Homework #4

1. The angular velocity component expressions for the Euler angle rates $\dot{\phi}, \dot{\theta},$ and $\dot{\psi}$,

$$\omega_1 = \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi$$
$$\omega_2 = \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi$$
$$\omega_3 = \dot{\psi} + \dot{\phi}\cos\theta$$

   are three linear equations in the three angular rates. Solve for these angular rates, and show that singularities occur as θ goes to 0.

2. Paralleling the development of section 4.3, write the inertial velocity vector of a mass element dm as

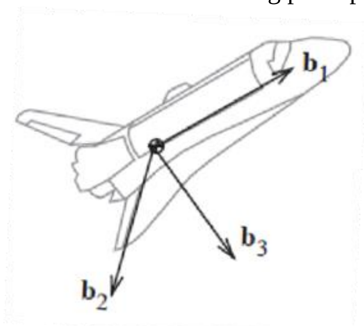
$$\vec{v} = \vec{v}_{\text{com}} + \vec{\omega}^{bi} \times \vec{r}$$

   Where $\vec{v}_{\text{com}}$ is the translational velocity of the center of mass and the second term on the right is the rotational velocity with respect to the center of mass. Show that the kinetic energy of the rigid body becomes,

$$T = \frac{1}{2}M(\vec{v}_{\text{com}} \cdot \vec{v}_{\text{com}}) + \frac{1}{2}(\vec{\omega}^{bi} \cdot [I]\vec{\omega}^{bi})$$

   where M is the total mass of the object.

3. The orbiter has the following principle axis frame and moment of inertia matrix:



$$I = \begin{bmatrix} 1.29 & 0 & 0 \\ 0 & 9.68 & 0 \\ 0 & 0 & 10.10 \end{bmatrix} * 10^6 kg\, m^2$$

   a. Describe the stable gravity gradient attitude for the orbiter in terms of alignment of axes with orbit vectors.
   b. Calculate the three oscillation frequencies for this vehicle in a 90 minute period orbit.