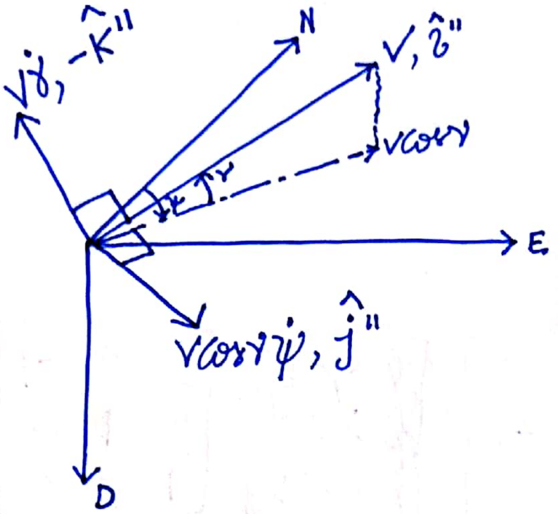

the unit vector of the Navigational axes are shown in the Picture.

$$\vec{F}^{Nav}_{ext} = m\,\vec{a}^{Nav}$$

$$\vec{F}^{Nav}_{ext} = R^{Nav/bod}\,\vec{F}^{bod}_{Thrust} + R^{Nav/NED}\,\vec{F}^{NED}_{gravity} + R^{Nav/wind}\,\vec{F}^{wind}_{Aerodynamic}$$

$$\vec{F}^{bod}_{Thrust} = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} ; \quad \vec{F}^{NED}_{Gravity} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} ; \quad \vec{F}^{wind}_{Aerodynamic} = \begin{bmatrix} -D \\ 0 \\ -L \end{bmatrix}$$

Based on the Equation for the Rotation matrices given

$$R^{Nav/bod} = R_1(-\phi)\,R_2(-\alpha)$$

$$R^{Nav/NED} = R_2(\gamma)\,R_3(\psi)$$

$$R^{wind/NED} = R_1(\phi)\,R_2(\gamma)\,R_3(\psi)$$

So we write $R^{Nav/wind}$ as

$$R^{Nav/wind} = R^{Nav/NED} R^{NED/wind} = R^{Nav/NED}\left[R^{wind/NED}\right]^T$$

$$= R_2(\gamma) R_3(\psi)\left[R_1(\phi) R_2(\gamma) R_3(\psi)\right]^T$$

$$= R_2(\gamma) R_3(\psi) R_3(\psi)^T R_2(\gamma)^T R_1(\phi)^T$$

Using the fact the $R_3(\psi)^T$ is inverse of $R_3(\psi)$ and $R_2(\gamma)^T$ is for $R_2(\gamma)$

$$R^{Nav/wind} = R_1(\phi)^T$$

also $R_1(\phi)^T$ is undoing the Rotation, so this is Equivalent to $R_1(-\phi)$

$$\boxed{\therefore R^{Nav/wind} = R_1(-\phi)}$$

So we substitute these to $\vec{F}_{ext}^{Nav}$

$$\vec{F}_{ext}^{Nav} = R_1(-\phi) R_2(-\alpha)\begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} + R_1(-\phi)\begin{bmatrix} -D \\ 0 \\ -L \end{bmatrix} + R_2(\gamma) R_3(\psi)\begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$

$$\vec{F}_{ext}^{Nav} = R_1(-\phi)\left\{R_2(-\alpha)\begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -D \\ 0 \\ -L \end{bmatrix}\right\} + R_2(\gamma) R_3(\psi)\begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$

$$R_2(-\alpha)\begin{bmatrix}T\\0\\0\end{bmatrix} = \begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}\begin{bmatrix}T\\0\\0\end{bmatrix} = \begin{bmatrix}T\cos\alpha\\0\\-T\sin\alpha\end{bmatrix}$$

$$R_1(-\phi)\left\{R_2(-\alpha)\begin{bmatrix}T\\0\\0\end{bmatrix} + \begin{bmatrix}-D\\0\\-L\end{bmatrix}\right\} = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}T\cos\alpha - D\\0\\-T\sin\alpha - L\end{bmatrix}$$

$$= \begin{bmatrix}T\cos\alpha - D\\ (T\sin\alpha + L)\sin\phi\\ -(T\sin\alpha + L)\cos\phi\end{bmatrix}$$

$$\vec{F}^{Nav}_{ext} = \begin{bmatrix}T\cos\alpha - D\\ (T\sin\alpha + L)\sin\phi\\ -(T\sin\alpha + L)\cos\phi\end{bmatrix} + R_2(\gamma)R_3(\psi)\begin{bmatrix}0\\0\\mg\end{bmatrix}$$

$$R_3(\psi)\begin{bmatrix}0\\0\\mg\end{bmatrix} = \begin{bmatrix}\cos\psi & \sin\psi & 0\\ -\sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\mg\end{bmatrix} = \begin{bmatrix}0\\0\\mg\end{bmatrix}$$

$$R_2(\gamma)R_3(\psi)\begin{bmatrix}0\\0\\mg\end{bmatrix} = \begin{bmatrix}\cos\gamma & 0 & -\sin\gamma\\ 0 & 1 & 0\\ \sin\gamma & 0 & \cos\gamma\end{bmatrix}\begin{bmatrix}0\\0\\mg\end{bmatrix} = \begin{bmatrix}-mg\sin\gamma\\0\\mg\cos\gamma\end{bmatrix}$$

$$\boxed{\vec{F}^{Nav}_{ext} = \begin{bmatrix}T\cos\alpha - D - mg\sin\gamma\\ (T\sin\alpha + L)\sin\phi\\ -(T\sin\alpha + L)\cos\phi + mg\cos\gamma\end{bmatrix}}$$

based of the definitions of $\hat{i}''$, $\hat{j}''$, $\hat{k}''$ and

$\dot{V}$, $V\dot{\gamma}$, $V\cos\gamma\,\dot{\psi}$

$$m\vec{a}^{\,NAV}_{Ext} = m\begin{bmatrix} \dot{V} \\ V\cos\gamma\,\dot{\psi} \\ -V\dot{\gamma} \end{bmatrix} = \vec{F}^{\,NAV}_{Ext}$$

So we show that

$$\boxed{\begin{aligned} m\dot{V} &= T\cos\alpha - D - mg\sin\gamma \\ mV\dot{\gamma} &= [T\sin\alpha + L]\cos\phi - mg\cos\gamma \\ mV\cos\gamma\,\dot{\psi} &= (T\sin\alpha + L)\sin\phi \end{aligned}}$$

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