Hypothesis Testing I

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02/13/2024

Project Proposal

- 2 pages (must include at least 5 references)
- 1 submission per team
- Follow the Heilmeier catechism

Project Proposal

• What is the objective of your project? Write using absolutely no jargon.
• What is the current practice and what are its limitations?
• What is the novelty in your project? Why would your project be successful?
• If you're successful, who will benefit?
• What are the costs/risks associated with your project?
• How will you evaluate your progress/success?

Some good project ideas

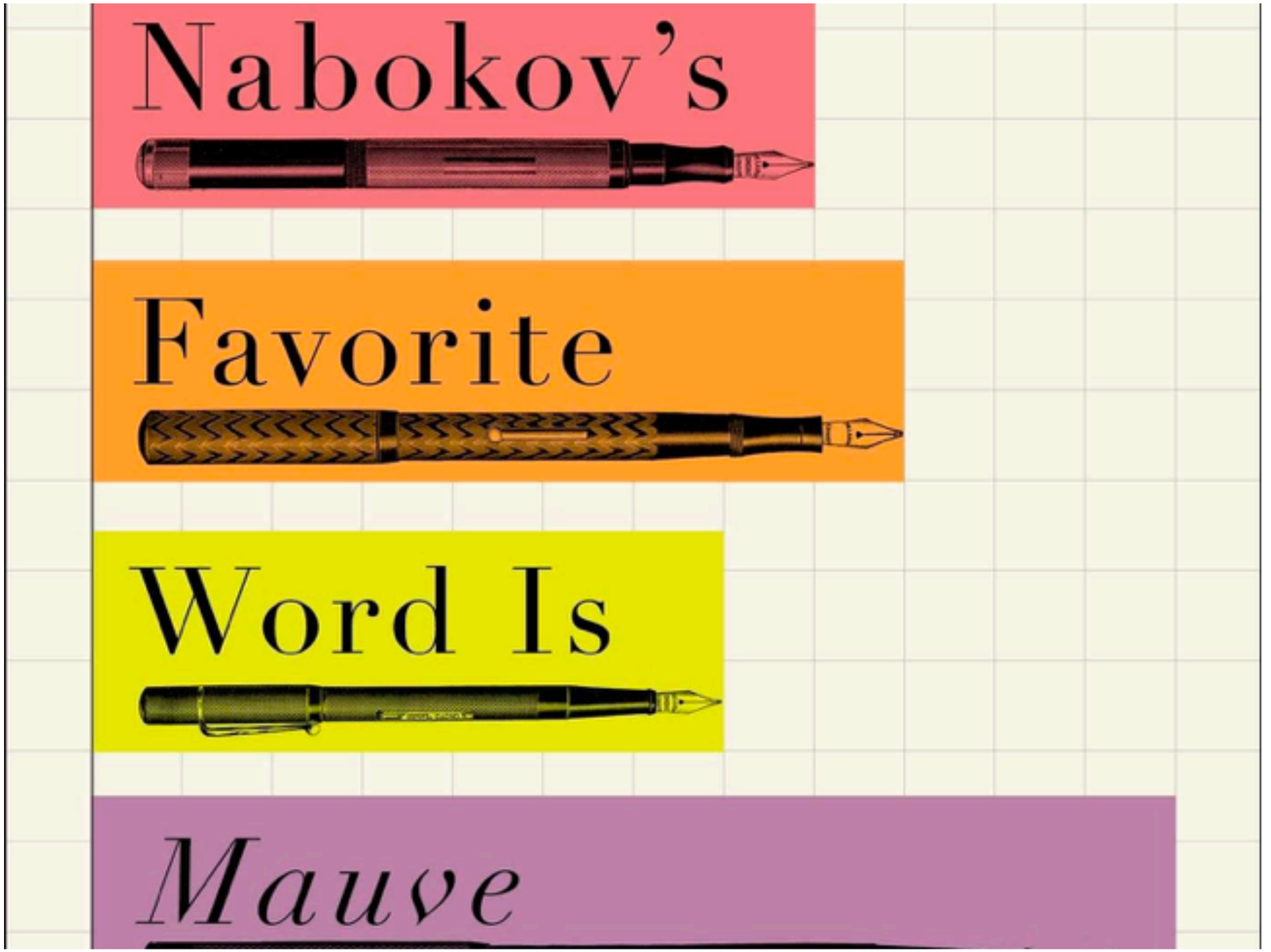
- Is innovation stagnating in research?
- What characteristics of music make it popular?
- Survey the impact of large language models on climate change

Some BAD project ideas

- Predict stock market price
  - Predict stock market price from social media
- Predict presidential election winner
- Sentiment analysis

Question for the day

"How to statistically test a claim using text?"



Cover page of the book written by Ben Platt

Hypotheses

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- Many claims can be framed as hypotheses

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Examples
Gender bias is decreasing in books
Reframing a tweet can increase its retweet rate
Institutional mistrust is predictive of hate speech
Classifier A is better than classifier B

Null hypothesis

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Hypothesis (H)	H_0
Gender bias is decreasing over time in books	Gender bias remains the same over time in books
Reframing a tweet can increase its retweet rate	Reframing a tweet has no effect on retweet rate
Institutional mistrust is predictive of hate speech	Institutional mistrust is not predictive of hate speech
Classifier A is better than classifier B	Classifier A is the same as classifier B

Hypothesis testing

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- Null hypothesis (H_0) gives us an expected result

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- Null hypothesis (H_0) gives us an expected result
- When testing if hypothesis H holds, what we're really testing is that if H_0 is assumed to be true, how likely does the observed data match our expectation?

Hypothesis testing

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According to the null hypothesis, I expect the statistic to be normally distributed with some mean and variance

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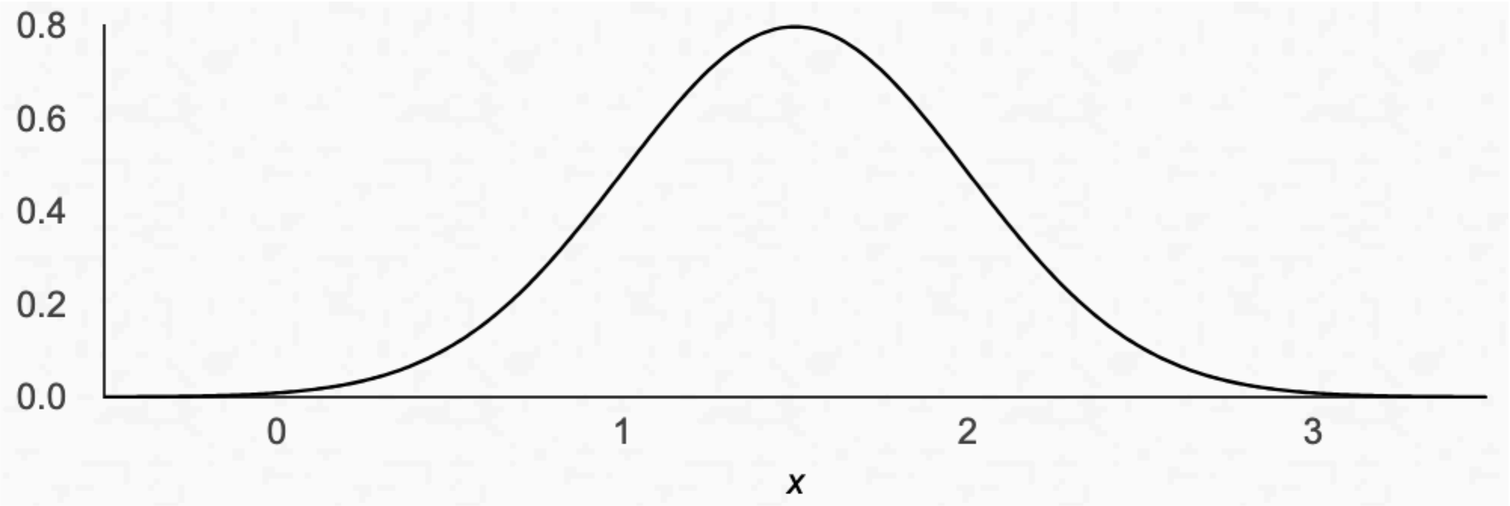

Hypothesis testing

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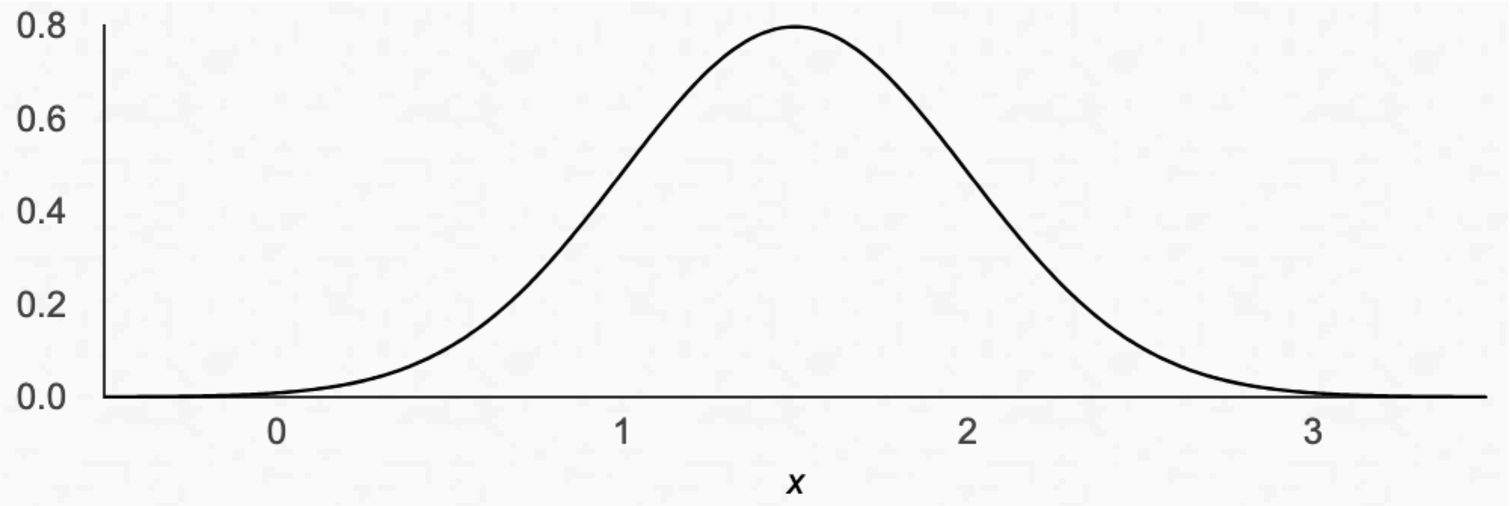

This sets up expectation about the shape and position of the distribution

Hypothesis testing

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If we observe x to be 1.3, how likely are we to see that if the null hypothesis holds?

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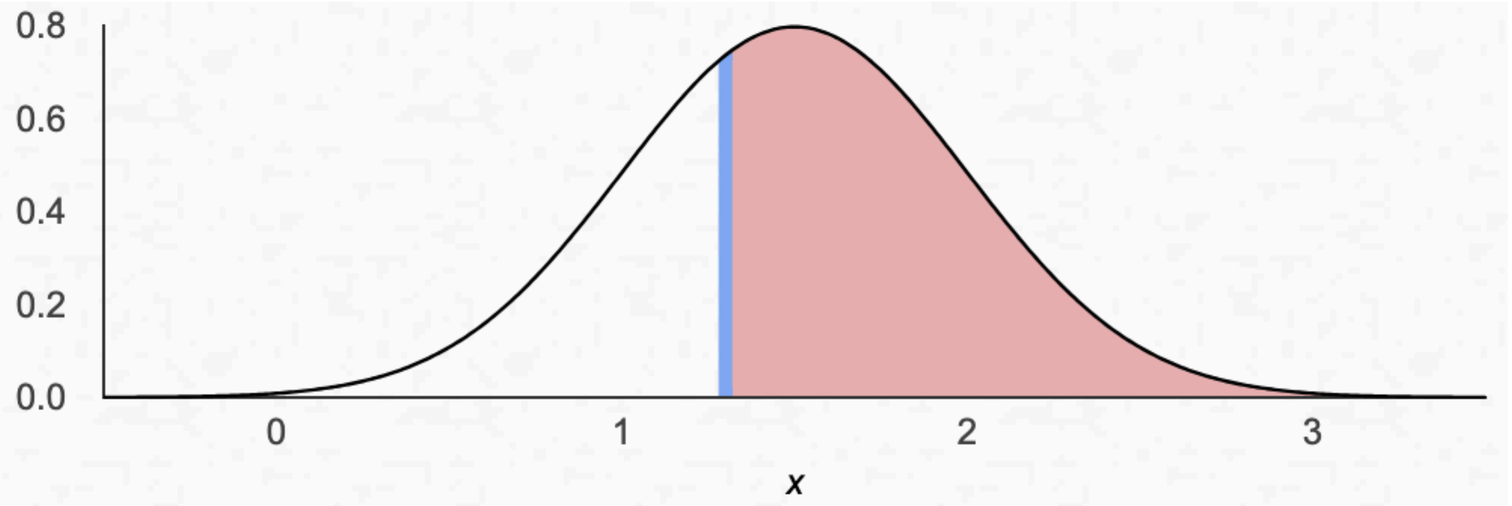

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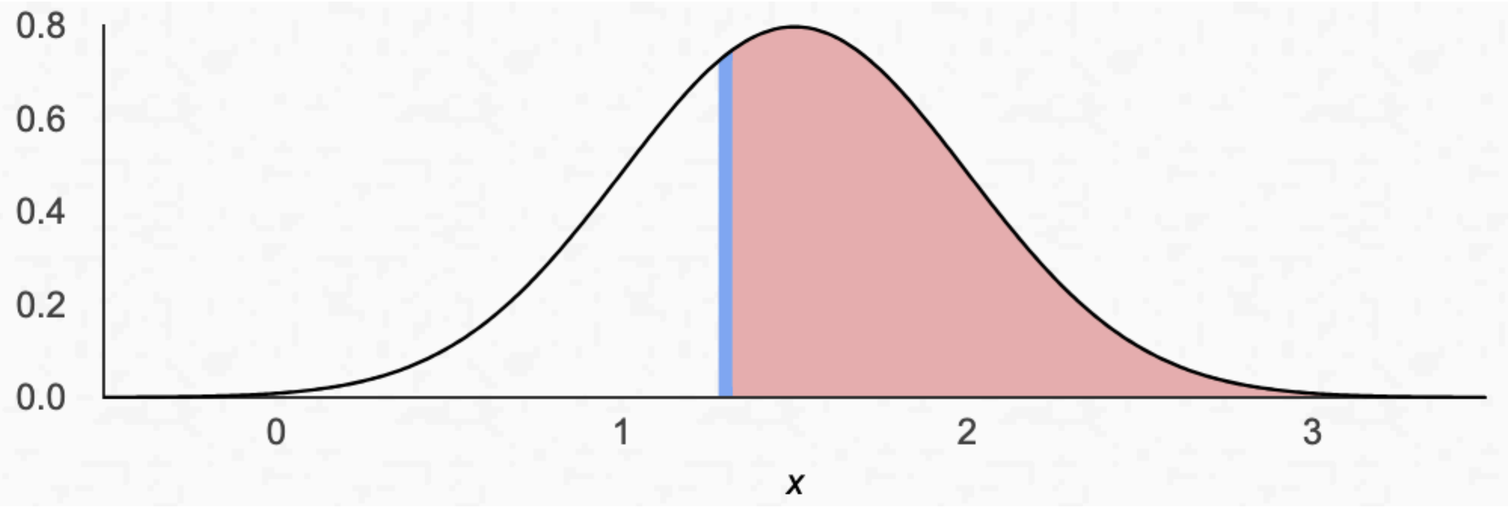

We can quantify this by calculating the probability of the shaded area

Hypothesis testing

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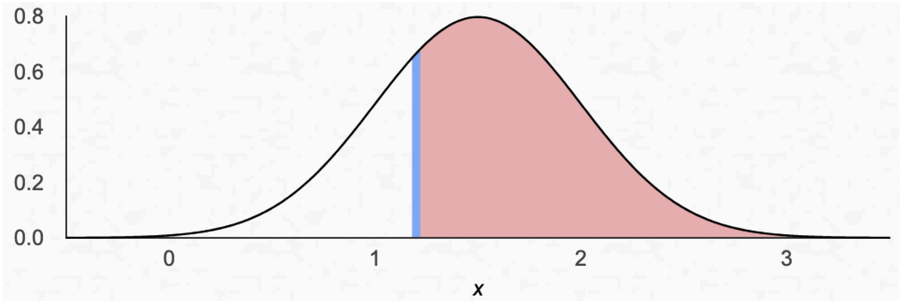
Which observation would you say is surprising if the null distribution holds?

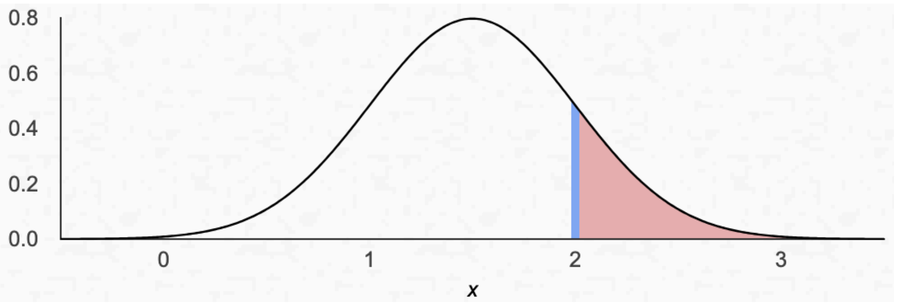
Hypothesis testing

Which observation would you say is surprising if the null distribution holds?

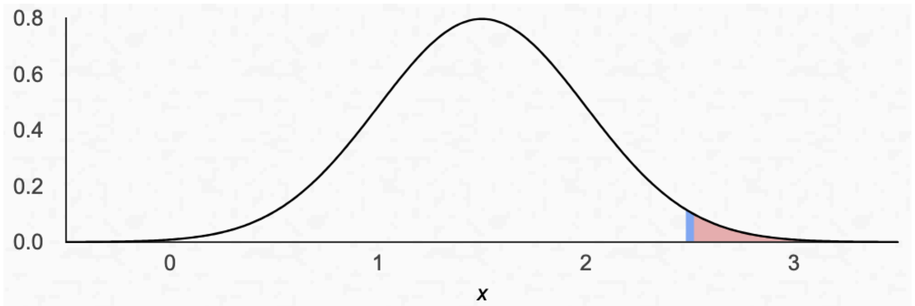
x=1.25



x=2



x=2.5



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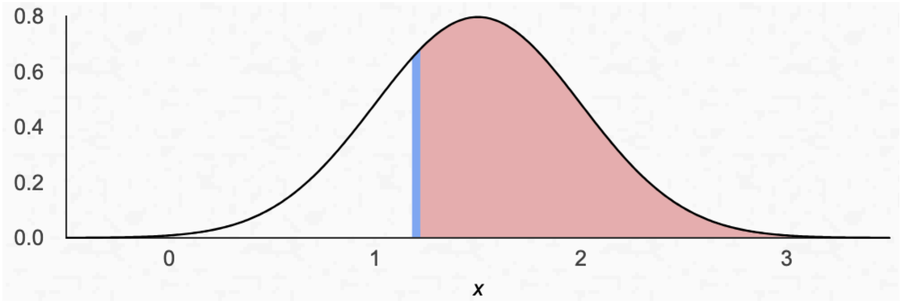
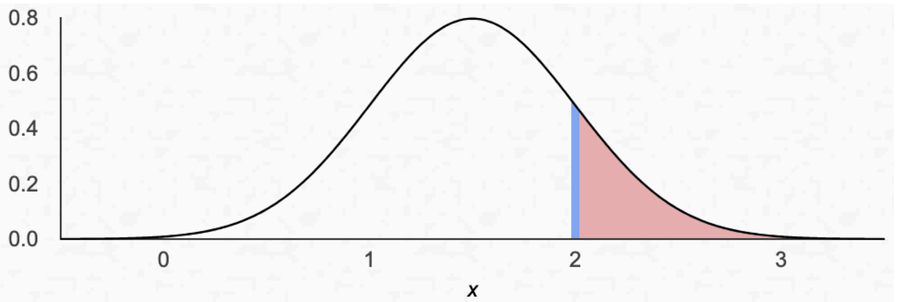


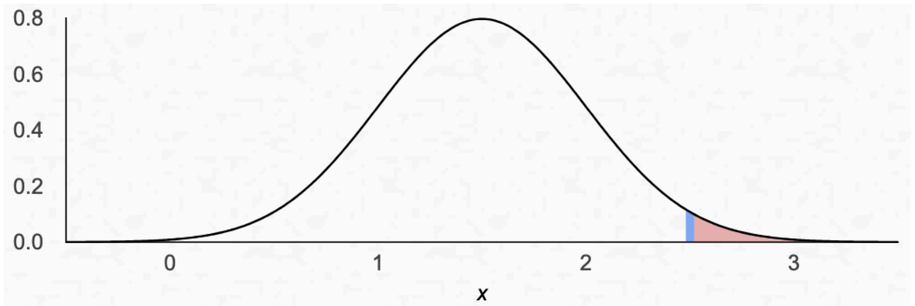
p=0.69

x=2



p=0.16

x=2.5



p=0.02

Significance level

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To make a decision, set up a rejection region by choosing the value of α

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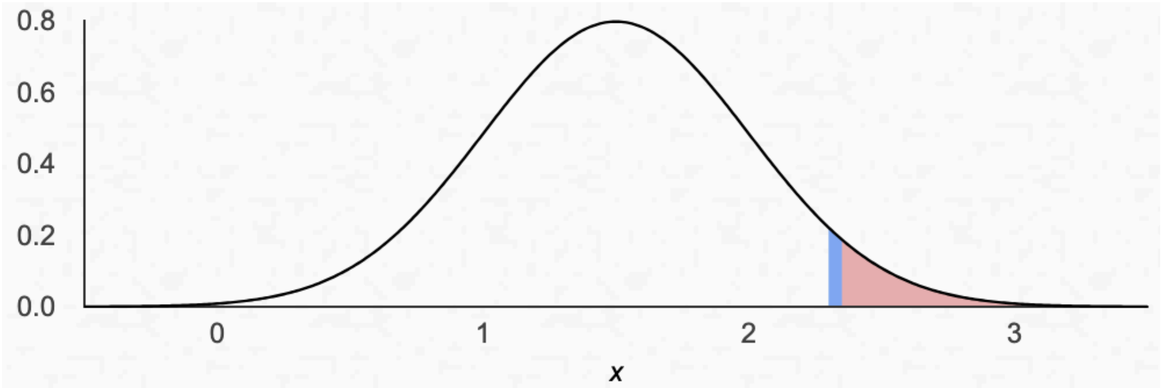

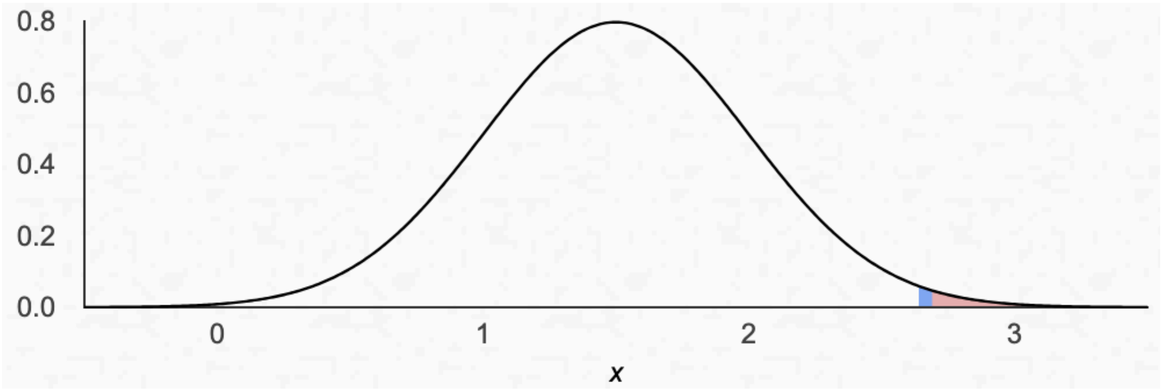

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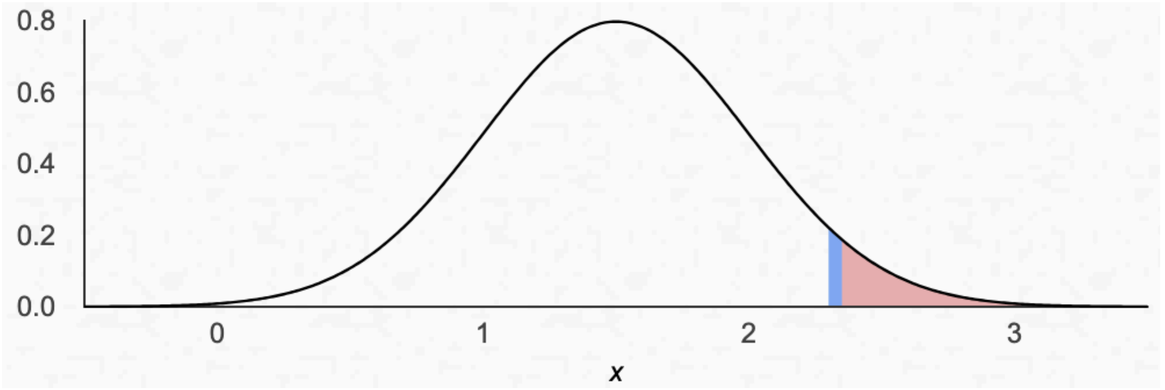
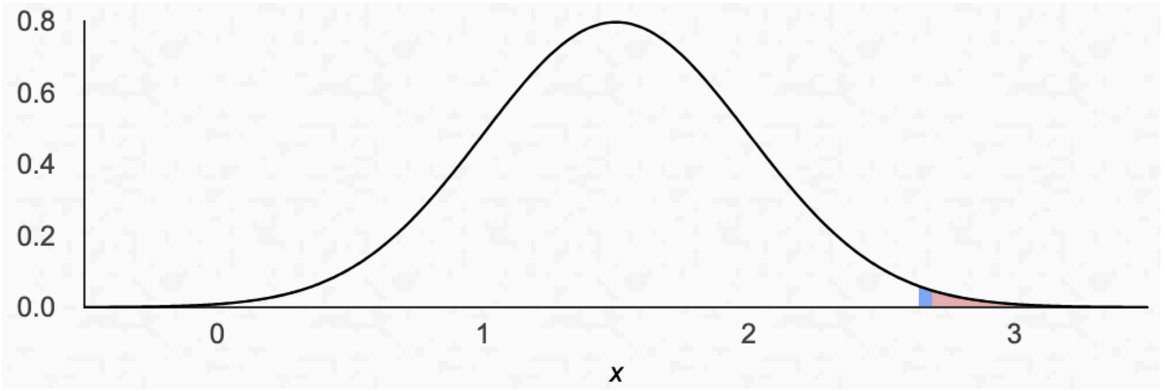
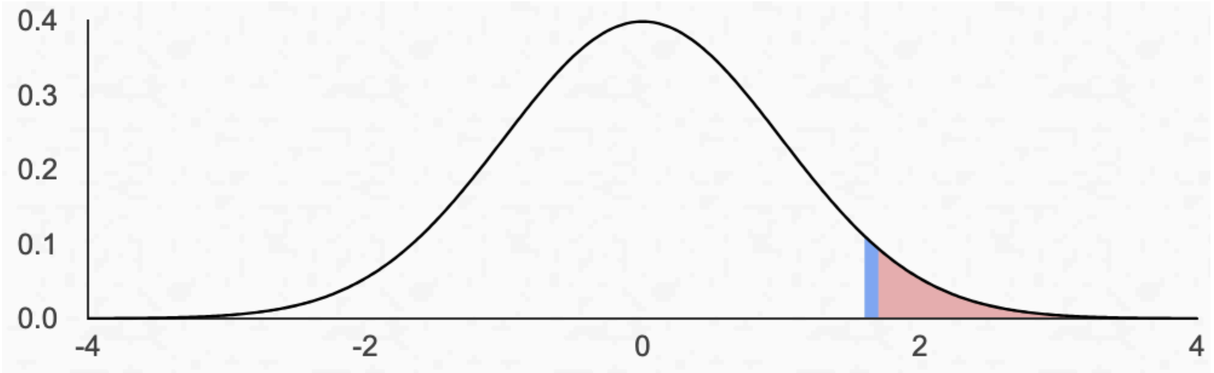


α=0.05



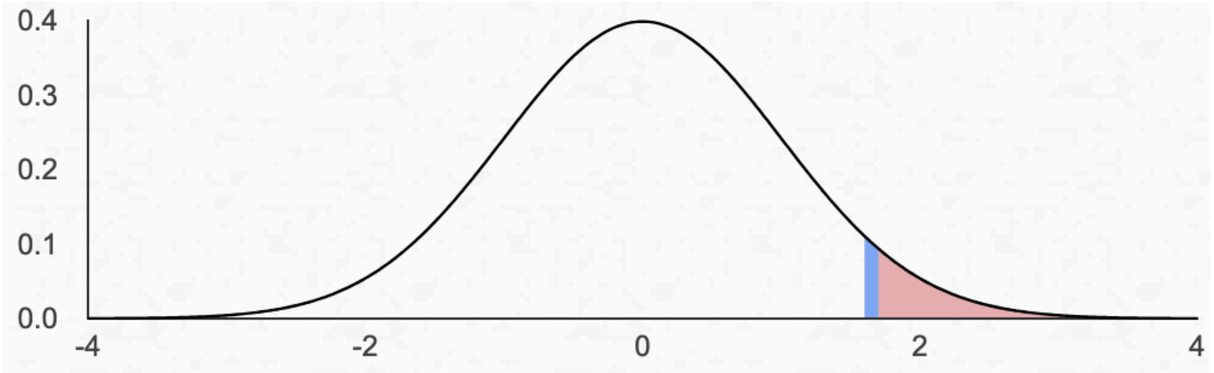
α=0.01

Z-score



Z-score

Calculating probabilities and comparing with the significance level can be tedious



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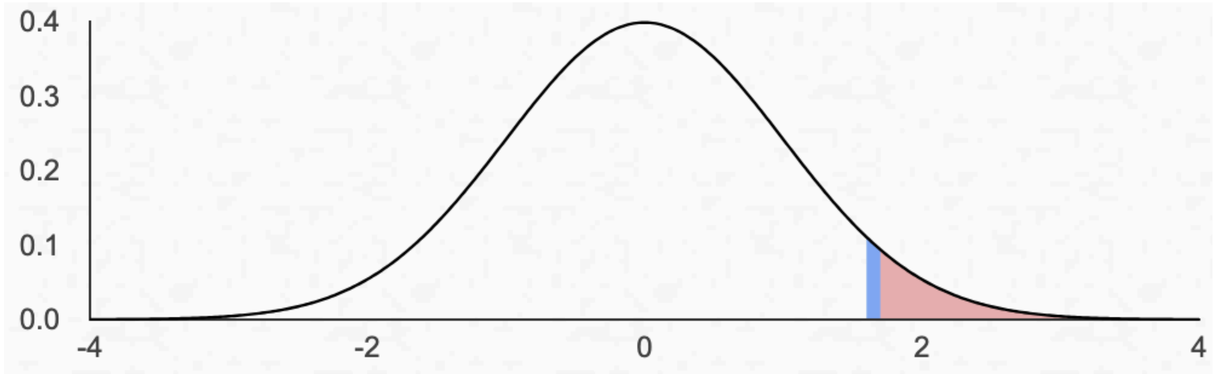
$$z = \frac{x - \mu}{\sigma/\sqrt{n}}$$



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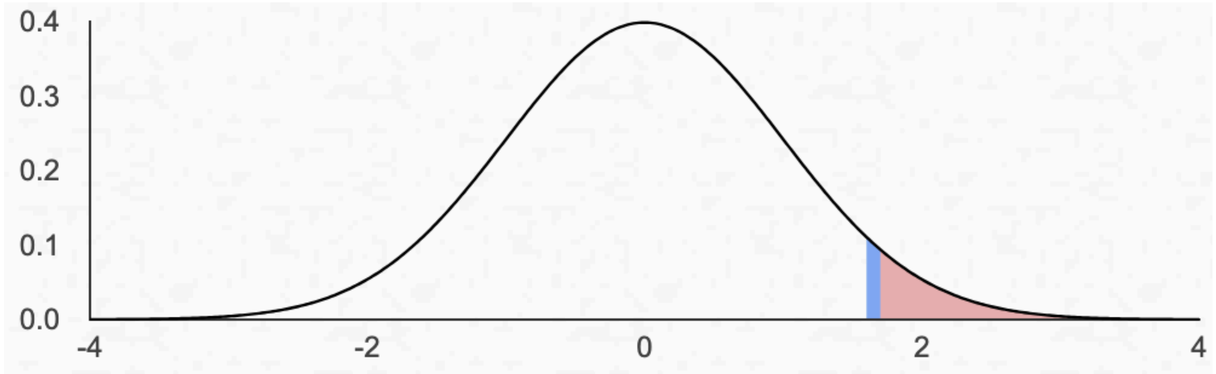

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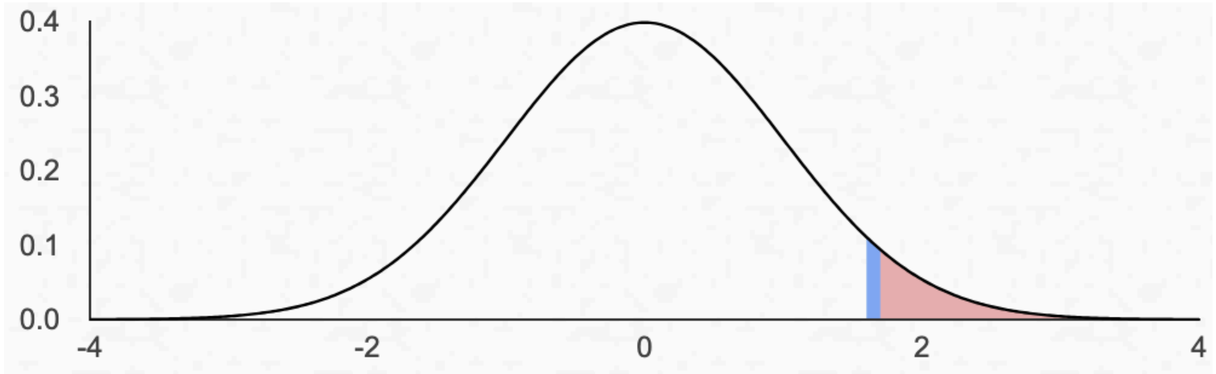

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We can simply calculate the z-score of the statistic and compare it to the z-score for the significance levels

Tails

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What is considered surprising is dependent on the problem at hand

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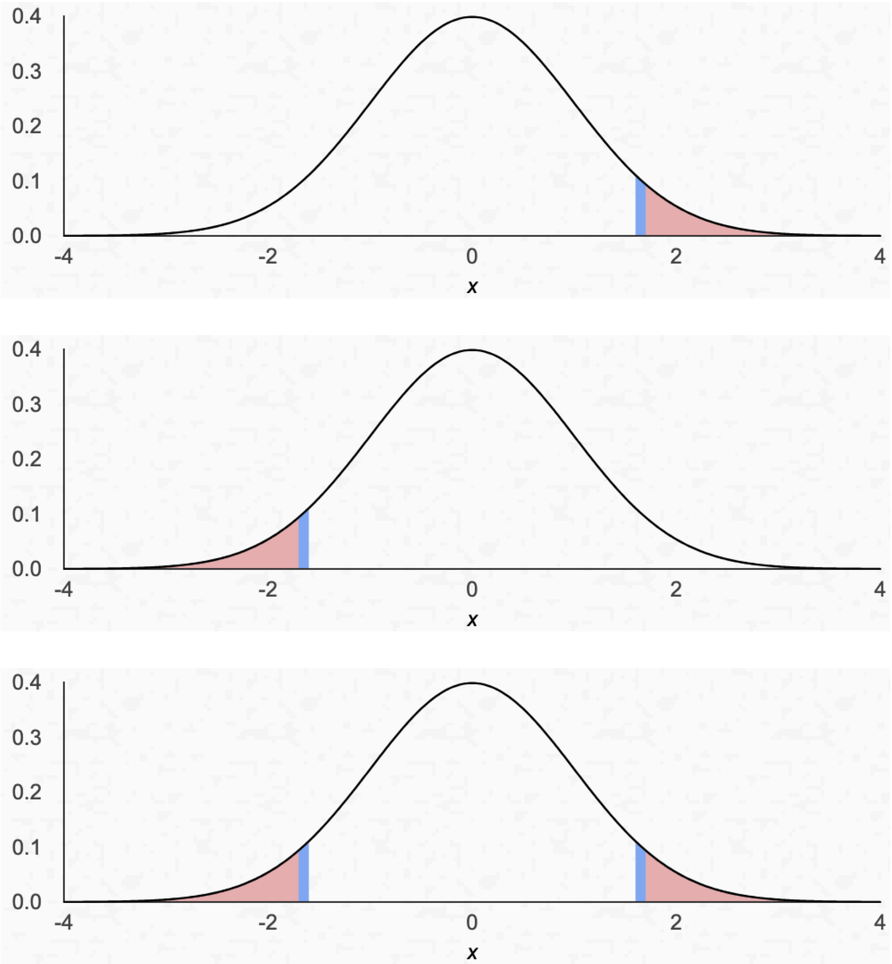

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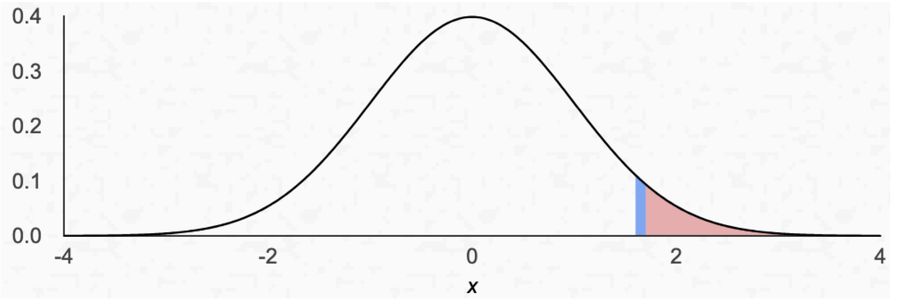
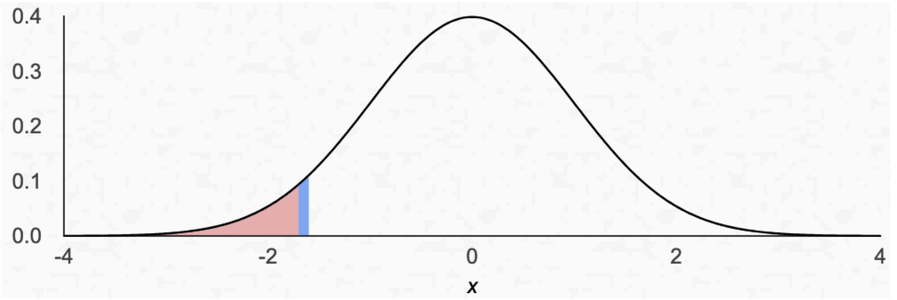
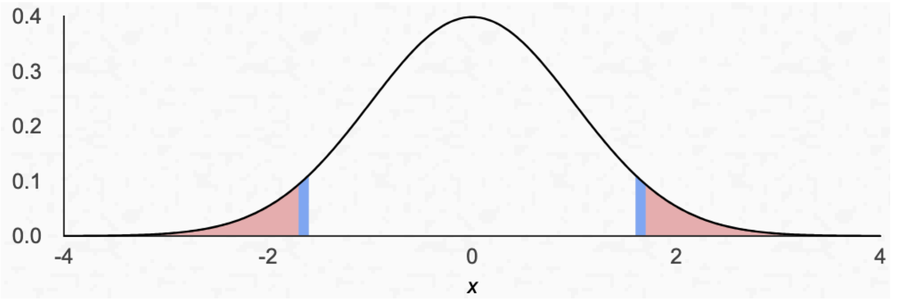


upper-tail: is x significantly greater?



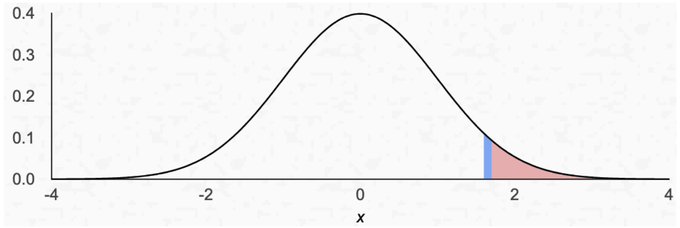
lower-tail: is x significantly smaller?



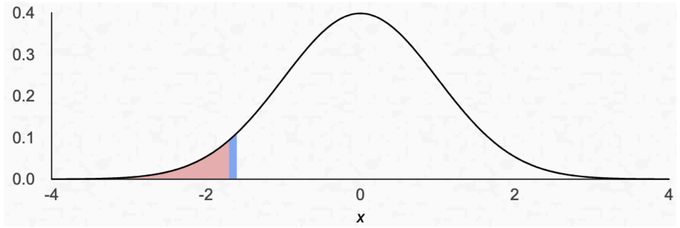
two-tail: is x significantly different?

P VALUES

A p value is the probability of observing a statistic at least as extreme as the one we did if the null hypothesis were true.



p-value (x) = $P(X \geq x \mid H_0) = 1 - P(X \leq x \mid H_0)$



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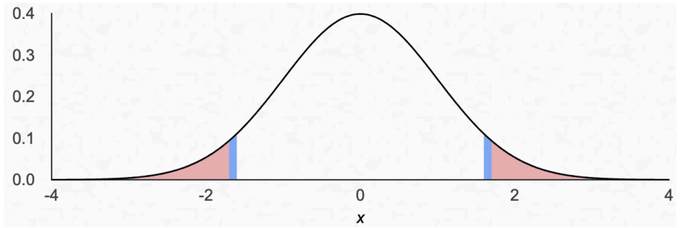

p-value (x) = $2 \times P(X \leq -\,|x|\,|H_0)$

Recipe for hypothesis testing

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- Assume significance level α; state H_0 and $H_{alternative}$

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- Calculate some statistic x whose conditional distribution is given by $P(X|H_0)$
- Calculate p-value
- If p-value falls in rejection region then null hypothesis can be rejected; else null hypothesis cannot be rejected

Errors

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- Type I error: We incorrectly rejected the null hypothesis

Errors

- Type I error: We *incorrectly rejected* the null hypothesis
- Type II error: We *incorrectly failed to reject* the null hypothesis

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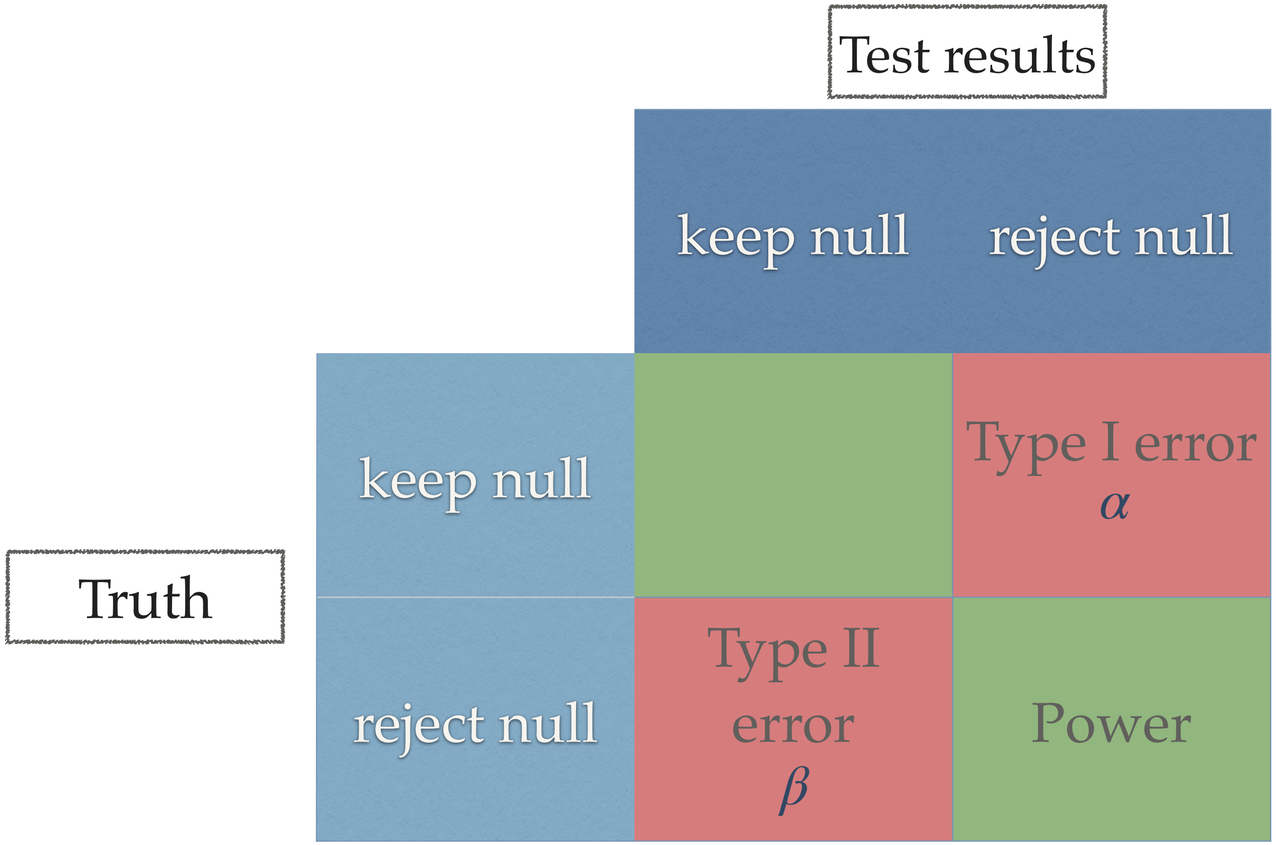

The Boy who Cried Wolf

- In the Aesop's fable, assuming that there typically is no wolf, the villagers make two types of errors
  - They believe there was a wolf when there was none
  - They believe there was no wolf when there was one

The Boy who Cried Wolf

The Boy who Cried Wolf

- In the Aesop's fable – The Boy who Cried Wolf – the villagers make two types of errors
  - They believe there was a wolf when there was none [TYPE I]
  - They believe there was no wolf when there was one [TYPE II]

The Boy who Cried Wolf

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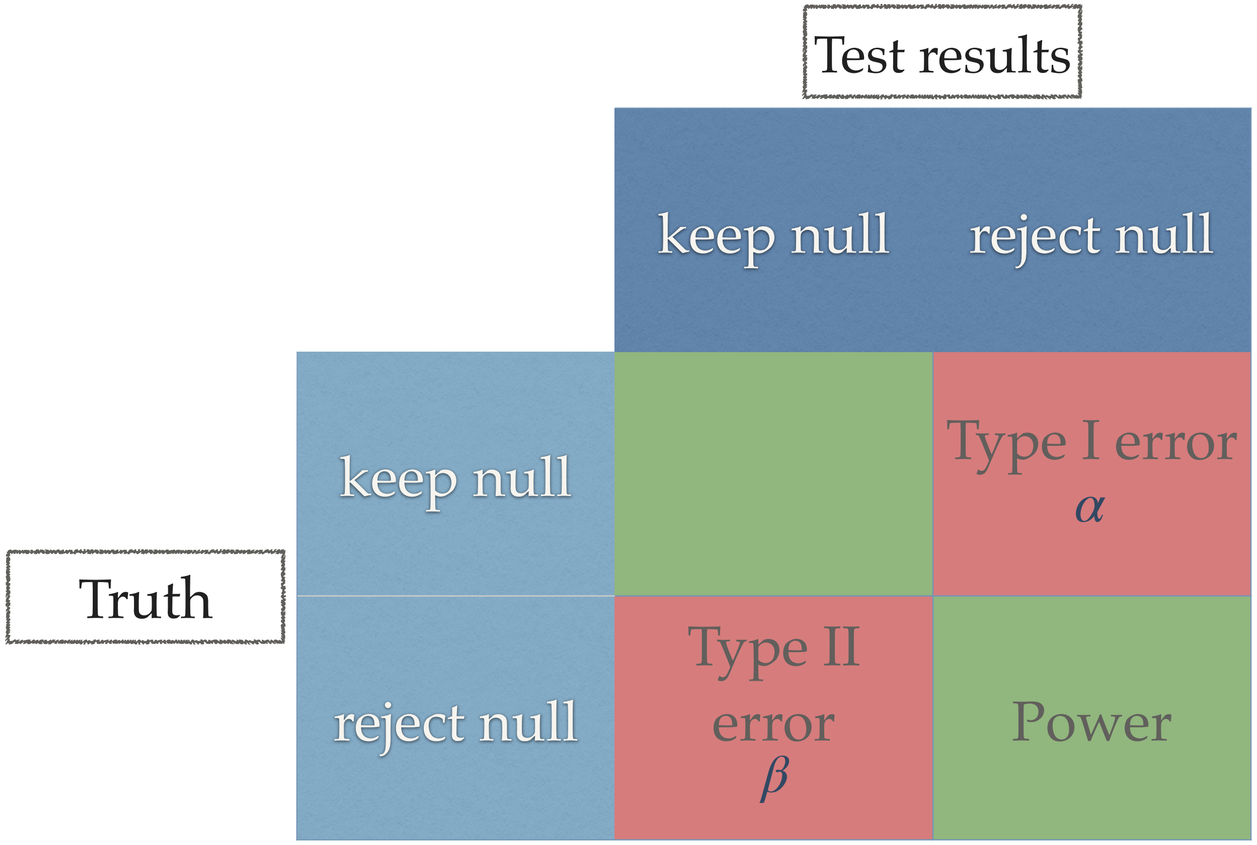
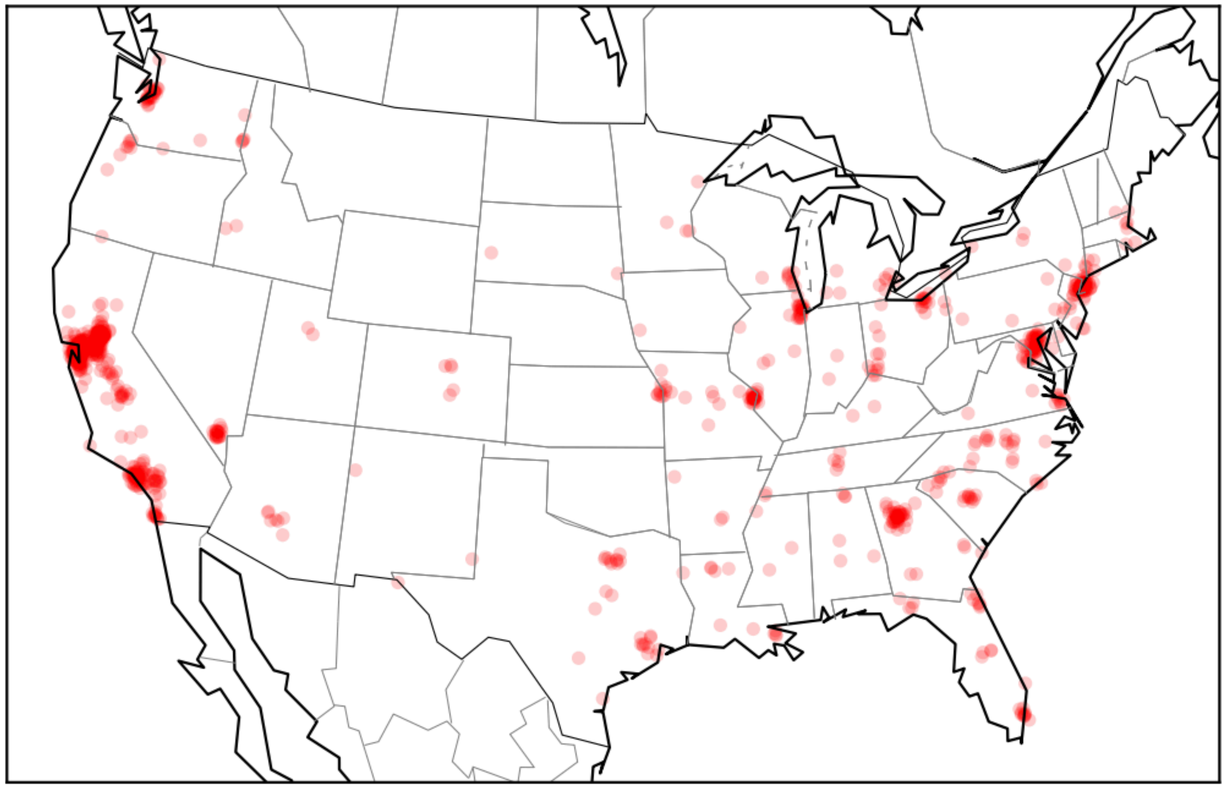
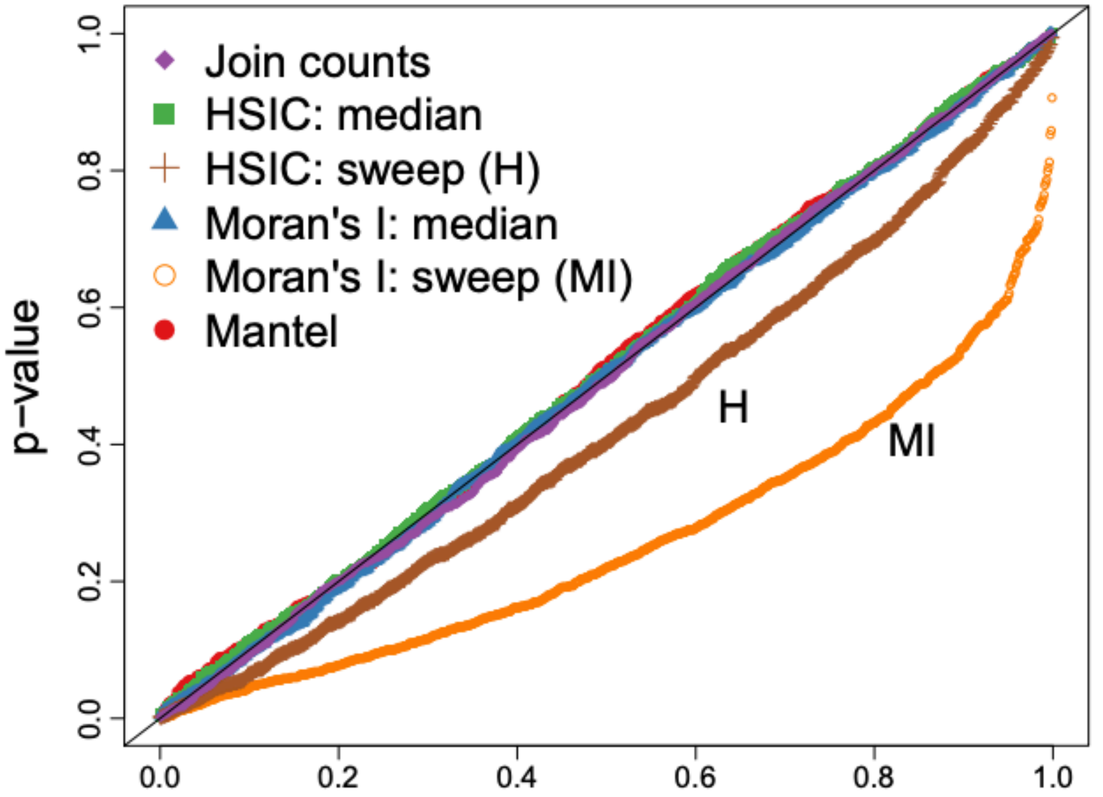


Test if lexical variation is independent of geography





Dong Nguyen and Jacob Eisenstein. 2017. A Kernel Independence Test for Geographical Language Variation. Computational Linguistics, 43(3):567–592.

Multiple hypothesis corrections

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$$\alpha = \frac{\alpha_0}{n}$$

Here the significance level α_0 is adjusted

Confidence intervals

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- Confidence intervals helps us quantify the range in which the observed metric will lie for the unobserved population

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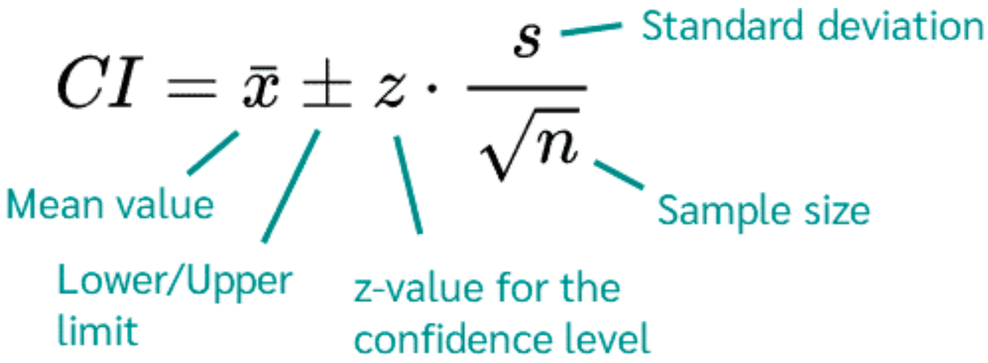
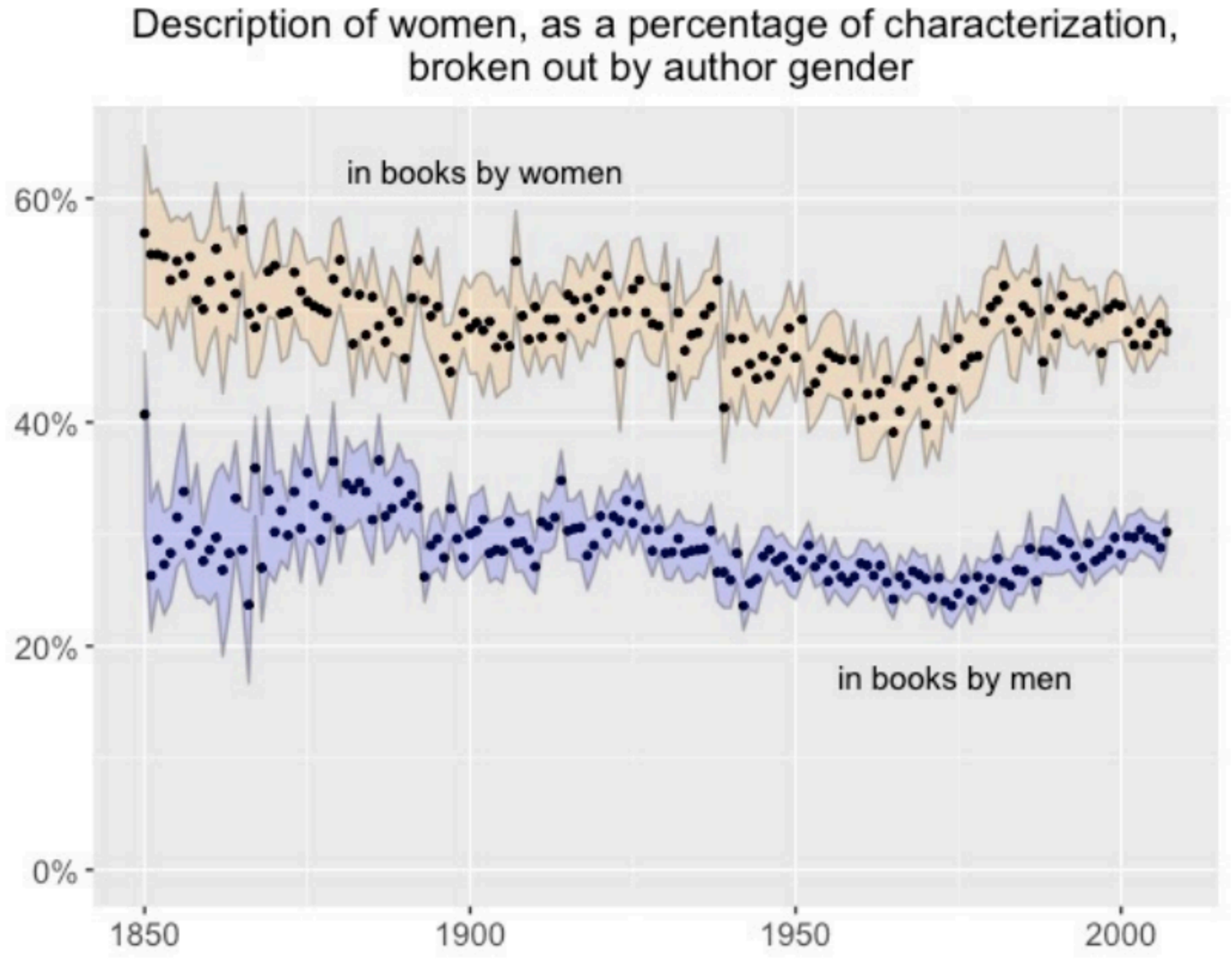
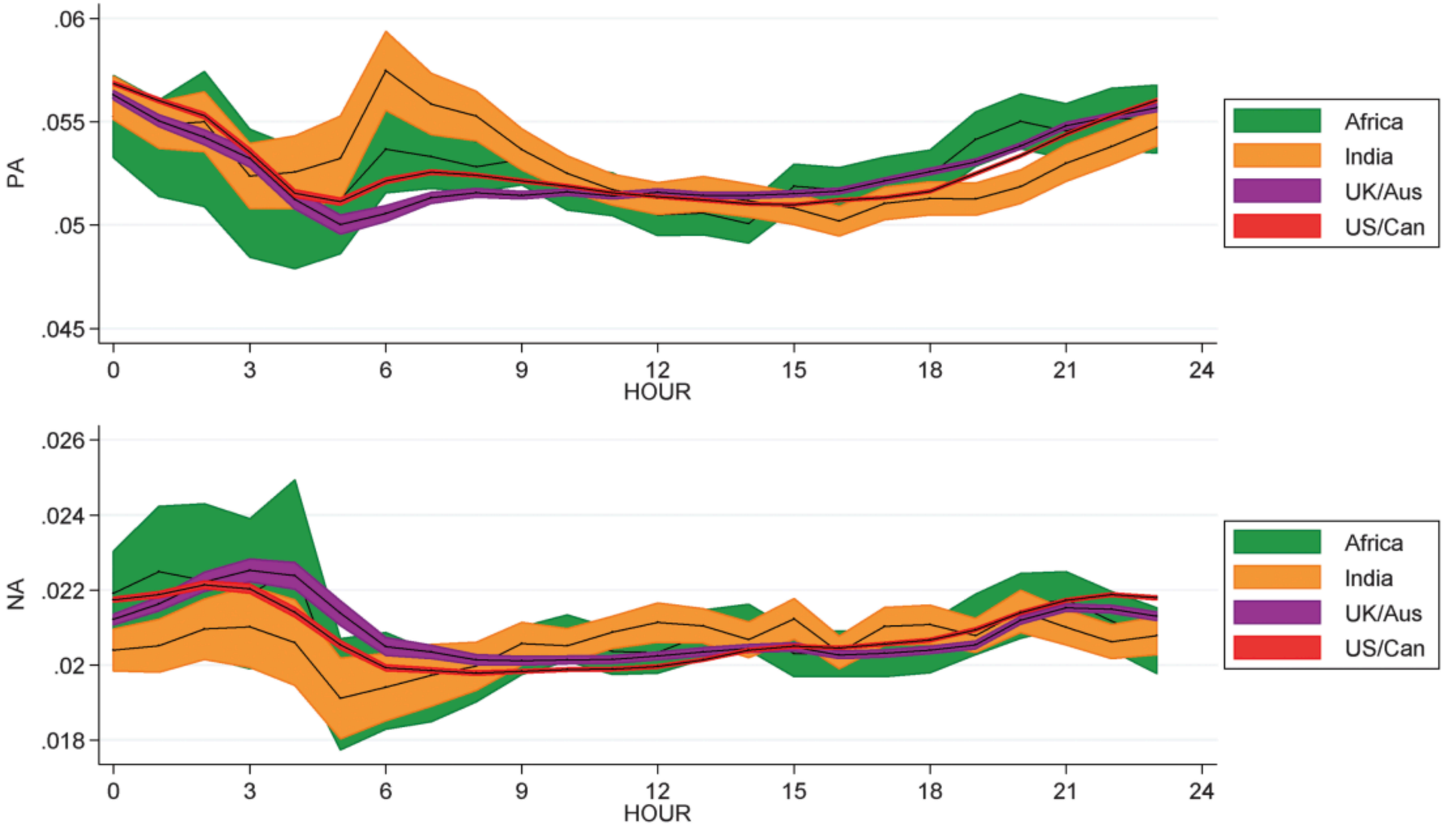
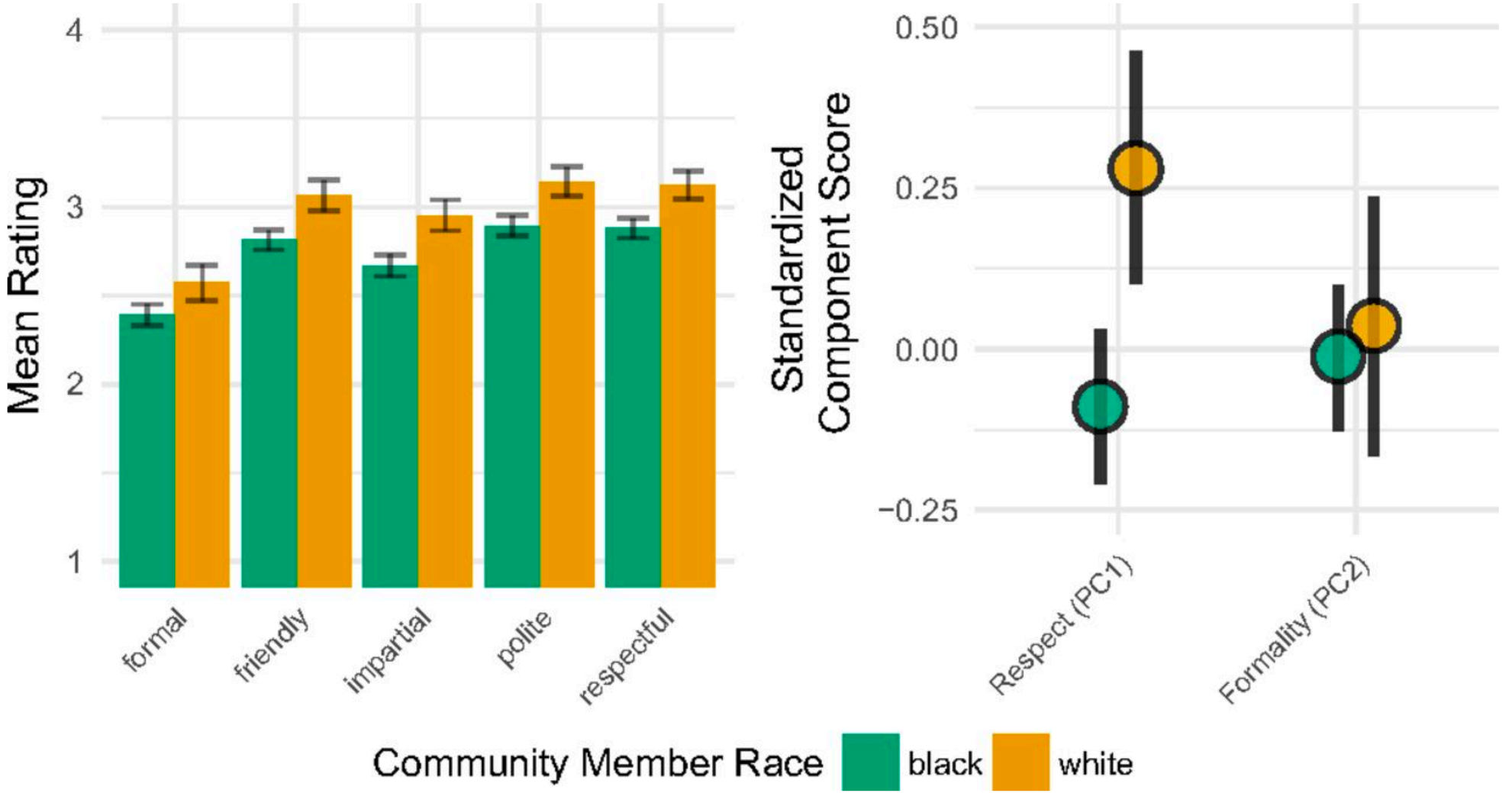




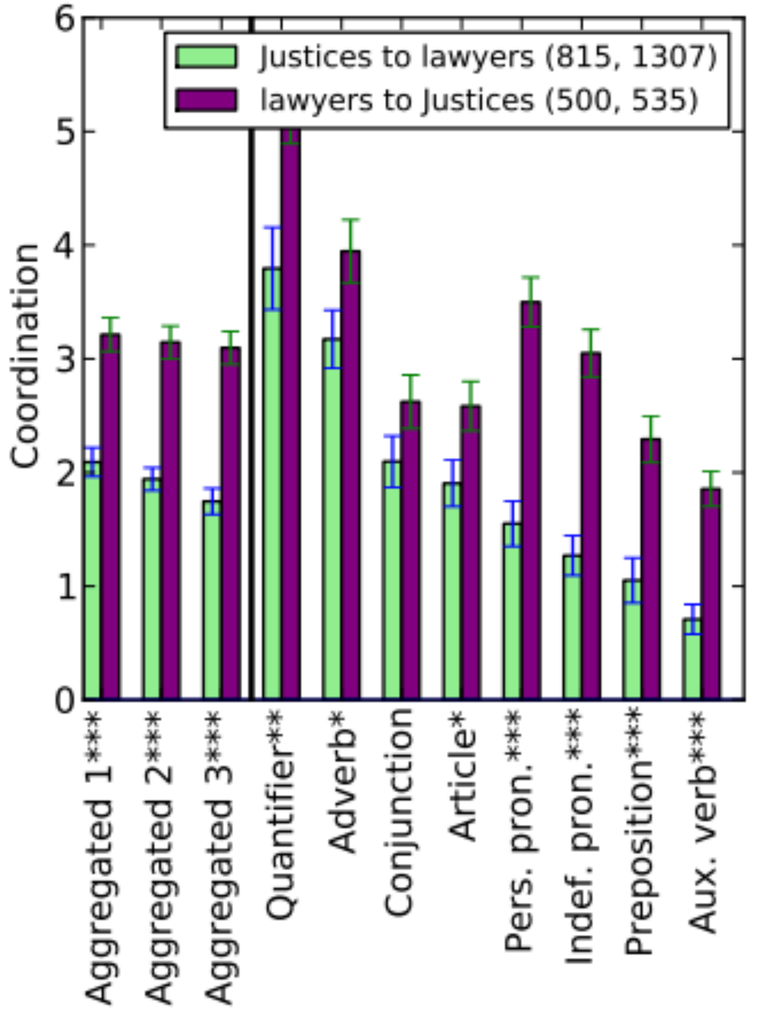
Underwood, Ted, David Bamman, and Sabrina Lee. "The transformation of gender in English-language fiction." Journal of Cultural Analytics 3.2 (2018).

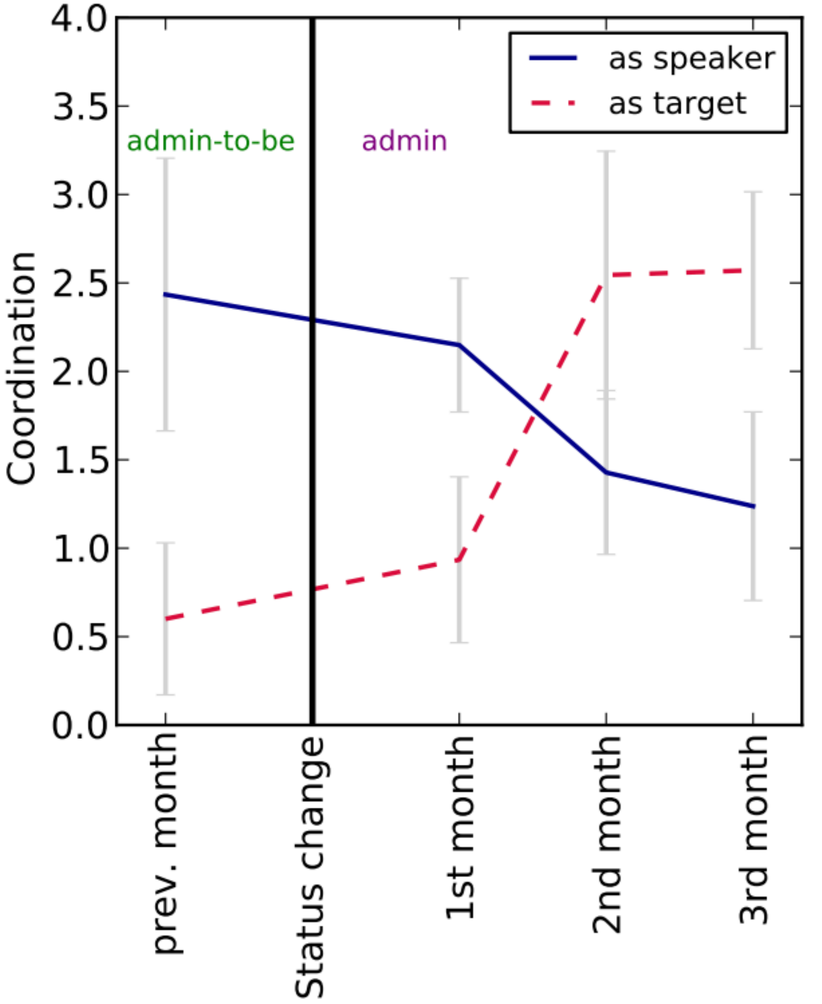


Golder, Scott A., and Michael W. Macy. "Diurnal and seasonal mood vary with work, sleep, and daylength across diverse cultures." Science 333.6051 (2011): 1878-1881.



Voigt, Rob, et al. "Language from police body camera footage shows racial disparities in officer respect." Proceedings of the National Academy of Sciences 114.25 (2017): 6521-6526.





Danescu-Niculescu-Mizil, Cristian, et al. "Echoes of power: Language effects and power differences in social interaction." Proceedings of the 21st international conference on World Wide Web. 2012.

Ablation testing

Ablation testing

- Are a set of features really important for the model?

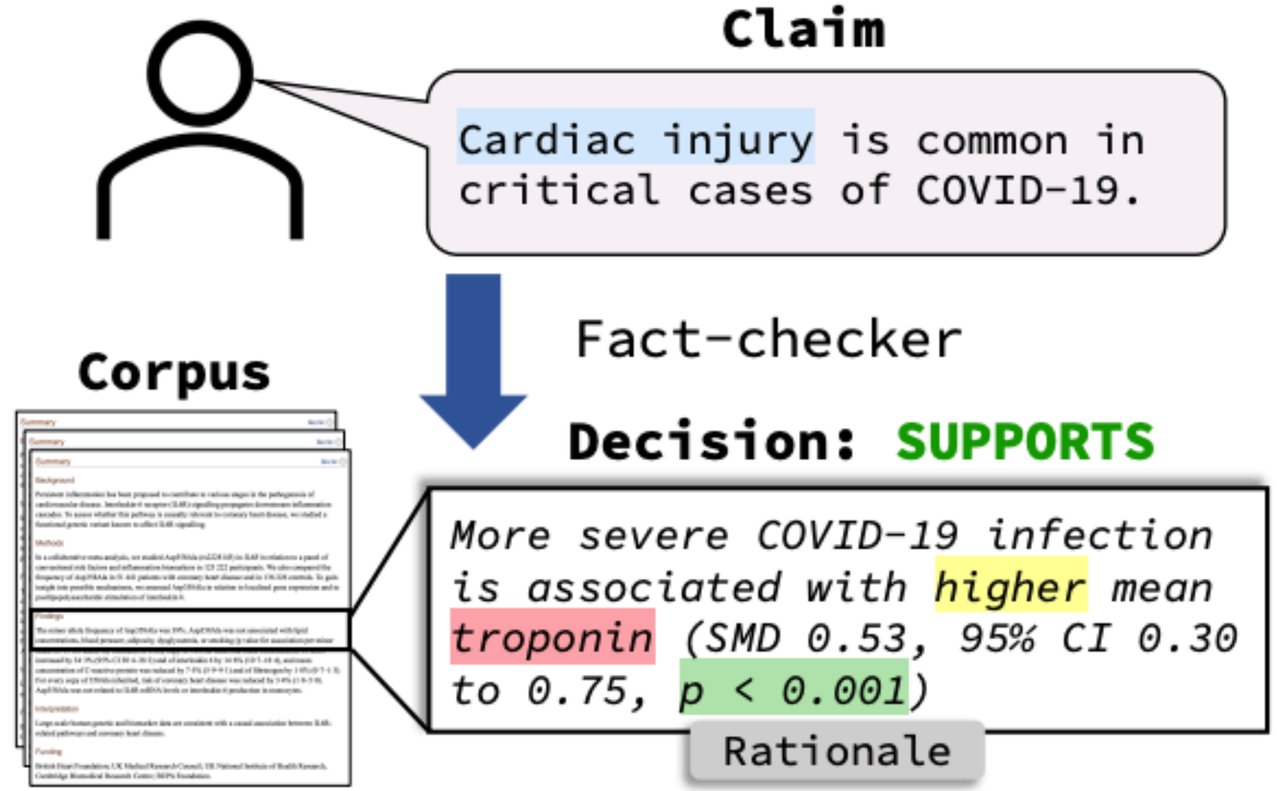
Ablation testing

- Are a set of features really important for the model?
- One can statistically test this:
  - Create model with all the features and evaluate
  - Create another model with test features removed and evaluate
  - Compare the difference in performance as a statistic and check for its statistical significance

	Features	# of features	frequency or presence?	NB	ME	SVM
(1)	unigrams	16165	freq.	**78.7**	N/A	72.8
(2)	unigrams	"	pres.	81.0	80.4	**82.9**
(3)	unigrams+bigrams	32330	pres.	80.6	80.8	**82.7**
(4)	bigrams	16165	pres.	77.3	**77.4**	77.1
(5)	unigrams+POS	16695	pres.	81.5	80.4	**81.9**
(6)	adjectives	2633	pres.	77.0	**77.7**	75.1
(7)	top 2633 unigrams	2633	pres.	80.3	81.0	**81.4**
(8)	unigrams+position	22430	pres.	81.0	80.1	**81.6**

Bo Pang, Lillian Lee, and Shivakumar Vaithyanathan. 2002. Thumbs up? Sentiment Classification using Machine Learning Techniques. In *Proceedings of the 2002 Conference on Empirical Methods in Natural Language Processing (EMNLP 2002)*, pages 79–86. Association for Computational Linguistics.

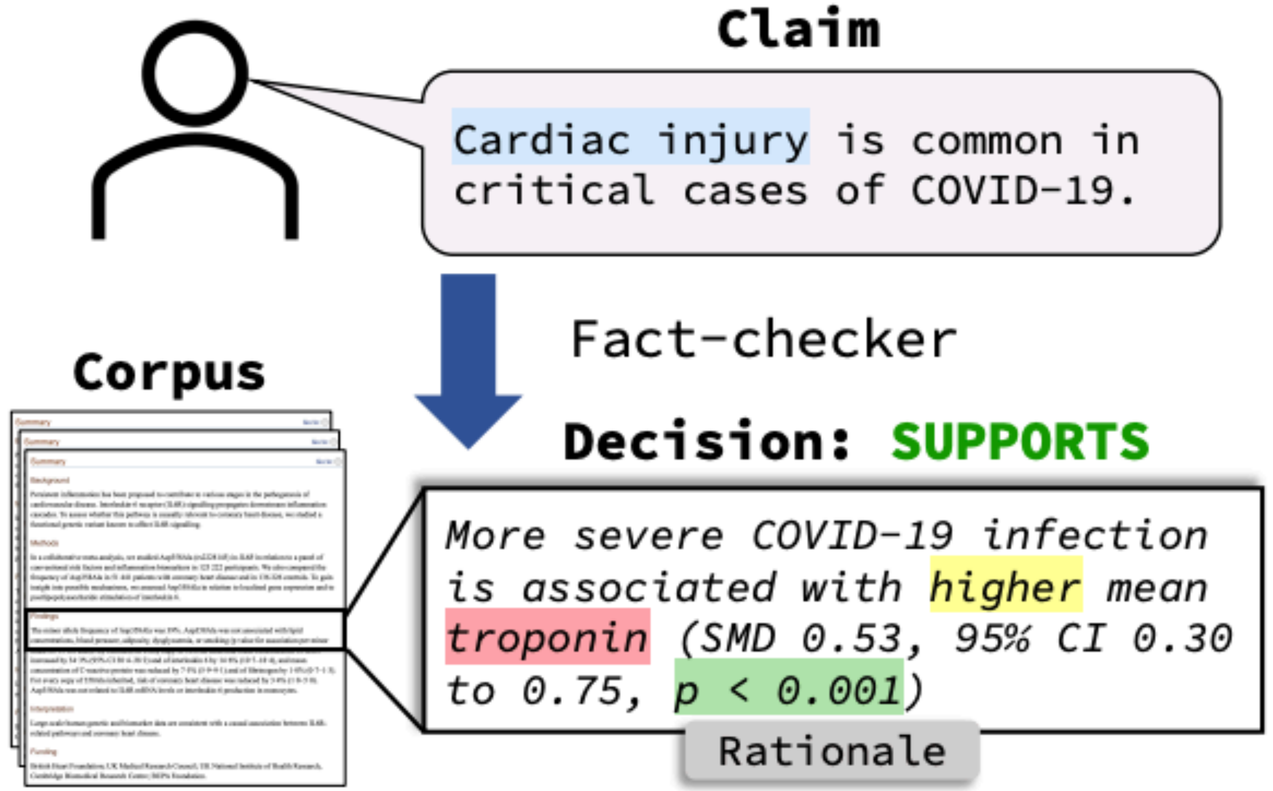
Extracting evidence for claims



Wadden, David, et al. "Fact or Fiction: Verifying Scientific Claims." Proceedings of the 2020 Conference on Empirical Methods in Natural Language Processing (EMNLP). 2020

Extracting evidence for claims

- For verifying some claims, we may need to extract data that supports or refutes a claim



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Summary

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- Use ablation testing to assess the importance of model components