Problem 1

```
%script_simaircraft05.m
%
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%
%  This Matlab script simulates the motion of an
%  aircraft by using the design and control input
%  data in maneuver02_data.mat
%  and the point-mass translational aircraft
%  dynamics model in ffunctaircraft03.m.
%
%  This script uses N-point Euler numerical
%  integration in order to do the numerical
%  integration.
%
%  This script also makes plots of the flight time history.
%
%  Clear the Matlab workspace.
%
   clear
%
%  Load the aircraft parameters, the thrust, angle-of-attack,
%  and roll/bank-angle input time histories, and the initial
%  state vector.
%
   load maneuver02_data
%
%  Define the aircraft dynamics function handle
%  in a form that is suitable for input to ode45.m
%  or to an Euler numerical integration.
%
   ffunctode45_03 = @(tdum,xdum) ...
             ffunctaircraft03(tdum,xdum,m,S,CLalpha,CD0,oneoverpiARe,...
                                tinhist,Thist,alphahist,phihist);
%
%  Define the time span of the simulation, computing outputs
%  every half second.
%
   t0 = tinhist(1,1);
   tf = tinhist(end,1);
%
%  Compute the ode45.m results using a very
%  precise relative tolerance.
%
   tspan = (t0:0.5:tf)';
   optionsode45 = odeset('RelTol',1.e-12);
   tic
   [thist03,xhist03] = ode45(ffunctode45_03,tspan,x0,optionsode45);
   timetoode45 = toc
%
%  Set up 3 different N values and prepare to store 3 different
%  time histories generated by Euler integration.
%
   Nvec = [1000;10000;100000];
   thist03_euler_cell = cell(3,1);
   xhist03_euler_cell = cell(3,1);
```

```
   timetoeuler_vec = zeros(3,1);
%
%  Select N and perform N steps of Euler numerical integration
%  to go from time tmin to time tmax.
%
   n = size(x0,1);
   for jj = 1:3
      N = Nvec(jj,1)
      deltat = (tf-t0)/N;
      Np1 = N + 1;
      thist03_euler = zeros(Np1,1);
      xhist03_euler = zeros(Np1,n);
      thist03_euler(1,1) = t0;
      xhist03_euler(1,:) = x0';
      clear Np1
      tic
      tkp1 = t0;
      xkp1 = x0;
      for k = 0:(N-1);
         tk = tkp1;
         xk = xkp1;
         fk = ffunctode45_03(tk,xk);
         tkp1 = tk+deltat;
         xkp1 = xk+deltat*fk;
         kp2 = k + 2;
         thist03_euler(kp2,1) = tkp1;
         xhist03_euler(kp2,:) = xkp1';
      end
      clear k tk xk fk tkp1 xkp1 kp2
      timetoeuler = toc
      timetoeuler_vec(jj,1) = timetoeuler;
      thist03_euler_cell{jj,1} = thist03_euler;
      xhist03_euler_cell{jj,1} = xhist03_euler;
   end
   clear jj N deltat thist03_euler xhist03_euler timetoeuler
%
%  Plot the ground track.
%
   figure(1)
   hold off
   plot(xhist03(:,2)*0.001,xhist03(:,1)*0.001,'b-','LineWidth',3)
   hold on
   plot(xhist03_euler_cell{1,1}(:,2)*0.001,...
        xhist03_euler_cell{1,1}(:,1)*0.001,'k:','LineWidth',1.5)
   plot(xhist03_euler_cell{2,1}(:,2)*0.001,...
        xhist03_euler_cell{2,1}(:,1)*0.001,'g--','LineWidth',1.5)
   plot(xhist03_euler_cell{3,1}(:,2)*0.001,...
        xhist03_euler_cell{3,1}(:,1)*0.001,'r-.','LineWidth',1.5)
   hold off
   grid
   axis('equal')
   xlabel('Eastward Displacement (km)')
   ylabel('Northward Displacment (km)')
   title('Ground Tracks for simaircraft05.mat')
   legend('ode45.m',...
          ['Euler integration w/',int2str(Nvec(1,1)),' steps'],...
          ['Euler integration w/',int2str(Nvec(2,1)),' steps'],...
          ['Euler integration w/',int2str(Nvec(3,1)),' steps'])
%
```

```
% Plot the altitude, airspeed, flight-path angle,
% and heading angle time histories.
%
   figure(2)
   subplot(411)
   hold off
   plot(thist03,-xhist03(:,3),'b-','LineWidth',3)
   hold on
   plot(thist03_euler_cell{1,1},-xhist03_euler_cell{1,1}(:,3),...
        'k:','LineWidth',1.5)
   plot(thist03_euler_cell{2,1},-xhist03_euler_cell{2,1}(:,3),...
        'g--','LineWidth',1.5)
   plot(thist03_euler_cell{3,1},-xhist03_euler_cell{3,1}(:,3),...
        'r-.','LineWidth',1.5)
   hold off
   grid
   ylabel('Altitude above Airport (m)')
   title('State time histories for simaircraft05.mat')
   legend('ode45.m',...
          ['Euler integration w/',int2str(Nvec(1,1)),' steps'],...
          ['Euler integration w/',int2str(Nvec(2,1)),' steps'],...
          ['Euler integration w/',int2str(Nvec(3,1)),' steps'])
   subplot(412)
   hold off
   plot(thist03,xhist03(:,4),'b-','LineWidth',3)
   hold on
   plot(thist03_euler_cell{1,1},xhist03_euler_cell{1,1}(:,4),...
        'k:','LineWidth',1.5)
   plot(thist03_euler_cell{2,1},xhist03_euler_cell{2,1}(:,4),...
        'g--','LineWidth',1.5)
   plot(thist03_euler_cell{3,1},xhist03_euler_cell{3,1}(:,4),...
        'r-.','LineWidth',1.5)
   hold off
   grid
   ylabel('Airspeed (m/sec)')
   subplot(413)
   hold off
   plot(thist03,xhist03(:,5)*(180/pi),'b-','LineWidth',3)
   hold on
   plot(thist03_euler_cell{1,1},...
        xhist03_euler_cell{1,1}(:,5)*(180/pi),'k:','LineWidth',1.5)
   plot(thist03_euler_cell{2,1},...
        xhist03_euler_cell{2,1}(:,5)*(180/pi),'g--','LineWidth',1.5)
   plot(thist03_euler_cell{3,1},...
        xhist03_euler_cell{3,1}(:,5)*(180/pi),'r-.','LineWidth',1.5)
   hold off
   grid
   ylabel('Flight Path Angle (deg)')
   subplot(414)
   hold off
   plot(thist03,xhist03(:,6)*(180/pi),'b-','LineWidth',3)
   hold on
   plot(thist03_euler_cell{1,1},...
        xhist03_euler_cell{1,1}(:,6)*(180/pi),'k:','LineWidth',1.5)
   plot(thist03_euler_cell{2,1},...
        xhist03_euler_cell{2,1}(:,6)*(180/pi),'g--','LineWidth',1.5)
   plot(thist03_euler_cell{3,1},...
        xhist03_euler_cell{3,1}(:,6)*(180/pi),'r-.','LineWidth',1.5)
   hold off
```

```
   grid
   ylabel('Heading Angle (deg)')
   xlabel('Time (seconds)')
%
%  Plot the thrust, angle-of-attack, and roll/bank-angle
%  time histories.
%
   figure(3)
   subplot(311)
   hold off
   plot(tinhist,Thist,'LineWidth',1.5)
   grid
   ylabel('Thrust (N)')
   title('Control input time histories for simaircraft05.mat')
   subplot(312)
   hold off
   plot(tinhist,alphahist*(180/pi),'LineWidth',1.5)
   grid
   ylabel('Angle-of-Attack (deg)')
   subplot(313)
   hold off
   plot(tinhist,phihist*(180/pi),'LineWidth',1.5)
   grid
   ylabel('Roll/Bank-Angle (deg)')
   xlabel('Time (seconds)')
%
%  Display final state error.
%
   format long
   errorxfinal_1000 = xhist03_euler_cell{1,1}(end,:)' - xhist03(end,:)'
   errorxfinal_10000 = xhist03_euler_cell{2,1}(end,:)' - xhist03(end,:)'
   errorxfinal_100000 = xhist03_euler_cell{3,1}(end,:)' - xhist03(end,:)'
%
%  Save the results.
%
   textcommands = ['These data have been generated by the',...
                   ' commands in script_simaircraft05.m'];
   save simaircraft05
   disp('errorxfinal_10000./errorxfinal_100000')
   disp(errorxfinal_10000./errorxfinal_100000)
```

Output

```
timetoode45 =    1.5511


N =        1000
timetoeuler =    0.2197


N =       10000
timetoeuler =    2.1907


N =      100000
timetoeuler =   21.5372
```

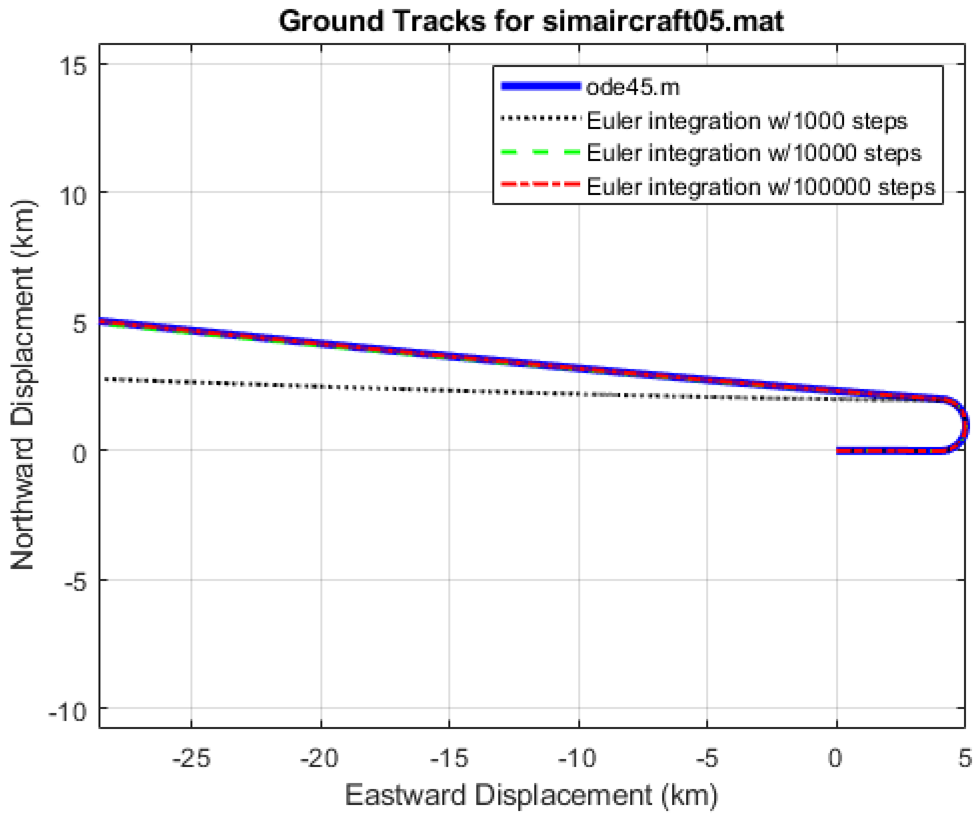
```
errorxfinal_1000 =

   1.0e+03 *

  -2.249895849628666
   0.251280539653530
   0.152176155278537
   0.003678238682550
   0.000056219697832
  -0.000066881570922


errorxfinal_10000 =

   1.0e+02 *

  -1.056444894361602
  -0.038021493036576
   0.020757723483021
   0.000410954780697
   0.000011519644391
  -0.000031502241030


errorxfinal_100000 =

  -9.952207111369717
  -0.365993885796343
   0.177905214820271
   0.003264838969415
   0.000100452336104
  -0.000296849328265

errorxfinal_10000./errorxfinal_100000
  10.615181964558259
  10.388559621384907
  11.667855551053442
  12.587290967402469
  11.467771520357749
  10.612198859979557
```

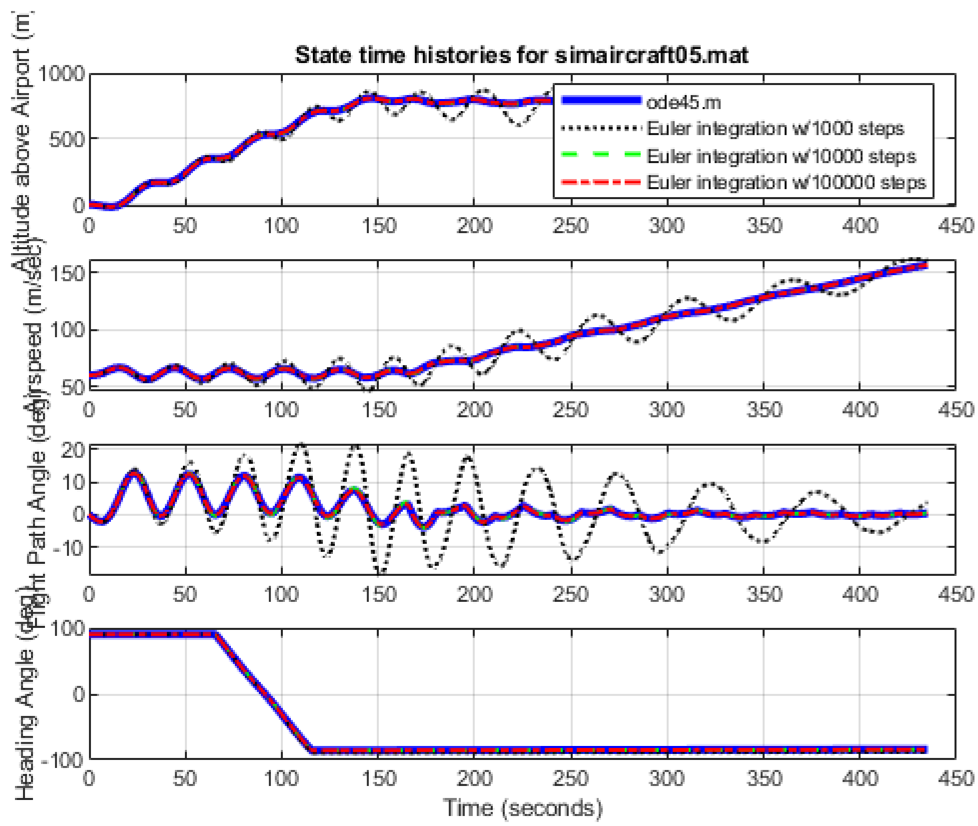
Q) How does Euler integration compare to `ode45.m` in terms of execution speed?

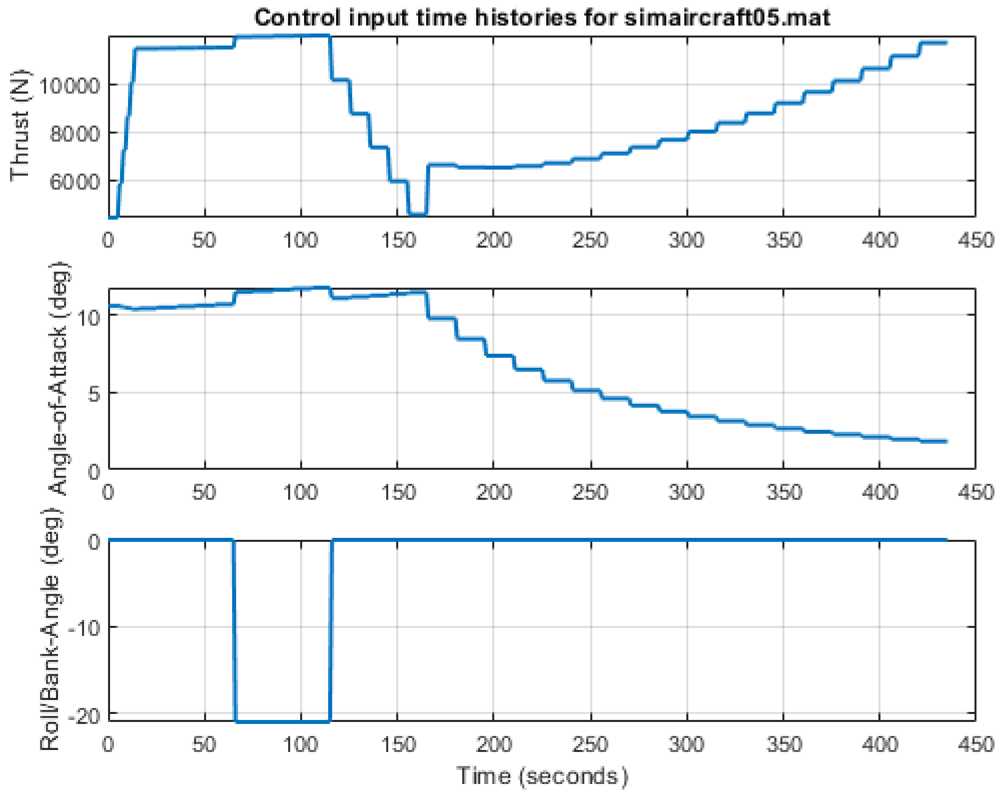
Ans) Given the accuracy considered in the ode45.m versus the accuracy achieved by Euler integration, the ode45 is very fast. The 1000 step Euler is fast but it is inaccurate.

Q) The theory of Euler's method predicts that these ratios should be about 10. Is that true?

Ans) Yes. It is close to 10, or at least 10.


Ground Tracks for simaircraft05.mat
ode45.m
Euler integration w/1000 steps
Euler integration w/10000 steps
Euler integration w/100000 steps
Northward Displacment (km)
Eastward Displacement (km)
15
10
5
0
-5
-10
-25
-20
-15
-10
-5
0
5


State time histories for simaircraft05.mat
ode45.m
Euler integration w/1000 steps
Euler integration w/10000 steps
Euler integration w/100000 steps
Altitude above Airport (m)
Airspeed (m/sec)
Flight Path Angle (deg)
Heading Angle (deg)
Time (seconds)
0
50
100
150
200
250
300
350
400
450


Control input time histories for simaircraft05.mat
Thrust (N)
10000
8000
6000
0
50
100
150
200
250
300
350
400
450
Angle-of-Attack (deg)
10
5
0
Roll/Bank-Angle (deg)
-10
-20
Time (seconds)