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Multi-scenario Design Optimization using ADMM of a Thermal Energy Storage System

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Abstract

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1. Introduction

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2. Decomposing two-stage Dynamic Optimization problems using ADMM

The optimal design problem for a dynamic system under time varying uncertainty can be represented as a two-stage dynamic optimization problem. The design decisions being the first stage variables (w^{des}) and the operations decisions as the second stage variables (w^{oper}). In this representation, it is assumed that the uncertainty is fully realized after the first stage design decisions are made, and hence the operations are optimal for the realized uncertainty. We can represent the uncertainty by a set of discrete scenarios $\mathcal{S} := \{1,\ldots,S\}$ with the cost weight ω_s to represent the likelihood of scenario *s* being realized. The dynamics of the system can be discretized into N equally spaced sampling intervals represented by the set $\mathcal{K} := \{0,\ldots,N-1\}$. The optimal design problem can be then cast as a nonlinear programming problem (NLP) of the form

$\min\limits_{w^{des},w^{oper}}$	$\phi^{capex}\left(w^{des}\right)+\sum\limits_{s\in\mathcal{S}}\omega_s\left[\sum\limits_{k=0}^{N-1} l\left(x_{k,s},z_{k,s},u_{k,s}\right)\right]$		(1a)
$s.t$	$g\left(x_{k,s},z_{k,s},u_{k,s},\theta_{k,s},w^{des}\right)\leq 0$	$\forall k\in\mathcal{K},\forall s\in$	(1b)
	$h\left(x_{k,s},x_{k+1,s},z_{k,s},u_{k,s},\theta_{k,s},w^{des}\right)=0$	$\forall k\in\mathcal{K},\forall s\in$	(1c)
	$x_{0,s}=x_0$		(1d)
	$w^{oper} := \left[x_{0,s},x_{k+1,s},z_{k,s},u_{k,s}\right]$	$\forall k\in\mathcal{K},\forall s\in$	(1e)

where $(.)_{k,s}$ represents the s^{th} scenario at timestep *k*. vectors *x* and *u* represent the differential states and the control inputs applied to the plant during operations respectively. The initial condition during operations is given by x_0 for all the scenarios. In the interest of simplifying notation, we collect the second stage variables for all the scenarios together into the single vector w^{oper}. The time varying parameters are represented by the vector θ. The function $\phi^{capex}(.)$ is used to represent the capital cost and $l(.)$ to represent the operating cost. The objective of the optimal design problem is then simply defined as the sum of capital costs and the expected costs during operations. Function $g(.)$ represent the nonlinear inequality constraints and $h(.)$ used to represent the dynamics of the system.

2.1. Reformulating the design problem as general form consensus optimization problem

Solving the optimal design problem as formed in (1) can become computationally intractable due to memory constraints when having to consider many scenarios for uncertainties and long horizon times. We explore the option of dividing the problem into smaller partition problems which can then be solved separately as a way to handle the memory limitations. We can divide the two-stage dynamic optimization problem into P partitions denoted by the set $\mathcal{P} := \{1, \ldots, P\}$ in a very flexible manner. An illustrative example of a design problem with 2 scenarios divided into 6 partitions is shown in

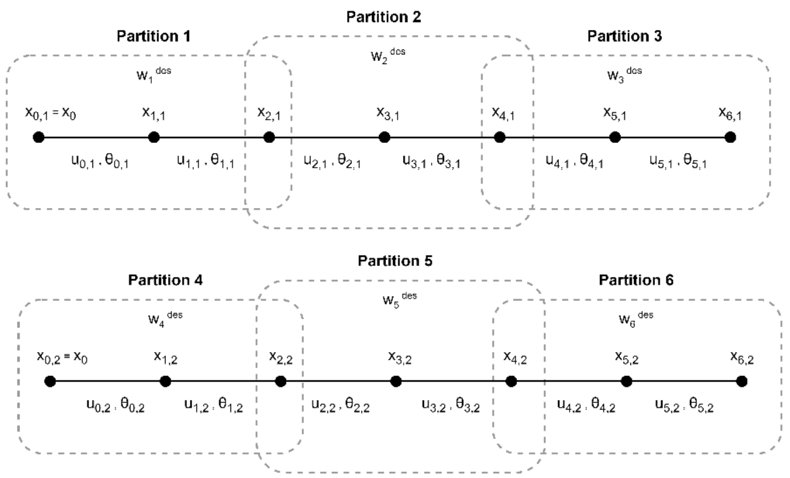

Figure 1: Illustration of dividing the optimal design problem with 2 scenarios into 6 partitions.

In the partition problems, each partition *p* has its own local variables for design (w_p^{des}) and operations (w_p^{oper}). The sum of the individual partition problems together is similar to the central problem in (1) when,

- All the partitions are in consensus of the value of their local design variables
- Adjacent partitions are in consensus of the value of the differential state variable shared between them (at the edge of the partitions)

We can introduce a global copy of all the variables that must reach consensus into a vector denoted as v. The consensus requirements can then be imposed as an additional constraint in each partition p. We can thus write the optimal design problem as a sum of smaller partition problems that are linked with consensus constraints as,

			(2a)
			(2b)

			(2c)
			(2d)
			(2e)
			(2f)

Where A_p and B_p are selection matrices used to link a subset of the local variables of partition p to the corresponding sections in the global copy ν. The objective function terms in the partition problems are chosen appropriately to add up to the original objective in (1). For the sake of compact notation, we collect the local variables in the partitions as w_p and can represent the constraints (2b) and (2c) which only involve the local variables w_p as $w_p \in \mathcal{W}_p$.

2.2. Applying ADMM to get a Distributed algorithm

The individual partition problems are not trivially separable due to the presence of constraints (2d) and (2e) that enforces the consensus condition by using the global variable. We can solve problem (2) in a distributed approach using ADMM as described below. The augmented Lagrangian (AL) function of (2) can be formed by taking the constraints (2d) and (2e) to the objective as,

$$L_\rho\left(w_p, \nu, \lambda_p\right) = \sum_{p \in \mathcal{P}} \left[\begin{array}{l} \Phi^{capex}\left(w_p^{des}\right) + \Phi_p^{oper} + \left(\lambda_p^{des}\right)^T \left(w_p^{des} - \nu^{des}\right) \\ + \dfrac{\rho^{des}}{2} \left\| w_p^{des} - \nu^{des} \right\|_2^2 + \left(\lambda_p^{oper}\right)^T \left(A_p w_p^{oper} - B_p \nu^{oper}\right) \\ + \dfrac{\rho^{des}}{2} \left\| A_p w_p^{oper} - B_p \nu^{oper} \right\|_2^2 \end{array} \right]$$

where $\lambda_p^T = \left[\left(\lambda_p^{des}\right)^T, \left(\lambda_p^{oper}\right)^T\right]$ are the lagrange multipliers associated with the consensus constraints and $\rho^T = \left[\left(\rho^{des}\right)^T, \left(\rho^{oper}\right)^T\right]$ is a vector of chosen penalty parameters. It can be seen that the AL is additively separable except for the quadratic penalty terms. The i^{th} iteration of the ADMM algorithm involves solving the partition problems while keeping ν and λ fixed and then updating them by keeping the local variables w_p fixed in an alternating fashion until convergence. The i^{th} iteration of the ADMM algorithm thus takes the form

		(3a)

		(3b)
		(3c)
		(3d)

We can see that step (3a) involves solving an optimization problem for each partition separately while the global variable is kept constant. Thus each partition problem can be solved in separate machines in parallel. Step (3b) is the minimization of the AL function while the local variables in partitions are kept constant. This step for consensus optimization problems reduces to finding the minimum of a quadratic function and can be shown to be the averaging operator which can be defined as (Rodriguez)

$$\nu^{i+1}(j) = \frac{1}{|\mathcal{P}_j|}\sum_{p\in\mathcal{P}} w_p^{i+1}(j)$$

where $\mathcal{P}_j \subseteq \mathcal{P}$ denotes the set of partitions connected to the global variable $\nu(j)$. Steps (3c) and (3d) are the updates to the lagrange multipliers of the consensus constraints and can also be carried out in each partition separately and in parallel. The termination criteria used for the ADMM iterations are that primal residual (r^{i+1}) and dual residuals (s^{i+1}) are reasonably close to zero as explained in (Boyd), which for our case is defined as

$r_p^{i+1} = \begin{bmatrix} \left(w_p^{des}\right)^{i+1} - \left(\nu^{des}\right)^{i+1} \\ A_p\left(w_p^{oper}\right)^{i+1} - B_p\left(\nu^{oper}\right)^{i+1} \end{bmatrix}$	$\forall p \in \mathcal{P}$	
$s_p^{i+1} = \begin{bmatrix} \left(\lambda_p^{des}\right)^{i+1} - \left(\lambda_p^{des}\right)^{i} \\ \left(\lambda_p^{oper}\right)^{i+1} - \left(\lambda_p^{oper}\right)^{i} \end{bmatrix}$	$\forall p \in \mathcal{P}$	

3. The TES Design problem

We use the flowsheet in Figure 2 to represent the heating section of a district heating network where water is used in a closed loop to satisfy the heating requirement in an area. The volumetric flowrate of water in the loop (q^{dh}) is determined by the number of consumers and their heating demands at any particular time. The temperature of the water returned to the heating section ($T^{dh,\mathrm{Ret}}$) is determined by the heat losses in the system and is assumed to be correlated to the ambient weather conditions. The operational objective of the heating system is to heat this water upto a temperature (T^{phb}) which must be above a contractually specified temperature ($T^{wh,\min\mathrm{Ret}}$). This stream can loose heat to the district heating system using the heat exchanger (WHB), and any additional cooling required in the process stream satisfied by the utility heat exchanger (DC) which uses some external cooling utility. In a similar manner, any additional heating required in the district heating side is satisfied by using the peak heat boiler (PHB). There is a temporal mismatch in the supply of heat from the process stream and the demand for heating from the consumers. In order to better manage this mismatch and decrease the reliance on external utilities, a thermal energy storage system in the form of a simple buffer tank is being considered to be installed. This simple TES system can charge/ discharge by raising/ lowering the temperature of the tank by manipulating the flow split (α).

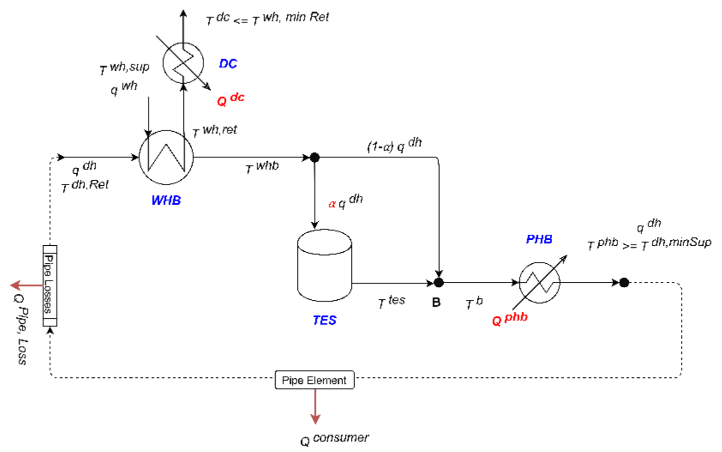

Figure 2: Simplified flowsheet of a district heating system.

The TES tank is modelled as a well mixed tank with perfect level control with the volume maintained at V^{tes}. The WHB is modelled as a purely counter current heat exchanger with the hot and cold sides discretized into a series of N_{cell} cells. Each cell is assumed well mixed with the heat transfer occurring only across the cell wall. Heat exchangers PHB and DC are modelled simply as a well mixed tank with the heat duty added or removed directly. The mass and energy balance equations can thus be written out for the system to model the dynamics of the system. The differential states thus consist of $x^T = \left[T^{whb}, T^{wh,\min \mathrm{Ret}}, T^{tes}, T^{phb}\right]$ while the control inputs are $u^T = \left[\alpha, Q^{phb}, Q^{dc}\right]$. In our model, we consider the uncertain parameters at each timestep k as $\theta_k = \left[q_k^{dh}, T_k^{dh,Ret}, q^{wh}, T^{wh,\mathrm{Sup}}\right]$. The uncertainties in future profiles can then be modelled by building representative scenarios of these uncertain parameters.

The design problem is then to find the optimal volume of the TES tank (V^{tes}) and the area of the heat exchanger WHB (A^{whb}) that must be installed, given the uncertain profiles of future supply and demand of thermal energy. We can thus form it as the large two-stage dynamic optimization problem of the form in (1), partition it into smaller problems of the form in (2), which can then be solved in a distributed manner using ADMM as described in section 2.2.

4. Results and Discussions

We present a simple simulation study to demonstrate the approach in Section 2 on the TES model described in Section 3 and compare the solution (and some convergence metrics) against solving the design problem as a single NLP (1). We consider a simple case with 2 equally likely scenarios with N=60 discretizations to represent the future operations. The profiles for q^{wh} and $T^{wh,Sup}$ both have a step change at timesteps k = 20 and k = 40, while all the other parameters are held at their steady state values. The design variables (w^{des}) and operations variables (w^{oper}) were all scaled to be within zero and one while formulating the optimization problem. Quadratic functions are used to represent the capital cost ($\phi^{capex} = 0.001\left(V^{tes}\right)^2 + 0.001\left(A^{whb}\right)^2$) and the operating cost ($l = \left(Q^{phb}\right)^2 + \left(Q^{dc}\right)^2$). Scaling facilitated selecting the AL penalty parameter ρ to tune the convergence speed of our distributed approach more easily. The penalty parameters used are $\rho^{des} = 0.001$ and $\rho^{oper} = 0.1$ in the partition problems to roughly balance the magnitude of the objective and penalization terms. We solve the problem using the distributed approach from Section 2.2 and show the results against the optimal design solution by solving the problem centrally.

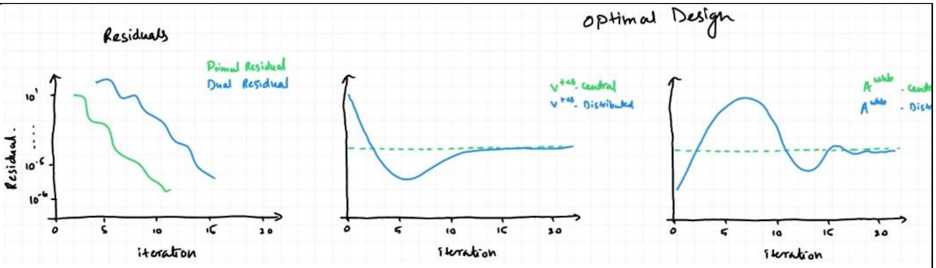

Figure 3: Results from solving the design problem centrally vs using the distributed approach.

From Figure 3a, we can see that the primal and dual residuals are sufficiently small around 20 iterations of the ADMM algorithm, indicating convergence in our distributed approach. From Figures 3b and 3c, the optimal design variables can be seen to converge to the same solution as the one found by solving the problem centrally. An important point to note is that this behaviour is not guaranteed by ADMM in the case of nonconvex problems. When applied to nonconvex problems , ADMM need not converge and even when it does converge, it need not converge to an optimal point and must be hence considered just as another local optimization method (Boyd). Although the convergence guarantees for ADMM in the case of complex nonconvex NLPs are poorly understood, it has been shown to perform satisfactorily in practice (Rodriguez). Three snapshots of the optimal differential state variables in the partition problems at iterations i = 1, 10 and 20 of the ADMM algorithm are shown in Figure 4.

An interesting observation in our approach is that all partitions apart from the leftmost partition has the initial condition of the dynamic optimization problem as a free variable.

Thus in Figure 4 we can see that during the initial iterations (left subplot), the optimal solution is to have the initial condition for the TES tank at the maximum allowable temperature. The penalization terms added are then able to implicitly close the gap between the partitions and achieve consensus (middle and right subplot). In this aspect our approach shares similarities to multiple shooting (MS) approach in dynamic optimization. The key difference is that MS is solved centrally and the state continuity (consensus constraint) is enforced explicitly as an equality constraint by the solver. In ADMM, we solve it in a distributed way where the consensus constraint is relaxed by forming the AL. It is then enforced implicitly by minimizing the AL.

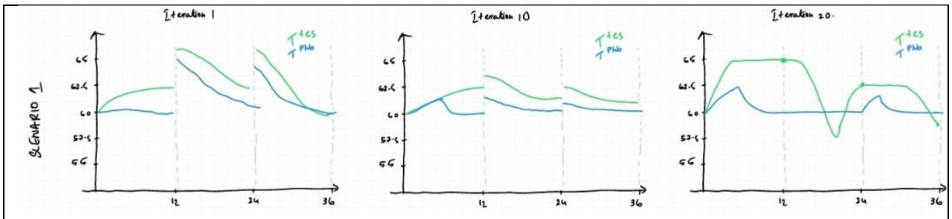

Figure 4: Snapshots at ADMM iterations (i = 1,10, 20) in the partition problems

5. Conclusions

In this paper, we presented the optimal design of a simple TES system under uncertainty as a two-stage nonlinear dynamic optimization problem. Due to limitations in memory of solving the problem centrally in a single machine, an approach for forming smaller partition problems in a general fashion was shown. The ADMM algorithm was applied to coordinate between the subproblems which could be solved separately and in parallel. A simple simulation exercise was used to demonstrate the approach.

References

Z. Allen, Year, Article or Chapter Title, Journal or Book Title, Volume, Issue, Pages

Y. Brown, Year, Article or Chapter Title, etc.

X. Cullem, Year, etc.