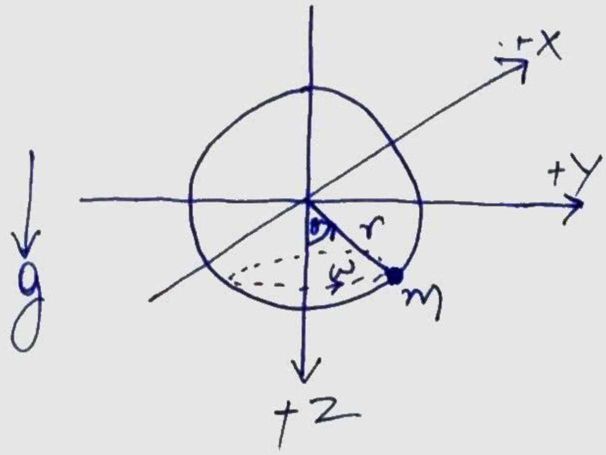
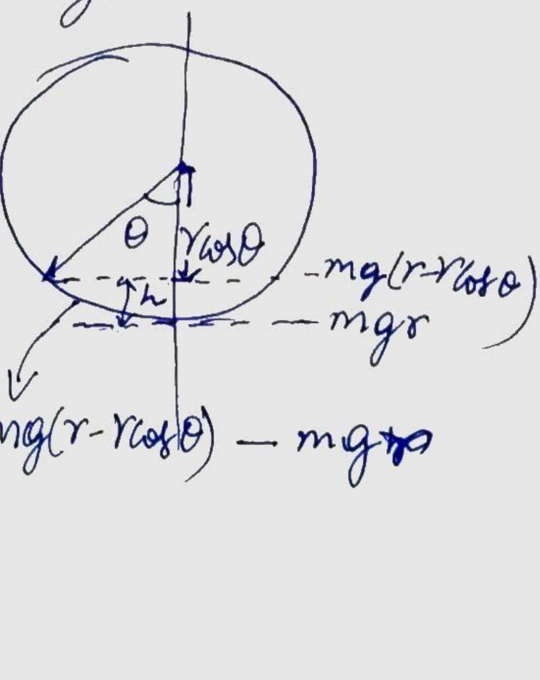
1) Though the System is constrained its a Holonomic Constraint and we donot write r, the radius as a generalized co-ord. We only deal with θ, a single generalized co-ord.



The mass m is alway rotating by ω. The angle θ is measured from the +z axis

to determine the PE we datum as the lowest point, $\theta = 0$. As PE is always a Relative quantity we get

$$PE = -mgr\cos\theta$$



$\Delta = mg(r - r\cos\theta) - mg\cancel{r}$

There are two Rotational Kinetic Energies assciated with two angular veloutis in two $\perp$ axes

about Z

$$KE_z = \frac{1}{2} I_{zz} \omega^2$$

about X

$$KE_x = \frac{1}{2} I_{xx} \dot{\theta}^2$$

$$I_{zz} = m \underbrace{(r\sin\theta)}_{\text{dist from } Z}{}^2$$ from MoI of a point mass

$$I_{xx} = mr^2$$

Total KE

$$T = \frac{1}{2} mr^2 \sin^2\theta\, \omega^2 + \frac{1}{2} mr^2 \dot{\theta}^2$$

the Lagrangian $L = T - V$

$$L = \frac{1}{2} mr^2 \sin^2\theta\, \omega^2 + \frac{1}{2} mr^2 \dot{\theta}^2 + mgr\cos\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = m r^2 \dot{\theta} \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = m r^2 \ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = m r^2 \omega^2 \sin\theta \cos\theta - m g r \sin\theta$$

The dynamics are,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$m r^2 \ddot{\theta} - m r^2 \omega^2 \sin\theta \cos\theta + m r g \sin\theta$$

$$\boxed{\ddot{\theta} - \omega^2 \sin\theta \cos\theta + \frac{g}{r} \sin\theta = 0}$$

Q2) There are two generalize co-ord y_1 & y_2
The Total KE is

$$T = \frac{1}{2} m_2 (\dot{y}_1 + \dot{y}_2)^2 + \frac{1}{2} m_1 \dot{y}_1^2$$

The potential Energy is stored in the springs, & the Gravitational PE of the masses relative to a zero y_1 & y_2 is

$$V = m_1 g y_1 + m_2 g (y_2 + y_1) + \frac{1}{2} k_1 y_1^2 + \frac{1}{2} k_2 y_2^2$$

Lagrangian, $L = T - V$

$$L = \frac{1}{2} m_2 \dot{y}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2 + m_2 \dot{y}_1 \dot{y}_2 + \frac{1}{2} m_1 \dot{y}_1^2 \ldots$$
$$- m_1 g y_1 - m_2 g (y_2 + y_1) - \frac{1}{2} k_1 y_1^2 - \frac{1}{2} k_2 y_2^2$$

$$\frac{\partial L}{\partial \dot{y}_1} = m_2 \dot{y}_1 + m_2 \dot{y}_2 + m_1 \dot{y}_1$$

$$\frac{\partial L}{\partial y_1} = - m_1 g - m_2 g - k_1 y_1$$

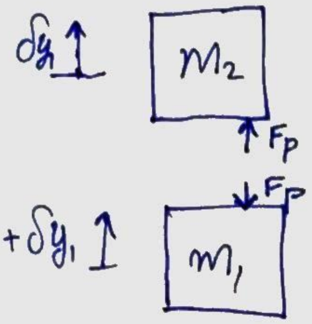
$$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_1}\right) - \frac{\partial L}{\partial y_1} = m_2 \ddot{y}_1 + m_2 \ddot{y}_2 + m_1 \ddot{y}_1 + m_1 g + m_2 g + k_1 y_1$$

$$\frac{\partial L}{\partial \dot{y}_2} = m_2 \dot{y}_2 + m_2 \dot{y}_1 \; ; \; \frac{\partial L}{\partial y_1} = -m_2 g - K_2 y_2$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_2}\right) - \frac{\partial L}{\partial y_2} = m_2 \ddot{y}_2 + m_2 \ddot{y}_1 + m_2 g + K_2 y_2}$$

There is non-conservative force F_p.

<u>for incremental δy_1:</u> y_2 is constant



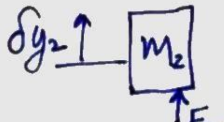
the total workdone

$$\delta W_1 = \underbrace{\delta y_1 F_p}_{\text{mass } m_2} - \underbrace{\delta y_1 F_p}_{\text{mass } m_1}$$

$$\delta W_1 = Q_1 \, \delta y_1 = 0$$

$$\therefore \boxed{Q_1 = 0}$$

<u>for incremental δy_2:</u> y_1 is constant, m_1 is stationary



$$\delta W_2 = F_p \, \delta y_2$$

$$\boxed{\therefore Q_2 = F_p}$$

The dynamics when there are non-conservative forces

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i$$

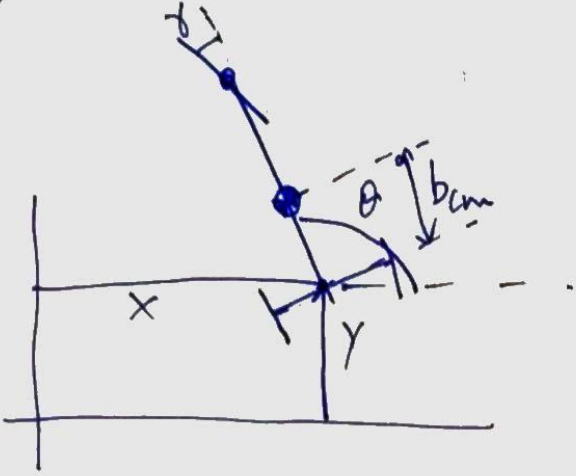
So the final dynamics are

$$\boxed{\begin{aligned} &(m_1+m_2)\ddot{y}_1 + m_2\ddot{y}_2 + (m_1+m_2)g + k_1 y_1 = 0 \\ &m_2(\ddot{y}_1+\ddot{y}_2) + m_2 g + k_2 y_2 = F_p(t) \end{aligned}}$$

3) The Total KE

$$T = \frac{1}{2} m \dot{X}_{cm}^2 + \frac{1}{2} m \dot{Y}_{cm}^2 + \frac{1}{2} I_{33} \dot{\theta}^2$$

Where X_{cm} & Y_{cm} are position ~~cos~~ of the Center of Mass



where
$q_1 = X$
$q_2 = X$
$q_3 = \theta$
the generlized co-ord of the system

$$X_{cm} = X + b_{cm} \cos\theta$$

$$Y_{cm} = Y + b_{cm} \sin\theta$$

$$\dot{X}_{cm} = \dot{X} - b_{cm} \sin\theta \, \dot{\theta}$$

$$\dot{Y}_{cm} = \dot{Y} + b_{cm} \cos\theta \, \dot{\theta}$$

$$\dot{X}_{cm}^2 = \dot{X}^2 + b_{cm}^2 \dot{\theta}^2 \sin^2\theta - 2\dot{X}\dot{\theta} b_{cm} \sin\theta$$

$$\dot{Y}_{cm}^2 = \dot{Y}^2 + b_{cm}^2 \dot{\theta}^2 \cos^2\theta + 2\dot{Y}\dot{\theta} b_{cm} \cos\theta$$

$$T = \tfrac{1}{2} m\dot{x}^2 + \tfrac{1}{2} m b_{cm}^2 \dot{\theta}^2 \sin^2\theta - m\dot{x}\dot{\theta} b_{cm} \sin\theta$$
$$+ \tfrac{1}{2} m\dot{y}^2 + \tfrac{1}{2} m b_{cm}^2 \dot{\theta}^2 \cos^2\theta + m\dot{y}\dot{\theta} b_{cm} \cos\theta$$
$$+ \tfrac{1}{2} I_{33} \dot{\theta}^2$$

can be written as $\tfrac{1}{2} m b_{cm}^2 \dot{\theta}^2$

There is no P.E. $\therefore V = 0 \;\therefore L = T$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} - m\dot{\theta} b_{cm} \sin\theta$$

$$\frac{\partial L}{\partial \dot{y}} = m\dot{y} + m\dot{\theta} b_{cm} \cos\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = \overbrace{m b_{cm}^2 \dot{\theta}}^{\text{there is } \sin^2\theta + \cos^2\theta} - m\dot{x} b_{cm} \sin\theta + m\dot{y} b_{cm} \cos\theta + I_{33}\dot{\theta}$$

$$\frac{\partial L}{\partial x} = 0 \;;\; \frac{\partial L}{\partial y} = 0$$

$$\frac{\partial L}{\partial \theta} = -m\dot{x}\dot{\theta} b_{cm} \cos\theta - m\dot{y}\dot{\theta} b_{cm} \sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x} - m\ddot{\theta} b_{cm}\sin\theta - m\dot{\theta}^2 b_{cm}\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = m\ddot{y} + m\ddot{\theta} b_{cm}\cos\theta - m\dot{\theta}^2 b_{cm}\sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \left(m b_{cm}^2 + I_{33}\right)\ddot{\theta} - m\ddot{x} b_{cm}\sin\theta + m\ddot{y} b_{cm}\cos\theta$$
$$- m\dot{x} b_{cm}\dot{\theta}\cos\theta - m\dot{y} b_{cm}\dot{\theta}\sin\theta$$

Recall,

$$\frac{\partial L}{\partial \theta} = -m\dot{x}\dot{\theta} b_{cm}\cos\theta - m\dot{y}\dot{\theta} b_{cm}\sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = m\ddot{x} - m\ddot{\theta} b_{cm}\sin\theta - m\dot{\theta}^2 b_{cm}\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = m\ddot{y} + m\ddot{\theta} b_{cm}\cos\theta - m\dot{\theta}^2 b_{cm}\sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \left(m b_{cm}^2 + I_{33}\right)\ddot{\theta} - m\ddot{x} b_{cm}\sin\theta + m\ddot{y} b_{cm}\cos\theta$$

There are two non-holonomic constraint Eq

Eq 1

$$-\sin\theta\,\dot{x} + \cos\theta\,\dot{y} = 0$$

Eq 2

$$-\sin(\theta+\gamma)\,\dot{x} + \cos(\theta+\gamma)\,\dot{y} + b_w\cos\gamma\,\dot{\theta} = 0$$

The System will be of the for

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \sum_{k=1}^{2} \lambda_k(t)\, a_{ki}(q_1, q_2, q_3, t)$$

$$i = 1 \text{ to } 3$$

The final dynamics are

$$m\ddot{x} - m\ddot{\theta} b_{cm}\sin\theta - m\dot{\theta}^2 b_{cm}\cos\theta = -\lambda_1(t)\sin\theta \ldots$$
$$- \lambda_2(t)\sin(\theta+\gamma)$$

$$m\ddot{y} + m\ddot{\theta} b_{cm}\cos\theta - m\dot{\theta}^2 b_{cm}\sin\theta = \lambda_1(t)\cos\theta \ldots$$
$$+\lambda_2(t)\cos(\theta+\gamma)$$

$$(m b_{cm}^2 + I_{33})\ddot{\theta} - m\ddot{x} b_{cm}\sin\theta + m\ddot{y} b_{cm}\cos\theta$$
$$= \lambda_2(t)\, b_w\cos\gamma$$