Quantifying the Impact of Air Drag Models Considering a Rotating Atmosphere in Resident Space Objects (RSO) Lifetime Predictions

AOE 5234
Orbital Mechanics
Final Presentation

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Motivation

- **Near-Earth space is getting increasingly congested.**
  The number of satellites at Low Earth orbit (LEO) is considerably increased by emerging mega-constellations like Starlink and OneWeb.
- Defunct satellites and space debris **pose risks to ongoing and future space operations**.
- The drag force imparted by the atmosphere **over long periods of time** at low altitudes becomes significant (especially for **high area-to-mass ratio** objects).

In this work, we are **leveraging the effects of atmospheric drag for reentry of spacecraft** after their useful lifetime has expired.

Objectives

1. **Analyze the long-term evolution of satellite trajectories under the influence of atmospheric drag**
    a. Express the secular variation of the Milankovitch elements analytically through averaging method.
    b. Present and compare two different formulations of the averaged equations with the non-averaged dynamics.
2. **Provide insight into the effects of atmospheric drag on orbit decay**
    a. Quantify the impact of considering a **rotating atmosphere** versus neglecting it.
    b. Show that atmospheric drag affects out-of-orbit-plane elements secularly.

Non-Averaged Dynamics (reference trajectory)

Modeling the nonlinear relative equation of motion (two-body problem).

$$\ddot{\mathbf{r}} = -\mu \frac{\mathbf{r}}{r^3} + \mathbf{f}$$

RSO position vector in ECI

Perturbation vector in ECI

$$\mathbf{f} = \mathbf{a}_{J_2} + \mathbf{a}_{S,M} + \mathbf{a}_d$$

Luni-Solar   Atmospheric Drag

When $\mathbf{f} = \mathbf{0}$ $\longrightarrow$ Orbit is Keplerian and we have closed form solution for $\mathbf{r}$

When $\mathbf{f} \neq \mathbf{0}$ $\longrightarrow$ Orbit is not Keplerian and we solve for $\mathbf{r}$ numerically

Perturbation Models

Earth oblateness

$$\mathbf{a}_{J_2} = -\frac{\mu J_2 R^2}{2r^5}\left[\left(1 - \frac{5r_z^2}{r^2}\right)\mathbf{r} + 2r_z\hat{\mathbf{z}}\right]$$

second spherical harmonic term

Component of $\mathbf{r}$ in $\hat{\mathbf{z}}$ direction

Earth spin axis

S/M third-body gravity

$$\mathbf{a}_{S/M} = -\mu_{S/M}\left(\frac{\mathbf{d}_{S/M}}{d_{S/M}^3} + \frac{\mathbf{r}_{S/M}}{r_{S/M}^3}\right)$$

S/M Gravity parameter

Relative position of RSO w.r.t S/M

Position of S/M in ECI

Atmospheric drag

$$\mathbf{a}_d = -\frac{1}{2}B\rho|\mathbf{v} - \mathbf{v}_{atm}|\left(\mathbf{v} - \mathbf{v}_{atm}\right)$$

Ballistic coefficient

Air Density

RSO velocity vector in ECI

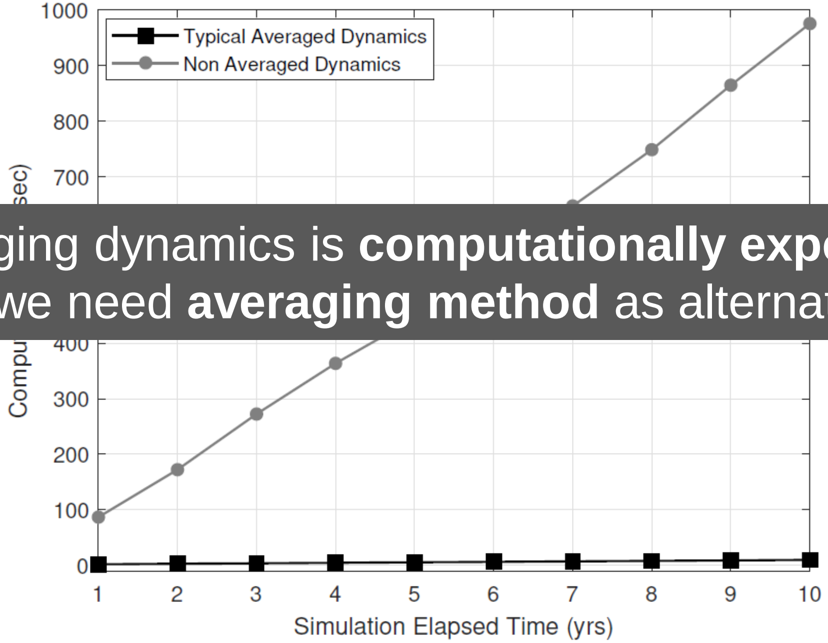
$$\boxed{\mathbf{v}_{atm} = \omega_a\hat{\mathbf{z}} \times \mathbf{r}}$$

Atmospheric velocity

Atmos angular velocity

$\omega_a = \omega_{earth}$

Non-averaged vs averaged dynamics

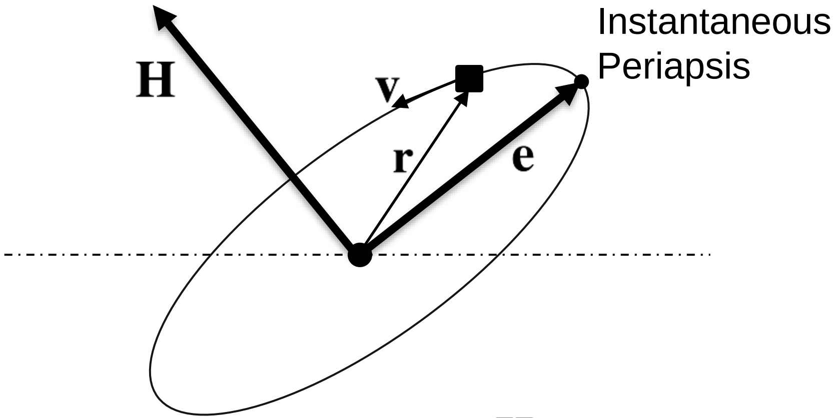


Non-averaging dynamics is **computationally expensive** and we need **averaging method** as alternative

Systems specification: CPU: 4.00GHz Intel(R) RAM: 32.0GB OS: x64 Windows

Averaged Dynamics in Milankovitch Elements

$$\begin{aligned} \dot{\mathbf{H}} &= \mathbf{r} \times \mathbf{f} \\ \dot{\mathbf{e}} &= \frac{1}{\mu} (\mathbf{v} \times \mathbf{r} - \mathbf{H}) \times \mathbf{f} \end{aligned}$$



$$\mathbf{H} = \mathbf{r} \times \mathbf{v}$$

$$e = |\mathbf{e}|$$

Instantaneous
Eccentricity

Averaged Dynamics over a single orbital period

$$\begin{aligned} \dot{\bar{\mathbf{H}}} &= \frac{1}{2\pi} \int_0^{2\pi} \mathbf{r} \times \mathbf{f} \, \mathrm{d}M \\ \dot{\bar{\mathbf{e}}} &= \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{\mu} (\mathbf{v} \times \mathbf{r} - \mathbf{H}) \, \mathbf{f} \, \mathrm{d}M \end{aligned}$$

[Wang 2016] Y. Wang and P. Gurfil, “Dynamical Modeling and Lifetime Analysis of Geostationary Transfer Orbits,” Acta Astronautica. 2016.

Averaged Drag with Still Atmosphere

Formulation 1

Drag Model

$$\mathbf{a}_d = -\frac{1}{2} B\rho \left|\mathbf{v} - \cancelto{0}{\mathbf{v}_{atm}}\right| \left(\mathbf{v} - \cancelto{0}{\mathbf{v}_{atm}}\right)$$

Averaging

$$\begin{aligned} \dot{\mathbf{H}}_d &= -\frac{1}{2} B\mathbf{H} \frac{1}{2\pi} \int_0^{2\pi} \rho v \mathrm{d}M \\ \dot{\mathbf{e}}_d &= \frac{1}{\mu} B\mathbf{H} \times \left(\frac{1}{2\pi} \int_0^{2\pi} \rho v \mathbf{v} \mathrm{d}M \right) \end{aligned}$$

$$\rho = \rho_{p_0} \exp\left(\frac{r_{p_0} - r_p}{H_{\rho_0}} \right)$$

Exponentially varying density model, function of relative perigee

Averaged Drag Perturbation

$$\dot{\mathbf{H}}_d = -\frac{1}{2} B \sqrt{\frac{\mu\left(1-e^2\right)}{2a\pi z}} \rho_{p_0} \exp\left(\frac{r_{p_0} - r_p}{H_{\rho_0}} \right) \left(1 + \frac{1+3e^2}{8z\left(1-e^2\right)} \right) \mathbf{H}$$

$$\dot{\mathbf{e}}_d = -B \frac{1+e}{a\sqrt{2\pi z}} \rho_{p_0} \exp\left(\frac{r_{p_0} - r_p}{H_{\rho_0}} \right) \left(1 + \frac{3e^2 - 4e - 3}{8z\left(1-e^2\right)} \right) H\hat{\mathbf{e}}$$

$$z = \frac{ae}{H_{\rho_0}}$$

A dimensionless orbital shape dependent parameter

[Wang 2016] Y. Wang and P. Gurfil, “Dynamical Modeling and Lifetime Analysis of Geostationary Transfer Orbits,” Acta Astronautica. 2016.

Averaged Drag with a Rotating Atmosphere

Formulation 2

Drag Model

$$\mathbf{a}_d = -\frac{1}{2}B\rho|\mathbf{v}-\mathbf{v}_{atm}|\,(\mathbf{v}-\mathbf{v}_{atm})$$

$I_0(z),\ I_1(z),\ I_2(z),$

are modified Bessel functions of first kind, with the order appearing subscript, which are shortened as;

$I_0,\ I_1,\ I_2$

Angular Momentum Vector

$$\dot{\mathbf{H}}_d = -\frac{BH^2\rho_{p_0}}{2a}\exp\left(\frac{r_{p_0}-a}{H_{\rho_0}}\right)\left[I_0+\frac{H_{\rho_0}e}{2a(1-e^2)}I_1-\frac{2\omega_a a^2\cos i}{H}\left[(1+e^2)I_0-2eI_1\right]\right]\hat{\mathbf{H}}$$
$$+\frac{BH\omega_a a\rho_{p_0}}{2}\exp\left(\frac{r_{p_0}-a}{H_{\rho_0}}\right)\left[\left[(1+e^2)I_0-2eI_1\right](\hat{\mathbf{e}}_\perp\cdot\hat{\mathbf{z}})\hat{\mathbf{e}}-\frac{1}{2}(1-e^2)(I_0-I_2)(\hat{\mathbf{e}}\cdot\hat{\mathbf{z}})\hat{\mathbf{e}}_\perp\right]\times\hat{\mathbf{H}}$$

Eccentricity Vector

$$\dot{\mathbf{e}}_d = -\frac{BH\rho_{p_0}}{a}\exp\left(\frac{r_{p_0}-a}{H_{\rho_0}}\right)\left[\left(1-\frac{H_{\rho_0}(2-e^2)}{2a(1-e^2)}\right)I_1+\left(1-\frac{H_{\rho_0}}{2a(1-e^2)}\right)eI_0-\left(\frac{2\omega_a a^2(1-e^2)\cos i}{H}\right)(I_1-eI_0)\right]\hat{\mathbf{e}}$$
$$+\frac{eB\omega_a a\rho_{p_0}}{2}\exp\left(\frac{r_{p_0}-a}{H_{\rho_0}}\right)\left[\frac{1}{2}(1-e^2)(I_0-I_2)(\hat{\mathbf{e}}\cdot\hat{\mathbf{z}})\hat{\mathbf{e}}_\perp\right]\times\hat{\mathbf{H}}$$

[Ward 1962] G. N. Ward, “On the Secular Variations of the Elements of Satellite Orbits,” Royal Society of London. 1962.

Numerical Stability Analysis of Formulation 2

Factoring density out:

$$\rho_{p_0} \exp\left(\frac{r_{p_0} - a}{H_{\rho_0}}\right) = \rho \ \exp\left(-\frac{ae}{H_{\rho_0}}\right)$$

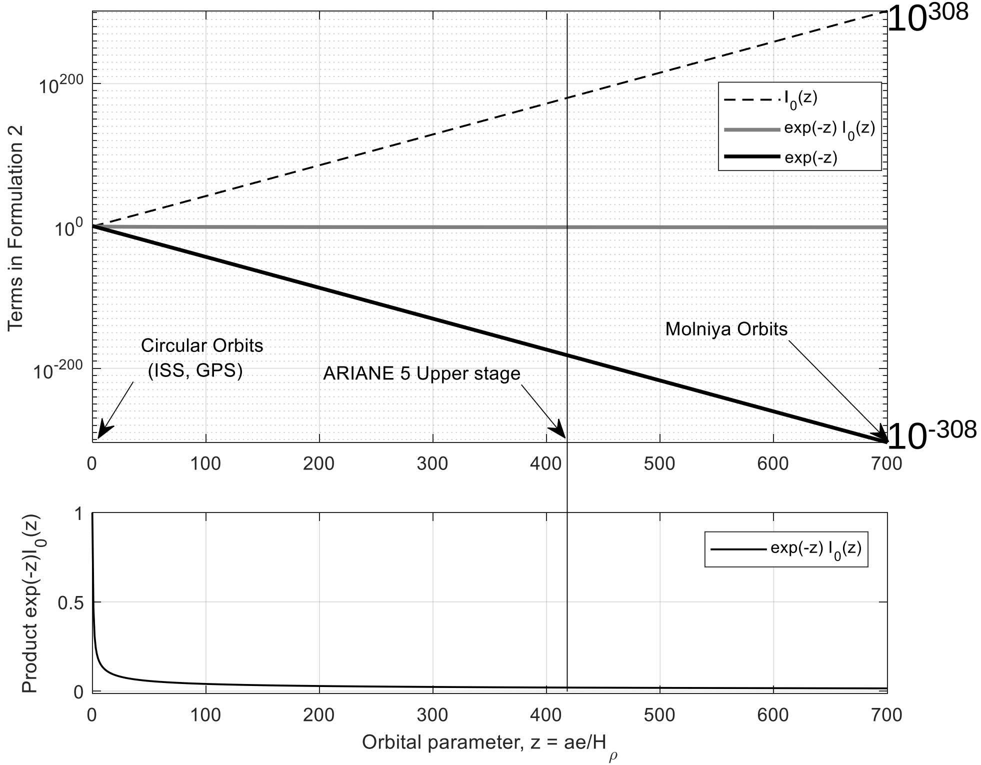
Identifying unstable factors:

$$\exp(-z)\ I_0(z) \qquad \exp(-z)\ I_1(z) \qquad \exp(-z)\ I_2(z)$$

Proposed Asymptotic expansions:

$$\exp(-z) I_\nu(z) \sim \frac{1}{\sqrt{2\pi z}}\left[1 - \frac{\alpha - 1}{8z} + \frac{(\alpha-1)(\alpha-3)}{2!(8z)^2} - \frac{(\alpha-1)(\alpha-9)(\alpha-25)}{3!(8z)^3} + \ldots\right]$$

$$\alpha = 4\nu^2$$



[Abramowitz 1988] Abramowitz, M., Stegun, I. A., and Romer, R. H., “Handbook of mathematical functions with formulas, graphs, and mathematical tables,”,1988

Averaged J2 and Luni-Solar Perturbations

$$\dot{\mathbf{H}} = \dot{\mathbf{H}}_{J_2} + \dot{\mathbf{H}}_S + \dot{\mathbf{H}}_M$$
$$= -\frac{3\mu J_2 R^2}{2a^3 h^5}(\hat{\mathbf{z}}\cdot\mathbf{h})\,\hat{\mathbf{z}}\times\mathbf{h} + \frac{3a^2\mu_S}{2d_S^3}\left[5\left(\hat{\mathbf{d}}_S\cdot\mathbf{e}\right)\mathbf{e}\times\hat{\mathbf{d}}_S - \left(\hat{\mathbf{d}}_S\cdot\mathbf{h}\right)\mathbf{h}\times\hat{\mathbf{d}}_S\right]$$
$$+ \frac{3a^2\mu_M}{2d_M^3}\left[5\left(\hat{\mathbf{d}}_M\cdot\mathbf{e}\right)\mathbf{e}\times\hat{\mathbf{d}}_M - \left(\hat{\mathbf{d}}_M\cdot\mathbf{h}\right)\mathbf{h}\times\hat{\mathbf{d}}_M\right]$$

$$\dot{\mathbf{e}} = \dot{\mathbf{e}}_{J_2} + \dot{\mathbf{e}}_S + \dot{\mathbf{e}}_M$$
$$= -\frac{3n\mu J_2 R^2}{4a^2 h^5}\left\{\left[1 - \frac{5}{h^2}(\hat{\mathbf{z}}\cdot\mathbf{h})^2\right]\mathbf{h}\times\mathbf{e} + 2(\hat{\mathbf{z}}\cdot\mathbf{h})\,\hat{\mathbf{z}}\times\mathbf{e}\right\}$$
$$+ \frac{3\mu_S}{2nd_S^3}\left[5\left(\hat{\mathbf{d}}_S\cdot\mathbf{e}\right)\mathbf{h}\times\hat{\mathbf{d}}_S - \left(\hat{\mathbf{d}}_S\cdot\mathbf{h}\right)\mathbf{e}\times\hat{\mathbf{d}}_S\right]$$
$$+ \frac{3\mu_M}{2nd_M^3}\left[5\left(\hat{\mathbf{d}}_M\cdot\mathbf{e}\right)\mathbf{h}\times\hat{\mathbf{d}}_M - \left(\hat{\mathbf{d}}_M\cdot\mathbf{h}\right)\mathbf{e}\times\hat{\mathbf{d}}_M\right]$$

$\mathbf{d}_S$ and $\mathbf{d}_M$

Vector Distances from Earth's center to the center of Sun and Moon. From JPL Ephemeris Solution.

And

$$\mathbf{h} = \mathbf{H}/\sqrt{\mu a}$$

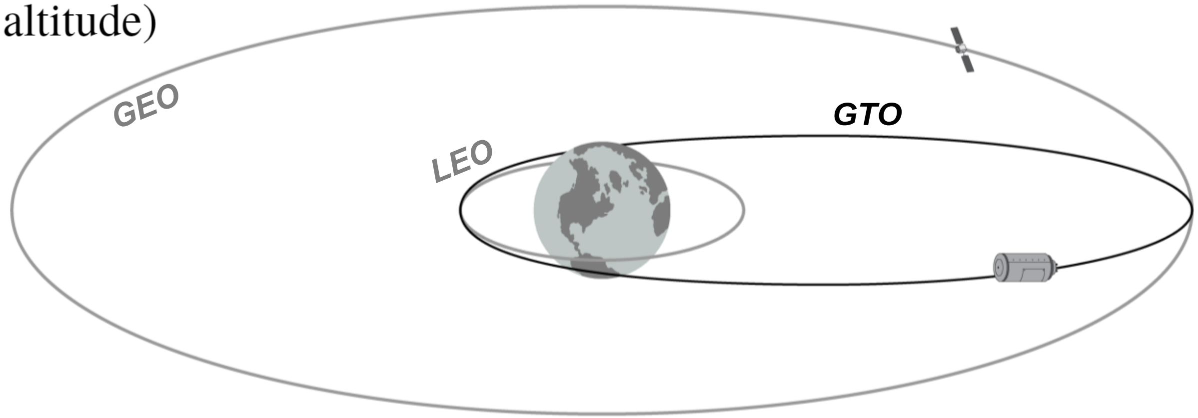
[Wang 2016] Y. Wang and P. Gurfil, "Dynamical Modeling and Lifetime Analysis of Geostationary Transfer Orbits," Acta Astronautica. 2016.

Simulation

Geo-synchronous Transfer Orbit (GTO)

Initial Conditions for Ariane 5 R/B:

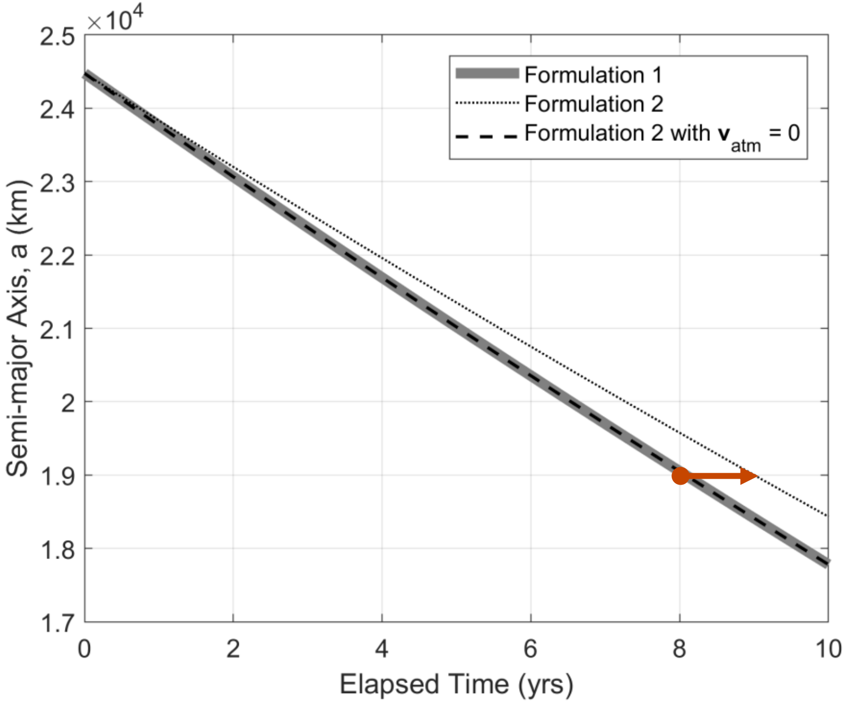
$$
\begin{aligned}
h_{a_0} &= 35,943 \text{ km (apogee altitude)} \\
h_{p_0} &= 250 \text{ km (perigee altitude)} \\
i_0 &= 6^\circ \\
\Omega_0 &= 60^\circ \\
\omega_0 &= 178^\circ \\
\text{AMR} &= 0.02 \text{ m}^2/\text{kg} \\
C_d &= 2.2
\end{aligned}
$$

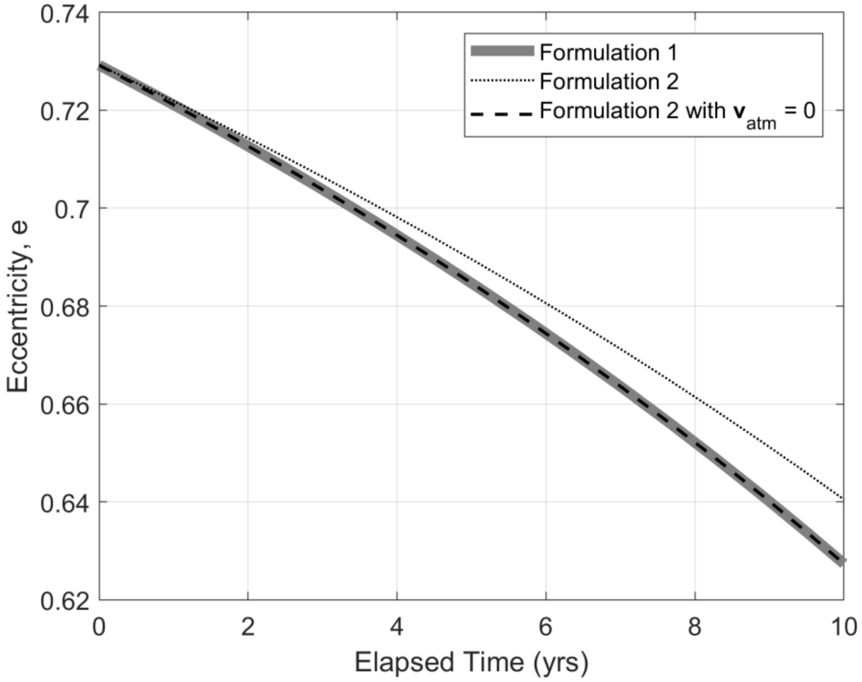


Formulation 1: Averaged dynamics with a *still atmosphere*

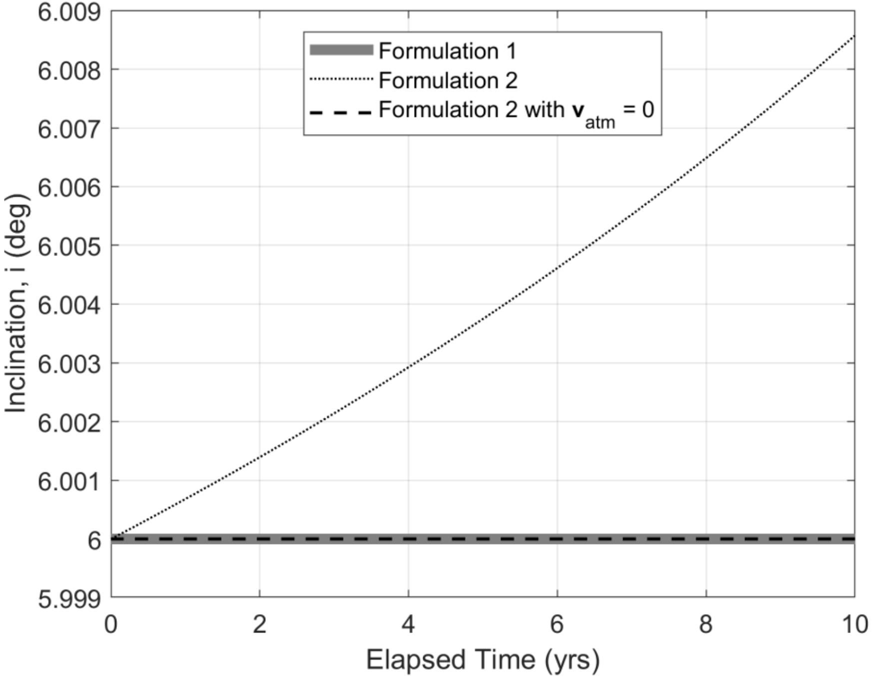
Formulation 2: Averaged dynamics with a *rotating atmosphere*

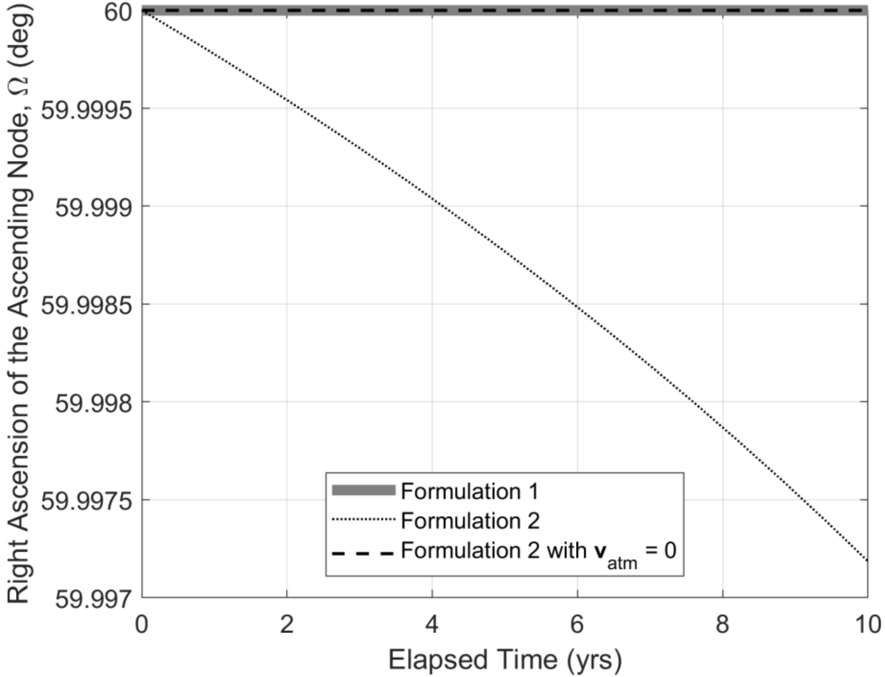
Atmospheric Drag Only



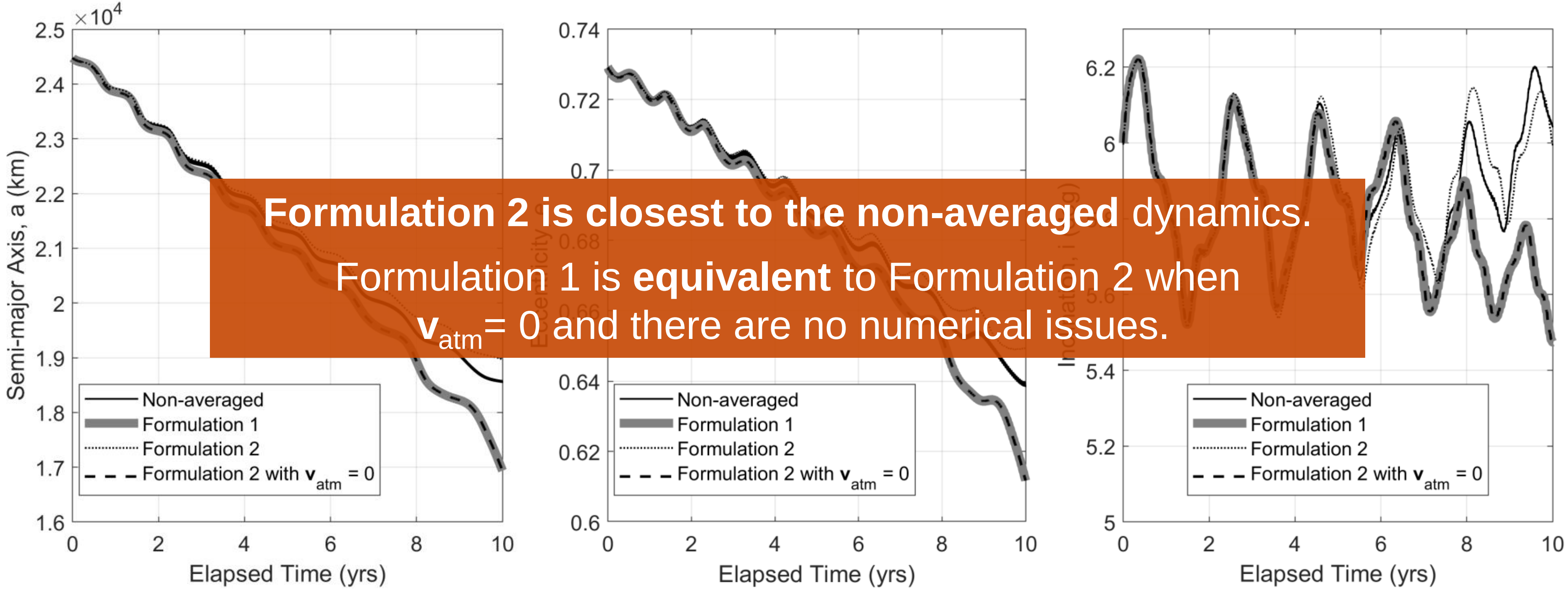


Atmospheric Drag Only





Drag + J_2 + Luni-Solar

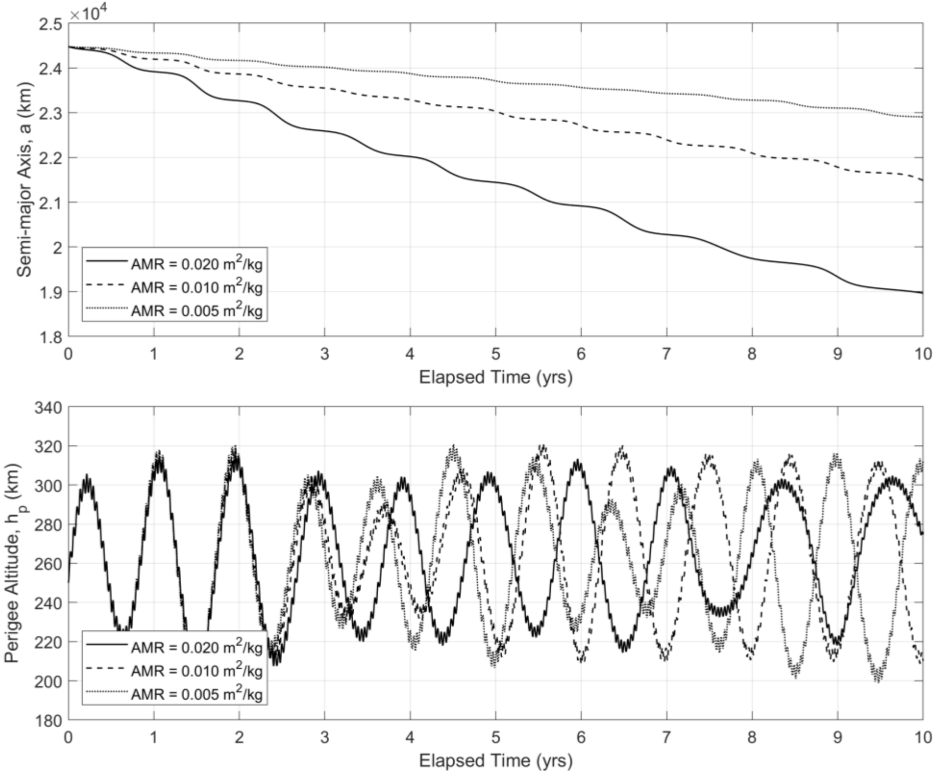


Formulation 2 is closest to the non-averaged dynamics.

Formulation 1 is **equivalent** to Formulation 2 when $\mathbf{v}_{atm} = 0$ and there are no numerical issues.

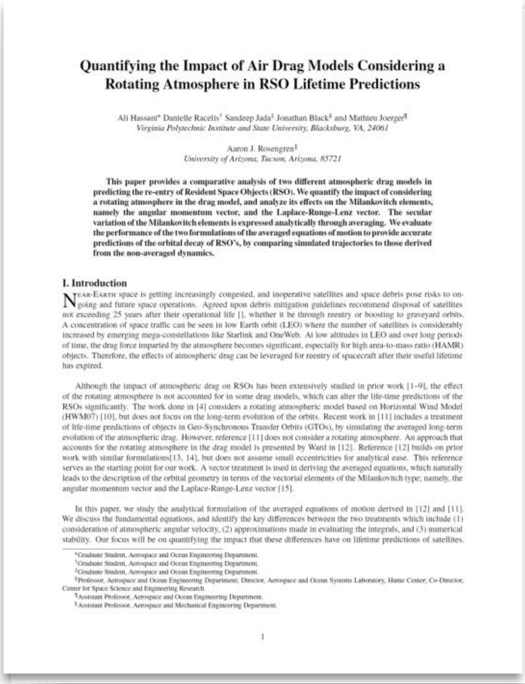
Formulation 2 (AMR Sensitivity)

Drag + J_2 + Luni-Solar



CONCLUSIONS

- Considering atmospheric rotation changes the out-of-orbit-plane elements secularly.
- Formulation 1
  - neglects atmospheric rotation
  - more numerically stable for GTO
- Formulation 2
  - closer to the non-averaged simulations
  - analytically ill-conditioned
- Formulation 1, 2 are equivalent when $\mathbf{v}_{atm} = 0$, and there are no numerical issues.
- Ideal model: Formulation 2 combined with **newly proposed approximations** with numerically stable terms



Quantifying the Impact of Air Drag Models Considering a Rotating Atmosphere in RSO Lifetime Predictions

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This paper provides a comparative analysis of two different atmospheric drag models in predicting the re-entry of Resident Space Objects (RSO). We quantify the impact of considering a rotating atmosphere in the drag model, and analyze its effects on the Milankovitch elements, namely the angular momentum vector, and the Laplace-Runge-Lenz vector. The secular variation of the Milankovitch elements is expressed analytically through averaging. We evaluate the performance of the two formulations of the averaged equations of motion to provide accurate predictions of the orbital decay of RSO's, by comparing simulated trajectories to those derived from the non-averaged dynamics.

I. Introduction

Near-Earth space is getting increasingly congested, and inoperative satellites and space debris pose risks to on-going and future space operations. Agreed upon debris mitigation guidelines recommend disposal of satellites not exceeding 25 years after their operational life [], whether it be through reentry or boosting to graveyard orbits. A concentration of space traffic can be seen in low Earth orbit (LEO) where the number of satellites is considerably increased by emerging mega-constellations like Starlink and OneWeb. At low altitudes in LEO and over long periods of time, the drag force imparted by the atmosphere becomes significant, especially for high area-to-mass ratio (HAMR) objects. Therefore, the effects of atmospheric drag can be leveraged for reentry of spacecraft after their useful lifetime has expired.

Although the impact of atmospheric drag on RSOs has been extensively studied in prior work [1–9], the effect of the rotating atmosphere is not accounted for in some drag models, which can alter the life-time predictions of the RSOs significantly. The work done in [4] considers a rotating atmospheric model based on Horizontal Wind Model (HWM07) [10], but does not focus on the long-term evolution of the orbits. Recent work in [11] includes a treatment of life-time predictions of objects in Geo-Synchronous Transfer Orbits (GTOs), by simulating the averaged long-term evolution of the atmospheric drag. However, reference [11] does not consider a rotating atmosphere. An approach that accounts for the rotating atmosphere in the drag model is presented by Ward in [12]. Reference [12] builds on prior work with similar formulations[13, 14], but does not assume small eccentricities for analytical ease. This reference serves as the starting point for our work. A vector treatment is used in deriving the averaged equations, which naturally leads to the description of the orbital geometry in terms of the vectorial elements of the Milankovitch type; namely, the angular momentum vector and the Laplace-Runge-Lenz vector [15].

In this paper, we study the analytical formulation of the averaged equations of motion derived in [12] and [11]. We discuss the fundamental equations, and identify the key differences between the two treatments which include (1) consideration of atmospheric angular velocity, (2) approximations made in evaluating the integrals, and (3) numerical stability. Our focus will be on quantifying the impact that these differences have on lifetime predictions of satellites.

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