ME-5339 Simulation Techniques for Dynamic Systems
Course Project

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Part 1 : Modeling a Micro Gas Turbine System

Micro Gas Turbines (MGT) are power generation devices that provide combined heat and power at relatively low-cost for small installation. MGT systems have many advantages over reciprocating engine generators, such as higher power-to-weight ratio, low emissions and reduced maintenance requirements.
Figure 1 shows the schematic of a MGT connected to an electric generator for stationary power applications.

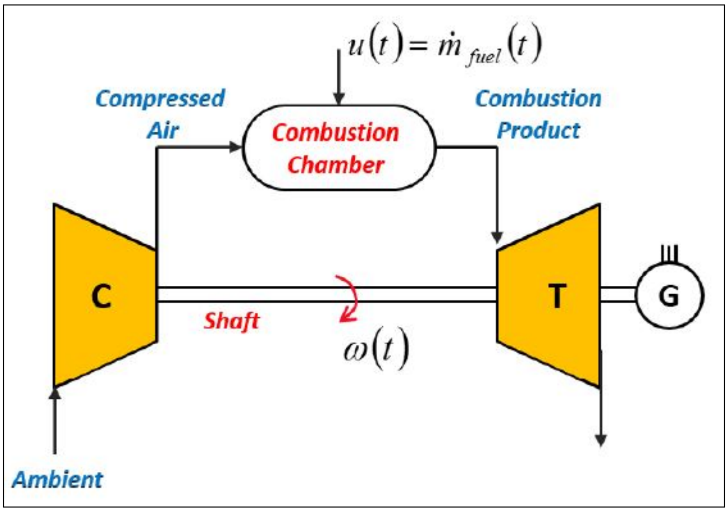


Figure 1: Schematic of the Gas Turbine Model

Ambient air is pressurized by a centrifugal compressor and fed to a combustion chamber,where it is mixed with methane and ignited to produce hot gases. The combustion products expand in the turbine and are then released in the ambient. The turbine is mechanically connected to the compressor and generator by a rotating shaft, and provides power to the whole system.
The parameters of the gas turbine model are summarized in Table 1.

Parameter Description	Value	Units
Shaft Inertia	0.25	$kg - m^2$
Comb. chamber vol.	0.2	m^3
Ambient pressure	1	bar
Ambient temperature	300	K
Specific gas constant	310	J/kg-K
Specific heat for air	1200	J/kg-K
Specific heat for methane	2300	J/kg-K
Methane LHV	50000	kJ/kg

Table 1: Parameters of Gas Turbine Model

Assumptions:

- The air and combustion products are assumed as an ideal gas with constant specific heat
- The combustion is assumed as an ideal heat addition to the system, from an outside heat source
- Pressure losses in the combustion chamber and connecting elements are negligible
- Heat losses (due to convection and radiation) are negligible

Terminology used:

- $\dot{m}$ = mass flow rate of mixture inside the combustion chamber
- $\dot{m}_f$ = mass flow rate of fuel
- $\dot{m}_t$ = mass flow rate to the turbine
- $\dot{m}_c$ = Mass flow rate of air at compressor output
- T_{amb} = Temperature of Air (Ambient)
- p_{Air} = Pressure of Air (Ambient)
- p_c = Pressure of air at compressor output
- N_{rpm} = Shaft rotation speed
- P_c = Power required to run Compressor
- T_c = Temperature of air at compressor output
- T_f = Temperature of fuel (Ambient)
- T_p = Temperature of combustion products at combustion chamber output
- T_t = Temperature of combustion products at Turbine output
- Pd = Power demand
- P_t = Power generated from turbine

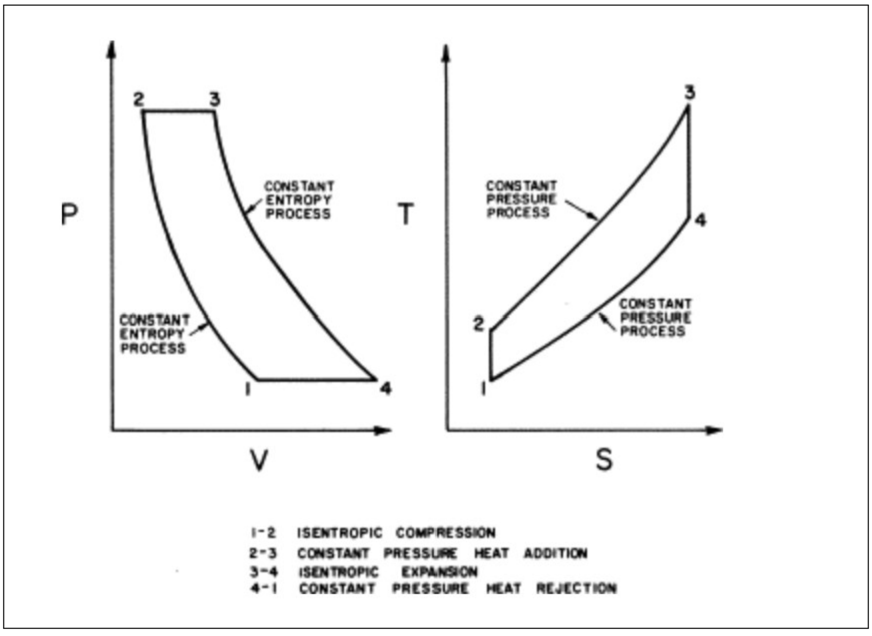


Figure 2: Brayton Cycle

1. Create block diagrams representing the four key components of the system (compressor, combustion chamber, turbine, shaft dynamics). For each block, define the input and output variables. Specify which components would you model as reservoirs (i.e., with differential equations), and specify the corresponding state variables.

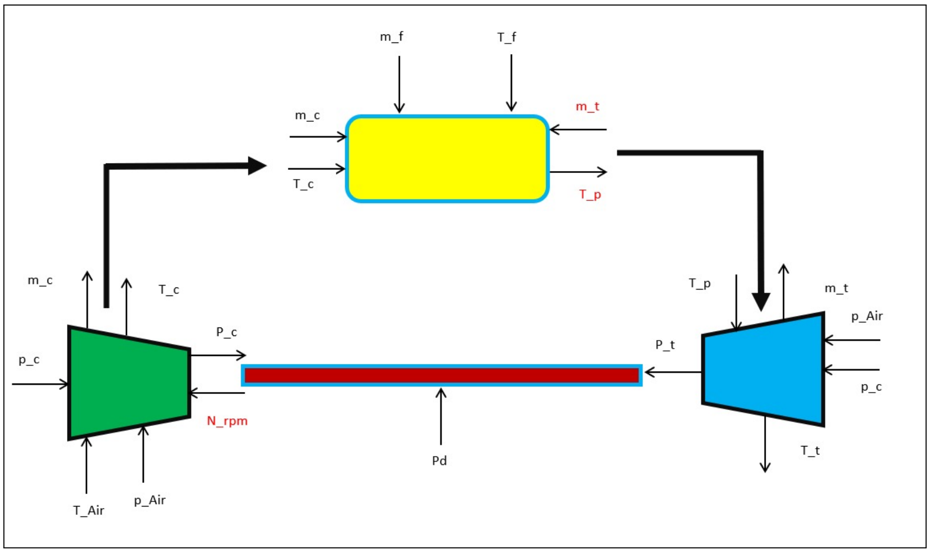


Figure 3: Block Diagram of the Micro Gas Turbine System

Compressor and turbine are already modelled in the provided Simulink Model.
The combustion chamber and Shaft can be modelled as reservoirs in the model.
The state variables for the combustion chamber are mass and temperature inside the chamber.
The state variable for the shaft is angular speed.

2. Write a set of equations that describes the dynamics of the combustion chamber.

Assumptions:

- Heat addition to the combustion chamber is a constant pressure heat addition process
- Combustion is considered to be an instantaneous process
- The temperature of the combustion products leaving the chamber is considered to be equal to the temperature inside the combustion chamber
- Gases inside the combustion chamber follow ideal gas law
- The fuel enters the combustion chamber at ambient temperature

Assuming that the micro gas turbine model follows the Brayton cycle, the heat addition process to the combustion chamber is considered to be a isobaric heat addition process
The conservation of mass equation can be written as,

$$\dot{m} = \dot{m}_c + \dot{m}_f - \dot{m}_t \tag{1}$$

The conservation of energy equation can be written as,

$$\frac{dE}{dt} = \dot{m}_f Q_{LHV} + \dot{m}_c C_p T_c + \dot{m}_f C_p T_{amb} - \dot{m}_t C_p T \tag{2}$$

where,
T_c = Outlet temperature at compressor
T = Temperature inside combustion chamber

T_{amb} = Temperature of air

$$\frac{dE}{dt} = C_v \left[\frac{dm}{dt} T + \frac{dT}{dt} m \right] \tag{3}$$

Substituting equation (3) in (2) and rearranging,

$$\frac{dT}{dt} m(t) = \frac{m_f Q}{C_v} + \dot{m}_c \gamma T_c + m_f \gamma T(t) - \dot{m}_t \gamma T(t) - \dot{m}_a T - \dot{m}_f T_{amb} + \dot{m}_t T \tag{4}$$

Assuming that the combustion chamber follows ideal gas laws,

$$P_c = \frac{mRT_c}{V} \tag{5}$$

The input, output and state variables are,

	Value	Units
Input Variables	Pressure of compressed air	Pa
	Temperature of compressed air	K
	Mass flow rate of fuel	kg/s
Output Variables	Temperature of combustion products	K
	Mass flow rate of combustion products	kg/s
	Pressure of combustion products	Pa
State Variables	Mass in combustion chamber	kg
	Temperature in combustion chamber	K

Table 2: Variables for Combustion Chamber

3. Write an equation that describes the dynamics of the turbine shaft speed, explaining the assumptions and listing the input and output variables with the corresponding units

   Assumptions:

   - Entire power generated by the turbine goes to the shaft. This power is split to power the compressor, run the shaft and rest to the generator
   - Shaft is considered to be a rigid body

$$J\omega_c \frac{d\omega_c}{dt} = P_{eff} - P_{comp} \tag{6}$$

$$P_{eff} = P_{turb} - P_{demand} \tag{7}$$

The input, output and state variables are,

	Value	Units
Input Variables	Turbine Power	W
	Compressor Power	W
	Power demand	W
Output Variables	Speed of Shaft	rpm
State Variables	Angular welocity of shaft	rad/s

Table 3: Variables for Shaft

4. Write a set of equations that predicts the outlet temperature of the compressed air and the power absorbed by the compressor. Repeat the process for the turbine to find expressions for the outlet gas temperature and the power produced.

   Assumptions:

   - Adiabatic air compression is considered for the compressor

Compressor:
The power absorbed by the compressor is given by the equation

$$P_{comp} = \frac{1}{\eta_c} \dot{m}_c C_p T_0 \left[\left(\frac{p}{p_0} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \tag{8}$$

The outlet temperature of the compressed air is

$$\frac{p_{amb}}{p} = \left(\frac{T_{amb}}{T_c} \right)^{\frac{\gamma}{\gamma-1}} \tag{9}$$

$$T_c = \left[\frac{p}{p_{amb}} \right]^{\frac{\gamma-1}{\gamma}} T_{amb} \tag{10}$$

	Value	Units
Input Variables	Ambient pressure	Pa
	Ambient temperature	K
	Mass flow rate of air	kg/s
Output Variables	Outlet pressure	Pa
	Mass flow rate of compressor	kg/s
	Temperature of compressed air	K
	Compressor power	W

Table 4: Variables for Compressor

Turbine:
Assumptions:

- Isentropic air expansion is considered for the compressor

The power generated by the turbine is

$$P_{turb} = \frac{1}{\eta_t} \dot{m}_t C_p T_p \left[1 - \left(\frac{p}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \right] \tag{11}$$

The outlet temperature of the turbine is

$$T_t = \left[\frac{p}{p_{amb}} \right]^{\frac{\gamma-1}{\gamma}} T_p \tag{12}$$

	Value	Units
Input Variables	Turbine Power	W
	Compressor Power	W
	Power demand	W
Output Variables	Speed of Shaft	rpm

Table 5: Variables for Turbine

5. Finally, create a block diagram of the MGT system, showing how the inputs and outputs of each component should be concatenated to "close" the model, such that the causality rules are satisfied.

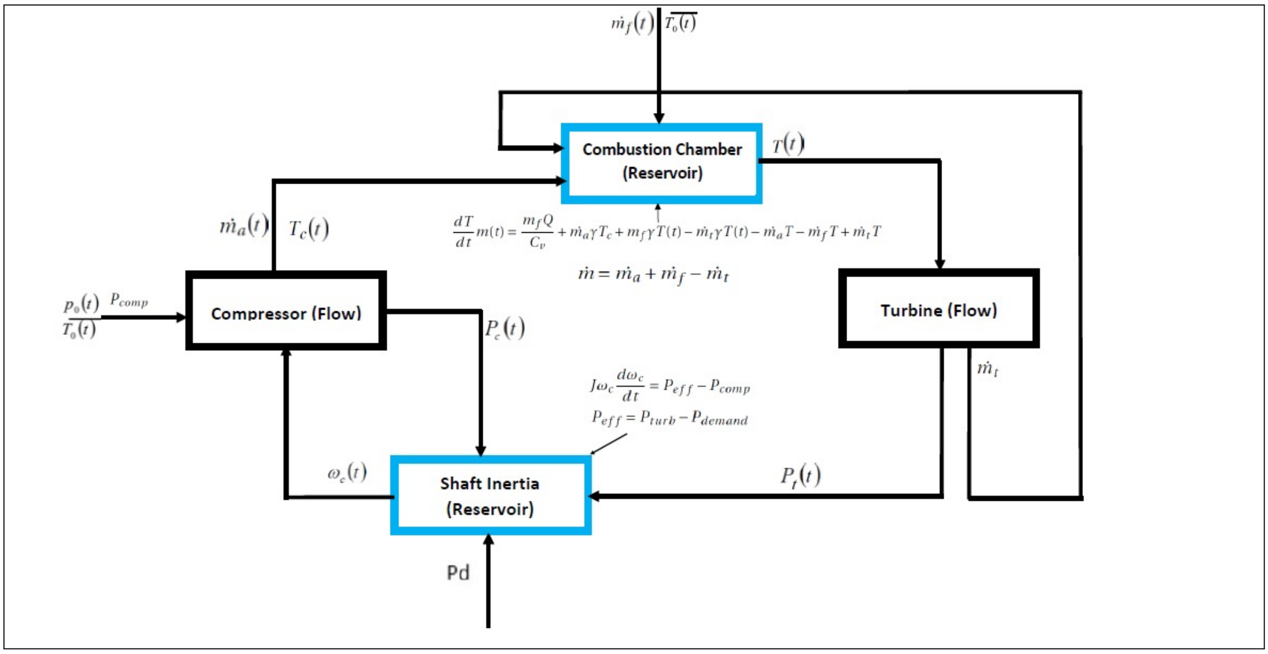


Figure 4: Causality Diagram

The equations for mass, temperature of combustion chamber and shaft speed are Ordinary Differential Equations. The equations of compressor and turbine are algebraic which are implemented in he model through maps.

Part 2 : Simulation and Analysis

1. Build the model of the combustion chamber and the shaft dynamics, and connect the corresponding inputs and outputs to the compressor and turbine models

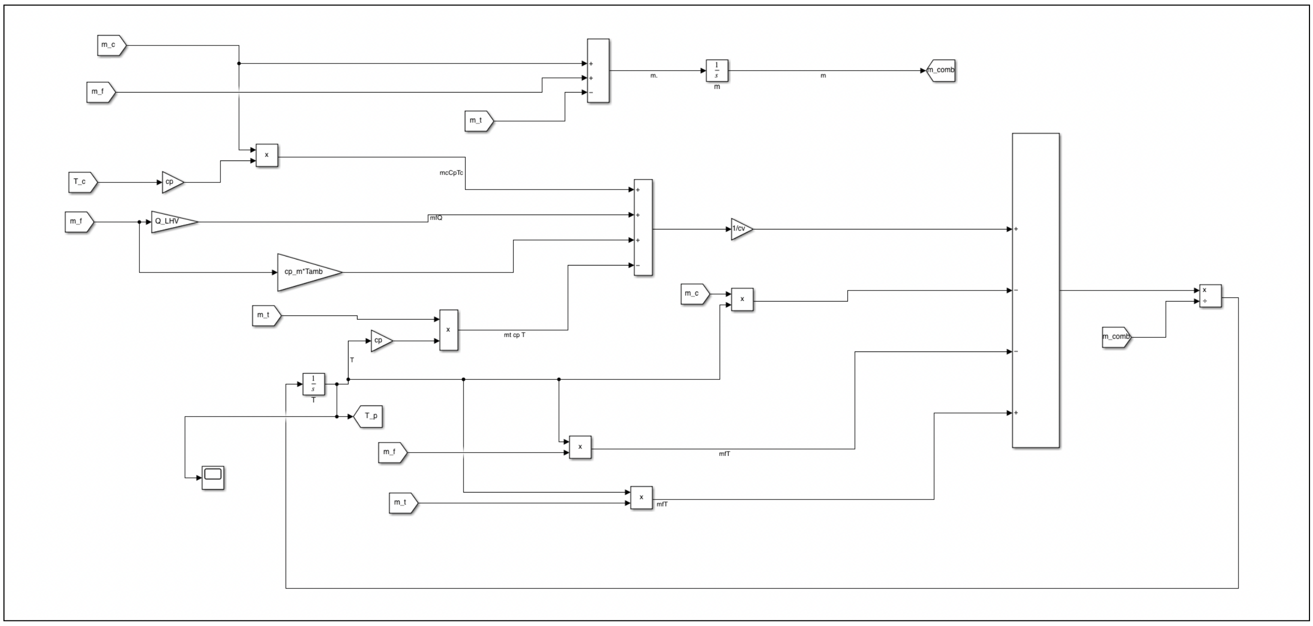


Figure 5: Simulink model for Combustion Chamber

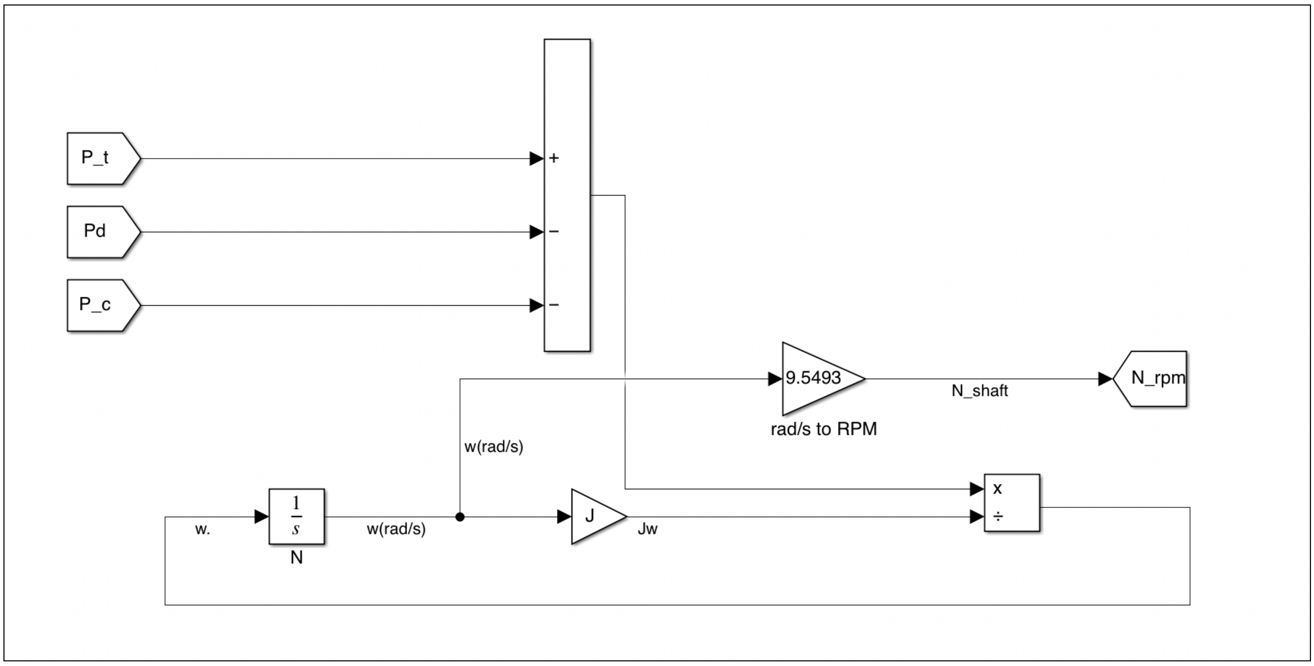


Figure 6: Simulink model for Shaft Dynamics

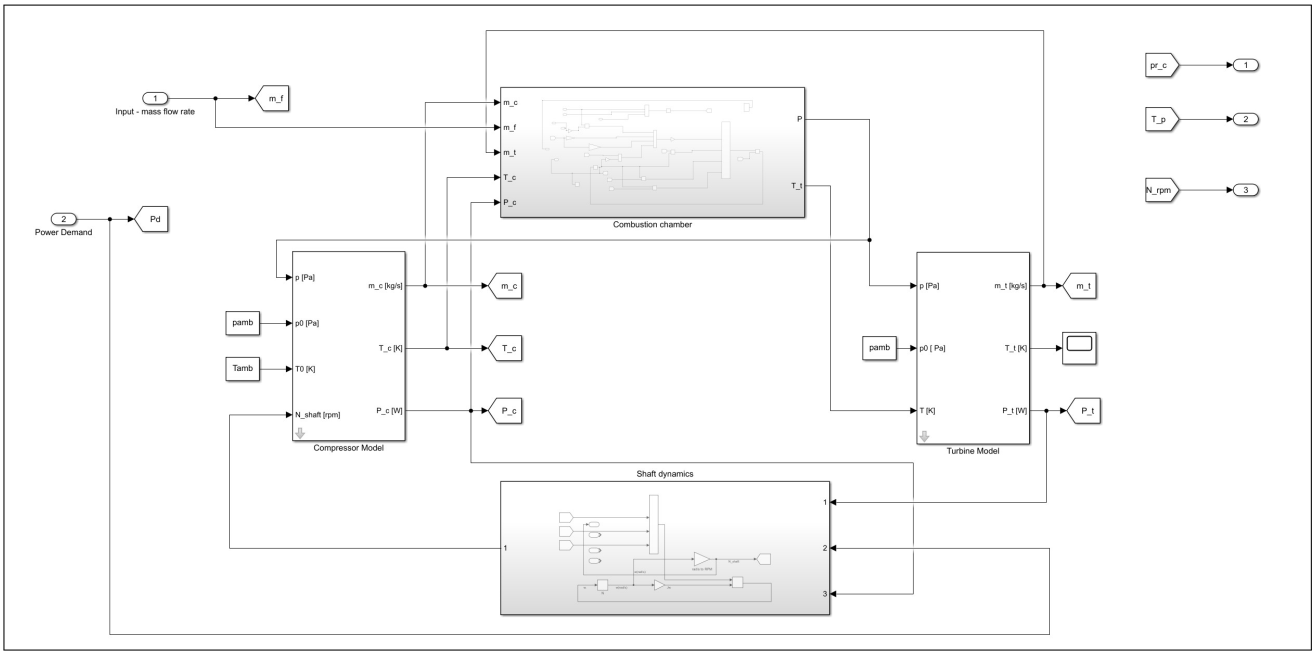


Figure 7: Simulink model for Micro Gas Turbine System

2. Simulate the free response of your system to verify that equilibrium conditions are reached at approximately p_c = 13bar, T_c = 1400K, ω = 90000rpm

   Given Conditions

   - $\dot{m}_f$ = 0.035 kg/s
   - Constant load = 200 kW

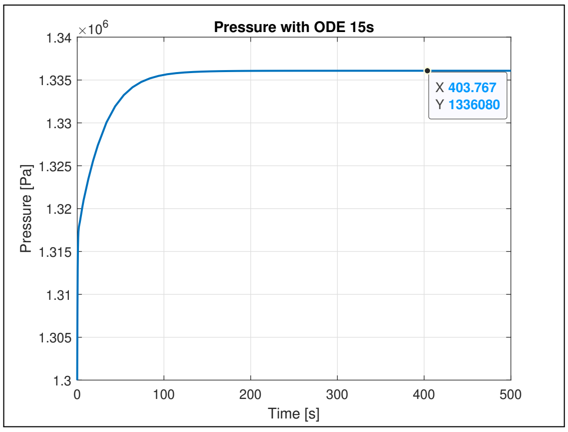
   Initial Conditions

   - T0 = 1400 K
   - N0 = 90000 * $\frac{2\pi}{60}$ rad/s

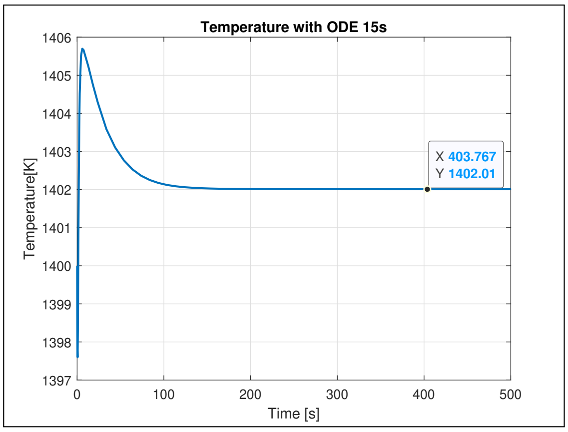
- m0= $\frac{p0*V}{(R*T0)}$ = 5.9908 kg

The equilibrium conditions obtained are:
p = 13.36 bar
T = 1402.01 K
N_{rpm} = 90725.5 rpm

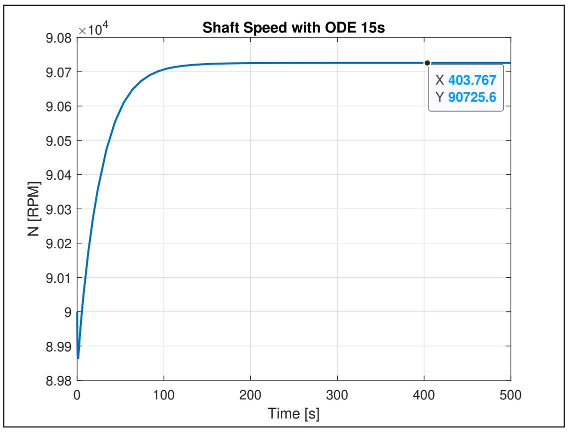
The following results are obtained for ODE15s solver



(a) Pressure



(b) Temperature

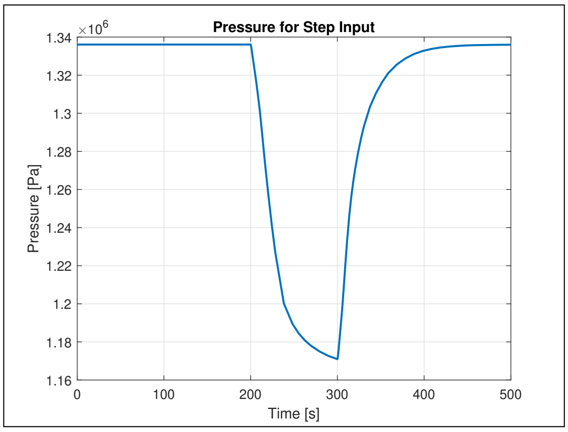


(c) Shaft Speed

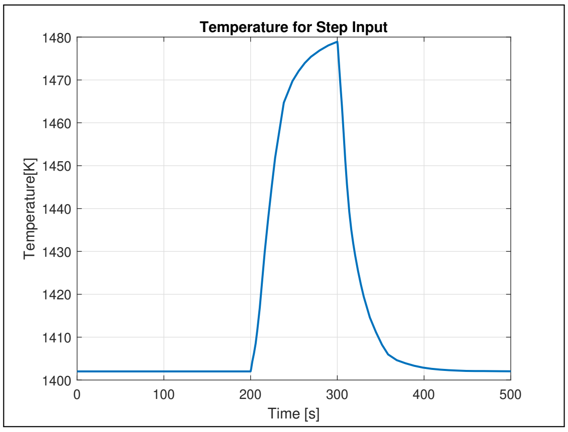
Figure 8: Simulating for constant inputs m_f = 0.035 and P = 200kW

3. Simulate a step response where the load is increased from 200kW to 250kW at t = 200s, and then decreased back to 200kW at t = 300s while keeping m_{fuel} = 0.035kg/s

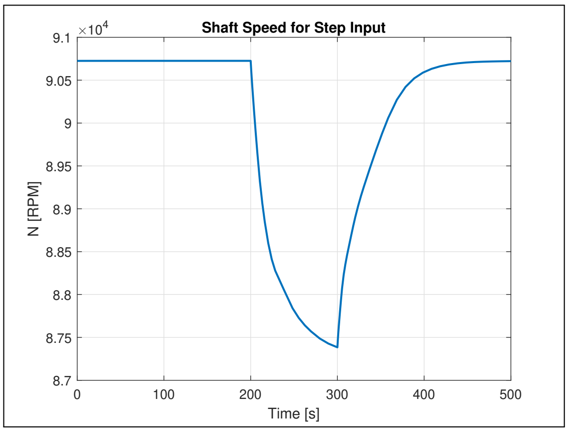
   From the above sub-question, the initial conditions for the system are:
   p = 13.36 bar
   T = 1402.01 K
   N_{rpm} = 90725.5 rpm
   The following results are obtained



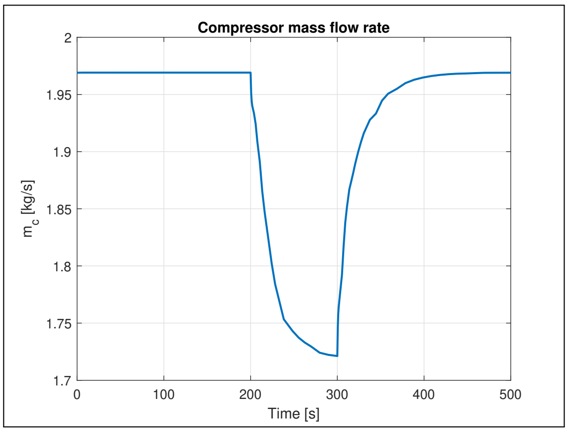
(a) Pressure



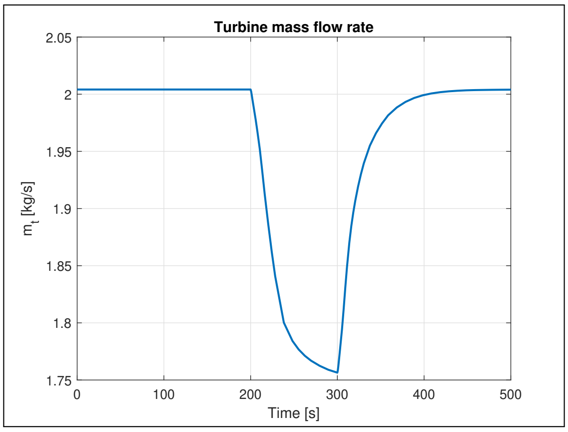
(b) Temperature



(c) Shaft Speed



(d) Mass flow rate of compressor



(e) Mass flow rate of turbine

Figure 9: Simulating for step response

The turbine efficiency can be calculated as

$$\eta_t = 1 - \frac{1}{\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}} \tag{13}$$

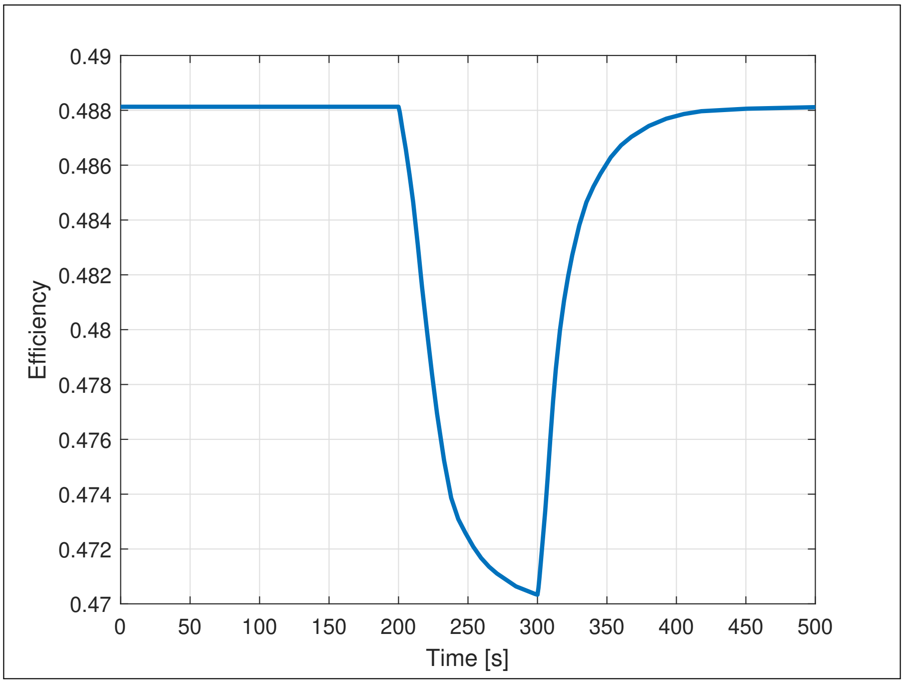


Figure 10: Turbine efficiency

The heat losses in the turbine can be calculated as

$$Q_{loss} = mC_{p,air}(T_t - T_{amb}) \tag{14}$$

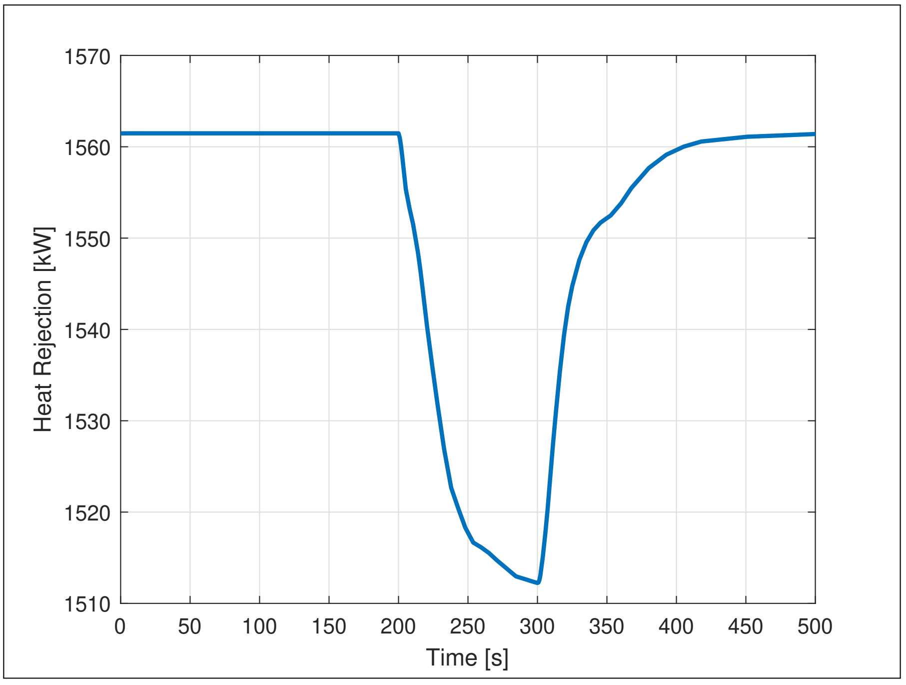


Figure 11: Heat Losses in Turbine

Part 3 : Model Linearization

1. Build a "Subsystem" block around your Simulink model to obtain a system whose inputs are u = [m_{fuel}; P_{load}] and outputs are: y = [p; T; N], where p and T are the pressure and temperature in the combustion chamber, and N is the shaft speed.

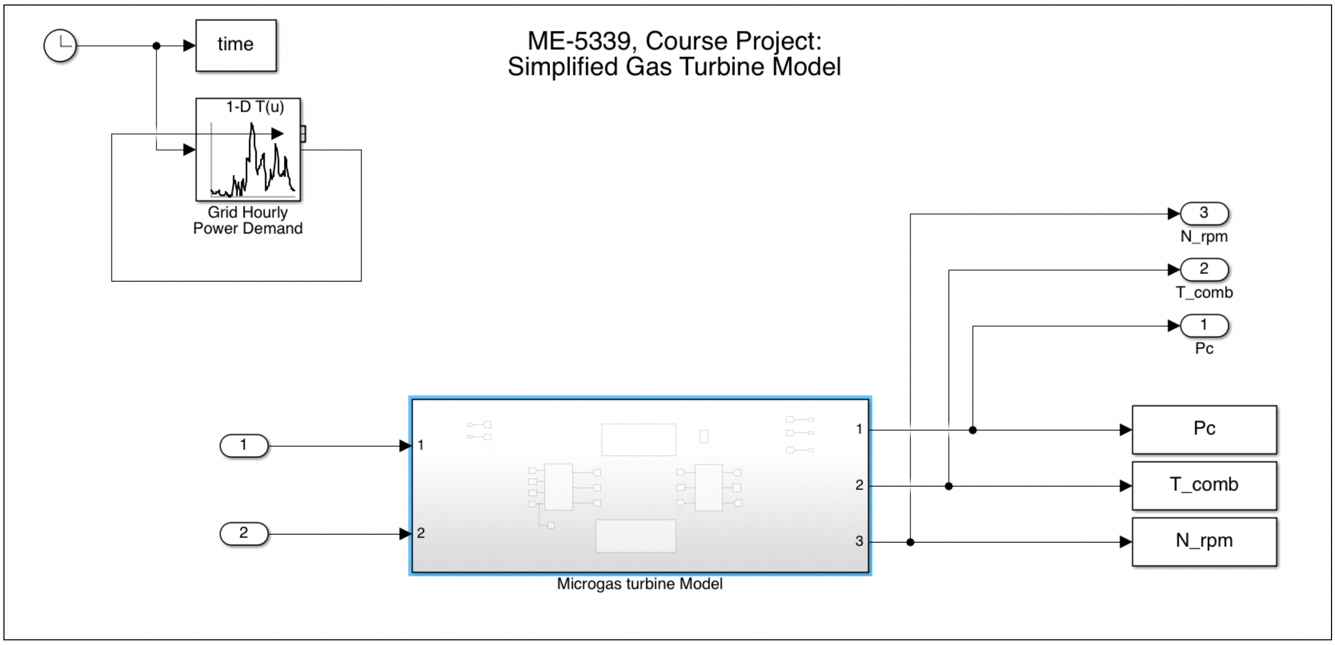


Figure 12: Simulink Model with Inports and Outport added

2. Using the MATLAB function trim.m, determine an equilibrium condition for the system

   A trim point is a point in the parameter space of a dynamic system at which the system is in a steady state. After multiple iterations to relax the inputs, states and outputs, the values of states given by the trim function are as follows
   m = 6.1482 kg/s
   T = 1402.00895 K
   N = 9500.75525 rpm

3. Linearize the gas turbine model and simulate the system response to a 50kW step in P_{load}

   Now to linearize the model, we pass the states and inputs x and u that we get from the trim function to the linmod command. We get the values of A,B,C,D as follows:

$$A = \begin{bmatrix} -2.3620 & -0.0104 & 0.0025 \\ 151.1239 & 0.2232 & -0.1921 \\ 449.0961 & 2.2977 & -0.5102 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 9034.774 & 0 \\ 0 & -0.00042 \end{bmatrix}$$

$$C = \begin{bmatrix} 217311.4 & 952.9758 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9.5493 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

   Now, the obtained linearized model is implemented in Simulink as follows:

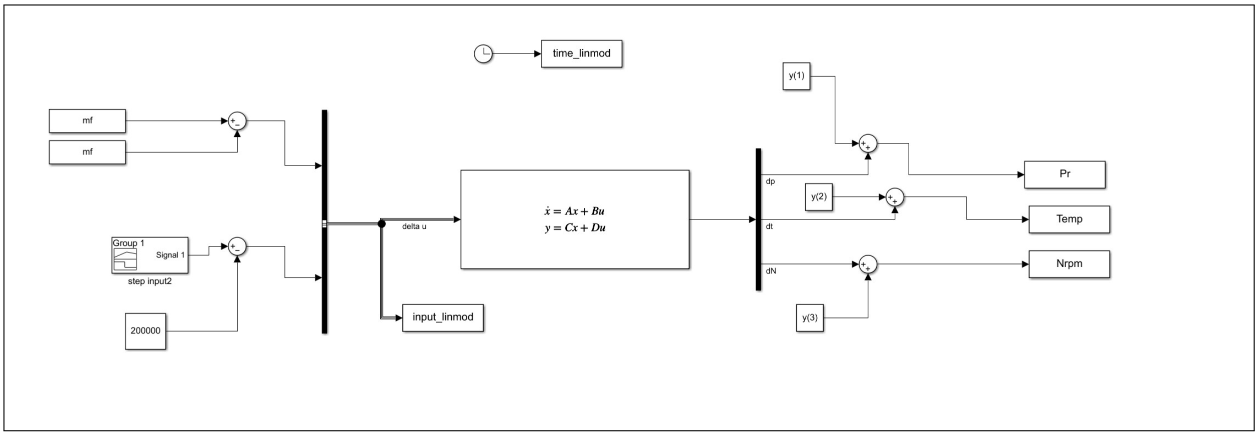
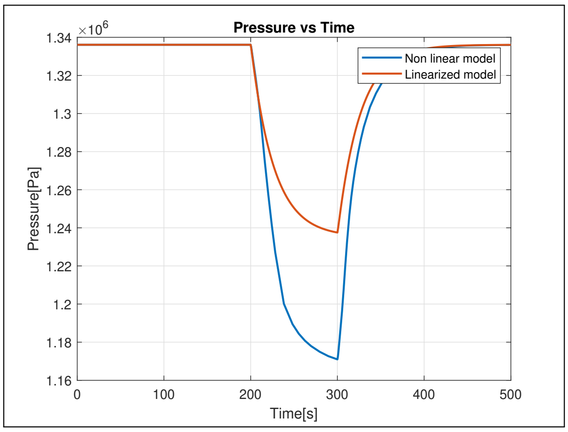
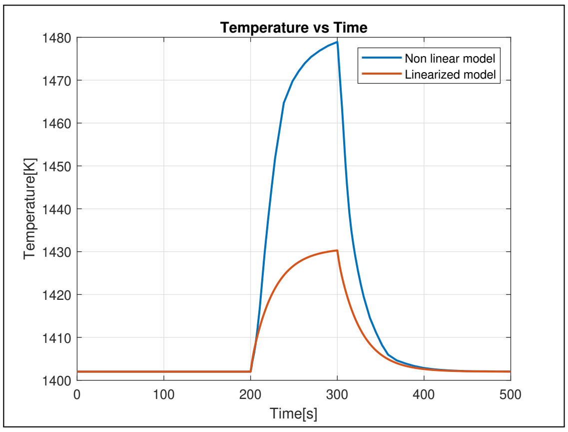


Figure 13: Linearized Model

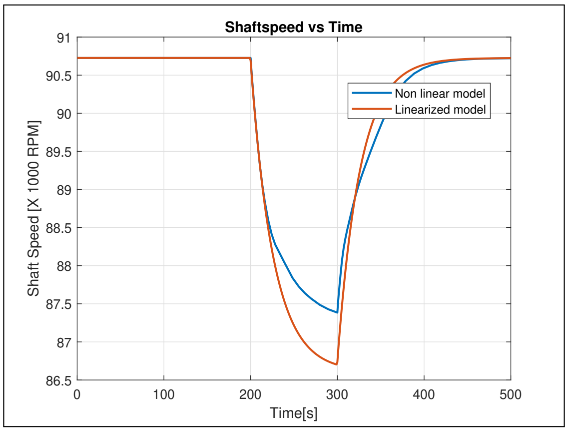
Simulating the model for a P_{load} step response of 50kW,



(a) Pressure



(b) Temperature

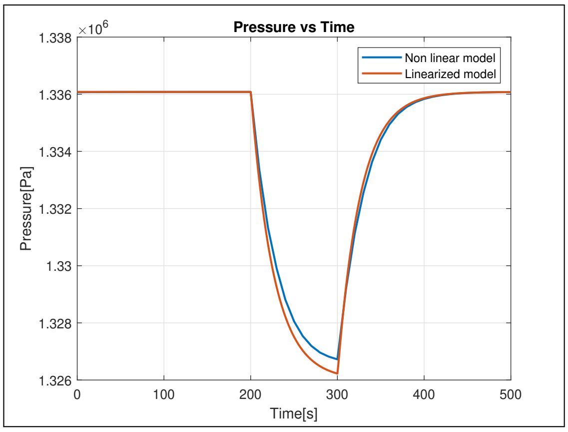


(c) Shaft Speed

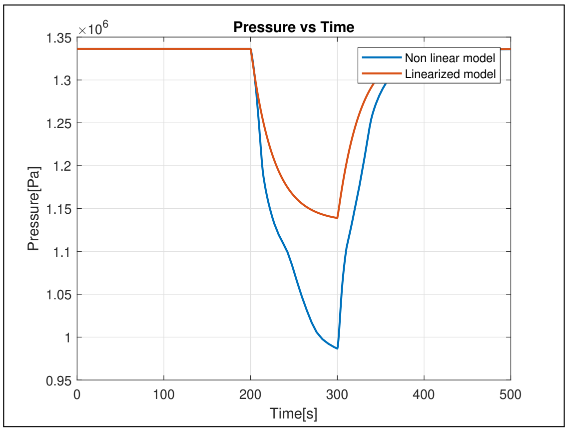
Figure 14: Comparison between the system response of the linearised and non-linearised models

As seen in the plots, we see that the linear model approximately follows the curve of the non linear system as expected. We see a slight variation during the power step.

4. Extend the comparison between the nonlinear and linearized model to several values of the step in power demand

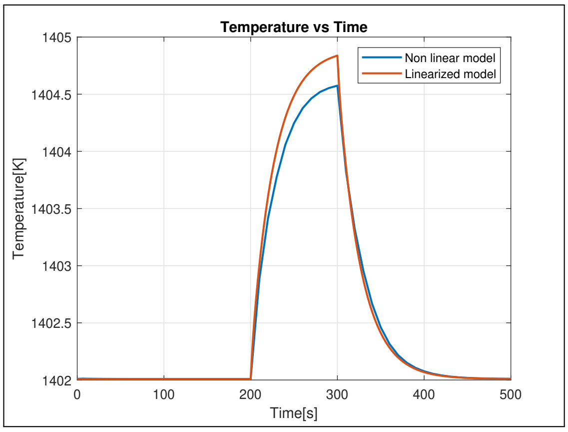


(a) Step = 5 kW

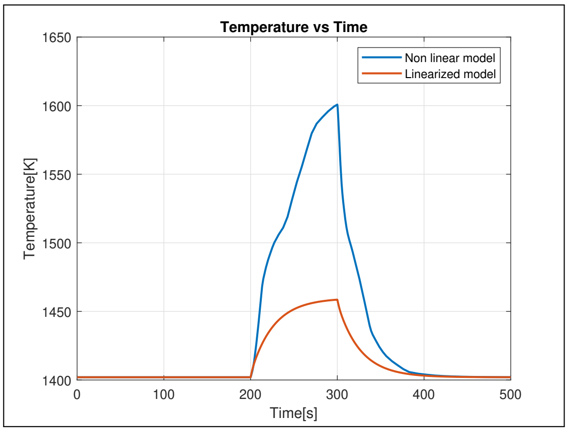


(b) Step = 10 kW

Figure 15: Pressure

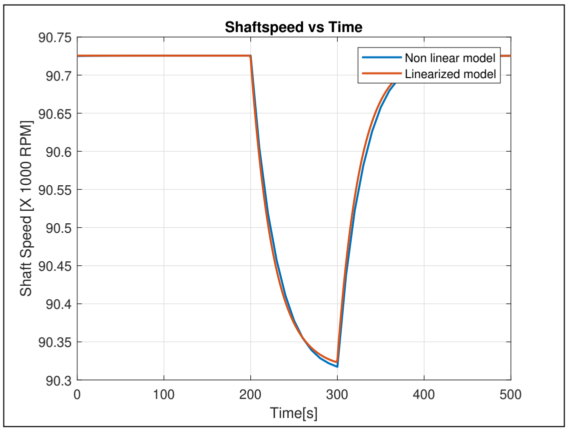


(a) Step = 5 kW

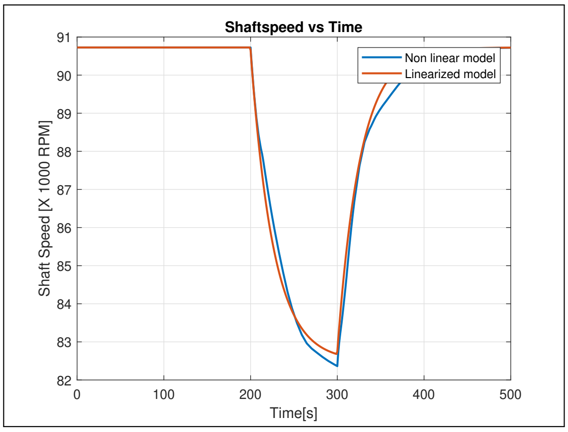


(b) Step = 10 kW

Figure 16: Temperature



(a) Step = 5 kW



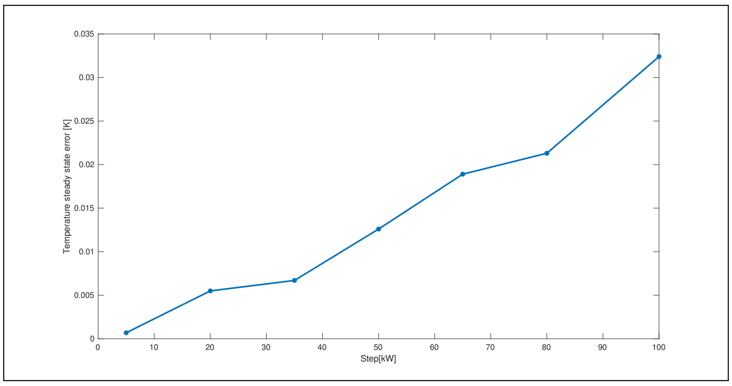
(b) Step = 10 kW

Figure 17: Shaft Speed

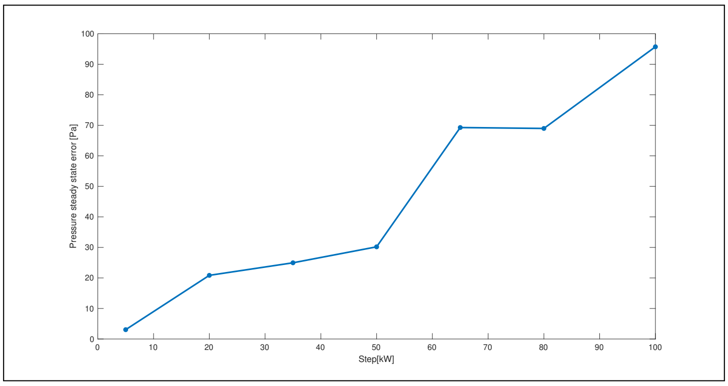
Steady State Error

Step (kW)	Steady State Error (After 500 s)		
	Pressure (Pa)	Temp (K)	Shaft Speed (RPM)
5	3.0556	6.9192e-04	0.1287
20	20.8511	0.0055	0.8675
35	24.9639	0.0067	1.0097
50	30.1936	0.0126	1.2413
65	69.2703	0.0189	2.7761
80	68.9786	0.0213	2.6711
100	95.7260	0.0324	3.7029

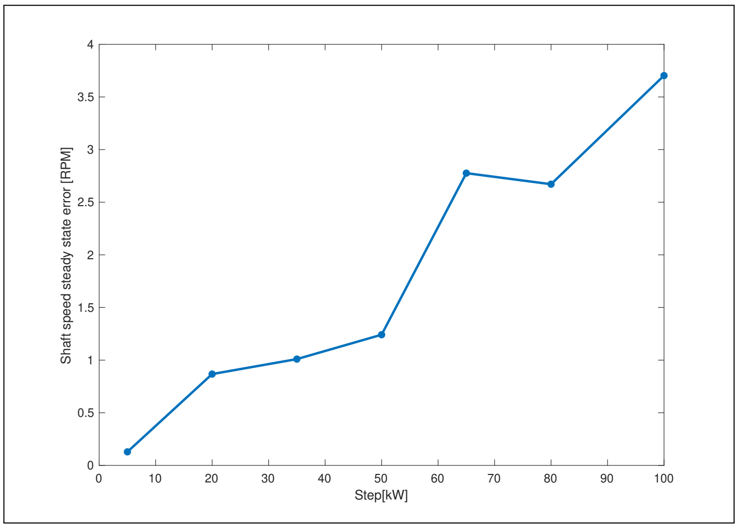
Table 6: Steady State error between linear and non linear models



(a) Temperature



(b) Pressure



(c) Shaft Speed

Figure 18: Steady State Error

As the step power magnitude increases, the steady state error between the linear and nonlinear model increases for all the output parameters pressure, temperature, and shaft speed.

5. Analyze the linearized model and determine if, at the given equilibrium condition, the system is asymptotically stable.

   We can check the stability of the linearized model by checking the eigenvalues of matrix A. The eigenvalues of A are found to be : - -1.9175, -0.6934, -0.0381.
   As all the eigen values are negative, it can be concluded that the system is asymptotically stable.

Part 4 : Control Design

1. Using the linearized model, design a PI controller that updates the fuel flow rate to maintain the turbine shaft velocity at the constant value N_{ref} = 90000rpm. The PI controller should add a correction term to the constant value m_{fuel} = 0:035kg/s. Your controller should be designed to achieve a settling time of less than 50s, with a maximum overshoot of 1000rpm.

   The values for the PI controller which satisfy the given conditions are:
   P = -6.19982666191308e-06
   I = -7.95045258926056e-07 The settling time is around 40 sec and maximum change in shaft speed is 264 rpm on either side for 50 kW step.

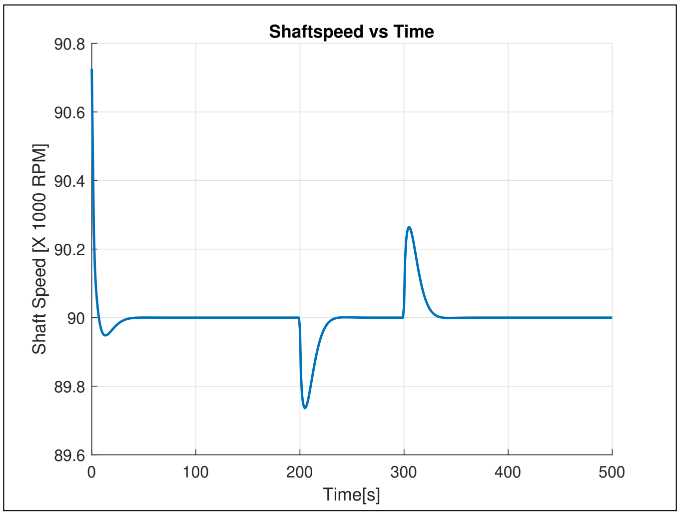
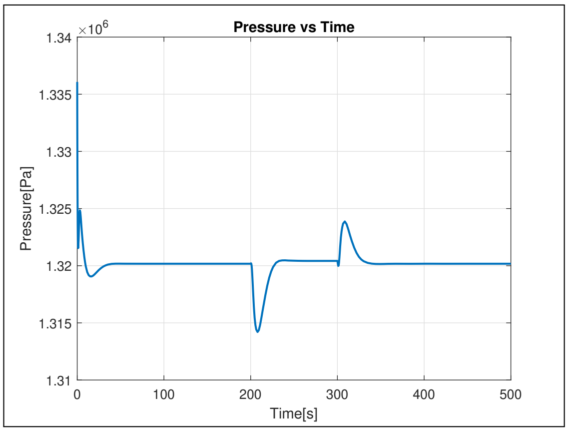


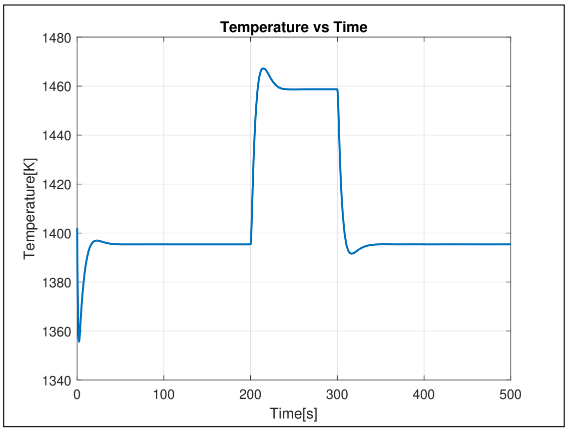
Figure 19: Shaft Speed

2. Implement the controller into the nonlinear MGT model, and simulate the system behavior in closed-loop

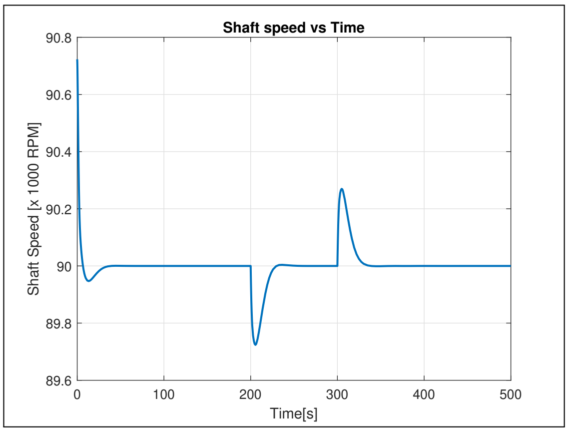
   We implement the PI controller to the non-linear model to keep the shaft speed constant at 90000rpm.



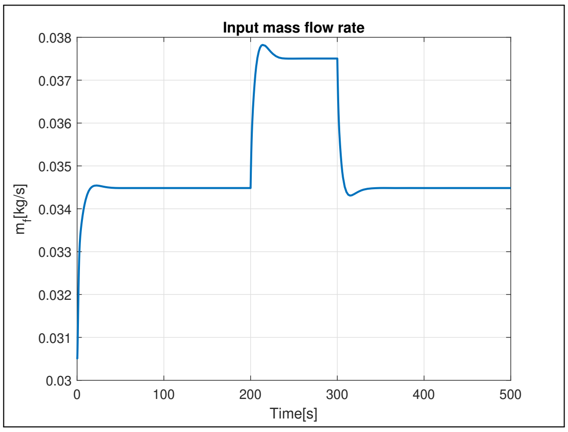
(a) Pressure



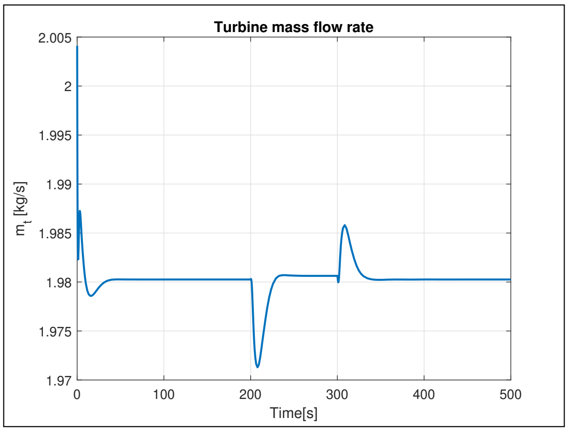
(b) Temperature



(c) Shaft Speed



(d) Mass flow rate of fuel

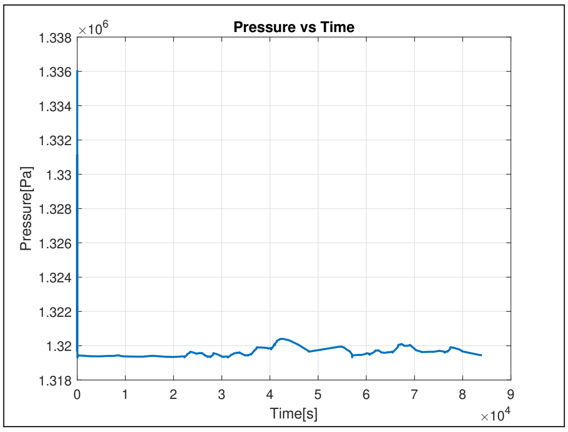


(e) Mass flow rate of turbine

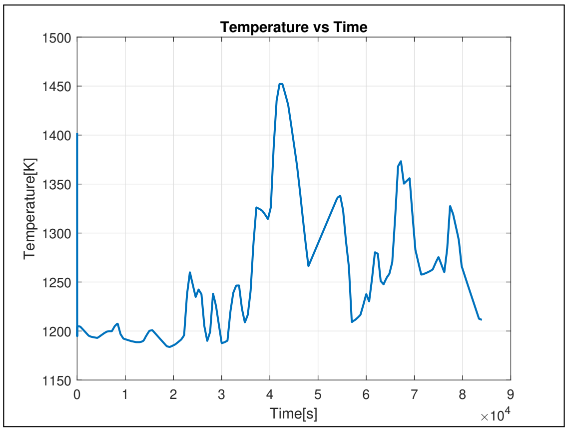
Figure 20: Results after implementation of the PID controller in the non linear MGT model

3. Test your feedback controller with a time-varying load profile.

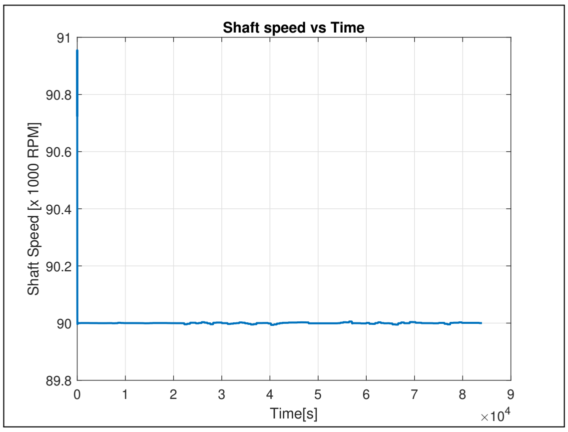
   Now we simulate the model with the power demand grid for 24 hours timeframe. We can see that the PI controller helps to keep the shaft speed constant and thus the variation is power demand will be met by changing the input mass flow rate of fuel. We can see the variation in the mass flow rates, Temperatures and pressure of the system in the plots below:



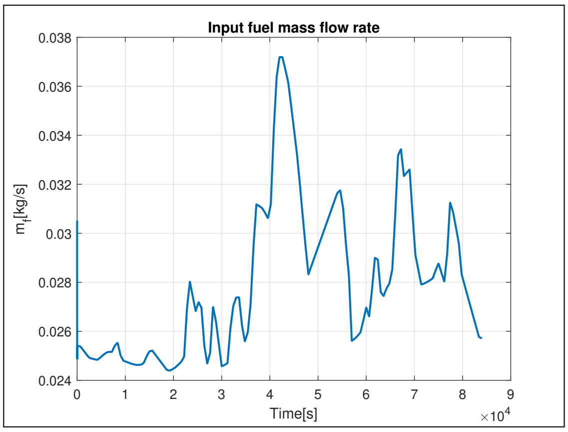
(a) Pressure



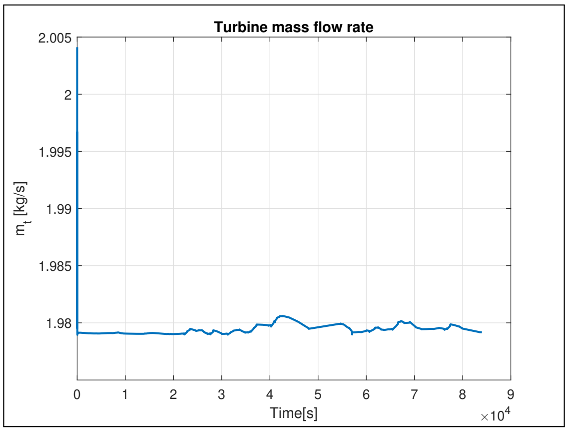
(b) Temperature



(c) Shaft Speed



(d) Mass flow rate of fuel



(e) Mass flow rate of turbine

Figure 21: Final Simulation Results with Grid Load

We can see that the PI controller maintains the speed of the shaft around 90000 rpm and changes the mass flow rate to meet the variation in power demand.

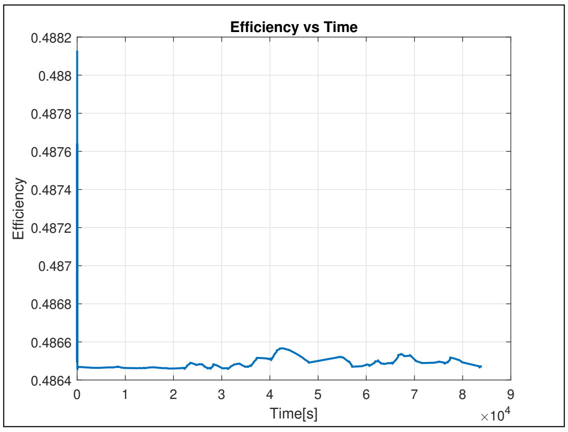


Figure 22: Efficiency

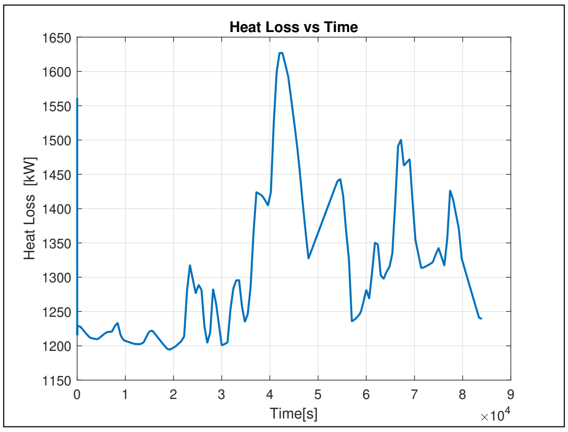


Figure 23: Heat Losses

The efficiency is around 48.6% and heat losses vary between 1200 and 1500 kW approximately.