#1: Gr07_2 Geo--Chapter 3. Isosceles and Equilateral Triangles

#2: Definitions and Angles in Isosceles and Equilateral Triangles

Definition

Definition: An **angle bisector** of a triangle is a segment drawn from a vertex to the opposite side so that it bisects the angle at that vertex into two congruent angles.

Definition

Definition: An **altitude** of a triangle is a perpendicular segment drawn from a vertex to the line of the opposite side.

Definition

Definition: A **median** of a triangle is a segment drawn from a vertex to the midpoint of the opposite side.

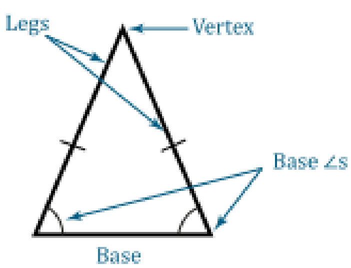
Definition

Definition: A triangle with at least two congruent sides is called **an isosceles triangle**.

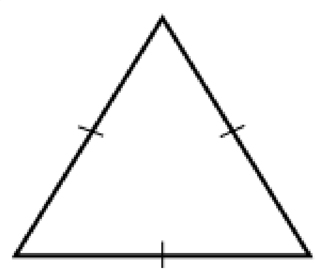
Definition

Definition: A triangle with all its sides congruent is **an equilateral triangle**.

Isosceles △



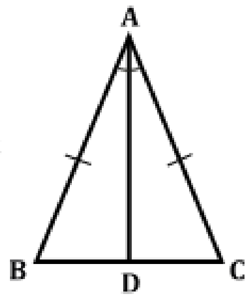
Equilateral △



Theorem | **Theorem (Base ∠s Th.):** The base angles of an isosceles triangle are congruent.

Given: $\triangle ABC$, $AB = AC$

Prove: $m\angle B = m\angle C$

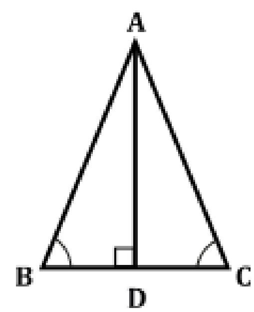


1. Draw $\overline{AD}$ – ∠ bisector of ∠A	1. Construction
2. $m\angle BAD = m\angle DAC$	2. Definition of ∠ bisector
3. $AB = AC$	3. Given
4. $AD = AD$	4. Reflexive property
5. $\triangle BAD \cong \triangle CAD$	5. SAS
6. $m\angle B = m\angle C$	6. CPCTC

Theorem | **Theorem (Base ∠s Converse Th.):** If two angles in a triangle are congruent then the sides opposite to those angles are congruent (the triangle is isosceles).

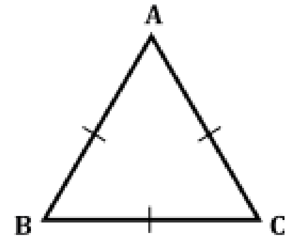
Proof:

1. $m\angle B = m\angle C$	1. Given
2. Draw $\overline{AD}$ – altitude to $\overline{BC}$	2. Construction
3. $m\angle ADB = m\angle ADC = 90°$	3. Def. of altitude, Def. of ⊥
4. $AD = AD$	4. Reflexive property
5. $\triangle BAD \cong \triangle CAD$	5. AAS (1, 3, 4)
6. $AB = AC$	6. CPCTC



Theorem | **Theorem (∠s in equilateral △):** The angles of equilateral triangle are congruent with measure of 60°.

Proof:
Let △ABC be an equilateral triangle. By definition, AC = BC and respectively m∠A = m∠B by Base Angles Theorem.
Similarly, AB = CB and m∠A = m∠C.
Then, m∠A = m∠B = m∠C = 60° by transitivity and Sum of ∠*s* in △.



Theorem | **Theorem (∠s in equilateral △ Conv.):** If all the angles in a triangle are congruent then it is equilateral.

Main Problem (∠s in right iso-△):
Prove that the angle measures of a right isosceles triangle are 45°, 45°, 90°.

Main Problem (Iso-△ with a 60° ∠):
Prove that an isosceles triangle with a 60° angle is an equilateral triangle.

#3: EXERCISES

#4: Part A

#5: Part B

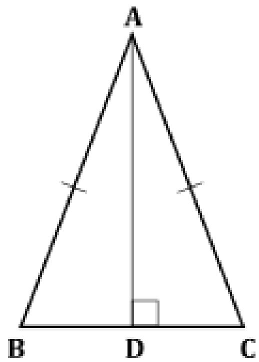
#6: Angle Bisectors, Altitudes, and Medians in Isosceles Triangles

Theorem | **Theorem (∠ bis. to base):** The angle bisector to the base of an isosceles triangle is also median and altitude.

Theorem | **Theorem (Median to base):** The median to the base of an isosceles triangle is also altitude and angle bisector.

Theorem | **Theorem (Altitude to base):** The altitude to the base of an isosceles triangle is also median and angle bisector.

Proof of Altitude to base Th:



1. AB = AC	1. Definition of isosceles Δ
2. m∠B = m∠C	2. Base ∠s Theorem
3. m∠ADB = m∠ADC = 90°	3. Definition of altitude
4. ΔBAD ≅ ΔCAD	4. SAA
5. BD = CD	5. CPCTC
6. $\overline{AD}$ − median to base	6. Definition of median
7. m∠BAD = m∠CAD	7. CPCTC
8. $\overline{AD}$ −∠ bisector of ∠A	8. Definition of ∠ bisector

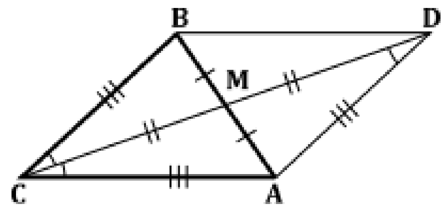
Converse Theorems

Theorem | **Theorem (Median ≡ Altitude):** If a median of a triangle is also an altitude, then the triangle is isosceles.

Theorem | **Theorem (∠ bisector ≡ Altitude):** If an angle bisector of a triangle is also an altitude, then the triangle is isosceles.

Theorem | **Theorem (∠ bisector ≡ Median):** If an angle bisector of a triangle is also its median, then the triangle is isosceles.

Given: $\triangle ABC$,
$\overline{CM}$ − median to $\overline{AB}$,
$\overline{CM}$ − angle bisector of $\angle C$
Prove: $\triangle ABC$ isosceles



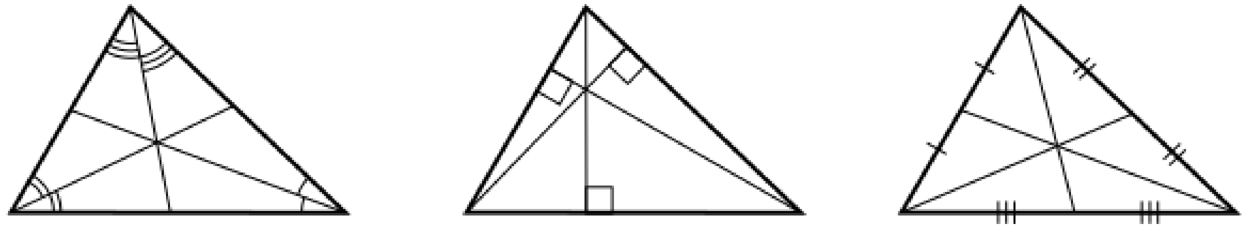
Proof:

1. Draw $\overline{MD}$ on $\overrightarrow{CM}$ so that CM = MD	1. Construction
2. CM = MD	2. By construction
3. BM = AM	3. Definition of median
4. m∠CMB = m∠DMA	4. Vertical angles ≅
5. △CMB ≅ △DMA	5. SAS
6. BC = AD	6. CPCTC
7. m∠BCM = m∠ADM	7. CPCTC
8. m∠BCM = m∠ACM	8. Definition of ∠ bisector
9. m∠ACM = m∠ADM	9. Transitivity
10. AC = AD	10. Base ∠s Theorem Conv.
11. AC = BC	11. Transitivity, 6, 10
12. △ABC isosceles	12. Definition of iso-△

Example:
Let $\overline{AD}$ be the altitude from vertex A in equilateral triangle △ABC. Find the angle measures of △ABD and △ACD. Proof as Class Exercise: Use properties of altitude in isosceles/ equilateral triangle.

Recall:

In every triangle, the angle bisectors are concurrent, i.e. they intersect at one point called **the incenter**. The lines containing the altitudes intersect at one point called **the orthocenter** and the medians intersect at one point called **the centroid**.



Incenter – ⋂ of ∠ bisectors   Orthocenter – ⋂ of altitudes   Centroid – ⋂ of medians

#7: EXERCISES

#8: Part A

#9: Part B