#1: Gr07_2 Geo--Chapter 4. Right Triangles

#2: Congruence of Right Triangles

Definition

Definition: A **right triangle** is a triangle with a right angle.
The side opposite to the right angle is called **hypotenuse.** The other two sides are called **legs.**

Theorem

Right triangles congruence theorems (and equivalencies):
Two right triangles are congruent if:

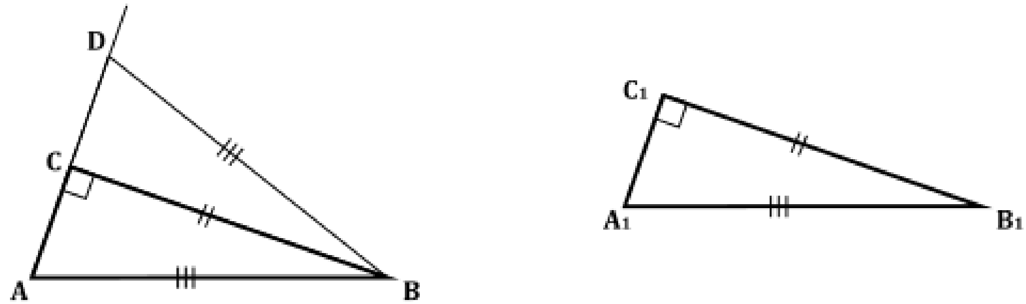
- Their corresponding legs are congruent **(LL ~ SAS).**
- Their corresponding leg and acute angle are congruent **(LA ~ ASA).**
- Their corresponding hypotenuse and acute angle are congruent **(HA ~ AAS).**
- Their corresponding leg and hypotenuse are congruent **(LH).**

NOTE: Recall that SSA is not a rule for congruence of all triangles but it works in the case of right triangles since the given right angle is opposite to the longer side, namely the hypotenuse.

Theorem | **LH Theorem:** Two right triangles are congruent if their corresponding leg and hypotenuse are congruent.

Given: $\triangle ABC$, $\triangle A_1B_1C_1$, $m\angle C = m\angle C_1 = 90°$, $AB = A_1B_1$, $BC = B_1C_1$

Prove: $\triangle ABC \cong \triangle A_1B_1C_1$



Proof:

1. On $\overrightarrow{AC}$ draw $\overline{CD}$, so that $CD = A_1C_1$	1. Construction
2. $CB = C_1B_1$	2. Given
3. $m\angle DCB = 180° - m\angle ACB = 90° = m\angle C_1$	3. Supplementary ∠s, Trans.
4. $\triangle DBC \cong \triangle A_1B_1C_1$	4. SAS
5. $DB = A_1B_1$	5. CPCTC
6. $DB = A_1B_1 = AB$	6. Transitivity
7. $\triangle ADB$ is isosceles	7. Definition of iso-Δ
8. $\angle BDC \cong \angle BAC$	8. Base ∠s Theorem
9. $\angle DCB \cong \angle ACB$	9. Supplementary ∠s
10. $BC = BC$	10. Reflexive property
11. $\triangle DBC \cong \triangle ABC$	11. SAA/ASA
12. $\triangle ABC \cong \triangle A_1B_1C_1$	12. Transitivity, 4.

Theorem | **Theorem (Point on Angle Bisector):** Any point that belongs to the angle bisector of an angle is equidistant to the sides of that angle.

Theorem | **Theorem (Point on Angle Bisector Converse):** If a point is in the interior of an angle and is equidistant to its sides then the point belongs to the angle bisector of that angle.

Definition | **Definition:** The line that goes through the midpoint of a segment, perpendicular to it, is called the **perpendicular bisector** of that segment.

Theorem | **Theorem (Perpendicular Bisector):** If a point belongs to the perpendicular bisector of a segment then it is equidistant from the endpoints of that segment.

Theorem | **Theorem (Perpendicular Bisector Converse):** If a point is equidistant from the endpoints of a segment then it belongs to the perpendicular bisector of that segment.

#3: EXERCISES

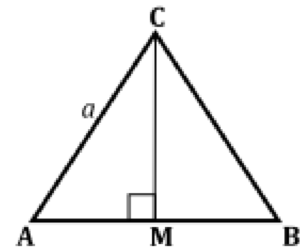
#4: Part A

#5: Part B

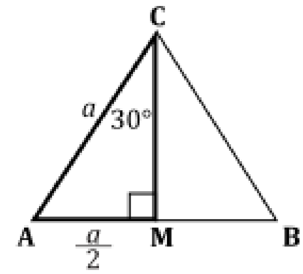
#6: Part C Additional Problems

#7: Leg Opposite to a 30°-angle in a Right Triangle

Now, consider the equilateral $\triangle ABC$ with length of its sides equal to a and altitude $\overline{CM}$ ($M \in \overline{AB}$) drawn to side $\overline{AB}$. Find the measures of the angles in triangles $\triangle AMC$ and $\triangle BMC$ and determine the length of the sides. What is the relation between the angles and the shorter leg in right $\triangle AMC$ or $\triangle BMC$?



Theorem | **Theorem (Leg Opposite to 30° ∠):** In a right triangle with a 30° angle, the leg opposite to the 30° angle is half of the hypotenuse.

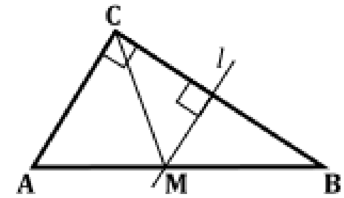


Proof:
Extend $\overline{AM}$ to point B so that AM = BM. Then, $\triangle AMC \cong \triangle BMC$ by SAS and AC = BC by CPCTC. But $m\angle A = 60°$ and $\triangle ABC$ is equilateral (as isosceles triangle with 60° angle). Thus, if $AC = a$, $AM = \frac{a}{2}$.

Theorem | **Theorem (Leg Opposite to 30° ∠ Converse):** If one of the legs in a right triangle is half of the hypotenuse then the angle opposite to that leg is with 30° measure.

Median to hypotenuse in a right triangle

Theorem

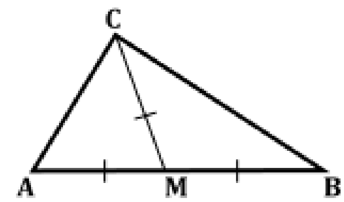


Theorem (Median to Hypotenuse): The median to the hypotenuse in a right triangle is half of the hypotenuse.

Proof:

Let l be the perpendicular bisector of $\overline{BC}$ and $l \cap \overline{AB} = M$.
Then BM = CM and respectively, m∠MBC = m∠MCB = β.
But m∠ACM = 90° − m∠MCB = 90° − β and m∠A = 90° − m∠B = 90° − β.
Then, m∠ACM = m∠A and from iso-△ △AMC, AM = CM = BM. Thus, CM = $\frac{1}{2}$ AB.

Theorem



Theorem (Median to Hypotenuse Converse): If the length of the median from a vertex in a triangle is half the length of the side towards which it is drawn then the angle at that vertex is a right angle.

Proof:

Triangles △AMC and △BMC are isosceles as AM = BM = CM.
Then the base angles m∠A = m∠ACM = x and m∠B = m∠BCM = y.
But m∠A + m∠ACM + m∠B + m∠BCM = $2x + 2y$ = 180°.
And thus, m∠ACB = $x + y$ = 90°.

#8: EXERCISES

#9: Part A

#10: Part B