix} = I_{MoI}^{c} = R_{pr}^{\mathrm{T}} I_{MoI}^{b} R_{pr}$$

   Suppose, also, that the rigid body consists the N particles with masses m_i and frame $\mathcal{F}_b$ position vectors $\vec{r}_i^{\,b}$ for $i = 1, \ldots, N$. Then the corresponding frame $\mathcal{F}_c$ position vectors are $\vec{r}_i^{\,c} = R_{pr}^{\mathrm{T}} \vec{r}_i^{\,b}$ for $i = 1, \ldots, N$. Develop formulas for I_{pr1}, I_{pr2}, and I_{pr3} in terms of the components of the $\mathcal{F}_c$ position vector offsets from the center of mass $[\Delta X_i^c; \Delta Y_i^c; \Delta Z_i^c] = \vec{r}_i^{\,c} - \vec{r}_{cm}^{\,c}$. Use your I_{pr1}, I_{pr2}, and I_{pr3} formulas to prove the desired result.

2. Complete the MATLAB template file `momentofinertia01_temp.m` by completing the parts of the code where ???? appears. The result will be the MATLAB function `momentofinertia01.m`. This function must compute the total mass, total center of mass, and total moment-of-inertia matrix about the center of mass of a set of rigid bodies that together form a composite rigid body.

3. Use your `momentofinertia01.m` function from Part 2 and the MATLAB data files `moicalcs01_data.mat` and `moicalcs02_data.mat` to compute the total system mass, center of mass, and moment-of-inertia matrix for two sets of rigid bodies. Hand in the final results to 15 significant digits for the second case, the one whose input data are contained in `moicalcs02_data.mat`. You can print the needed results in the MATLAB command prompt window by executing the following four MATLAB commands:

```
>> format long
>> Mtot
>> rcmtot
>> IMoItot
```

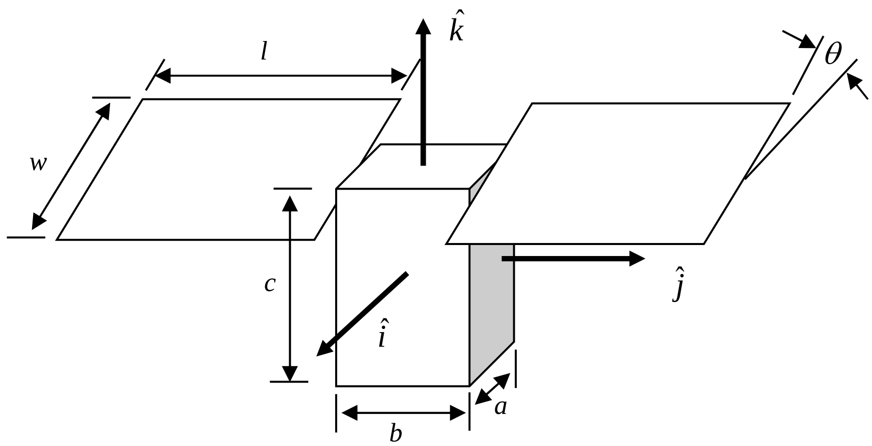
   As a way of checking your work, these operations produce the following results for the case where the inputs come from the file `moicalcs01_data.mat`:

```
Mtot =

   2.058306536059932e+02
```

```
rcmtot =
  0.409101895949622
  0.526651850364819
  0.058154823743388
IMoItot =
  1.0e+04 *
  1.047260719697208   0.028550325166975   0.040407207761532
  0.028550325166975   1.063729452526229   0.010880405279456
  0.040407207761532   0.010880405279456   1.146269471862332
```

4. Consider the following composite rigid body:



It is a model of a main spacecraft bus – the central rectangular parallelepiped – and two canted rectangular solar panels. The central rectangular parallelepiped has mass M, and its center of mass is located at the origin of the axis system shown in the figure. Its mass is uniformly distributed over it, and its sides have the following lengths, respectively, along the $\hat{i}$, $\hat{j}$, and $\hat{k}$ axes: a, b, and c. The two solar panels each have mass m, and they can be modeled as being flat rectangular plates with their masses uniformly distributed over their areas. These plates have length l along the $\hat{j}$ axis and width w along an axis that lies in the $\hat{i}$-$\hat{k}$ plane. This width axis is rotated away from the $\hat{i}$ axis by the rotation angle θ about an axis that is parallel to the $+\hat{j}$ axis but that passes through the center of the $+\hat{k}$ face of the central rectangular parallelepiped, as shown in the figure. The centers of mass of the two solar panels are located at their geometric centers. Thus, there locations in the $\hat{i}$-$\hat{j}$-$\hat{k}$ axes system are [0; +0.5(b+l); +0.5c] and [0; -0.5(b+l); +0.5c].

Derive appropriate formulas and write MATLAB software for determining the total mass, the center-of-mass location, and the moment-of-inertia matrix about the center of mass for the entire rigid-body satellite. Your results must be given in the $\hat{i}$-$\hat{j}$-$\hat{k}$ axes system. Run your code for the case M = 15 kg, m = 0.8 kg, a = 0.3 m, b = 0.4 m, c = 0.6 m, l = 1.1m, w = 0.5 m, and θ = 0.34906585 rad (very near to 20 deg). Hand in your analysis, your MATLAB code, and your results for the mass, the center-of-mass location, and the composite moment-of-inertia matrix, all given to 15 significant digits.

Hints: Except for the presence of the cant angle θ, this is a fairly straightforward application of the formulas for the moment-of-inertia matrix of a rectangular parallelepiped and the composition of several rigid bodies into a single rigid body. The moment-of-inertia matrices for the solar panels are straightforward to calculate along the $\hat{i}'$-$\hat{j}'$-$\hat{k}'$ axes that have $\hat{j}'$ parallel to $\hat{j}$ while $\hat{k}'$ points normal to the plane of the solar panel and $\hat{i}'$ is defined by the right-hand rule so that $\hat{i}' = \hat{j}' \times \hat{k}'$. If R is the direction cosines matrix that transforms from $\hat{i}'$-$\hat{j}'$-$\hat{k}'$ axes to $\hat{i}$-$\hat{j}$-$\hat{k}$ axes and if I'_{spMoI} is a solar-panel's moment-of-inertia matrix about its center of mass and defined along the $\hat{i}'$-$\hat{j}'$-$\hat{k}'$ axes, then $I_{spMoI} = RI'_{spMoI}R^{\mathrm{T}}$ is the moment-of-inertia matrix about its center of mass and defined along the $\hat{i}$-$\hat{j}$-$\hat{k}$ axes. It may be helpful to use your `momentofinertia01.m` function from Part 2 as part of the MATLAB code that you use to calculate answers for this question.

As an aid to checking your analysis and MATLAB code, the following results are obtained if M = 20 kg, m = 0.6 kg, a = 0.4 m, b = 0.2 m, c = 0.8 m, l = 2.1m, w = 0.6 m, and θ = 0.523599 rad (very near to 30 deg):

```
Mtot =
 21.200000000000003
rcmtot =
                  0
                  0
  0.022641509433962
IMoItot =
  3.351465415801186                   0   0.015588461307349
                  0   1.550465408805032                   0
  0.015588461307349                   0   2.388333326337180
```