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Prob 1:

$$\omega_{3_{init}} = 0.005 \text{ rad/sec}$$

$$\omega_{3_{final}} = 0.525 \text{ rad/sec}$$

from the three Scalar Euler Eq, the third give

$$M_3 = C\dot{\omega}_3 + \cancel{(B-A)}^{\to 0}\,\omega_1\omega_2$$

B = A becomes its a cylinder

$$\int_0^t M_3\,dt = C\,\omega_3(t) - C\,\omega_3(t=0)$$

$$= C\left(\omega_{3_{final}} - \omega_{3_{init}}\right)$$

where, $C = 1{,}320 \text{ kg/m}^3$

$$\int_0^t M_0\,dt = \underset{\substack{\downarrow \\ \text{number of} \\ \text{Pulses}}}{n} \times \underset{\substack{\downarrow \\ \text{two} \\ \text{thrusters}}}{2} \times \underset{\substack{\downarrow \\ \text{thrust per} \\ \text{Pulse}}}{12.5} \times \underset{\substack{\downarrow \\ \text{arm}}}{1.25}$$

$\int_0^{t_{pulse}} F\,dt$

Solving for n gives

$$\boxed{n = 21.9648 \approx 22 \text{ pulses}}$$

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The Stability depends on the value of the inertias A, B, C

a) If $C > A = B$

then the System is Stable.

b) If $C < (A = B)$

→ with dissipation = loss of Energy

The Cylinder settels to Spinning about the A or B axis

→ without any dissipation = No Energy loss

The cylinder wobbles like a top and this motion Stays like that

Neutrally Stable

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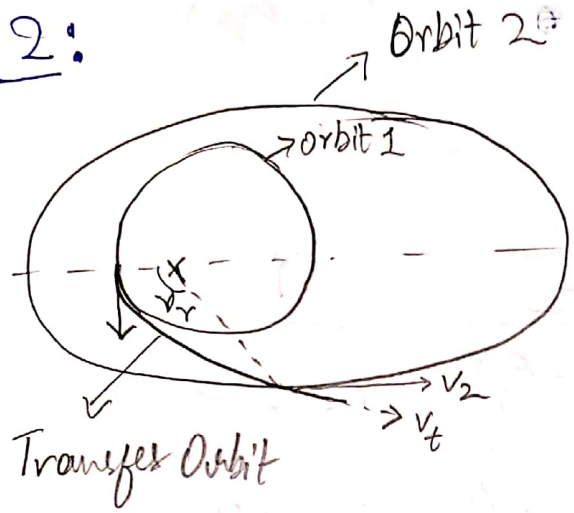
Prob 2:



Eccentricities Exagerated.
$\rightarrow \nu_r = 120°$

the orbits 2 & transfer meet at $\nu_r = 120$ (true anomaly)
in the peri-focal frame, the radius at this point is same

$$r_{at\ \nu=120} = r_2 = r_t$$

$$\frac{P_2}{1 + e_2 \cos 120} = \frac{P_t}{1 + e_t \cos 120} \quad \text{—— (1)}$$

$$P = a(1-e^2) = a(1-e)(1+e)$$

$$a(1-e) = r_{at\ perigee}$$

$$r_{at\ perige} = r_1 = r_t$$

$$\therefore P_t = r_{1\ at\ perige} = a_1(1+e_1)(1+e_t)$$

a_1 for orbit 1, e_1 ecc of orbit 1, e_t ecc of transfer orbit

$$\frac{P_2}{1+e_2\cos 120} = \frac{a_1(1-e_1)(1+e_t)}{1+e_t\cos 120}$$

$$\underbrace{\frac{1+e_t}{1+e_t\cos 120}}_{\text{unkown in } e_t} = \underbrace{\frac{P_2}{a_1(1-e_1)} \quad \frac{1}{1+e_2\cos 120}}_{\text{all known}}$$

Solving for e_t give

$$\boxed{e_t = 0.5154}$$

$$a_1(1-e_1) = a_t(1-e_t)$$

Solving for a_1

$$\boxed{a_t = 2.1564 \text{ DU}}$$

The velocities change at perigee can be found from energy Equation

$\underline{\Delta V_{tangent}}$:

$$\mathcal{E} = -\frac{\mu}{2a} = \frac{V^2}{2} - \frac{\mu}{r}$$

$$V_{t\,\text{at perigee}} - V_{1\,\text{at perigee}} = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a_t}} - \sqrt{\frac{2\mu}{r} - \frac{\mu}{a_1}}$$

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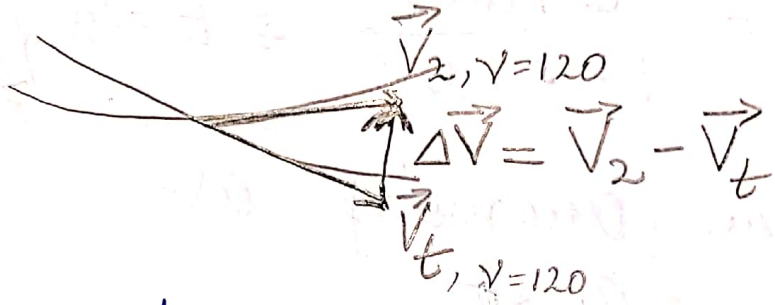
giving a ΔV

$$\boxed{\Delta V_{tangent} = 0.2018 \ DU/TU}$$

The velocity change at t to 2 orbits need ~~to~~ be computed via the vector subtraction, at $\nu = 120°$;



Since we know the true anomaly the eq for velocity in the perifocal frame can be written as

$$\vec{V} = \sqrt{\frac{\mu}{p}}\left(-\sin\nu \, \hat{p} + (e + \cos\nu)\hat{q}\right)$$

$$\overrightarrow{\Delta V}_{\nu=120} = \vec{V}_{2\,\nu=120} - \overline{V}_{t\,\nu=120}$$

Using matlab the vector subtraction gives

$$\Delta\vec{V}_{\nu=120} = 0.0632\,\hat{p} - 0.2287\,\hat{q} \ DU/TU$$

the mag of $\Delta\vec{V}$ is one we are after

$$\Delta V_{\nu=120} = 0.2373 \ DU/TU$$

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The total $\Delta V = \Delta V_{tangent} + \Delta V_{\nu=120}$

$$\boxed{\Delta V = 0.4391 \ DU/TU}$$

for TOF calc we compute the Eccentric anomoly &

$$\cos(E) = \frac{e + \cos\nu}{1 + e\cos\nu} \Rightarrow E = \cos^{-1}\left\{\frac{e+\cos\nu}{1+e\cos\nu}\right\}$$

The Mean anomoly as

$$M = E - e\sin E$$

$$M = n(t - T_0) \quad \text{when, } n = \sqrt{\frac{\mu}{a^3}}$$

$T_0 = 0$

$$t = ToF = \frac{M}{n}$$

for the transfer orbit at $\nu = 120°$

$E = 1.5501$ Rad

$M = 1.0348$

$$\boxed{ToF = 3.2767 \ TU}$$

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Prob 3: a)

The CW Eqs given as

$$\begin{bmatrix} \delta r(t) \\ r_0 \delta\theta(t) \\ \delta z(t) \\ \delta \dot{r}(t) \\ r_0 \delta\dot{\theta}(t) \\ \delta \dot{z}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi_{rr}(n,t) & \Phi_{rv}(n,t) \\ \Phi_{vr}(n,t) & \Phi_{vv}(n,t) \end{bmatrix}}_{STM} \begin{bmatrix} \delta r_0 \\ r_0 \delta\theta_0 \\ \delta z_0 \\ \delta \dot{r}_0 \\ r_0 \delta\dot{\theta}_0 \\ \delta \dot{z}_0 \end{bmatrix} \Bigg\} \text{delta initial position, delta init vel}$$

for rendezvous final position is zero

$$\delta \vec{v}(t_0) = -\left(\Phi_{rv}(n,t)\right)^{-1} \Phi_{rr}(n,t)\, \delta \vec{r}(t_0)$$

using the STM in lecture 16 & solving in matlab gives

$$\boxed{\underbrace{\delta \vec{v}(t_0)}_{\text{initial delta velocity}} = \begin{bmatrix} 0.29 & -0.566 & 0.09845 \end{bmatrix}^T \text{ m/s}}$$

Prob 3: b) Out-of plane — δz

the out of plane motion is a bounded harmonic oscillation with amplitude dictated by initial conditions δz_0 & $\delta \dot{z}_0$.

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Radial — δr

is the same as δz, but its dictated by the initial condition $(\delta r_0, \delta \dot{r}_0, \delta \dot{\theta}_0)$

In-Track — $r_0 \delta\theta$

There are harmonic terms & there is one term that is a coefficient of t. If this coeff is not zero then the motion is not bounded it grows, so

$$3\delta\dot{\theta}_0 + 6n\frac{\delta r_0}{r_0} = 0$$ (required for bounded motion)

More generally, the initial condition that lead to a closed path are bounded.

$$\underset{\text{STM}}{\Phi}\left[\underbrace{\delta r_{\text{initia}}}_{\text{initial } \delta\vec{r}_0 \,\&\, \delta\vec{v}_0}\right] \rightarrow \text{Closed path}$$

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Prob 4:

The following procedure was followed in Solving this

Step 1: Convert $\vec{r}_0$ & $\vec{V}_0$ to the Classical Orbital Elements $a, e, i, \Omega, \omega, \nu_0$

The lecture 6 Eqs were used

the COEs are

a — 2.5428 DU

e — 0.74633

i — 136.12 deg

Ω — -33.63 (OS) 326.36 deg

ω — 326.99 deg (Quadrant check) $\vec{e}\cdot\vec{k}$

ν_0 — 191.77 deg (Quadrant check) $\vec{r}\cdot\vec{v}$

↓ at Epoch

Step 2: Compute the Mean anomaly at Epoch

$$E_0 = 2\,\mathrm{atan}\left\{\sqrt{\frac{1-e}{1+e}}\,\tan\frac{\nu_0}{2}\right\}$$

$$M_0 = E_0 - e\sin E_0 = -2.2369 \text{ Rad}$$

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Step 4: Propogate the Mean Anomoly

$$M_{final} = M_0 + \underbrace{n}_{\sqrt{\frac{\mu}{a^3}}} \underbrace{(t - T_0)}_{2.75\ Hrs}$$

$$M_{final} = 0.7896$$

Step 5: Compute the final true Anomoly ν_f, by solving Kepler Eq

$$M_{final} = E_{final} - e \sin E_{final}$$

interatively Solve for E_{final}

$$\boxed{E_{final} = 1.5355 \text{ Rad}}$$

$$\nu_{final} = 2 \operatorname{atan}\left\{\sqrt{\frac{1+e}{1-e}} \tan\left(\frac{E_{final}}{2}\right)\right\}$$

$$\boxed{\nu_{final} = 136.9 \text{ deg}}$$

Step 6: Compute $\vec{r}_{fin}$ & $\vec{V}_{fin}$ in peri-focal frame

$$r_{final} = \frac{p}{1 + e\cos\nu_{final}} \; ; \; \vec{r}_{final} = r_{final}\cos\nu_{final}\,\hat{p} + r_{final}\sin\nu_{final}\,\hat{q}$$

$$\vec{V}_{final} = \sqrt{\frac{\mu}{p}}\left(-\sin\nu_{final}\,\hat{p} + (e + \cos\nu_{final})\,\hat{q}\right)$$

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$$\vec{r}^{\,peri}_{final} = -1.8080\,\hat{p} + 1.6914\,\hat{q} \quad DU$$

$$\vec{V}^{\,peri}_{final} = -0.6437\,\hat{p} + 0.0151\,\hat{q} \quad DU/TU$$

<u>Step 7:</u> Rotate from peri-focal frame $\vec{r}^{\,P}_f$ & $\vec{V}^{\,P}_f$ to ECI intertial

$$\vec{r}^{\,i}_{final} = R_3(-\Omega)\,R_1(-i)\,R_3(-\omega)\,\vec{r}^{\,P}_f$$

$$\vec{V}^{\,i}_{final} = (\quad '' \quad)\,\vec{V}^{\,P}_f$$

$$\boxed{\vec{r}^{\,i}_{fina} = \begin{bmatrix} -1.4566 \\ -1.1128 \\ 1.6658 \end{bmatrix} DU \;;\; \vec{V}^{\,i}_{fina} = \begin{bmatrix} -0.5876 \\ 0.07635 \\ 0.25187 \end{bmatrix} DU/TU}$$

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Prob 5: a)

To solve for the Lagrange points we solve for the position x & y when in equillibrium, ie, $\ddot{x}, \ddot{y}, \ddot{z}, \dot{x}, \dot{y}$, & $\dot{z}$ turns to zero. hence

$$-x = -\frac{(1-\mu)(x-\mu)}{r_1^3} - \frac{\mu(x+1-\mu)}{r_2^2}$$

$$-y = -\frac{(1-\mu)y}{r_1^3} - \frac{\mu y}{r_2^3}$$

where,

$$r_1 = \sqrt{(x-\mu)^2 + y^2 + z^2}$$

$$r_2 = \sqrt{(x+1-\mu)^2 + y^2 + z^2}$$

The out-of-plane motion, 'z' donot give equillibrium conditions, so all the Lagrange points are in plane.

Solving numerically for x, y for a $\mu = 0.01213$, guess values of $x_g = -1; y_g = 1$

$$\boxed{\begin{aligned} x &= -0.48787 \\ y &= 0.866025 \end{aligned}}$$

DU

$-y$ will give L_3

Prob 5.b

The delta V from a rocket

$$\Delta V = -I_{sp}\, g_{sealevel} \ln\left(\epsilon + \pi(1-\epsilon)\right)$$

$\epsilon = 0.05$, Structural mass ratio $= \dfrac{m_s}{m_s + m_p}$

$\pi = \dfrac{m_*}{m_0}$ payload ratio

$m_* = m_{sat} = 1000\ kg$

$m_0 = m_{upperstage} = 1300\ kg$ (Best guess from the question description)

$g_{sealevel} = 9.806\ m/sec^2$

$I_{sp} = 400\ s$

$$\boxed{\Delta V_{park} = 970.698\ m/s}$$ Available with rocket.

Assuming a Hohmann transfer to elliptic transfer orbit

$$\Delta V_1 = V_{transfer} - V_{park}$$

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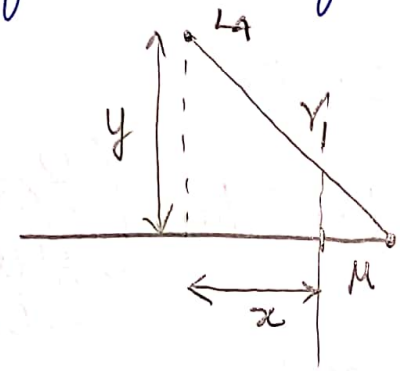
where

$$\Delta V_1 = \sqrt{\frac{\mu_{earth}}{r_{park}}}\left(\sqrt{\frac{2r_1}{r_1 + r_{park}}} - 1\right)$$

where

$r_{park} = 1/60$ DU

r_1 is from the following diagram



$r_1^2 = (x+\mu)^2 + y^2$

absolute dist

$r_1 \approx 1$ DU (very close to 1)

$$\boxed{\Delta V_1 = 3190.08 \text{ m/s}}$$

as $\Delta V_{rock} = 970.698$ m/s

$\boxed{\text{Not possible to get to } L_4}$

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Prob 6:

The Answer is <u>NO</u>

1) The Out-of-plane vs in-plane orbits will have $\vec{V}_{ellip}$ & $\vec{V}_{circ}$ pointing in different direction, so its not possible to have same $\vec{V}$

2) So we look a $\vec{r}$ & $\vec{v}$ in peri-focal frame

for a common point, v is same

$$\vec{r}_{circ} = \vec{r}_{ellipse} \quad \& \Rightarrow r_{circ} = r_{ellipse}$$

$$\boxed{\frac{P}{1+e\cos v} = r_{circ}}$$

the velocity in perifocal frame

$$\vec{V}_{circ} = \sqrt{\frac{\mu}{r_{circ}}}\left(-\sin v\,\hat{p} + (0+\cos v)\,\hat{q}\right)$$

And the components have to be the same

$$\vec{V}_{ellipse} = \sqrt{\frac{\mu}{P}}\left(-\sin v\,\hat{p} + (e+\cos v)\,\hat{q}\right)$$

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Equating components $\hat{p}$ & $\hat{q}$

$$\sqrt{\frac{\mu}{r_{circ}}} - \sin\nu = \sqrt{\frac{\mu}{p}} - \sin\nu$$

$$\sqrt{\frac{\mu}{r_{circ}}} - \sin\nu = \sqrt{\frac{\mu(1+e\cos\nu)}{r_{circ}}} - \sin\nu$$

$$-\sin\nu = \sqrt{1+e\cos\nu} - \sin\nu$$

$$\boxed{\left(1-\sqrt{1+e\cos\nu}\right)\sin\nu = 0} \quad \text{— Eq ①}$$

now, $\hat{q}$

$$\sqrt{\frac{\mu}{r_{circ}}}\cos\nu = \sqrt{\frac{\mu(1+e\cos\nu)}{r_{circ}}}\,(e+\cos\nu)$$

$$\cos\nu = \sqrt{1+e\cos\nu}\,(e+\cos\nu)$$

$$\boxed{(e+\cos\nu)\sqrt{1+e\cos\nu} + \cos\nu = 0} \quad \text{Eq - ②}$$

Eq① when $\nu = 0$ or π $\sin\nu = 0$ & $\cos\nu = 1, -1$

let $\cos\nu = 1$

Eq ②

$$(e+1)\sqrt{1+e} + 1 = 0$$

$$(e+1)\sqrt{1+e} \neq -1$$

Fallacy, because LHS is alway positive

let $\cos\nu = -1$

Eq ②

$$(e-1)\sqrt{1-e} - 1 = 0$$

$$(e-1)\sqrt{1-e} = 1$$

$$(1-e)\sqrt{1-e} \neq -1$$

Fallacy, becauses LHS is always positive

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So we conclude $\sin\nu \neq 0$

from Eq ① we write

$$1 - \sqrt{e\cos\nu + 1} = 0$$

$$\sqrt{e\cos\nu + 1} = 1$$

$$e\cos\nu + 1 = 1$$

$$e\cos\nu = 0$$

Either $e = 0$, then we disproved that common points with same velocity exists

(or)

$$\cos\nu = 0$$

we substitute this in Eq ②

$$(e + \cos\nu)\sqrt{1 + e\cos\nu} + \cos\nu = 0$$

$$(e + 0)\sqrt{1 + 0} + 0 = 0$$

$$\boxed{e = 0} \rightarrow \text{Implying a circle}$$

$\therefore$ There are <u>no points</u> on that are common to an elliptical & circular orbit around a body, with the same velocity, $\vec{v}$

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