Posting Date: Thursday Oct. 15th.

You need not hand in anything. Instead, be prepared to answer any of these problems on an upcoming take-home prelim exam. You may discuss your solutions with classmates up until the time that the prelim becomes available.

1. This problem refers back to Problems 5 & 6 of Assignment #5. Re-do Problem 6 using the Kalman filter problem matrices of Problem 5. Note that in Problem 5 there are 3 possible Q values. Run your truth-model simulation using the largest of the three Q values, but run your Kalman filter using the smallest of the three Q values. What does this do to your consistency evaluation? Is this what you expect?

2. There are two Kalman filtering problems defined by the two MATLAB scripts "kf_example03a.m" and "kf_example03b.m". Solve each of these Kalman filtering problems in two ways. First, use a standard Kalman filter. Second, use a square-root information filter (SRIF). The two filters should work about equally well for the problem in "kf_example03a.m", but the Kalman filter will not work as well as the SRIF for the problem in "kf_example03b.m". This is true because of the small $R(k)$ value, which causes the computed covariance matrix to be ill-conditioned. Compare the state estimates and the covariances for the two filters at the terminal time. Is there a significant difference for the second filtering problem but not for the first?

   Note: this is a relatively benign case. The improvement due to use of the SRIF is not extremely significant. There are, however, known practical situations where a Kalman filter completely breaks down while an SRIF functions well.

3. Prove that the smoother backwards state propagation formula

   $$\underline{x}^*(k) = F^{-1}(k)[\underline{x}^*(k+1) - G(k)\underline{u}(k) - \Gamma(k)\underline{v}^*(k)]$$

   is equivalent to the formula

   $$\underline{x}^*(k) = \underline{\hat{x}}(k) + P(k)F^{\mathrm{T}}(k)\bar{P}^{-1}(k+1)[\underline{x}^*(k+1) - \underline{\bar{x}}(k+1)]$$

   Hints: Add $\underline{\hat{x}}(k)$ to the right-hand side of the first equation and subtract $F^{-1}(k)F(k)\underline{\hat{x}}(k)$ from the right-hand side, which yields no net change. The latter term can be included in the bracketed expression. Substitute into the right-hand side the formula for $\underline{v}^*(k)$ in terms of $[\underline{x}^*(k+1) - \underline{\bar{x}}(k+1)]$ that was given in lecture. Some clever matrix and vector substitutions and manipulations fill allow you to finish the proof from this point.

4. Prove that the smoother backwards covariance propagation formula

   $$P^*(k) = F^{-1}(k)[P^*(k+1) - P_{vx}^{*\mathrm{T}}(k+1)\Gamma^{\mathrm{T}}(k) - \Gamma(k)P_{vx}^*(k+1) + \Gamma(k)P_{vv}^*(k)\Gamma^{\mathrm{T}}(k)]F^{-\mathrm{T}}(k)$$

   is equivalent to the formula

$$P^*(k) = P(k) - P(k)F^{\mathrm{T}}(k)\bar{P}^{-1}(k+1)[\bar{P}(k+1) - P^*(k+1)]\bar{P}^{-1}(k+1)F(k)P(k)$$

<u>Hints</u>: Substitute into the right-hand side the formulas for $P^*_{vx}(k+1)$ and $P^*_{vv}(k)$ from lecture and then start combining terms. It may be helpful to form terms that contain $[\bar{P}(k+1) - P^*(k+1)]$. You will need to make some matrix substitutions. At some point it may be helpful to effectively add and subtract $P(k)$ from the right-hand side of an intermediate form of your equation.

5. Calculate the smoothed estimates for the problem in "kf_example03a.m". Compare $\underline{\hat{x}}(10)$ with $\underline{x}^*(10)$ and compare $P(10)$ with $P^*(10)$. Is $P^*(10) \leq P(10)$? Do the smoothed state time history estimate plots look "smoother" than the filtered state time history estimate plots?