Problem 2

```
%script_simaircraft06.m
%
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%
%  This Matlab script simulates the motion of an
%  aircraft by using the design and control input
%  data in maneuver02_data.mat
%  and the point-mass translational aircraft
%  dynamics model in ffunctaircraft03.m.
%
%  This script uses N-point trapezoidal numerical
%  integration in order to do the numerical
%  integration.
%
%  This script also makes plots of the flight time history.
%
%  Clear the Matlab workspace.
%
   clear
%
%  Load the aircraft parameters, the thrust, angle-of-attack,
%  and roll/bank-angle input time histories, and the initial
%  state vector.
%
   load maneuver02_data
%
%  Define the aircraft dynamics function handle
%  in a form that is suitable for input to ode45.m
%  or to a trapezoidal numerical integration.
%
   ffunctode45_03 = @(tdum,xdum) ...
             ffunctaircraft03(tdum,xdum,m,S,CLalpha,CD0,oneoverpiARe,...
                              tinhist,Thist,alphahist,phihist);
%
%  Define the time span of the simulation, computing outputs
%  every half second.
%
   t0 = tinhist(1,1);
   tf = tinhist(end,1);
%
%  Compute the ode45.m results using a very
%  precise relative tolerance.
%
   tspan = (t0:0.5:tf)';
   optionsode45 = odeset('RelTol',1.e-12);
   tic
   [thist03,xhist03] = ode45(ffunctode45_03,tspan,x0,optionsode45);
   timetoode45 = toc
%
%  Set up 3 different N values and prepare to store 3 different
%  time histories generated by trapezoidal integration.
%
   Nvec = [500;2000;8000];
   thist03_trapez_cell = cell(3,1);
   xhist03_trapez_cell = cell(3,1);
```

```
   timetotrapez_vec = zeros(3,1);
%
%  Select N and perform N steps of trapezoidal numerical integration
%  to go from time tmin to time tmax.
%
   n = size(x0,1);
   for jj = 1:3
      N = Nvec(jj,1)
      deltat = (tf-t0)/N;
      Np1 = N + 1;
      thist03_trapez = zeros(Np1,1);
      xhist03_trapez = zeros(Np1,n);
      thist03_trapez(1,1) = t0;
      xhist03_trapez(1,:) = x0';
      clear Np1
      tic
      tkp1 = t0;
      xkp1 = x0;
      for k = 0:(N-1);
         tk = tkp1;
         xk = xkp1;
         tak = tk;
         xak = xk;
         fak = ffunctode45_03(tak,xak);
         tbk = tk+deltat;
         xbk = xk+deltat*fak;
         fbk = ffunctode45_03(tbk,xbk);
         tkp1 = tk+deltat;
         xkp1 = xk+(deltat/2)*(fak+fbk);
         kp2 = k + 2;
         thist03_trapez(kp2,1) = tkp1;
         xhist03_trapez(kp2,:) = xkp1';
      end
      clear k tk xk tak xak fak tbk xbk fbk tkp1 xkp1 kp2
      timetotrapez = toc
      timetotrapez_vec(jj,1) = timetotrapez;
      thist03_trapez_cell{jj,1} = thist03_trapez;
      xhist03_trapez_cell{jj,1} = xhist03_trapez;
   end
   clear jj N deltat thist03_trapez xhist03_trapez timetotrapez
%
%  Plot the ground track.
%
   figure(1)
   hold off
   plot(xhist03(:,2)*0.001,xhist03(:,1)*0.001,'b-','LineWidth',3)
   hold on
   plot(xhist03_trapez_cell{1,1}(:,2)*0.001,...
        xhist03_trapez_cell{1,1}(:,1)*0.001,'k:','LineWidth',1.5)
   plot(xhist03_trapez_cell{2,1}(:,2)*0.001,...
        xhist03_trapez_cell{2,1}(:,1)*0.001,'g--','LineWidth',1.5)
   plot(xhist03_trapez_cell{3,1}(:,2)*0.001,...
        xhist03_trapez_cell{3,1}(:,1)*0.001,'r-.','LineWidth',1.5)
   hold off
   grid
   axis('equal')
   xlabel('Eastward Displacement (km)')
   ylabel('Northward Displacment (km)')
   title('Ground Tracks for simaircraft06.mat')
```

```
   legend('ode45.m',...
          ['Trapezoidal integration w/',int2str(Nvec(1,1)),' steps'],...
          ['Trapezoidal integration w/',int2str(Nvec(2,1)),' steps'],...
          ['Trapezoidal integration w/',int2str(Nvec(3,1)),' steps'])
%
%  Plot the altitude, airspeed, flight-path angle,
%  and heading angle time histories.
%
   figure(2)
   subplot(411)
   hold off
   plot(thist03,-xhist03(:,3),'b-','LineWidth',3)
   hold on
   plot(thist03_trapez_cell{1,1},-xhist03_trapez_cell{1,1}(:,3),...
        'k:','LineWidth',1.5)
   plot(thist03_trapez_cell{2,1},-xhist03_trapez_cell{2,1}(:,3),...
        'g--','LineWidth',1.5)
   plot(thist03_trapez_cell{3,1},-xhist03_trapez_cell{3,1}(:,3),...
        'r-.','LineWidth',1.5)
   hold off
   grid
   ylabel('Altitude above Airport (m)')
   title('State time histories for simaircraft06.mat')
   legend('ode45.m',...
          ['Trapezoidal integration w/',int2str(Nvec(1,1)),' steps'],...
          ['Trapezoidal integration w/',int2str(Nvec(2,1)),' steps'],...
          ['Trapezoidal integration w/',int2str(Nvec(3,1)),' steps'])
   subplot(412)
   hold off
   plot(thist03,xhist03(:,4),'b-','LineWidth',3)
   hold on
   plot(thist03_trapez_cell{1,1},xhist03_trapez_cell{1,1}(:,4),...
        'k:','LineWidth',1.5)
   plot(thist03_trapez_cell{2,1},xhist03_trapez_cell{2,1}(:,4),...
        'g--','LineWidth',1.5)
   plot(thist03_trapez_cell{3,1},xhist03_trapez_cell{3,1}(:,4),...
        'r-.','LineWidth',1.5)
   hold off
   grid
   ylabel('Airspeed (m/sec)')
   subplot(413)
   hold off
   plot(thist03,xhist03(:,5)*(180/pi),'b-','LineWidth',3)
   hold on
   plot(thist03_trapez_cell{1,1},...
        xhist03_trapez_cell{1,1}(:,5)*(180/pi),'k:','LineWidth',1.5)
   plot(thist03_trapez_cell{2,1},...
        xhist03_trapez_cell{2,1}(:,5)*(180/pi),'g--','LineWidth',1.5)
   plot(thist03_trapez_cell{3,1},...
        xhist03_trapez_cell{3,1}(:,5)*(180/pi),'r-.','LineWidth',1.5)
   hold off
   grid
   ylabel('Flight Path Angle (deg)')
   subplot(414)
   hold off
   plot(thist03,xhist03(:,6)*(180/pi),'b-','LineWidth',3)
   hold on
   plot(thist03_trapez_cell{1,1},...
        xhist03_trapez_cell{1,1}(:,6)*(180/pi),'k:','LineWidth',1.5)
```

```
   plot(thist03_trapez_cell{2,1},...
        xhist03_trapez_cell{2,1}(:,6)*(180/pi),'g--','LineWidth',1.5)
   plot(thist03_trapez_cell{3,1},...
        xhist03_trapez_cell{3,1}(:,6)*(180/pi),'r-.','LineWidth',1.5)
   hold off
   grid
   ylabel('Heading Angle (deg)')
   xlabel('Time (seconds)')
%
%  Plot the thrust, angle-of-attack, and roll/bank-angle
%  time histories.
%
   figure(3)
   subplot(311)
   hold off
   plot(tinhist,Thist,'LineWidth',1.5)
   grid
   ylabel('Thrust (N)')
   title('Control input time histories for simaircraft06.mat')
   subplot(312)
   hold off
   plot(tinhist,alphahist*(180/pi),'LineWidth',1.5)
   grid
   ylabel('Angle-of-Attack (deg)')
   subplot(313)
   hold off
   plot(tinhist,phihist*(180/pi),'LineWidth',1.5)
   grid
   ylabel('Roll/Bank-Angle (deg)')
   xlabel('Time (seconds)')
%
%  Display final state error.
%
   format long
   errorxfinal_500 = xhist03_trapez_cell{1,1}(end,:)' - xhist03(end,:)'
   errorxfinal_2000 = xhist03_trapez_cell{2,1}(end,:)' - xhist03(end,:)'
   errorxfinal_8000 = xhist03_trapez_cell{3,1}(end,:)' - xhist03(end,:)'
%
%  Save the results.
%
   textcommands = ['These data have been generated by the',...
                   ' commands in script_simaircraft06.m'];
   save simaircraft06
   disp('errorxfinal_2000./errorxfinal_8000')
   disp(errorxfinal_2000./errorxfinal_8000)
```

Output

```
timetoode45 =   1.379359100000000


N =   500
timetotrapez =   0.237054400000000


N =        2000
timetotrapez =   0.917606000000000
```

```
N =        8000
timetotrapez =   3.608178800000000


errorxfinal_500 =

   1.0e+02 *

   2.470874862165565
   0.312780779269015
  -0.020084760958699
  -0.001686353740498
   0.000002202438762
   0.000076213603099


errorxfinal_2000 =

   4.784498364122555
   0.478795131981315
  -0.085716895745350
  -0.007187758640072
   0.000037796464585
   0.000149039574289


errorxfinal_8000 =

   0.302894297890816
   0.029262601630762
  -0.004806028911162
  -0.000405483572820
   0.000002360428262
   0.000009412494139

errorxfinal_2000./errorxfinal_8000
  15.795934084725539
  16.362015176326043
  17.835285082508793
  17.726386768487977
  16.012545345590301
  15.834227580369248
```

Q) How do `errorxfinal_2000` and `errorxfinal_8000` for this run compare `errorxfinal_10000` and `errorxfinal_100000` for the Euler integration run?

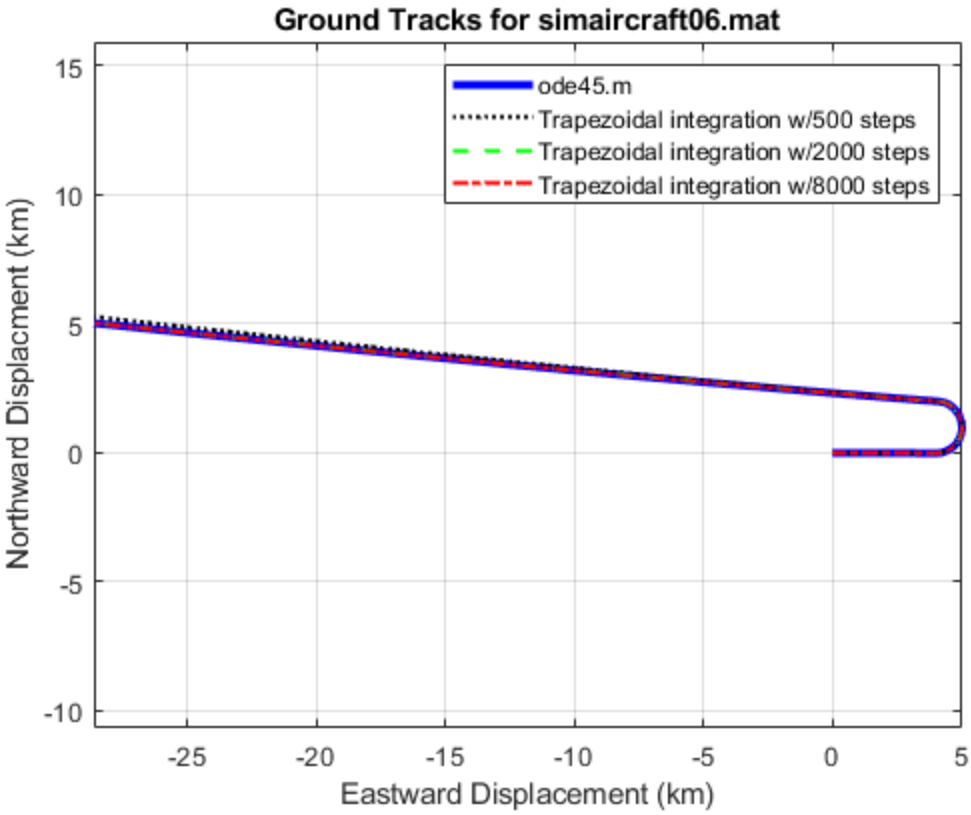
Ans) The are off by 2 orders of magnitude. About 100 to 40 times lower in Trapezoidal integration.

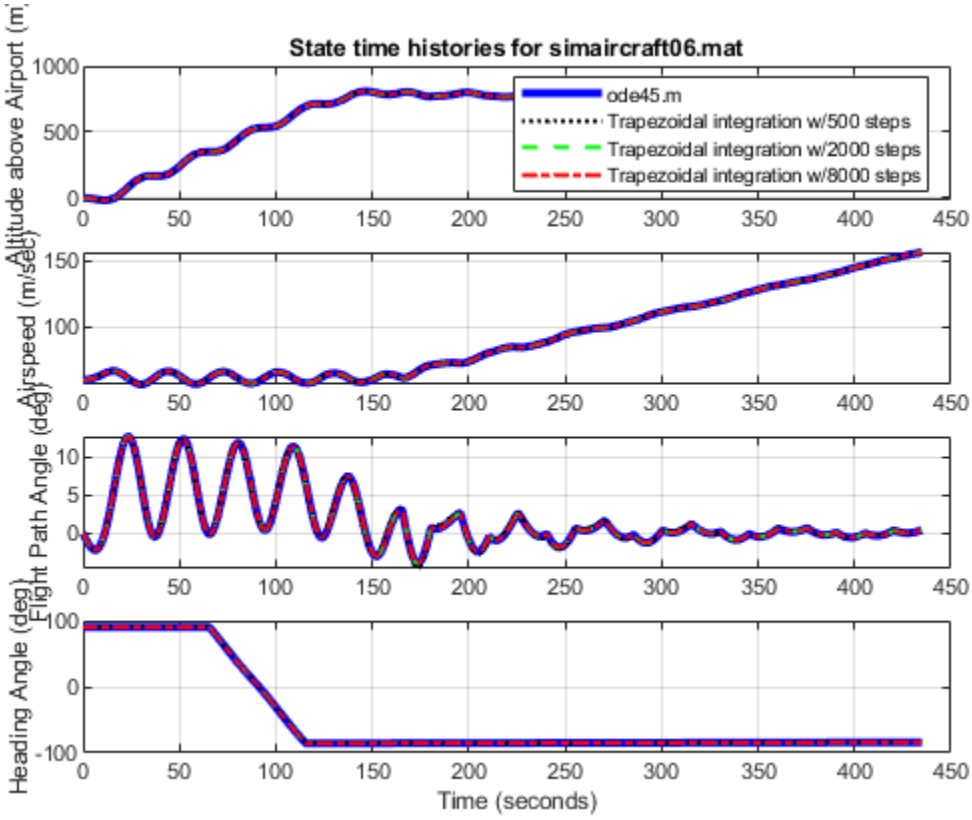
Q) How does trapezoidal integration compare to `ode45.m` in terms of execution speed?

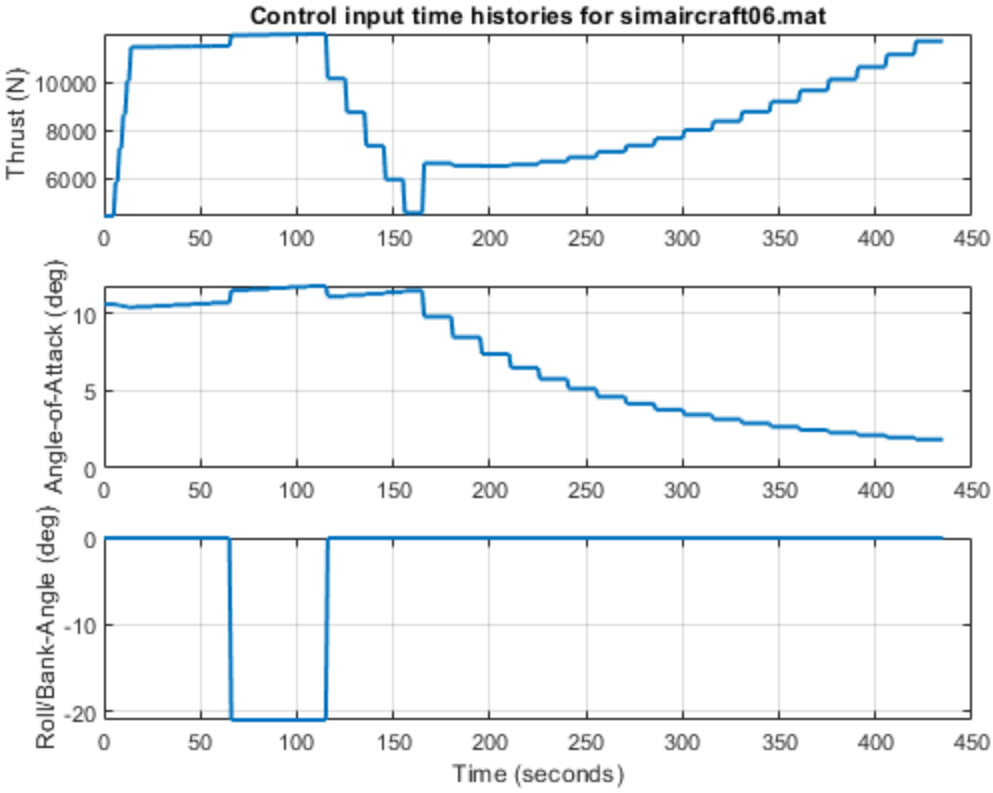
Ans) The speeds are comparable.

Q) The theory of Euler's method predicts that these ratios should be about 10. Is that true?

Ans) Yes. It is close to 16.


Ground Tracks for simaircraft06.mat
ode45.m
Trapezoidal integration w/500 steps
Trapezoidal integration w/2000 steps
Trapezoidal integration w/8000 steps
Northward Displacment (km)
Eastward Displacement (km)


State time histories for simaircraft06.mat
ode45.m
Trapezoidal integration w/500 steps
Trapezoidal integration w/2000 steps
Trapezoidal integration w/8000 steps
Altitude above Airport (m)
Airspeed (m/sec)
Flight Path Angle (deg)
Heading Angle (deg)
Time (seconds)



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