

Difference matrix :-

(Everything is zero base indexing)

Given "arr" of size "n".

Let us define "diff" of size n

Initializing diff :- $i \in [0, n-1]$

$$\text{diff}[i] = \begin{cases} \text{arr}[0] & \text{if } i=0 \\ \text{arr}[i] - \text{arr}[i-1] & \text{otherwise} \end{cases}$$

* Note that we can restore "arr" from diff.

i.e for unique arr there exist unique diff and vice versa.

algo to restore :-

$$\text{arr}[0] = \text{diff}[0]$$

for i from 1 to n-1 :-

$$\text{arr}[i] = \text{diff}[i] + \text{arr}[i-1]$$

* update operation. (zero base)

l to r

let arr be $a_0, a_1, a_2, \dots, a_l, \dots, a_r, \dots, a_{n-1}$ initially

after updation $a_0, a_1, a_2, \dots, a_{l+n}, \dots, a_{r+n}, \dots, a_{n-1}$

The difference matrix before updation would be let

$d_0, d_1, d_2, \dots, d_l, \dots, d_r, \dots, d_{n-1}$
from the definition.

let the new difference array be $u_0, u_1, u_2, \dots, u_l, \dots, u_r, \dots, u_{n-1}$

d_0	d_1	...	d_l	...	d_x	...	d_{n-1}
a_0	$a_1 - a_0$		$a_l - a_{l-1}$...	$a_x - a_{x-1}$		$a_{n-1} - a_{n-2}$

u_0	u_1	...	u_l	u_{l+1}	...	u_x	u_{x+1}	...
a_0	$a_1 - a_0$		$a_{l+n} - a_{l-1}$	$a_{l+1} - a_l$...	d_x	$a_{x+1} - a_{x-n}$...

$$\hookrightarrow u_i = \begin{cases} d_{i+n} & \text{if } i=1 \\ d_{i-n} & \text{else if } i=x+1 \text{ \& } x \neq n-1 \\ d_i & \text{else} \end{cases}$$

from u we can get update matrix.

Similarly for any number of updates this holds.