ARMA basics

Problem 1: ARMA basics

1. ARMA(1,1) model is given by:

$$r_t - \phi_1 r_{t-1} = \phi_0 + \epsilon_t - \theta_1 \epsilon_{t-1}$$

First order autocorrelation is given by:

$$\rho_1 = \phi_1 - \theta_1 \frac{\sigma_\epsilon^2}{\gamma_0}$$

Here, $\phi_1 = 0.95$, $\theta_1 = 0.9$, $\sigma_\epsilon^2 = 0.05^2$. Computation of γ_0:

$$\gamma_0 = \sigma_\epsilon^2 \frac{1 + \theta_1^2 - 2\phi_1\theta_1}{1 - \phi_1^2}$$

Substituting, we get $\gamma_0 = 0.002564$

Therefore, calculating ρ_1, we get $\rho_1 = 0.0725$

2. Second order autocorrelation of y_t is given by: $\rho_2 = \phi_1 \rho_1$

Therefore, $\rho_2 = 0.68875$

Ratio of second order to first order autocorrelation $= \phi_1 = 0.95$

This result is expected because ARMA models gets its correlation just like AR(p) models and hence the ratio is just ϕ_1. This also tells us that the autocorrelations decrease over time if $\phi_1 < 1$.

3. Irrespective of y_t, ARMA models have the same mean for the series since it is stationary, and hence

$$E[y_{t+1}] = E[y_{t+2}] = \frac{\phi_0}{1 - \phi_1} = 0$$

4. Given $\hat{x_t} = E[y_{t+1}]$. Therefore, $\hat{x_t}$ is always a constant that