Posting Date: Monday Sept. 16th.
Due Date: Monday Sept. 23rd.

1. The MATLAB files `mrvdata0_2019.mat` and `mrvdata1_2019.mat` contain mass, position, and velocity data for a collection of $N = 12$ particles at the respective times t_0 and t_1. The 1-by-12 array `mvec` is identical in the two files and contains the particle masses: m_i = `mvec(1,i)` for $i = 1, \ldots. N$. The 3-by-12 arrays `rmat0` and `rmat1` in the two files contain the respective position vectors so that $\vec{r}_i(t_0)$ = `rmat0(:,i)` and $\vec{r}_i(t_1)$ = `rmat1(:,i)` for $i = 1, \ldots. N$. Similarly, the 3-by-12 arrays `vmat0` and `vmat1` in the two files contain the respective velocity vectors so that $\vec{v}_i(t_0)$ = `vmat0(:,i)` and $\vec{v}_i(t_1)$ = `vmat1(:,i)` for $i = 1, \ldots. N$. The two respective times are contained in the scalar MATLAB variables `t0` and `t1`.

   Compute the center-of-mass positions of the collection of particles at the two times, `rcm0` and `rcm1`, the total linear momentum of the collection of particles at the two times, `ptot0` and `ptot1`, the center-of-mass velocity of the collection of particles at the two times, `vcm0` and `vcm1`, and the total angular momentum of the collection of particles about its center of mass at the two times, `h0` and `h1`.

   Hand in the MATLAB code that you used to generate these results along with your computed values for `rcm1`, `ptot1`, `vcm1`, and `h1` as displayed using MATLAB's "`format long`" command.

   As an aid to your programming, the results for time t_0 are:

   ```
   rcm0 =
     [ 4.135068659460397; 4.232438883625573; -2.649670988093459]

   ptot0 =
     [ -3.304955397587092; 2.615681258483552; -12.378512402556995]

   vcm0 =
     [ -0.448630861116144; 0.355065407616567; -1.680320007510681]

   h0 =
     [ 0.158538578051034; -0.117644152647727; -0.423922316388524]
   ```

   Hints: Suppose that one wants to form the sum

   $$\vec{y} = \sum_{i=1}^{N} a_i \vec{z}_i$$

   in MATLAB. Suppose that the relevant input variables are contained reside in the MATLAB variables `avec` (a 1-by-N row vector) and `zmat` (a 3-by-N matrix) such that a_i = `avec(1,i)` and $\vec{z}_i$ = `zmat(:,i)` for $i = 1, \ldots. N$. Then the following MATLAB code computes the required sum:

```
N = size(avec,2);
y = zeros(3,1);
for i = 1:N
   ai = avec(1,i);
   zi = zmat(:,i);
   y = y + ai*zi;
end
```

Suppose one wants to compute the cross product $\vec{y} = \vec{w} \times \vec{z}$ in MATLAB and suppose that the inputs are contained in the 3-by-1 MATLAB vectors `w` and `z`. Then this calculation can be performed by the following MATLAB assignment statement: `y = cross(w,z)`.

2. Compute the average total external force that acts on the system of particles from time t_0 to time t_1. Also, compute the average total external torque that acts about the center of mass of the system of particles from time t_0 to time t_1. You can compute these averaged values by using finite-difference approximations of appropriate time derivatives. Hand in the MATLAB code that you used to generate these results along with your computed values for `Fexttotavg` and `Texttotavg` as displayed using MATLAB's "`format long`" command.

3. Suppose you are given a position vector $\vec{r}$ and its representation in the $\mathcal{F}_a$ coordinate system [X^a; Y^a, Z^a] so that $\vec{r} = \hat{i}_a X^a + \hat{j}_a Y^a + \hat{k}_a Z^a$. Prove that the length of $\vec{r}$, which equals $\sqrt{\vec{r} \cdot \vec{r}}$, also equals $\sqrt{(X^a)^2 + (Y^a)^2 + (Z^a)^2}$

4. Suppose that you are given representations of the same position vector in two coordinate frames that share the same origin but that have different axes, $\hat{i}_a$, $\hat{j}_a$, and $\hat{k}_a$ for frame $\mathcal{F}_a$ and $\hat{i}_b$, $\hat{j}_b$, and $\hat{k}_b$ for frame $\mathcal{F}_b$. Thus, the two representations of the same position are related to each other as follows:

$$\begin{bmatrix} X^b \\ Y^b \\ Z^b \end{bmatrix} = R^{ba} \begin{bmatrix} X^a \\ Y^a \\ Z^a \end{bmatrix}$$

where R^{ba} is the 3-by-3 orthonormal rotation matrix that transforms from frame $\mathcal{F}_a$ to frame $\mathcal{F}_b$. Prove that $\sqrt{(X^b)^2 + (Y^b)^2 + (Z^b)^2} = \sqrt{(X^a)^2 + (Y^a)^2 + (Z^a)^2}$. That is, prove that the two representations have the same length.

Hints: It should be helpful to make use of the orthonormality of R^{ba} and the following facts from linear algebra:

$$X^2 + Y^2 + Z^2 = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$(AB)^{\mathrm{T}} = B^{\mathrm{T}} A^{\mathrm{T}}$$

The last expression is true for any matrices or vectors A and B for which the matrix-matrix, vector-matrix, or matrix-vector product AB makes sense.