Posting Date: Wednesday Oct. 9th.
Due Date: Monday Oct. 21st.

1. Complete the MATLAB template file `ffunctaircraft03_temp.m` by completing the parts of the code where ???? appears. The result will be the MATLAB function `ffunctaircraft03.m`. This function must compute the state rate dynamics model for the 6-element state vector of a 3-dimensional point-mass aircraft translational model. This model differs from the one you developed in Assignment 1 because it includes the effects of centrifugal and Coriolis accelerations caused by Earth rotation. It still works in local North/East/Down (ned) coordinates, but those coordinates are centered at the center of the runway of the Blacksburg, VA airport: latitude = 37.2076389 deg, longitude = -80.4078333 deg, altitude = 649.7 m. Hand in your completed code.

   There are three specific differences between the model in `ffunctaircraft03.m` and the one in `ffunctaircraft01.m` of Problem Set #1. They are the following: First, the local gravity magnitude has been reduced from 9.81 m/sec^2 to 9.79721 m/sec^2. This lowered number accounts for the main $1/r^2$ gravity term, the gravity perturbation due to Earth oblateness, and the centrifugal acceleration at the origin of the coordinate system. These three effects add up to give a net effective gravity term that points vertical relative to the Earth's oblate spheroidal surface, as expected. The second difference from the model in `ffunctaircraft01.m` is the inclusion of the full centrifugal and Coriolis effects of the Earth's rotation. Part of the centrifugal effect is already lumped into the modified value of gravitational acceleration that applies at the local-level coordinate system origin. This effectively amounts to accounting for the inertial acceleration of the local-level coordinate system's origin. The additional centrifugal effect is caused by the Earth's rotation rate and the vector from the coordinate frame origin to the aircraft. The third difference from the model in `ffunctaircraft01.m` is that the third element of its state vector is the (negative) altitude difference from the airport altitude of 649.7 m. Thus, `-x(3,1)+649.7` is the aircraft altitude in meters above sea level.

   Hint: The aircraft acceleration measured with respect to the rotating North/East/Down coordinate frame and expressed along navigation axes is:

$$\vec{a}^{nav} = \begin{bmatrix} \dot{V} \\ V\cos\gamma\,\dot{\psi} \\ -V\dot{\gamma} \end{bmatrix}$$

   The Coriolis acceleration and the incremental centrifugal acceleration must be added to this acceleration vector. The result must be multiplied by the aircraft mass and set equal to the sum of the thrust, lift, drag, and gravity forces expressed in navigation axes in order to set up valid equations of motion. These equations must be solved for $\dot{V}$, $\dot{\gamma}$, and $\dot{\psi}$ in order to derive expressions that will enable you to complete `ffunctaircraft03_temp.m`.

2. Use your `ffunctaircraft03.m` function from Part 1, the MATLAB scripts `script_simaircraft03.m` and `script_simaircraft04.m`, the MATLAB function `ffunctaircraft02.m` (provided), and the MATLAB data files

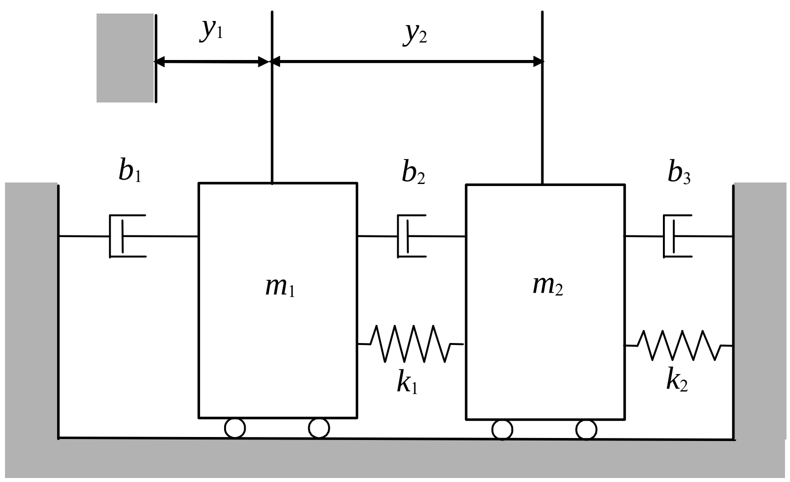
`maneuver01_data.mat` and `maneuver02_data.mat` to run simulations of the same two aircraft flight cases that have been presented in lecture. Hand in hardcopies of the plots of North displacement (X) vs. East displacement (Y) and of the plots of altitude above the airport ($-Z$), airspeed (V), flight-path angle (γ), and heading angle (ψ) for the two cases. Also, hand in the final state vector to 15 significant digits for the second case, the one run by `script_simaircraft04.m`. You can print the final state vector in the MATLAB command prompt window by executing the following two MATLAB commands:

```
>> format long
>> xhist03(end,:)'
```

As a way of checking your work, these operations produce the following result for the `script_simaircraft03.m` case:

```
ans =
  1.0e+04 *
  0.487433708638846
 -2.391929837861427
  0.057847195930909
  0.013769740920283
 -0.000008373096714
 -0.000145806918460
```

3. Consider the following mechanical system:



It consists of two masses on two carts. The right-most cart is connected to a rigid support by a spring k_2 and a damper b_3, the two masses are connected by a spring k_1 and a damper b_2, and the left-most mass is connected to another rigid support by a damper b_1. The displacements of the of the left-most mass from its equilibrium position is y_1. The displacement y_2 is a relative displacement between the two masses. Derive equations of motion that model the time evolution of y_1 and y_2. You need not solve these equations of motion. Be careful to draw free-body diagrams, define force directions, and get signs correct. Note that y_1 is increasing when mass m_1 is moving towards the right. The relative displacement y_2 is increasing when the distance between the two masses is increasing. Only the displacement y_1 is measured relative to an inertial coordinate system. If the two positions are simultaneously at $y_1 = 0$ and $y_2 = 0$, then the two springs are in their equilibrium states so that neither one produces a force. This is the equilibrium position of the system.

Hints: There is no need to consider vertical constraint forces because the vertical and horizontal motions are decoupled, and the vertical motion of both masses is known to be zero. In your constitutive laws for the springs and the dampers, be careful to use valid expressions for the relevant spring deflections or damper deflection rates. Also, be careful to use the correct signs for your constitutive law formulas. They must agree with the definitions of positive force that you have made by pointing the arrows on your free-body diagrams. A good way to get the correct sign is to do a little "thought experiment" for each constitutive law. A "thought experiment" might go as follows: "If I deflect this mass in this particular direction while leaving the other mass undeflected, will this defined spring force point in the same direction is its positive arrow (indicating that a positive value should result from the constitutive law) or in the opposite direction from the arrow (indicating that a negative value should result from the constitutive law)? Does the sign definition in my constitutive law agree with this conclusion?" Proper computation of the inertial acceleration of mass m_2 and proper computation of the deflection of spring k_2 can be tricky. Be careful.