AOE 5204 Problem Set #4 Fall 2019

Posting Date: Wednesday Oct. 2nd.
Due Date: Wednesday Oct. 9th.

1. Suppose you are given the position of a particle of mass m in spherical coordinates r, λ, and ϕ, as in lecture. Suppose that these three spherical coordinates have non-zero time derivatives. This particle's angular momentum about the coordinate system center can be expressed in the form

$$\vec{h} = h_r \hat{r} + h_\lambda \hat{\lambda} + h_\phi \hat{\phi}$$

What are the values of its components h_r, h_λ, and h_ϕ? Express your answers solely in terms of m, r, λ, ϕ, $\dot{r}$, $\dot{\lambda}$, and $\dot{\phi}$ or a subset of these quantities.

2. Suppose that a particle of mass m is attached to the center of an inertial coordinate system by a linear spring within a slider track and that this linear spring/slider-track mechanism rotates about the $\hat{k}$ axis of the coordinate system so that it points along the direction $\hat{r}$ = $\hat{\imath}\cos\lambda + \hat{\jmath}\sin\lambda$ with known longitude time history $\lambda(t)$ = ωt for some fixed rotation rate ω. The spring force acts in the $\hat{r}$ direction so that $F_r = -Kr$. The spring/slider-track holds the latitude constant at $\phi(t) = 0$. The slider mechanism applies whatever F_λ and F_ϕ force components are needed to maintain $\lambda(t) = \omega t$ and $\phi(t) = 0$. Write down an equation of motion that can be solved to determine $r(t)$. Describe qualitatively what happens when the spring stiffness K is very low and what happens when K is very large. At what value of K does the behavior of the particle change in a significant qualitative way between these two regimes? In the low-K regime, the total of spring potential energy plus particle kinetic energy grows with time. The spring force is conservative. Therefore, this energy growth must come from something other than the spring. Where does it come from?

3. Suppose that the rotation matrix from coordinate frame $\mathcal{F}_a$ to coordinate frame $\mathcal{F}_b$ is R and suppose that R is parameterized by the following 2-1-2 Euler angle representation:

$$R = R_2(\alpha) R_1(\beta) R_2(\gamma)$$

Suppose that $\vec{\omega}^b$ is the rotation rate vector of frame $\mathcal{F}_b$ relative to frame $\mathcal{F}_a$ as expressed in frame $\mathcal{F}_b$. Derive a formula for $\vec{\omega}^b$ in terms of α, β, γ, $\dot{\alpha}$, $\dot{\beta}$, and $\dot{\gamma}$ or a subset of these quantities.

4. Prove the validity of the following formula that was given in class.

$$\vec{\omega}^b = 2[\Xi(\underline{q})]^T \underline{\dot{q}}$$

Hint: A good way to start is by proving some helpful properties about the columns of the 4-by-3 $\Xi(\underline{q})$ matrix. You can then complete your proof by using these properties and the equation $\underline{\dot{q}} = \frac{1}{2}\Xi(\underline{q})\vec{\omega}^b$ that was derived in lecture.