A distributed optimization approach using ADMM for optimal design of Thermal Energy Storage system

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**Motivation**

Thermal energy storage (TES) systems help mitigate the issue of asynchronous supply and demand profiles of heat in industrial systems by providing additional degrees of freedom during operation. A systematic approach to design of these systems needs to account for uncertainties in future demand and supply profiles of thermal energy. Long time horizons and many scenarios need to be considered to adequately represent the uncertainty and assess the long-term profitability of the system.

Simple linear models can be used to represent the system dynamics and the physical design parameters then calculated from the solution of a two-stage stochastic LP based on some heuristics (Thombre *et al.,* 2020). The issue anticipated with such an approach is that these physical design parameters are not necessarily optimal and could even be infeasible due to the linear model approximations.

Nonlinear models would be needed to accurately represent the system dynamics and guarantee optimal physical design parameters and the design problem represented as a two-stage stochastic NLP. The solution of such a large NLP can become intractable due to memory limitations of the computing device used.

**Study Design**

Distributed optimization techniques have been used for solving similar large-scale optimization problems by dividing the centralized problem into smaller subproblems which can be then solved independently. The alternating direction method of multipliers (ADMM) can be used to coordinate between the subproblems (Boyd *et al.*, 2010).

(Martí *et al.*, 2015) presented how individual scenario could be represented as separate subproblems which are then linked by the first stage variables. (Rodriguez *et al.*, 2018) presented forming subproblems linked in time by splitting the prediction horizon for an optimal control problem. When considering very large prediction horizons and many scenarios, either of these approaches separately might not be sufficient to create subproblems that are within the available memory of the device.

We present a general formulation using the ADMM algorithm to create subproblems that are separated both by scenarios and in time (within each scenario). This lets us form much smaller subproblems as needed which we then solve in a distributed manner.

**Major Results**

Our distributed approach is shown to be able to reach the same local solution as compared to solving the problem centrally within few iterations (around 15 – 25) of the ADMM algorithm. The impact of varying the penalty parameter used in ADMM on the speed of convergence is also analysed.

**Conclusion**

The overall solution time for the distributed approach is expected to be larger compared to solving the problem centrally (if possible). But the distributed approach provides benefit in limited memory usage, thus allowing us to formulate and solve larger problems without being constrained by the available memory in a single machine. The approach also provides the ability to use multiple smaller machines in parallel that improve the solution times compared to a sequential implementation. Although ADMM cannot guarantee convergence for nonconvex problems, in practice these schemes have been shown to perform satisfactorily on complex nonconvex-NLPs.

**Keywords**: Distributed optimization, ADMM, Dynamic optimization, Two-stage stochastic program.

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